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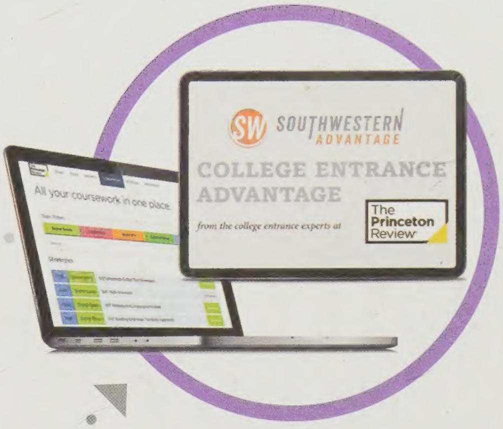
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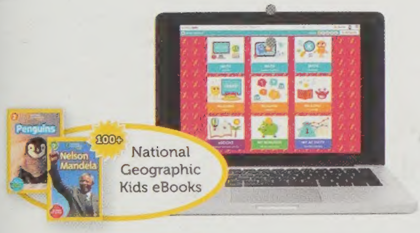
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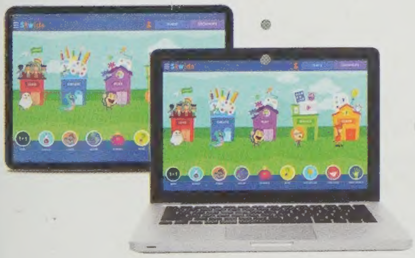

Skwids website & app
HOMER app

My Book of Colors
My Fun with Words

My Books series,
My Fun with Words set

Sharing the Advantage

SHARE THE
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ADVANTAGE

Our *Share the Advantage* mission is to share education, learning skills and opportunity with every child and every family, regardless of their circumstances through individual relationships, qualified nonprofit partnerships and local community involvement.

southwesternadvantage.com/share-the-advantage

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MATH I

SOUTHWESTERN
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MATH II

SOUTHWESTERN
ADVANTAGE

Southwestern Advantage

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Preface

Welcome to *Southwestern Advantage Math*. We are pleased to bring you these unique, user-friendly reference books. Designed in such a way that students can spend "more time learning, less time looking," the pages are open and inviting, and critical information is summarized in boxes, lists, and other easily usable and understandable pieces.

Problems are shown worked out step-by-step. Where a problem can be worked by more than one method, those methods are also shown. Additionally, simply by keying in the page number, you can access step-by-step videos of each problem from that page at SWadvantage.com.

Got to Know boxes summarize the most essential information; cross-references in the **Need More Help** boxes direct you to pages where you can find additional information or review material. **Try It This Way** suggests alternative ways people with various learning styles can use to more effectively approach, work, or visualize problems and concepts. **Watch Out** boxes alert students to things that might be easily confused or that might give students difficulty.

We hope you will find this book both useful and enjoyable. Every effort has been made to ensure that the information in this book is as accurate as possible. If errors should be found, however, we would appreciate hearing from you. Please share your comments or suggestions with us at 1-888-551-5901 or send them to Editor c/o Southwestern Advantage, P.O. Box 305142, Nashville, TN 37230.

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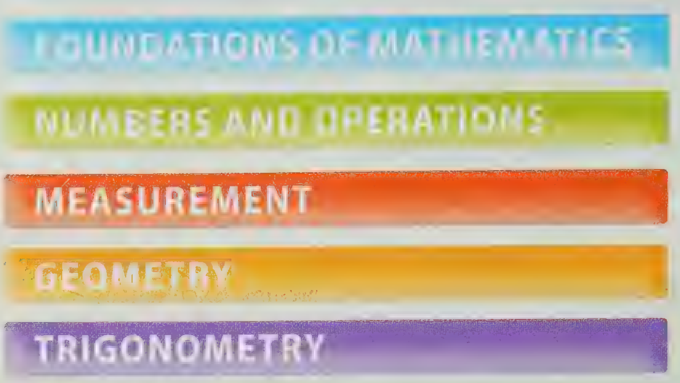
How to Use Southwestern Advantage

How to Use These Books

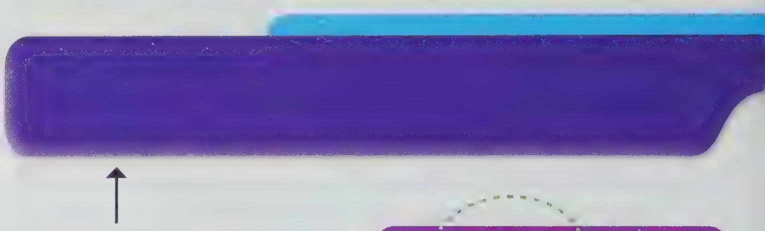
Designed in such a way that students can spend “more time learning, less time looking,” these books are divided into nine “strands”: Foundations of Mathematics; Numbers and Operations; Measurement; Geometry; Trigonometry; Statistics and Probability; Algebra; Advanced Algebra; and Calculus. Each strand is then divided into smaller units.

The first navigational tool is the detailed, color-coded table of contents. Each book contains a detailed listing of its own contents, plus a list of the strands covered in the other book. The contents pages also indicate separately where the special features of the book can be found, such as tables, charts, and glossaries.



Next, color bars above the heading on the right-hand text pages tell you exactly where you are in the book. The bar that extends all the way to the edge of the page is the unit color; the other bar denotes the strand. The strand color is repeated in a tab at the bottom of the page. When the book is closed, you can tell at a glance where each strand and unit begins and ends.

When the book is open, headings on the pages also help to tell you exactly where you are in the book, for example, the Infinite Limits section of Limits and Continuity in the Calculus strand.



SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Ways to REMEMBER

Think of the weights of items that are familiar to you. For example, a bird weighs a few ounces, flour comes in 1-, 5-, and 10-pound bags, and a truck weighs several tons. This will help you remember when to use the different units of weight.

How to Use Southwestern Advantage Online (www.SWadvantage.com)

An integral part of Southwestern Advantage is the accompanying Web site. Organized by subject areas, it is a comprehensive suite of online study helps, additional in-depth subject matter, tips for parents, and coaching for students on how to get better at life.

STATISTICS AND PROBABILITY

ALGEBRA

ADVANCED ALGEBRA

CALCULUS

SPECIAL ADVANTAGES

Math's New Methods:
What Parents Need to Know 14

Unit Tables and Charts 206, 470, 604, 884,
..............1124, 1380, 1746, 2102, and 2328

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and Charts ....1128 and 2330

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Limits and Continuity **CALCULUS**

Unit within Calculus

Name of strand Calculus strand

Unit color bar

Try It This Way

You can visualize and then use mental math to solve Example 4(b).

Add each measure mentally.

$$\begin{array}{r} 3 \text{ yd } 2 \text{ ft} \\ +\ 5 \text{ yd } 2 \text{ ft} \\ \hline 8 \text{ yd } 4 \text{ ft} \end{array}$$

Think: 3 ft = 1 yd

Think: 4 ft is 3 ft + 1 ft

$$8 \text{ yd} + 4 \text{ ft}$$

$$\underbrace{8 \text{ yd} + 1 \text{ yd}}_{9 \text{ yd}} + 1 \text{ ft}$$

9 yd 1 ft

Got to KNOW!

Bias

Bias can occur in different stages of the surveying process. The identified population, the sample chosen, or a survey question is biased if it in any way influences the results of the survey.

Need More HELP

When you write a unit in a different form, you rename the unit. The directions below mean the same thing.

Rename 5 feet as inches.

Express 5 feet as inches.

Convert 5 feet to inches.

Watch Out!

To make a general statement based on information from a sample, the sample must be *random*.

To learn more, see *Random Sampling* on page 1224.

Strand color tab

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Math Curricula and Programs

TODAY'S INSTRUCTIONAL APPROACHES In today's math class, it may not be so much what you know, but rather *how* you know it. While many students still learn math the same way their parents were taught, more and more are learning the basic operations of addition, subtraction, multiplication, and division through nontraditional methods. These methods have been introduced in a number of new math curricula and programs developed over the last 25 years. Three such programs are *Saxon Math*, *Everyday Mathematics*, and the Singapore Math curriculum.

Saxon Math

The *Saxon Math* program was developed by math educator John Saxon (1923–1996). Its emphasis is on incremental learning of math concepts. This incremental approach differs significantly from other math programs. In many traditional math programs, a concept is introduced, practiced, and assessed in a specified block of time. Once students are tested on the concept, the block is finished and students move on to a new concept.

Saxon felt that big math concepts were easier to learn and master when broken down into smaller concepts that are introduced separately and reviewed continually. Following this approach, the *Saxon Math* program introduces a new concept every day. The concept is practiced and reviewed continually throughout the year, and it is expanded on in subsequent lessons, enabling students to build on what they know.

An example is Saxon's approach to multiplication.

- Students are introduced to multiplication as repeated addition and then by making a multiplication table.
- Two days later, they learn how to multiply by one-digit numbers. The multiplication concept reappears a few lessons later with an explanation on how to multiply by multiples of 10 and 100.
- In the next appearance of the concept, students learn how to multiply by two-digit numbers and then three-digit numbers.
- When the concept appears again, students learn how to multiply fractions.
- By the end of the year, students are learning how to multiply decimals and mixed numbers. All along the way, students are continually reviewing and practicing the multiplication concepts they've already learned.

In contrast to many of the newer instructional approaches, *Saxon Math* relies heavily on the traditional, or standard, *algorithms* for teaching the basic operations of addition, subtraction, multiplication, and division.

Need More HELP

An *algorithm* is a step-by-step procedure for performing a math computation.

Everyday Mathematics

Everyday Mathematics (also called Everyday Math or Chicago Math) is a preK–6 math curriculum that was developed in the 1980's by the University of Chicago School Mathematics Project. The program is designed to encourage students to explore a variety of methods for working math problems. For example, to solve an addition problem, they might use a parts-and-total diagram, or they might work a multiplication problem by shading in the squares of an array.

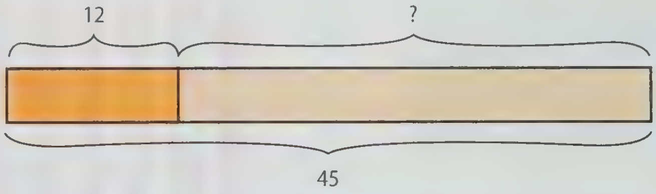
As with traditional math programs, students of *Everyday Mathematics* commonly use algorithms to work arithmetic problems. But these algorithms are not the standard algorithms you may be accustomed to seeing. As a parent, it is important for you to understand these "non-standard" algorithms and to be able to support your child in applying them. The following pages present a brief overview of some of these procedures.

Singapore Math

The term "Singapore math" refers to a math program developed in the early 1980's by the Ministry of Education in Singapore. This program has since been revised and expanded. In its present form, Singapore math aligns with the National Council of Teachers of Mathematics (NCTM) principles and standards. *Math in Focus: The Singapore Approach* is the U.S. edition of Singapore math.

A distinguishing characteristic of Singapore math is its use of graphic representations and modeling strategies to solve all types of problems, from simple to complex. The models, often called box models or bar models, enable students to represent a word problem visually so they can see the number relationships in the problem.

The box model below shows the relationship of two given numbers, 12 and 45, to an unknown number. It is clear from the model that you would subtract to find the unknown value.



The word problem section on pages 34–35 shows the use of box models to solve word problems.

Addition

Partial Sums

When using the **partial-sums addition method**, first add the largest place value, then the next place value, and the next, and so on until the ones are added. Then add the partial sums. This process allows for more mental math than paper-and-pencil computation does. It also reduces the need to regroup.

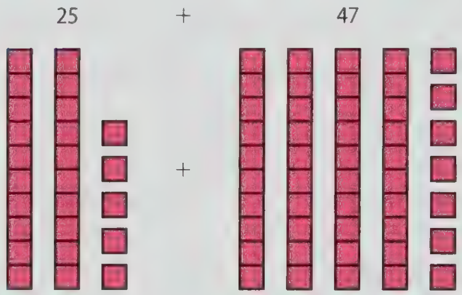
Need More HELP

The "partial sums" method is sometimes called the **left-to-right addition method** because you work from the largest to the smallest place value.

EXAMPLE 1

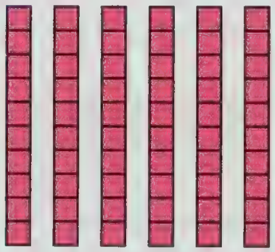
Add 25 + 47.

STEP 1 Use base ten blocks to model the addition.



STEP 2 Add the tens.

2 tens + 4 tens = 6 tens, or 60



STEP 3 Add the ones.

5 ones + 7 ones = 12 ones, or 12

STEP 4 Add the partial sums.

$$\begin{array}{r} 25 \\ +\ 47 \\ \hline 60 \\ 12 \\ \hline 72 \end{array}$$

The sum of 25 and 47 is 72.

Try It This Way

To see another method for adding 25 + 47, go to Example 4 on page 18.

Once you understand the method, you can use it without the models.

EXAMPLE 2

Add 347 + 236.

METHOD 1

Add horizontally.

	347 + 236
Add the hundreds.	300 + 200 = 500
Add the tens.	40 + 30 = 70
Add the ones.	7 + 6 = 13
Add the partial sums.	500 + 70 + 13 = 583

The sum of 347 and 236 is 583.

METHOD 2

Use a vertical format.

	347
	+ 236
Add the hundreds: 300 + 200	500
Add the tens: 40 + 30	70
Add the ones: 7 + 6	+ 13
Add the partial sums.	583

The sum of 347 and 236 is 583.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

To review basic addition facts, go to *Addition Facts* in *Foundations of Mathematics* (p. 56).

EXAMPLE 3

Add 173 + 458.

	173
	+ 458
Add the hundreds: 100 + 400	500
Add the tens: 70 + 50	120
Add the ones: 3 + 8	+ 11
Add the partial sums.	631

The sum of 173 and 458 is 631.

Try It This Way

To see two other methods for adding 173 + 458, go to Example 5 on page 18 and Example 7 on page 19.

Column Addition

When using the **column addition method**, draw lines to separate each place value. Add the digits in each place. Then, if there are 2 digits in the sum of a place, trade 10 in for 1 in the next column to the left.

EXAMPLE 4

Add 25 + 47.

Write the addition problem in a vertical format.

		Tens	Ones
		2	5
	+	4	7
Add the digits in each place.		6	12
Adjust the sum by trading 10 ones for 1 ten in the next column.		7	2

The sum of 25 and 47 is 72.

Try It This Way

You may see this method shown with a horizontal line between each of the steps.

	Tens	Ones
	2	5
+	4	7
	6	12
	7	2

EXAMPLE 5

Add 173 + 458.

Write the addition problem in a vertical format.

		Hundreds	Tens	Ones
		1	7	3
	+	4	5	8
Add the digits in each place.		5	12	11
Trade 10 ones for 1 ten.		5	13	1
Trade 10 tens for 1 hundred.		6	3	1

The sum of 173 and 458 is 631.

Try It This Way

To see two other methods for adding 173 + 458, go to Example 3 on page 17 and Example 7 on page 19.

EXAMPLE 6

Add 536 + 289.

Write the addition problem in a vertical format.

	Hundreds	Tens	Ones
	5	3	6
	+ 2	8	9
Add the digits in each place.	7	11	15
Trade 10 ones for 1 ten.	7	12	5
Trade 10 tens for 1 hundred.	8	2	5

The sum of 536 and 289 is 825.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Opposite-Change Addition

When using the **opposite-change addition method**, change one addend to a number that ends in zero (or a series of zeros) by adding or subtracting a number. Next, change the other addend using the same number but the opposite operation. Then add the resulting addends. The idea is to simplify a problem to reduce the number of times you need to regroup.

EXAMPLE 7

Add 173 + 458.

METHOD 1

Think: What number can I add or subtract to 173 to get a 0 in the ones place?

Subtract 3 from 173 to get a 0 as the last digit.	173	− 3 =	170
Do the opposite: Add 3 to 458.	+ 458	+ 3 =	+ 461
Add the new addends.			631

The sum of 173 and 458 is 631.

METHOD 2

Think: What number can I add or subtract to 458 to get a 0 in the ones place?

Add 2 to 458 to get 460.	173	− 2 =	171
Do the opposite: Subtract 2 from 173.	+ 458	+ 2 =	+ 460
Add the new addends.			631

The sum of 173 and 458 is 631.

Try It This Way

To see two other methods for adding 173 + 458, go to Example 3 on page 17 and Example 5 on page 18.

Subtraction

Need More **HELP**

A subtraction problem consists of these three parts:

85 minuend
− 12 subtrahend
73 difference

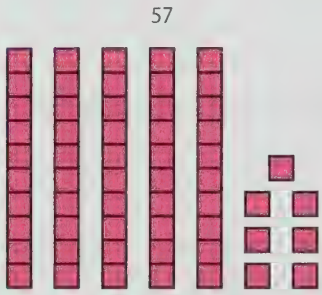
Trade-First Subtraction

When using the **trade-first subtraction method**, you must regroup (trade) when the digit in the minuend (top number) has a lesser value than the digit in the subtrahend (bottom number). You can use a model or the algorithm shown in Examples 2 and 3.

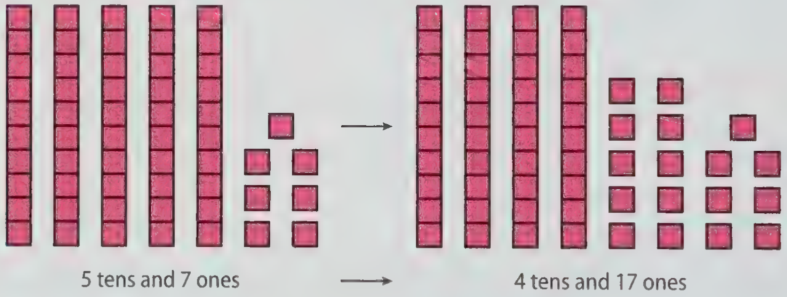
EXAMPLE 1

Subtract 57 − 28.

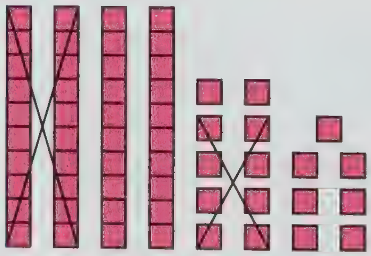
STEP 1 Use base ten blocks to model the minuend, 57.



STEP 2 Since 8 ones > 7 ones, you cannot subtract 28 until you regroup 57.



STEP 3 Subtract the ones by removing 8 ones. Subtract the tens by removing 2 tens.



STEP 4 Count the remaining bars and squares to find the difference.

4 tens − 2 tens = 2 tens, or 20
17 ones − 8 ones = 9 ones, or 9
29

The difference between 57 and 28 is 29.

To use the Trade First method without a model, first write the problem numbers with plenty of space between the columns and space above the columns. Then do all of the trading, from right to left. When you have completed the trading, then subtract.

EXAMPLE 2

Subtract 5,482 − 1,393.

STEP 1 Trade from right to left.

- Ones column: You cannot subtract 2 − 3. Trade 1 ten for 10 ones. Now you have 7 tens and 12 ones.

$$\begin{array}{rcccc} & 5 & 4 & \overset{7}{\cancel{8}} & \overset{12}{\cancel{2}} \\ - & 1 & 3 & 9 & 3 \\ \hline \end{array}$$

- Tens column: You cannot subtract 7 − 9. Trade 1 hundred for 10 tens. Now you have 3 hundreds and 17 tens.
- Hundreds column: You can subtract 3 − 3.
- Thousands column: You can subtract 5 − 1.

$$\begin{array}{rcccc} & 5 & \overset{3}{\cancel{4}} & \overset{\overset{17}{\cancel{7}}}{\cancel{8}} & \overset{12}{\cancel{2}} \\ - & 1 & 3 & 9 & 3 \\ \hline \end{array}$$

STEP 2 Subtract.

Because you have already done the trading, you can subtract from right to left or from left to right.

$$\begin{array}{rcccc} & 5 & \overset{3}{\cancel{4}} & \overset{\overset{17}{\cancel{7}}}{\cancel{8}} & \overset{12}{\cancel{2}} \\ - & 1 & 3 & 9 & 3 \\ \hline & 4 & 0 & 8 & 9 \end{array}$$

12 − 3 = 9   17 − 9 = 8
3 − 3 = 0   5 − 1 = 4

The difference between 5,482 and 1,393 is 4,089.

Need More HELP

To review basic subtraction facts, go to *Subtraction Facts* in *Foundations of Mathematics* (p. 62).

Try It This Way

To see other methods for subtracting 5,482 − 1,393, go to Example 7 on page 23 and Example 11 on page 25.

EXAMPLE 3

Subtract 503 − 129.

STEP 1 Trade from right to left.

You cannot subtract 3 − 9 but there are no tens to trade.
First trade 1 hundred for 10 tens.
Then trade 1 ten for 10 ones.
Now you have 4 hundreds, 9 tens, and 13 ones.

$$\begin{array}{rccc} & \overset{4}{\cancel{5}} & \overset{\overset{9}{\cancel{10}}}{\cancel{0}} & \overset{13}{\cancel{3}} \\ - & 1 & 2 & 9 \\ \hline & 3 & 7 & 4 \end{array}$$

STEP 2 Subtract.

13 − 9 = 4   9 − 2 = 7   4 − 1 = 3

The difference between 503 and 129 is 374.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

To review basic addition facts, go to *Addition Facts* in *Foundations of Mathematics* (p. 56).

Counting-Up Subtraction

When using the **counting-up subtraction method**, first write the subtrahend, the number being subtracted. Then, count up to get the next place value. Circle each number used to count up. Then add the circled numbers to find the difference.

EXAMPLE 4

Subtract 93 − 38.

Write the subtrahend.	38
Count up to the nearest ten.	+② 40
Count up to the tens place (9) in the minuend, 93.	+㊿ 90
Count up to the minuend.	+③ 93
Add the circled numbers.	2 + 50 + 3 = 55

The difference between 93 and 38 is 55.

EXAMPLE 5

Subtract 741 − 57.

Write the subtrahend.	57
Count up to the nearest ten.	+③ 60
Count up to one hundred.	+㊵ 100
Count up to the hundreds place (7) in the minuend, 741.	+(600) 700
Count up to the minuend.	+㊶ 741
Add the circled numbers.	3 + 40 + 600 + 41 = 684

The difference between 741 and 57 is 684.

Left-to-Right Subtraction

When using the **left-to-right subtraction method**, first subtract the leftmost digits (the digits with the greatest place value). Continue subtracting working toward the right until the ones have been subtracted. You can work horizontally or vertically.

EXAMPLE 6

Subtract 842 − 197.

The subtrahend 197 is the same as 100 + 90 + 7.

METHOD 1

Subtract the hundreds. $842 - 100 = 742$

Subtract the tens. $742 - 90 = 652$

Subtract the ones. $652 - 7 = 645$

The difference between 842 and 197 is 645.

METHOD 2

$$\begin{array}{lr} & 842 \\ \text{Subtract the hundreds.} & -\,100 \\ \cline{2-2} & 742 \\ \text{Subtract the tens.} & -\,90 \\ \cline{2-2} & 652 \\ \text{Subtract the ones.} & -\,7 \\ \cline{2-2} & 645 \end{array}$$

The difference between 842 and 197 is 645.

EXAMPLE 7

Subtract 5,482 − 1,393.

The subtrahend 1,393 is the same as 1,000 + 300 + 90 + 3.

Subtract the thousands. $5{,}482 - 1{,}000 = 4{,}482$

Subtract the hundreds. $4{,}482 - 300 = 4{,}182$

Subtract the tens. $4{,}182 - 90 = 4{,}092$

Subtract the ones. $4{,}092 - 3 = 4{,}089$

The difference between 5,482 and 1,393 is 4,089.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Try It This Way

To see other methods for subtracting 5,482 − 1,393, go to Example 2 on page 21 and Example 11 on page 25.

Partial Differences

The **partial-differences subtraction method** is similar to the partial-sums method for addition. Starting on the left, with the greatest place value, subtract from left to right until you have subtracted the ones. Then combine the partial differences to find the total difference.

SEARCH

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EXAMPLE 8

Subtract 493 − 361.

$$\begin{array}{r} 493 \\ -\ 361 \\ \hline \end{array}$$

Subtract the hundreds: 400 − 300	100
Subtract the tens: 90 − 60	30
Subtract the ones: 3 − 1	2
Add the partial differences.	100 + 30 + 2 = 132

493 − 361 = 132

EXAMPLE 9

Subtract 6,378 − 1,243.

$$\begin{array}{r} 6{,}378 \\ -\ 1{,}243 \\ \hline \end{array}$$

Subtract the thousands: 6,000 − 1,000	5,000
Subtract the hundreds: 300 − 200	100
Subtract the tens: 70 − 40	30
Subtract the ones: 8 − 3	5
Add the partial differences.	5,000 + 100 + 30 + 5 = 5,135

6,378 − 1,243 = 5,135

Same-Change Subtraction

When using the **same-change subtraction method**, the same number is added to or subtracted from *both* the minuend and the subtrahend. Then the resulting numbers are subtracted. The idea is to change the numbers so the subtrahend ends in zero or in a series of zeros. If you choose the number added or subtracted carefully, you may be able to get two zeros in one step, as shown in Example 11.

EXAMPLE 10

Subtract 352 − 171.

Think: How can I get the subtrahend 171 to end in zeros?

STEP 1 Subtract 1 to change the right-most 1 in 171 to 0.

Subtract 1 from both numbers. $(352 - 1) - (171 - 1)$

$351 - 170$

STEP 2 Add 30 to change the 7 in 170 to 0.

Add 30 to both numbers. $(351 + 30) - (170 + 30)$

$381 - 200$

Subtract the resulting numbers.

$$\begin{array}{r} 381 \\ -\ 200 \\ \hline 181 \end{array}$$

352 − 171 = 181

EXAMPLE 11

Subtract 5,482 − 1,393.

Think: 1,393 is almost 1,400.

STEP 1 Add 7 to 1,393 to get 1,400.

Add 7 to both numbers. $(5{,}482 + 7) - (1{,}393 + 7)$

$5{,}489 - 1{,}400$

STEP 2 Subtract the resulting numbers.

$$\begin{array}{r} 5{,}489 \\ -\ 1{,}400 \\ \hline 4{,}089 \end{array}$$

5,482 − 1,393 = 4,089

Try It This Way

To see other methods for subtracting 5,482 − 1,393, go to Example 2 on page 21 and Example 7 on page 23.

Multiplication

Need More HELP

To review basic multiplication facts, go to *Multiplication Facts* in *Foundations of Mathematics* (p. 68).

Partial Products

When using the **partial-products multiplication method**, first break down each factor to show the value of its digits. Then multiply each part of one factor by each part of the other factor to get the partial product. Finally, add all of the partial products.

EXAMPLE 1

Multiply 73 × 18.

Think: $(70 + 3) \times (10 + 8)$

Multiply 7 tens and 1 ten.	$70 \times 10 = 700$
Multiply 7 tens and 8 ones.	$70 \times 8 = 560$
Multiply 3 ones and 1 ten.	$3 \times 10 = 30$
Multiply 3 ones and 8 ones.	$3 \times 8 = 24$
Add the partial products.	$700 + 560 + 30 + 24 = 1{,}314$

The product of 73 and 18 is 1,314.

EXAMPLE 2

Multiply 132 × 46.

Think: $(100 + 30 + 2) \times (40 + 6)$

Multiply 1 hundred and 4 tens.	$100 \times 40 = 4{,}000$
Multiply 1 hundred and 6 ones.	$100 \times 6 = 600$
Multiply 3 tens and 4 tens.	$30 \times 40 = 1{,}200$
Multiply 3 tens and 6 ones.	$30 \times 6 = 180$
Multiply 2 ones and 4 tens.	$2 \times 40 = 80$
Multiply 2 ones and 6 ones.	$2 \times 6 = 12$
Add the partial products.	$4{,}000 + 600 + 1{,}200 + 180 + 80 + 12 = 6{,}072$

The product of 132 and 46 is 6,072.

You can also use a vertical format for the partial products method, as shown in Example 3 and Example 4 below.

EXAMPLE 3

Multiply 62 × 37.

STEP 1 Think: $(60 + 2) \times (30 + 7)$

STEP 2 Write the problem in vertical format. Then multiply each part of one factor by each part of the other factor.

$$\begin{array}{lcr} & & 62 \\ & & \times\ 37 \\ \cline{3-3} 60 \times 30 = 1800 & \longrightarrow & 1800 \\ 60 \times 7 = 420 & \longrightarrow & 420 \\ 2 \times 30 = 60 & \longrightarrow & 60 \\ 2 \times 7 = 14 & \longrightarrow & 14 \end{array}$$

STEP 3 Add the partial products.

$1{,}800 + 420 + 60 + 14 = 2{,}294$

The product of 62 and 37 is 2,294.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 4

Multiply 235 × 74.

STEP 1 Think: $(200 + 30 + 5) \times (70 + 4)$

STEP 2 Write the problem in vertical format. Then multiply each part of one factor by each part of the other factor.

$$\begin{array}{lcr} & & 235 \\ & & \times\ 74 \\ \cline{3-3} 200 \times 70 = 14{,}000 & \longrightarrow & 14000 \\ 200 \times 4 = 800 & \longrightarrow & 800 \\ 30 \times 70 = 2100 & \longrightarrow & 2100 \\ 30 \times 4 = 120 & \longrightarrow & 120 \\ 5 \times 70 = 350 & \longrightarrow & 350 \\ 5 \times 4 = 20 & \longrightarrow & 20 \end{array}$$

STEP 3 Add the partial products.

$14{,}000 + 800 + 2{,}100 + 120 + 350 + 20 = 17{,}390$

The product of 235 and 74 is 17,390.

Try It This Way

To see another method for multiplying 235 × 74, go to Example 6 on page 29.

Egyptian Multiplication

When using the **Egyptian multiplication method**, write the problem horizontally with a column of numbers below each factor.

- The left column *always* shows powers of 2 (the number 1 is doubled). Double until you reach a number that is greater than the factor above the column.
- The right column starts with the factor above it. Double this factor until there are the same number of entries as in the column on the left.

Find two or more numbers in the left column that equal the factor on the left. Then add the corresponding numbers in the right column to find the product.

EXAMPLE 5

Multiply 36 × 52.

STEP 1 Write the multiplication horizontally. Below the factor on the left, double the number 1 until you reach a number that is greater than the factor, 36. Since 64 > 36, stop with 32.

For this multiplication, there are six entries in the left column.

36	×	52
1		
2		
4		
8		
16		
32		
~~64~~		

STEP 2 Below the factor on the right, write the factor. Then double the factor until there are six entries in the column on the right.

36	×	52
1		52
2		104
4		208
8		416
16		832
32		1,664

STEP 3 Find two numbers in the column on the left whose sum is 36.

4 + 32 = 36

Find the corresponding numbers in the column on the right: 208 and 1,664.

36	×	52
1		52
2		104
4	→	**208**
8		416
16		832
32	→	**1,664**

STEP 4 Add the two numbers in the column on the right.

208 + 1,664 = 1,872

The product of 36 and 52 is 1,872.

Need More HELP

Egyptian Multiplication was developed by the ancient Egyptians although they used hieroglyphs rather than numerals. It is very similar to another method called Russian Peasant Multiplication.

Try It This Way

You can write the factors in any order. In Example 5, you could write the problem as 52 × 36 and double 36.

For any problem, look at the factors carefully and decide which will be easiest to double. Place that factor on the right.

Lattice Multiplication

The **lattice multiplication method** uses a grid of squares with each square divided by a diagonal. The factors are placed as follows.

- One factor is written along the top, left to right, with a digit above each square.
- The other factor is written along the right side of the grid, from top to bottom, with a digit to the right of each square.

Multiply each digit in one factor by each digit in the other factor. For each product, place the tens digit above the diagonal and the ones digit below it.

Then add the digits along each diagonal, starting at the bottom right corner. Place the sum at the bottom of each diagonal *outside* of the grid. If necessary, carry the tens digit to the next diagonal. Read the answer down the left side and then from left to right across the bottom.

EXAMPLE 6

Multiply 235 × 74.

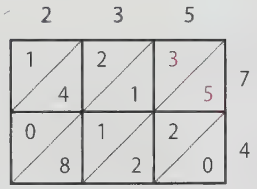
STEP 1 Place the multiplication on a grid.

First multiply 7 × 5 = 35. Place the 3 above the diagonal and the 5 below the diagonal. Continue multiplying, each time writing the partial product on the grid.

7 × 3 = 21 7 × 2 = 14

4 × 5 = 20 4 × 3 = 12

4 × 2 = 8



STEP 2 Add the partial products starting at the bottom right corner and moving left. If you extend the diagonal lines, you can see the diagonals better.

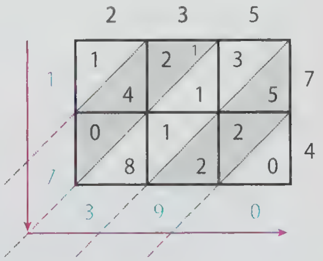
1st diagonal: 0

2nd diagonal: 5 + 2 + 2 = 9

3rd diagonal: 3 + 1 + 1 + 8 = 13
Write 3 in the diagonal and place the 1 ten in the next diagonal.

4th diagonal: 1 + 2 + 4 + 0 = 7

5th diagonal: 1



STEP 3 Find the product. Start with the top left digit outside of the grid, 1. Move down and then across the bottom of the grid. Write the digits in order: 1 7 3 9 0

The product of 235 and 74 is 17,390.

Try It This Way

To see another method for multiplying 235 × 74, go to Example 4 on page 27.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Lattice Method and Grids

Sometimes lattice multiplication is shown using a grid for all of the numbers, as shown in Example 7 and Example 8. The cells on the grid that show the products may have a heavier border, or they may be tinted. Although lattice styles may differ somewhat, the steps for finding the product remain the same.

EXAMPLE 7

Multiply 432 × 36.

STEP 1 Place the multiplication on a grid.

First multiply 3 × 2 = 6. There are no tens, so place a 0 above the diagonal and the 6 below the diagonal.

Continue multiplying, each time writing the partial product on the grid.

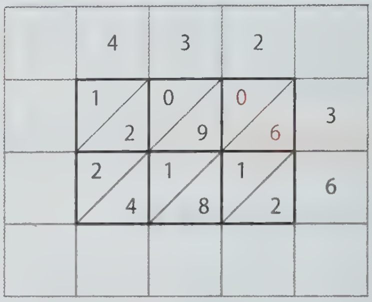
3 × 3 = 9

3 × 4 = 12

6 × 2 = 12

6 × 3 = 18

6 × 4 = 24



STEP 2 Add the partial products starting at the bottom right corner and moving left.

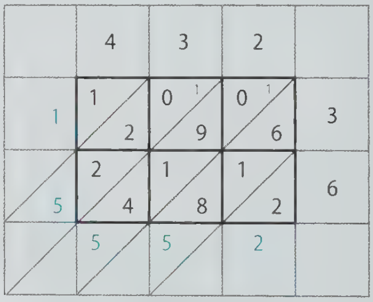
1st diagonal: 2

2nd diagonal: 6 + 1 + 8 = 15
Write 5 in the box and place the 1 ten in the next diagonal.

3rd diagonal: 1 + 9 + 1 + 4 = 15
Write 5 in the box and place the 1 ten in the next diagonal.

4th diagonal: 1 + 0 + 2 + 2 = 5

5th diagonal: 1



STEP 3 Find the product.

Start with the top left digit outside of the grid, 1. Move down and then across the bottom of the grid. Write the digits in order: 1 5 5 5 2

The product of 432 and 36 is 15,552.

EXAMPLE 8

Multiply 6,387 × 209.

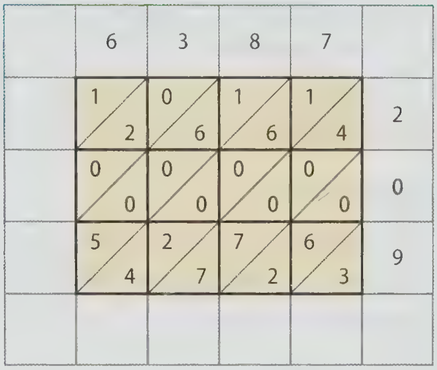
STEP 1 Place the multiplication on a grid.

Then multiply. Remember to place the tens digit above the diagonal and the ones digit below it.

First multiply each digit in the factor 6,387 by 2.

Next, multiply each digit by 0.

Finally, multiply each digit by 9.



	6	3	8	7	
	1/2	0/6	1/6	1/4	2
	0/0	0/0	0/0	0/0	0
	5/4	2/7	7/2	6/3	9

STEP 2 Add the partial products starting at the bottom right corner and moving left.

1st diagonal: 3

2nd diagonal: 0 + 6 + 2 = 8

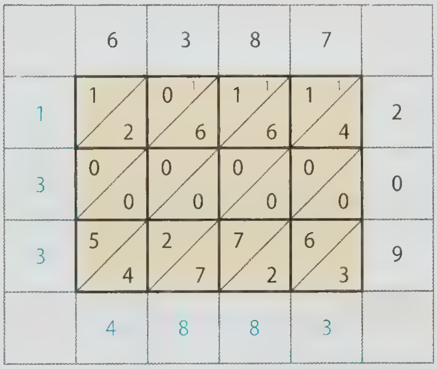
3rd diagonal: 4 + 7 + 7 = 18
Write 5 in the box and place the 1 ten in the next diagonal.

4th diagonal: 1 + 1 + 6 + 2 + 4 = 14
Write 4 in the box and place the 1 ten in the next diagonal.

5th diagonal: 1 + 1 + 6 + 5 = 13
Write 3 in the box and place the 1 ten in the next diagonal.

6th diagonal: 1 + 0 + 2 + 0 = 3

7th diagonal: 1



	6	3	8	7	
1	1/2	0/6 1	1/6 1	1/4 1	2
3	0/0	0/0	0/0	0/0	0
3	5/4	2/7	7/2	6/3	9
	4	8	8	3	

STEP 3 Find the product.

Start with the top left digit outside of the grid, 1. Move down and then across the bottom of the grid. Write the digits in order: 1 3 3 4 8 8 3

The product of 6,387 and 209 is 1,334,883.

SEARCH

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Division

Need More

HELP

A division problem consists of these three parts:

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

Partial Quotients

The **partial-quotients division method** involves making a series of estimates by multiplying the divisor by a small number and subtracting. Continue until no more divisions are possible. Add the partial quotients. Include any remainder in the final quotient.

EXAMPLE 1

Divide 14 ÷ 4.

Think: The divisor is 4. Since 4 × 2 = 8, there are at least two 4's in 14.

Write 4 × 2.
Then subtract 14 − 8 = 6

$$\begin{array}{rl} 4\overline{)14} & 4 \times 2 \\ \underline{-8} & \\ 6 & \end{array}$$

There is a remainder of 6, so there is another 4 in 6.

Write 4 × 1.
Then subtract 6 − 4 = 2.

$$\begin{array}{rl} 4\overline{)14} & 4 \times 2 \\ \underline{-8} & \\ 6 & 4 \times 1 \\ \underline{-4} & \\ 2 & \end{array}$$

In the number 14, there are 3 groups of 4 with a remainder of 2.

Write the quotient: 3 R2.

$$\begin{array}{rl} 3\text{ R2} & \\ 4\overline{)14} & 4 \times 2 \\ \underline{-8} & \\ 6 & 4 \times 1 \\ \underline{-4} & \\ 2 & \overline{3} \end{array}$$

The quotient 14 ÷ 4 is 3 R2.

EXAMPLE 2

Divide 137 ÷ 9.

Estimate the number of times the divisor 9 will go into the dividend 137.

$$\begin{array}{rr} 15\text{ R2} & \\ 9\overline{)137} & 9 \times 10 \\ \underline{-90} & \\ 47 & 9 \times 5 \\ \underline{-45} & \\ 2 & \overline{15} \end{array}$$

Think: Since 9 × 10 = 90, it will go at least 10 times. Write 9 × 10.

Subtract 137 − 90 = 47. There is a remainder of 47. Estimate again.

Think: Since 9 × 5 = 45, it will go 5 times. Write 9 × 5 = 45.

Subtract 47 − 45 = 2. There are no more groups of 9 so 2 is the remainder.

There are 15 groups of 9 with 2 left over, so the quotient 137 ÷ 9 is 15 R2.

Try It This Way

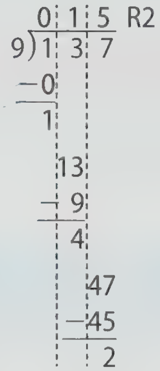
To see another method for dividing 137 ÷ 9, go to Example 3 on page 33.

Column Division

The **column division method** is a variation of the standard long division algorithm. Vertical lines, which form columns, are inserted to separate the digits in the dividend. Remainders are always repositioned before performing the next division. Example 4 shows a variation of the algorithm for this method.

EXAMPLE 3

Divide 137 ÷ 9.



STEP 1 First Division

- Determine how many times 9 will go into 1. Since 9 > 0, it will not go.
- Write 0 above the 1 in the dividend. Then subtract 1 − 0 = 1.
- Bring down the 3. Reposition the remainder 1 to the left of the 3 to show 13.

STEP 2 Second Division

- Determine how many times 9 will go into 13. Since 2 × 9 = 18, it will go 1 time.
- Write 1 above the 3 in the dividend. Then subtract 13 − 9 = 4.
- Bring down the 7. Reposition the remainder 4 to the left of the 7 to show 47.

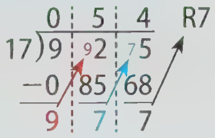
STEP 3 Third Division

- Determine how many times 9 will go into 47. Since 5 × 9 = 45, it will go 5 times.
- Write 5 above the 7 in the dividend. There are no more digits in the dividend to bring down. Subtract 47 − 45 = 2.

The quotient 137 ÷ 9 is 15 R2.

EXAMPLE 4

Divide 925 ÷ 17.



Determine how many times 17 will go into 9.

- Since 17 > 9, write 0 in the dividend. Then subtract 9 − 0 = 9.
- Place a 9 next to the next number in the dividend, 2, to get 92.
- Continue dividing in this way, as shown on the right.

The quotient 925 ÷ 17 is 54 R7.

Need More HELP

To review basic division facts, go to *Division Facts* in *Foundations of Mathematics* (p. 74).

Try It This Way

To see another method for dividing 137 ÷ 9, go to Example 2 on page 32.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Word Problems

Need More HELP

For more word problems involving whole numbers, go to *Word Problems* in *Foundations of Mathematics* (p. 104).

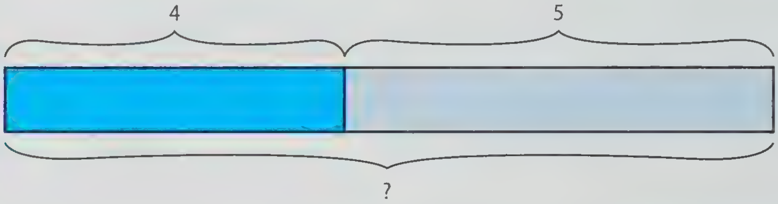
Box Models

In a **box model**, the "box" is a rectangular bar that shows the relationship between a known and an unknown quantity. This type of model can help you visualize what operation is needed to solve a problem. For more complex problems, two or more bars may be used to represent the situation.

EXAMPLE 1

Hector has 4 fish in his fish tank. He buys 5 more at the pet store. How many fish does he have now?

Draw a bar and divide it into two parts. Let one part represent 4 fish. Let the other part represent 5 more fish.

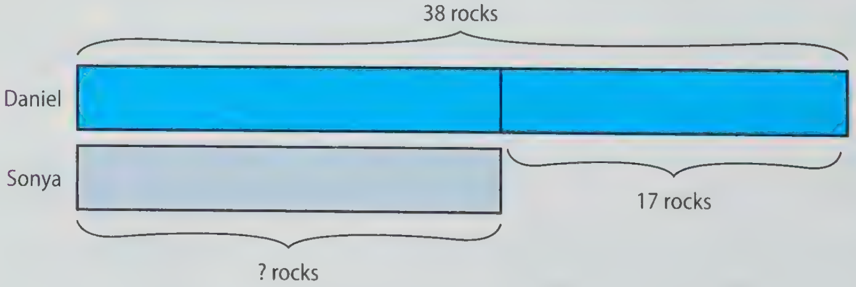


Add the two known parts to find the whole, or unknown, part: 4 + 5 = 9.

Hector has 9 fish now.

EXAMPLE 2

Daniel collected 38 rocks. Sonya found 17 fewer rocks than Daniel. How many rocks does Sonya have?



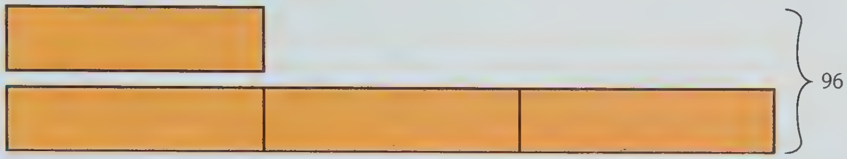
The whole is 38 rocks, the number Daniel collected.
A known part is 17 rocks.
Subtract to find the unknown part: 38 − 17 = 21

Sonya has 21 rocks.

EXAMPLE 3

The sum of two numbers is 96. One of the numbers is three times greater than the other number. What are the two numbers?

Represent the two numbers as rectangles. The rectangle for the larger number is three times the length of the rectangle for the smaller number.



There are 4 equal sections, so the smaller number is 96 ÷ 4 = 24. The larger number is 24 × 3 = 72. Since 24 + 72 = 96, these values must be correct.

The numbers are 24 and 72.

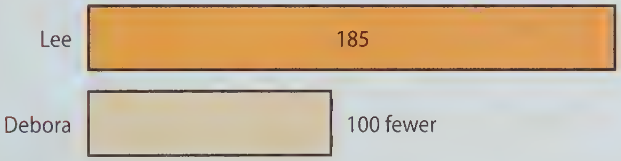
SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 4

Lee had 185 trading cards. Debora had 100 fewer trading cards. Lee gives Debora some of his cards. Now he has 2 times as many cards as Debora.

a. At first, how many trading cards did Debora have?

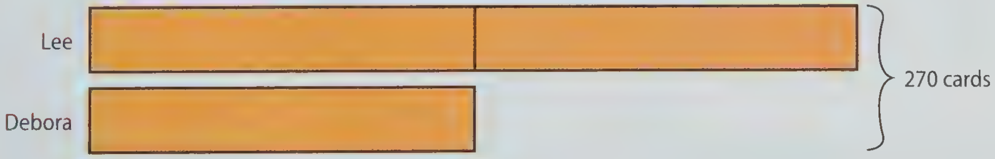


Debora: 185 − 100 = 85

At first, Debora had 85 trading cards.

b. How many trading cards do they both have now?

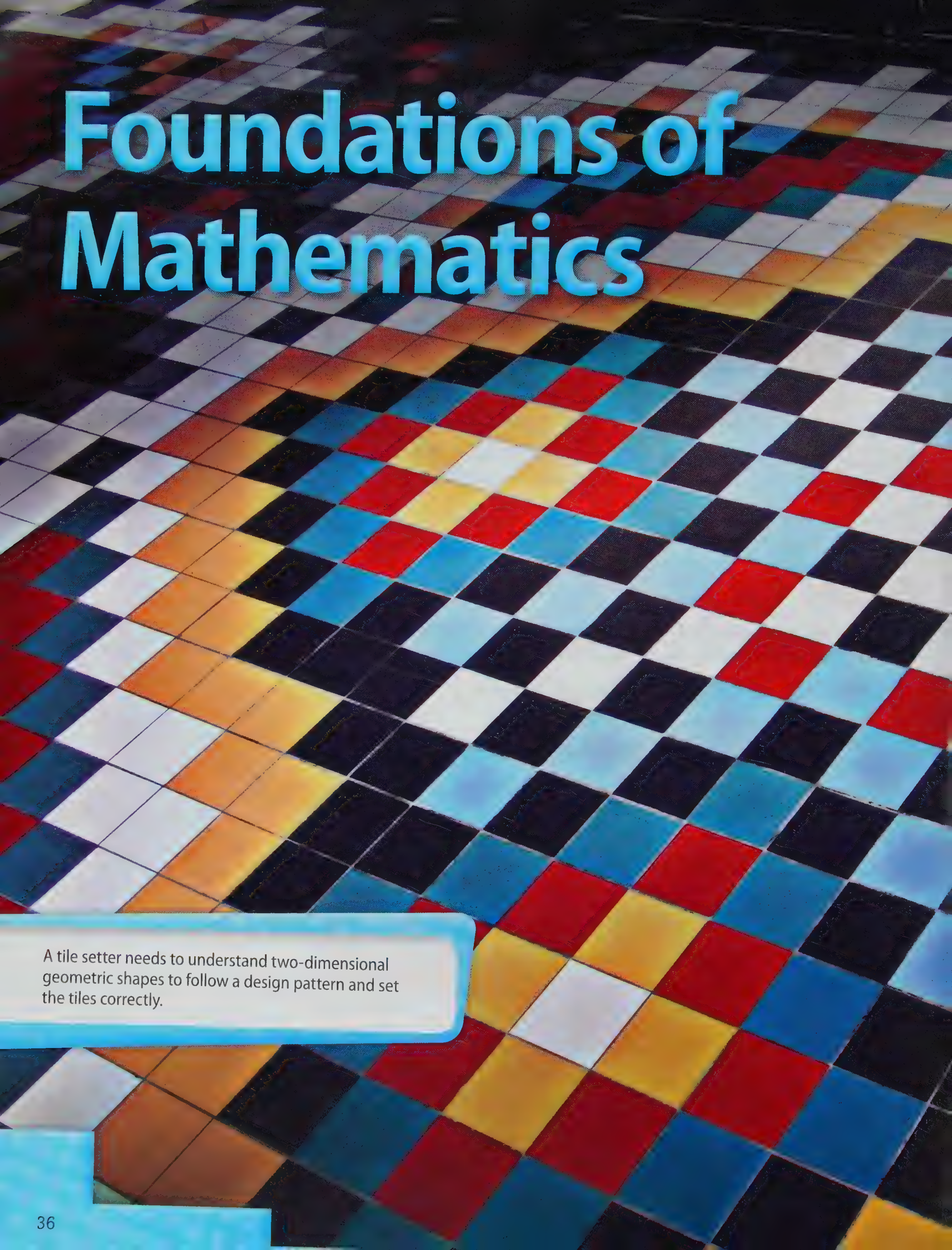
The total number of cards is 185 + 85 = 270. The 270 cards are shared between Lee and Debora. After giving Debora some cards, Lee has 2 × as many cards as she has.



Debora: 270 ÷ 3 = 90 Lee = 90 × 2 = 180 Total: 90 + 180 = 270

Now, Lee has 180 cards and Debora has 90 cards.

Foundations of Mathematics



A tile setter needs to understand two-dimensional geometric shapes to follow a design pattern and set the tiles correctly.

Place Value 38

Whole Numbers 52

Estimating with Whole Numbers 108

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Analyzing Shapes in a Plane 152

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Place Value to Billions

Using a Place-Value Chart

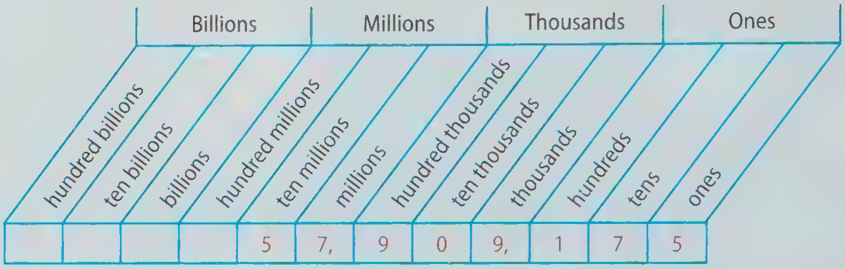
All whole numbers are formed by arranging the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 in different orders. The value of each digit depends on its place in a number. A *place-value chart* can help you find the value of the digits. Each group of three places on the chart is called a **period**. Commas separate each period into hundreds, tens, and ones.

Need More HELP

The *periods* on a place-value chart are sometimes called *families*. For example, the Thousands Family contains one thousands, ten thousands, and hundred thousands.

EXAMPLE 1

The planet Mercury is 57,909,175 kilometers (km) from the sun. Use the place-value chart to help you find the value of this number.


Billions
Millions
Thousands
Ones
hundred billions
ten billions
billions
hundred millions
ten millions
millions
hundred thousands
ten thousands
thousands
hundreds
tens
ones
5
7,
9
0
9,
1
7
5

Start with the period farthest to the left.

STEP 1 The digits in the *millions period* are 5 and 7. Say: 57 million.

STEP 2 The digits in the *thousands period* are 9, 0, and 9. Say: 909 thousand.

STEP 3 The digits in the *ones period* are 1, 7, and 5. Say: 175 as one hundred seventy-five. Do not say the name of the period.

Mercury is 57 million, 909 thousand, 175 kilometers from the sun.

GOT TO KNOW!

Values of Each Place

The value of each place on a place-value chart is 10 times greater than the place to its right. The value of a digit in a number is the product of that digit and the value of its place.

In 57,909,175, from left to right:

The first 5 has a value of 5 × 10,000,000, or 50,000,000.

The first 7 has a value of 7 × 1,000,000, or 7,000,000.

The first 9 has a value of 9 × 100,000, or 900,000.

The second 9 has a value of 9 × 1,000, or 9,000.

The second 7 has a value 7 × 10, or 70.

The second 5 has a value of 5 × 1, or 5.

Using Place Value

The same digit may appear in more than one place in a number. A place-value chart can help you compare the values of these digits.

EXAMPLE 2

a. The table shows gasoline sales over a three-month period. In which month was the least amount of gasoline sold?

Month	Gallons Sold
January	18,088,688
February	18,866,000
March	18,860,086

STEP 1 For each month, the digit in the *ten millions* place is 1, and the digit in the *millions* place is 8.

STEP 2 For January, the digit in the *hundred thousands* place is 0. For February and March, the digit in the *hundred thousands* place is 8.

This means that the number for January contains no hundred thousands, and the numbers for February and March contain 8 hundred thousands.

The least amount of gas was sold in January.

b. Look at the digit pairs in the number for January: 18,088,688. How do the values differ?

The red digit on the left has a value that is 10 × the value of the red digit to its right. 80,000 vs. 8,000

The green digit on the left has a value that is 10 × the value of the green digit to its right. 80 vs. 8

SEARCH

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EXAMPLE 3

A mystery number has: 1 in the *billions* and the *thousands* places; 0 in the *hundred millions, ten millions,* and *tens* places; 3 in the *ten billions* and *hundred thousands* places; 5 in the *millions* place; 4 in the *ones* place; 8 in the *ten thousands* place; and 7 in the *hundreds* place. Use a place-value chart to write the mystery number.

Billions			Millions			Thousands			Ones		
100 B	10 B	B	100 M	10 M	M	100 Th	10 Th	Th	H	T	O
	3	1,	0	0	5,	3	8	1,	7	0	4

Comparing Whole Numbers

Ways to REMEMBER

On a sheet of paper, write a 6 and a 3 several inches apart. Use the thumb and forefinger of your *right* hand to model the greater-than symbol, >. Now, place your hand between the number pair. Notice how the larger number could "swallow" the smaller number.

6 > 3

Next, write a 2 and a 7. Repeat the process using the thumb and forefinger of your *left* hand to form the less-than symbol, <.

Using Symbols to Compare

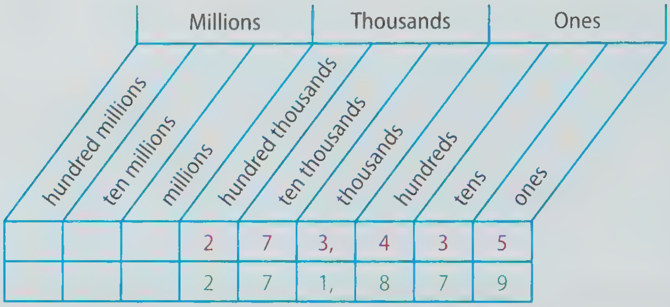
To compare two numbers, decide if their values are greater than, equal to, or less than each other. Use the comparison symbols to compare numbers.

Read > as "is greater than."	5 > 4
Read = as "is equal to."	3 = 3
Read < as "is less than."	8 < 12

EXAMPLE 1

Compare 271,879 and 273,435. Use >, <, or =.

STEP 1 Write the numbers one below the other on a place-value chart.



Millions			Thousands			Ones		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
			2	7	3,	4	3	5
			2	7	1,	8	7	9

STEP 2 Start at the greatest place-value position. Compare the digits on the place-value chart from left to right until you reach two digits that have a different value.

Compare the hundred thousands place: 2 = 2
Compare the ten thousands place: 7 = 7
Compare the thousands place: 3 > 1

STEP 3 The digits in the thousands place are different.

271,879 contains 1 thousand.
273,435 contains 3 thousands.

273,435 > 271,879

GOT TO KNOW!

Comparing Two Whole Numbers

Different number of digits:

- The number with the greater number of digits has the greater value.

Same number of digits:

- Compare the digits from left to right.
- Look for the first place in which the digits differ. The number with the greater digit in that place is the greater number.

Comparing Data in Tables

EXAMPLE 2

Use the population data in the table below.

City	Population
Calgary	1,230,248
Dallas	1,299,543
Philadelphia	1,547,297
Phoenix	1,601,587
San Diego	1,306,301

a. Which is greater, the population of Dallas or the population of Calgary?

STEP 1 Line up the numbers.

Calgary: 1,230,248
Dallas: 1,299,543

STEP 2 Start at the left. Compare digits.

The digits in the millions place are the same: $1 = 1$
The digits in the hundred thousands place are the same: $2 = 2$
The digits in the ten thousands place are different: $9 > 3$
$1{,}299{,}543 > 1{,}230{,}248$

The population of Dallas is greater than the population of Calgary.

b. Which is less, the population of San Diego or the population of Philadelphia?

STEP 1 Line up the numbers.

San Diego: 1,306,301
Philadelphia: 1,547,297

STEP 2 Start at the left. Compare digits.

The digits in the millions place are the same: $1 = 1$
The digits in the hundred thousands place are different: $3 < 5$
$1{,}306{,}301 < 1{,}547{,}297$

The population of San Diego is less than the population of Philadelphia.

c. Which city has the greatest population? How do you know?

All the digits in the millions place are 1. Look at the digits in the hundred thousands place. The greatest digit is 6, so Phoenix's population is the greatest.

Try It This Way

Use a sheet of paper to cover all the digits to the right of the column of digits you are comparing. This will help you compare only the digits in the same place-value position.

It's easier to compare the digits in the hundred thousands place when the rest are covered.

1,230,248
1,299,543
1,547,297
1,601,587
1,306,301

SEARCH

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Rounding Whole Numbers

Round Using Place Value

Sometimes it is helpful to know an approximate value instead of an exact value. You can find the approximate value of a number by **rounding** the exact value to a given place. A rounded number tells *about* how much or *about* how many.

Try It This Way

To better see the place to which you will round, write the number with extra space between the digits.

Then circle the digit in that place.

Then underline the digit to its right.

3 ⑥ $\underline{8}$, 4 9 0

EXAMPLE 1

Round 368,490 to the nearest ten thousand.

STEP 1 Identify the digit in the ten thousands place, 6. 368,490

STEP 2 Look at the digit in the place to its right, 8. 368,490

STEP 3 If this digit is greater than or equal to 5, increase the 6 ten thousands by 1.
If this digit is less than 5, leave the 6 in the ten thousands place.
Since $8 > 5$, increase the 6 by 1: $6 + 1 = 7$
Change all the digits to the right of 7 to zeros. $3\underline{7}0{,}000$

368,490 rounded to the nearest ten thousand is 370,000.

EXAMPLE 2

Round 14,359,808 to the nearest million.

STEP 1 Identify the digit in the millions place, 4. 14,359,808

STEP 2 Look at the digit in the place to its right, 3. 14,359,808

STEP 3 If this digit is greater than or equal to 5, increase the 4 millions by 1.
If this digit is less than 5, leave the 4 in the millions place.
Since $3 < 5$, leave the 4 in the millions place.
Change all the digits to the right of 4 to zeros. $1\underline{4}{,}000{,}000$

14,359,808 rounded to the nearest million is 14,000,000.

GOT TO KNOW!

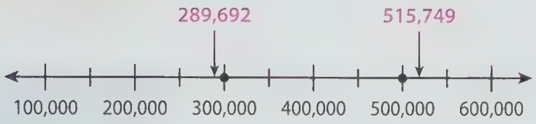
Rounding Rules

1. Underline the digit in the place you are rounding to.
2. Look at the digit to its right.
   - If the digit is greater than or equal to 5, add 1 to the underlined digit.
   - If the digit is less than 5, do not change the underlined digit.
3. Drop the digits to the right of the underlined digit and add zeros in their place.

Round Using a Number Line or Place-Value Chart

EXAMPLE 3

Use the number line to round 289,692 and 515,749 to the nearest hundred thousand.



STEP 1 Look at the labels below the number line. The scale goes from 100,000 to 600,000 in increments of 100,000. The unlabeled tick marks show the halfway points between the labeled whole numbers.

STEP 2 Look at the locations of the points and decide how to round each number.

Think: 289,692 is between 250,000 and 300,000, but closer to 300,000.

Think: 515,749 is between 500,000 and 550,000, but closer to 500,000.

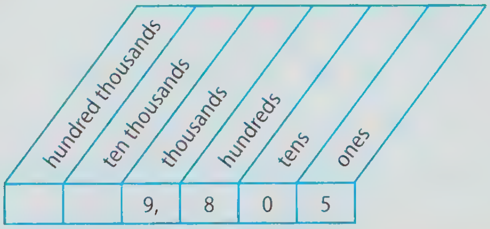
289,692 rounds to 300,000.
515,749 rounds to 500,000.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 4

Ben has a collection of pennies. He writes the number of pennies on a place-value chart. Then he rounds the number to the greatest possible place value. What is Ben's rounded number?



hundred thousands	ten thousands	thousands	hundreds	tens	ones
		9,	8	0	5

- The greatest place in the number is the thousands place.
- There is a 9 in the thousands place.
- There is an 8 to its right in the hundreds place.
- Since 8 > 5, Ben must have rounded 9 to the next higher digit, 10.
- The digits to the left of the 10 become zeros.

Ben's rounded number is 10,000.

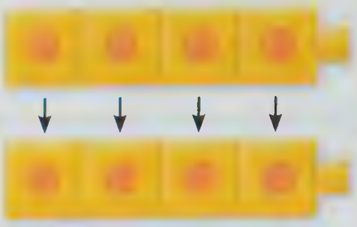
Watch Out!

The 805 in the number becomes 000. The zero in the thousands place is not one of the digits that were changed to zero.

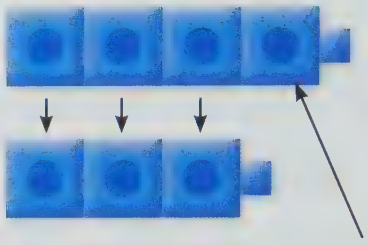
Odd and Even Numbers

What Are Odd and Even Numbers?

If a whole number is divisible by 2, it is an **even number**. An even number of things can be put into pairs.



Odd numbers are *not* divisible by 2. When you try to put an odd number of things into pairs, there is always 1 thing left over.

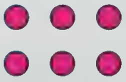

This cube is left over.

Ways to REMEMBER

You may have heard the phrase, "an odd sock." This means a sock without the sock that matches it. The term *odd* means "without a matching item."

EXAMPLE 1

Does the diagram at the right show an even or an odd number of dots?



METHOD 1

Check to see if the dots can be put into pairs with none left over.

The dots are in pairs with none left over.

There is an even number of dots.

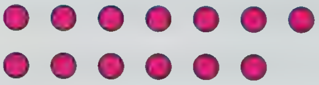
METHOD 2

Count the number of dots. Is the number divisible by 2?

There are 6 dots, and 6 is divisible by 2.

EXAMPLE 2

Does the diagram at the right show an even or an odd number of dots?



METHOD 1

Check to see if the dots can be put into pairs with none left over.

The dots are in pairs with one left over.

There is an odd number of dots.

METHOD 2

Count the number of dots. Is the number divisible by 2?

13 ends with 3, which is not divisible by 2.

EXAMPLE 3

Classify each number as even or odd. Explain your choice.

a. 23 Odd. It ends in 3, which is not divisible by 2.

b. 48 Even. It ends in 8, which is divisible by 2.

c. 580 Even. It ends in 0, which is divisible by 2.

d. 329 Odd. It ends in 9, which is not divisible by 2.

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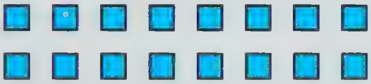
EXAMPLE 4

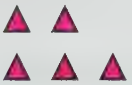
Decide if each group contains an even or an odd number of objects. Explain your choice.

a.



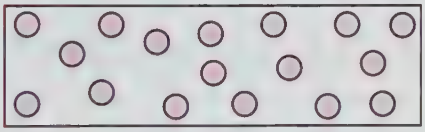
Even. The squares form 8 pairs with no squares left over.

b.



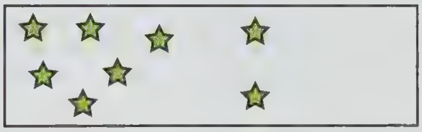
Odd. There is a triangle left over.

c.



Odd. There are 17 circles. 17 ends with 7, which is not divisible by 2.

d.



Even. There are 8 stars, and 8 is divisible by 2.

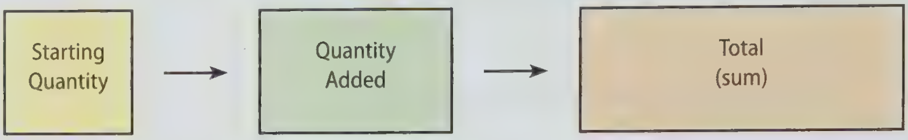
GOT TO KNOW!

Even Numbers	Odd Numbers
Even numbers are divisible by 2. They always end with 0, 2, 4, 6, or 8.	Odd numbers are not divisible by 2. They always end with 1, 3, 5, 7, or 9.
Since zero is divisible by 2, zero is an even number.	Odd numbers of items cannot be placed in pairs. There will always be one item left over.
Even numbers of items can be placed into pairs with none left over.	

Addition Facts

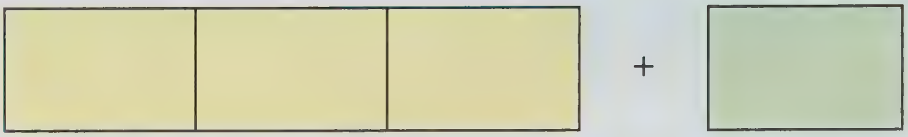
What Is Addition?

Addition is a mathematic operation in which you find the total of two or more numbers. The total is called the **sum**.



EXAMPLE 1

How many is 3 plus 1 more?



One way to find the answer is to count from left to right: 1, 2, 3, 4.
This method works fine for small numbers. It may not work well for large numbers.

$3 + 1 = 4$

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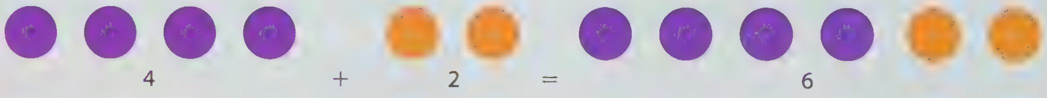
EXAMPLE 2

Solve the problem.

There are 4 beads in a bowl. Then, 2 more beads are placed in the bowl. How many beads are there in all?



There are 6 beads in all.

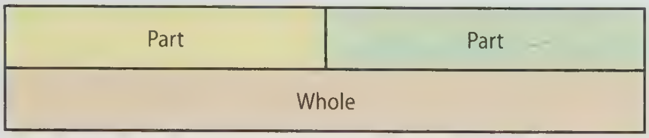
GOT TO KNOW!

Addition Is . . .

a mathematic operation.
Use addition to find the sum (total) of two or more quantities.
The addition sign is the plus symbol, +.

Addition as Part-Part-Whole

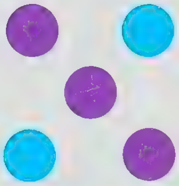
Sometimes addition describes two parts and the total is the whole.



EXAMPLE 3

Solve the problem.

There are 3 purple circles. There are 2 blue circles. How many circles are there in all?



$3 + 2 = 5$

There are 5 circles in all.

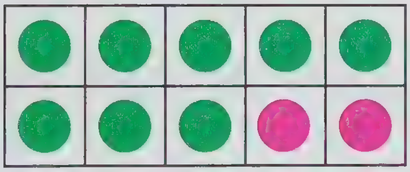
Models for Addition

There are many ways to model addition. You can use a ten frame or a number line. Both of the models below show $8 + 2 = 10$.

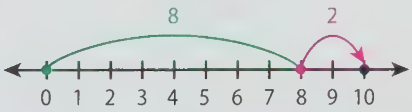
Ways to REMEMBER

"See" the ten frame in your head to remember simple addition facts.

ten frame



number line



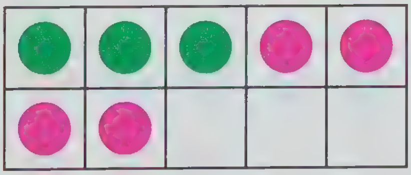
$8 + 2 = 10$

EXAMPLE 4

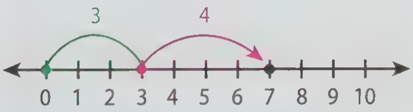
Use a model to solve the problem.

Kent painted for 3 hours. Then he painted for 4 more hours. How many hours did he paint in all?

ten frame



number line



$3 + 4 = 7$

Kent painted 7 hours in all.

Strategies for Finding Sums

There are many strategies to help find the sum for addition problems.

One way to find a sum is to **Count On**. Start with the greater quantity and then count on. This strategy is best for counting on 0, 1, 2, or 3, but it can work for any numbers.

EXAMPLE 5

Laura had 5 T-shirts. She got 3 more T-shirts. How many T-shirts in all?

Start with the greater number of T-shirts, 5.

Then count on 3 more: $5, \underbrace{6, 7, 8}_{\text{3 more}}$.

5 + 3 = 8, so, Laura has 8 T-shirts.

Another way to find a sum is to **Memorize the Addition Facts**. See pages 60 and 61 for ways to make this task easier.

Try It
This Way

Use the number line on page 57.

Or, look up the fact using the addition chart on page 61.

EXAMPLE 6

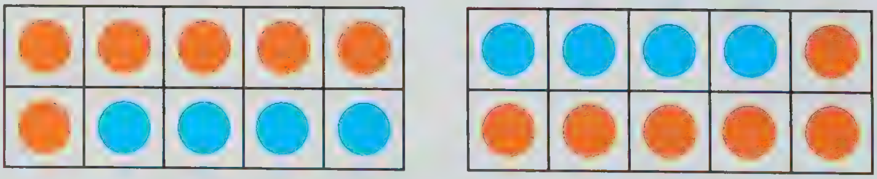
Find the missing numbers.
7 + 3 = ? ? + 3 = 11

If you have memorized the addition facts, you know that 7 + 3 = 10 and 8 + 3 = 11. If you only know that 7 + 3 = 10, you can reason that 11 is 10 + 1. So the missing number is one more than 7, or 7 + 1 = 8.

Another strategy is the **Switch the Order** of the numbers. You can add two numbers starting with either number. If you remember the fact in one order, you can use it to add in the other order.

EXAMPLE 7

Find the missing number. 4 + 6 = ?



If you know 6 + 4 = 10, you can use that to know that 4 + 6 = 10.

Another strategy is **Doubles and Near Doubles**. Facts such as 2 + 2 = 4 or 8 + 8 = 16 are easy to remember. Because the number is added to itself, these facts are known as *doubles*.

Some facts are called *near doubles*. These are facts in which the sum is one more or one less than doubles.

Double: 2 + 2 = 4
Near Double: 2 + 3 = 5
The sum is 1 more than the double.

Double: 8 + 8 = 16
Near Double: 8 + 7 = 15
The sum is 1 less than the double.

EXAMPLE 8

5 + 6 = ?

5 + 6 is a near doubles fact. Here are two ways to find the sum.

Double: 5 + 5 = 10
Near Double: 5 + 6 = 11

Double: 6 + 6 = 12
Near Double: 6 + 5 = 11

5 + 6 = 11

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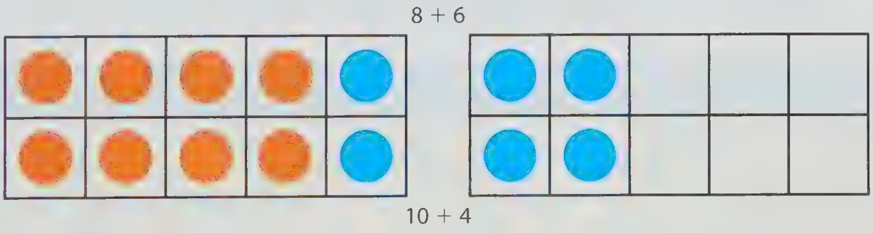
To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

When you know the facts that make ten, you can use the **Make Ten** strategy. In this strategy, you break apart numbers to make a ten. Then you add on the rest.

EXAMPLE 9

8 + 6 = ?

Rearrange the numbers in the ten frames. You have a group of 10 and 4 more.



8 + 6 = 14

GOT TO KNOW!

Powerful Addition Strategies

If you can use these strategies, you can add any two numbers that have a sum up to 20.

Memorize the Addition Facts (facts with sums up to 10)

Make Ten

Switch the Order

Strategies for Finding Sums

There are many strategies to help find the sum for addition problems.

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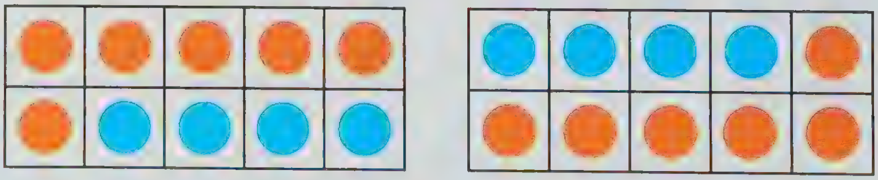
Another strategy is the **Switch the Order** of the numbers. You can add two numbers starting with either number. If you remember the fact in one order, you can use it to add in the other order.

EXAMPLE 7

Find the missing number. $4 + 6 = ?$



If you know $6 + 4 = 10$, you can use that to know that $4 + 6 = 10$.

Another strategy is **Doubles and Near Doubles**. Facts such as 2 + 2 = 4 or 8 + 8 = 16 are easy to remember. Because the number is added to itself, these facts are known as *doubles*.

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The sum is 1 less than the double.

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5 + 6 = ?

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Double: 5 + 5 = 10
Near Double: 5 + 6 = 11

Double: 6 + 6 = 12
Near Double: 6 + 5 = 11

5 + 6 = 11

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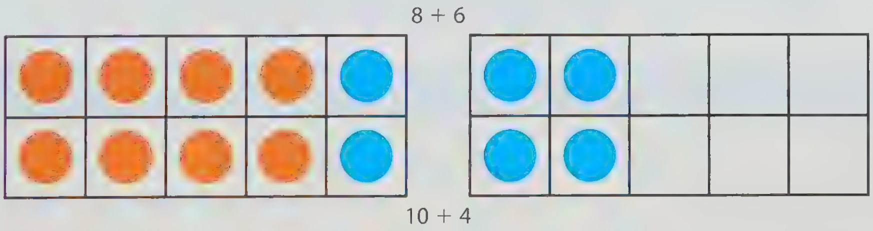
EXAMPLE 9

8 + 6 = ?

Rearrange the numbers in the ten frames. You have a group of 10 and 4 more.



8 + 6 = 14

GOT TO KNOW!

Powerful Addition Strategies

If you can use these strategies, you can add any two numbers that have a sum up to 20.

Memorize the Addition Facts (facts with sums up to 10)

Make Ten

Switch the Order

Memorizing Addition Facts

Although you can use strategies to figure out the sum of addition problems, you will be more powerful mathematically if you memorize the basic facts.

There are 121 basic addition facts. They are listed in the two charts below.

This chart organizes the numbers in columns and rows.

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Basic Addition Facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Use these steps to find a sum on the chart above.

- Find the first number (addend) in the column on the left. For example, 5.
- Find the second addend in the row along the top. For example, 3.
- Find the sum where the row and the column meet. The chart shows how to find 5 + 3 = 8.

This chart shows every fact as an equation.

Basic Addition Facts

0 + 0 = 0	1 + 0 = 1	2 + 0 = 2	3 + 0 = 3	4 + 0 = 4	5 + 0 = 5	6 + 0 = 6	7 + 0 = 7	8 + 0 = 8	9 + 0 = 0	10 + 0 = 10
0 + 1 = 1	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10	10 + 1 = 11
0 + 2 = 2	1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7	6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11	10 + 2 = 12
0 + 3 = 3	1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8	6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12	10 + 3 = 13
0 + 4 = 4	1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9	6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13	10 + 4 = 14
0 + 5 = 5	1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10	6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14	10 + 5 = 15
0 + 6 = 6	1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11	6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15	10 + 6 = 16
0 + 7 = 7	1 + 7 = 8	2 + 7 = 9	3 + 7 = 10	4 + 7 = 11	5 + 7 = 12	6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16	10 + 7 = 17
0 + 8 = 8	1 + 8 = 9	2 + 8 = 10	3 + 8 = 11	4 + 8 = 12	5 + 8 = 13	6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17	10 + 8 = 18
0 + 9 = 9	1 + 9 = 10	2 + 9 = 11	3 + 9 = 12	4 + 9 = 13	5 + 9 = 14	6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18	10 + 9 = 19
0 + 10 = 10	1 + 10 = 11	2 + 10 = 12	3 + 10 = 13	4 + 10 = 14	5 + 10 = 15	6 + 10 = 16	7 + 10 = 17	8 + 10 = 18	9 + 10 = 19	10 + 10 = 20

Memorize Fewer than 121

There are 121 basic facts, but here are some ways to cut down the task. Use math knowledge to make memorizing some facts very easy.

- 0 plus any number is that number.
- 1 plus any number is the next number when you count.
- 2 plus any even number is the next even number.
- 2 plus any odd number is the next odd number.
- 10 plus any one-digit number is a teen number with the 1 in the tens place and the one-digit number in the ones place.
- If you know 2 + 3, you know 3 + 2. This takes care of half of the 121 facts.
- Once you memorize the doubles, the near doubles are easy to remember.
- Any number plus 9 is one less than that number plus ten.
- This leaves the ten most difficult facts to memorize.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Hints on Memorizing Addition Facts

1. Memorize the facts by saying the facts aloud. When you see 3 + 8, say "Three plus eight equals eleven."
2. Memorize a few facts at a time, 2 or 3 facts per day. Start with easier facts.
3. Practice saying the facts in reverse order, so you don't have to memorize so may facts: 8 + 5 = 13, 5 + 8 = 13.
4. Play addition fact games. For example, toss two dice and add the dots that come up. Make or buy addition facts bingo to play with a friend.
5. Make flash cards and use them by yourself or with friends and family.
6. Every day, review the facts you know or usually know.

Try It
This Way

Practice your facts using a calculator.

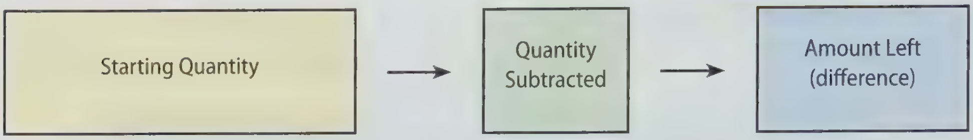
For example, as you push the matching keys, say "3 + 5". Think of the answer and say "equals 8" as you press ENTER.

The display will show whether you were right!

Subtraction Facts

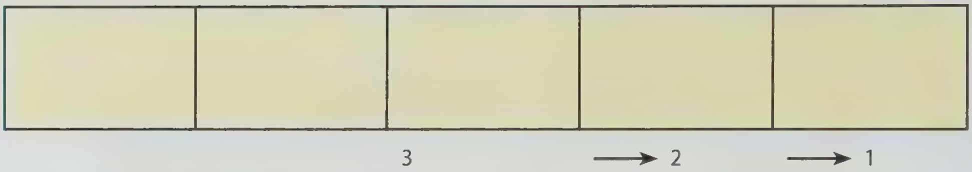
What Is Subtraction?

Subtraction is a mathematical operation in which you take a certain quantity away from another quantity. The amount left is called the **difference**.


Starting Quantity
Quantity Subtracted
Amount Left (difference)

EXAMPLE 1

How many is 5 take away 3?



One way to find the answer is to count from right to left three places: 1, 2, 3. There are two rectangles left.

This method works fine for small numbers. It may not work well for large numbers.

$5 - 3 = 2$

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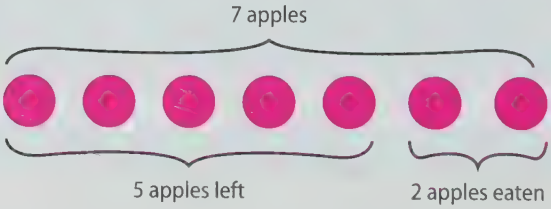
EXAMPLE 2

Amaya had 7 apples. She ate 2 of the apples. How many are left?


7 apples
5 apples left
2 apples eaten

$7 - 2 = 5$

There are 5 apples left.

GOT TO KNOW!

Subtraction is . . .

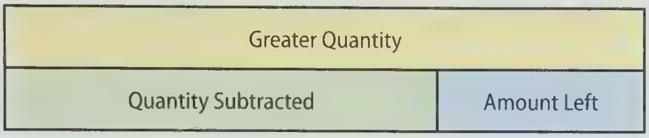
a mathematical operation.

Use subtraction to find the amount left (difference) when you take one quantity from another quantity.

The subtraction sign is the minus symbol, −.

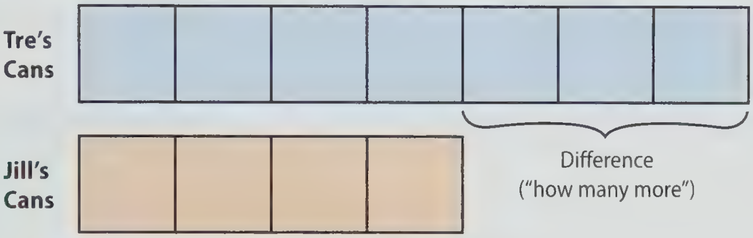
Subtract to Compare

When you compare quantities to find the amount left, or difference, you subtract. When a problem asks, “How many more?” you subtract.


Greater Quantity
Quantity Subtracted
Amount Left

EXAMPLE 3

Tre has 7 cans to recycle. Jill has 4 cans. How many more does Tre have?

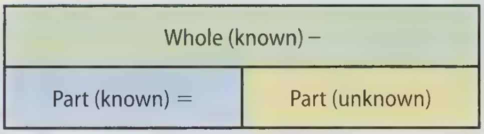

Tre’s Cans
Jill’s Cans
Difference
(“how many more”)

$7 - 4 = 3$ (or $7 = 4 + \underline{3}$)

Tre has 3 more cans than Jill.

Subtract to Find the Missing Part

Subtract when you know the whole and a part, and you want to find the missing part.

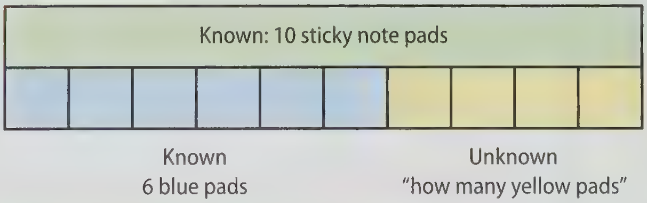

Whole (known) –
Part (known) =
Part (unknown)

EXAMPLE 4

Dhara has 10 sticky-note pads. Six of the pads are blue. The rest are yellow. How many pads are yellow?

$10 - 6 = 4$

Dhara has 4 yellow sticky-note pads.


Known: 10 sticky note pads
Known
6 blue pads
Unknown
“how many yellow pads”

Watch Out !

When a problem is about parts of a whole, it can be either addition or subtraction. Carefully identify what is missing. If it is a missing part, use subtraction. If it is a missing whole, use addition.

Strategies for Finding Differences

There are many strategies to help you figure the difference for subtraction.

One way to find a difference is to **Count Back**. Start with the quantity you are subtracting from. Then count backward starting with the next number.

Watch Out!

When you count back, start at the number you are subtracting from. Do not count that number, but the number one less as "one."

For example, to count back to solve 9 − 2, you think "8 (one), 7 (two). So 9 − 2 = 7.

EXAMPLE 5

***Count Back* to solve 11 − 3.**

Count back 3 numbers starting with 10 ⟶ 10, 9, 8.

$$11, \underbrace{10, 9, 8}_{\text{three numbers}}, \underset{\uparrow\ \text{difference}}{7}, 6, 5, 4, 3, 2, 1$$

11 − 3 = 8

Another way to find the difference is to **Think Addition**. Addition is putting together, and subtraction is taking apart. If you want to find 12 − 9 = ?, you can think 9 + 3 = 12. So the missing number is 3, and 12 − 9 = 3.

EXAMPLE 6

Use *Think Addition* to find 13 − 5 = ?

13 − 5 = ? and 5 + 8 = 13

13 − 5 = 8

There are a number of ways you can **Use Tens** to subtract. Examples 7 and 8 show two of the ways.

When you are subtracting from a number in the teens, Use Tens to take the number apart. First take the teen number apart into 10 and the number in the ones place. Subtract the number from the 10. Then add what's left to get the final answer.

EXAMPLE 7

***Use Tens* to find 15 − 8 = ?**

Think: 15 = 10 + 5
Subtract 8 from the 10.
Add the result to the 5.

$$\begin{array}{r} 10 + 5 \\ -\ 8 \\ \hline 2 + 5 = 7 \end{array}$$

15 − 8 = 7

When you are subtracting a number from 9 or 11, Use Tens and the One More or One Less method.

EXAMPLE 8

a. *Use Tens* to find 11 − 6 = ?

Think: Eleven is 1 more than 10.

Subtract: 10 − 6 = 4

Add the 1 more: 4 + 1 = 5

11 − 6 = 5

b. *Use Tens* to find 9 − 7 = ?

Think: Nine is 1 less than 10.

Subtract: 10 − 7 = 3

Subtract the 1 less: 3 − 1 = 2

9 − 7 = 2

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To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

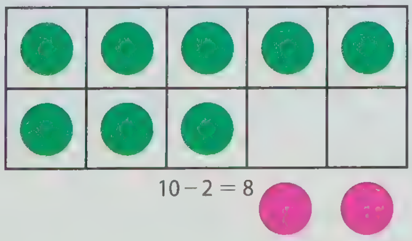
You can also **Use a Model**, such as a number line or ten frame, to subtract.

EXAMPLE 9

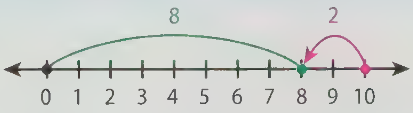
***Use a Model* to find 10 − 2.**

When you remove 2 counters from the ten frame, there are 8 counters left.
When you count back 2 places on the number line, you end up at 8.

ten frame



number line



10 − 2 = 8

GOT TO KNOW!

Powerful Subtraction Strategies

If you can use these strategies, you will be quick and accurate in subtraction.

Think Addition
Use Tens

Memorizing Subtraction Facts

Although you can use strategies to figure out the difference in subtraction problems, you will be more powerful mathematically if you memorize the basic facts.

There are 121 basic subtraction facts. They are listed in the two charts below.

This chart organizes the numbers in columns and rows.

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Basic Subtraction Facts

−	0	1	2	3	4	5	6	7	8	9	10
0	0										
1	1	0									
2	2	1	0								
3	3	2	1	0							
4	4	3	2	1	0						
5	5	4	3	2	1	0					
6	6	5	4	3	2	1	0				
7	7	6	5	4	3	2	1	0			
8	8	7	6	5	4	3	2	1	0		
9	9	8	7	6	5	4	3	2	1	0	
10	10	9	8	7	6	5	4	3	2	1	0

Use these steps to find a difference on the chart above.

- Find the first number (the whole) in the column on the left. For example, 8.
- Find the number subtracted in the row along the top. For example, 5.
- Find the difference where the row and the column meet. The chart shows how to find 8 − 5 = 3.

This chart shows the basic subtraction facts as equations.

Basic Subtraction Facts

0 − 0 = 0										
1 − 0 = 1	1 − 1 = 0									
2 − 0 = 2	2 − 1 = 1	2 − 2 = 0								
3 − 0 = 3	3 − 1 = 2	3 − 2 = 1	3 − 3 = 0							
4 − 0 = 4	4 − 1 = 3	4 − 2 = 2	4 − 3 = 1	4 − 4 = 0						
5 − 0 = 5	5 − 1 = 4	5 − 2 = 3	5 − 3 = 2	5 − 4 = 1	5 − 5 = 0					
6 − 0 = 6	6 − 1 = 5	6 − 2 = 4	6 − 3 = 3	6 − 4 = 2	6 − 5 = 1	6 − 6 = 0				
7 − 0 = 7	7 − 1 = 6	7 − 2 = 5	7 − 3 = 4	7 − 4 = 3	7 − 5 = 2	7 − 6 = 1	7 − 7 = 0			
8 − 0 = 8	8 − 1 = 7	8 − 2 = 6	8 − 3 = 5	8 − 4 = 4	8 − 5 = 3	8 − 6 = 2	8 − 7 = 1	8 − 8 = 0		
9 − 0 = 9	9 − 1 = 8	9 − 2 = 7	9 − 3 = 6	9 − 4 = 5	9 − 5 = 4	9 − 6 = 3	9 − 7 = 2	9 − 8 = 1	9 − 9 = 0	
10 − 0 = 10	10 − 1 = 9	10 − 2 = 8	10 − 3 = 7	10 − 4 = 6	10 − 5 = 5	10 − 6 = 4	10 − 7 = 3	10 − 8 = 2	10 − 9 = 1	10 − 10 = 0

Memorize Fewer than 121

There are 121 basic facts, but here are some ways to cut down the task. Use math knowledge to make memorizing subtraction facts very easy.

- Any number minus 0 is itself.
- Any number minus itself is 0.
- Any number minus 1 is the previous counting number.
- Any even number minus 2 is the previous even number.
- Any odd number minus 2 is the previous odd number.
- Any teen number minus 10 is the digit in the ones place of the teen number.
- If you know $8 - 3 = 5$, you also know $8 - 5 = 3$. This takes care of half of the subtraction facts.
- Nine minus any number is one less than 10 minus the same number.

Try It
This Way

Practice your facts using a calculator.

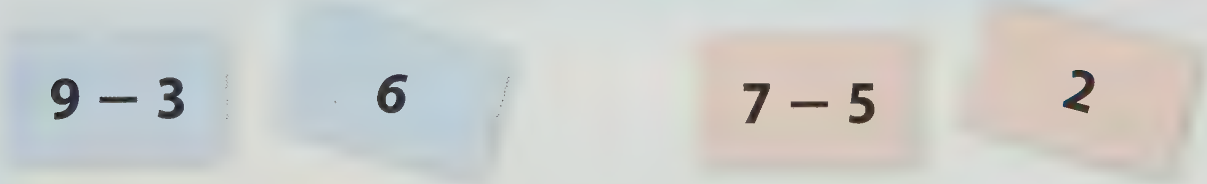
For example, as you push the matching keys, say "13 minus 7." Think of the answer and say "equals 6" as you press ENTER.

The display will show whether you were right!

Hints on Memorizing Subtraction Facts

1. Memorize the facts by saying the whole fact aloud. When you see $8 - 3$, say "Eight minus three equals five."
2. Memorize a few facts at a time. Work on 2 or 3 facts per day. Start with the easier facts.
3. Practice saying the facts in reverse order, so you don't have to memorize so many facts: $9 - 4 = 5$, $9 - 5 = 4$.
4. Play subtraction fact games. For example, toss two dice and add the dots that come up. Then have a friend toss two dice and add the dots. Then subtract the smaller sum from the larger sum.

5. Make flash cards and use them by yourself or with friends and family.



6. Every day, review the facts you know or usually know. Work on two new facts and play games to practice them.

Multiplication Facts

What Is Multiplication?

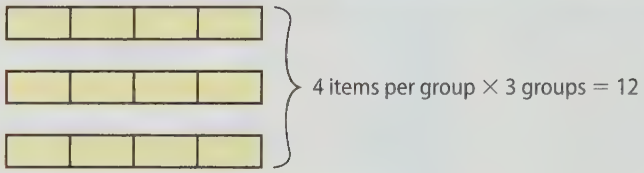
Multiplication is a mathematical operation in which a particular number of equal-sized groups are combined. The result is called the *product*. In the multiplication below, there are 3 groups of 4 rectangles. When the groups are combined, there are 12 rectangles.

Watch Out!

Unlike addition and subtraction, the numbers in a multiplication problem do not mean two different amounts of the same category.

One number tells the number of equal groups. The other number tells the size of each equal group.

Number of Items per Group × Number of Equal Groups = Total Amount (product)



3 × 4 means the same thing as: 4 + 4 + 4 = 12.

EXAMPLE 1

Kimi, Joe, and Bob each have two books. How many books do they have in all?

2 books per person × 3 people = 6 books

2 × 3 = 6

There are 6 books in all.

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GOT TO KNOW!

Multiplication is . . .

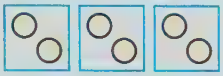
a mathematical operation. The product gives the same result as the sum of adding the number of items in each group. You can change any multiplication problem into an addition problem.

Multiplication	Addition
5 × 3 = 15	5 + 5 + 5 = 15
5 items per group × 3 groups = 15	5 items per group + 5 items per group + 5 items per group = 15

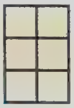
Models for Multiplication

Three ways to model multiplication are: use counters, make an *array,* and skip count on a number line. An **array** is an arrangement of objects in rows and columns. Each row has the same number of objects.

3 groups of 2 counters

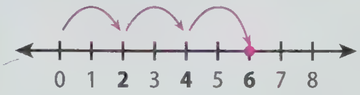


3 rows of 2



3 × 2 = 6

3 jumps of 2 spaces



Watch Out!

In multiplication, all of the groups must be the same size.

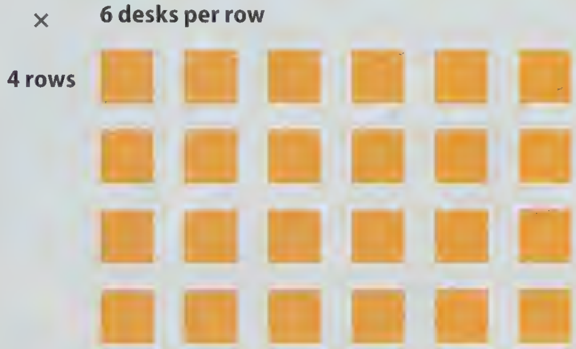
7 + 7 + 7 + 7 + 7 = 5 × 7

7 + 7 + 7 + 7 + 8 = 4 × 7, plus 8

EXAMPLE 2

Javier's classroom has 4 rows of desks. There are 6 desks in each row. How many desks in all?

There are the same number of desks in each row. This is an equal groups problem, so you can multiply. Model the problem using an array.



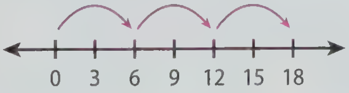
6 × 4 = 24

There are 24 desks in all.

EXAMPLE 3

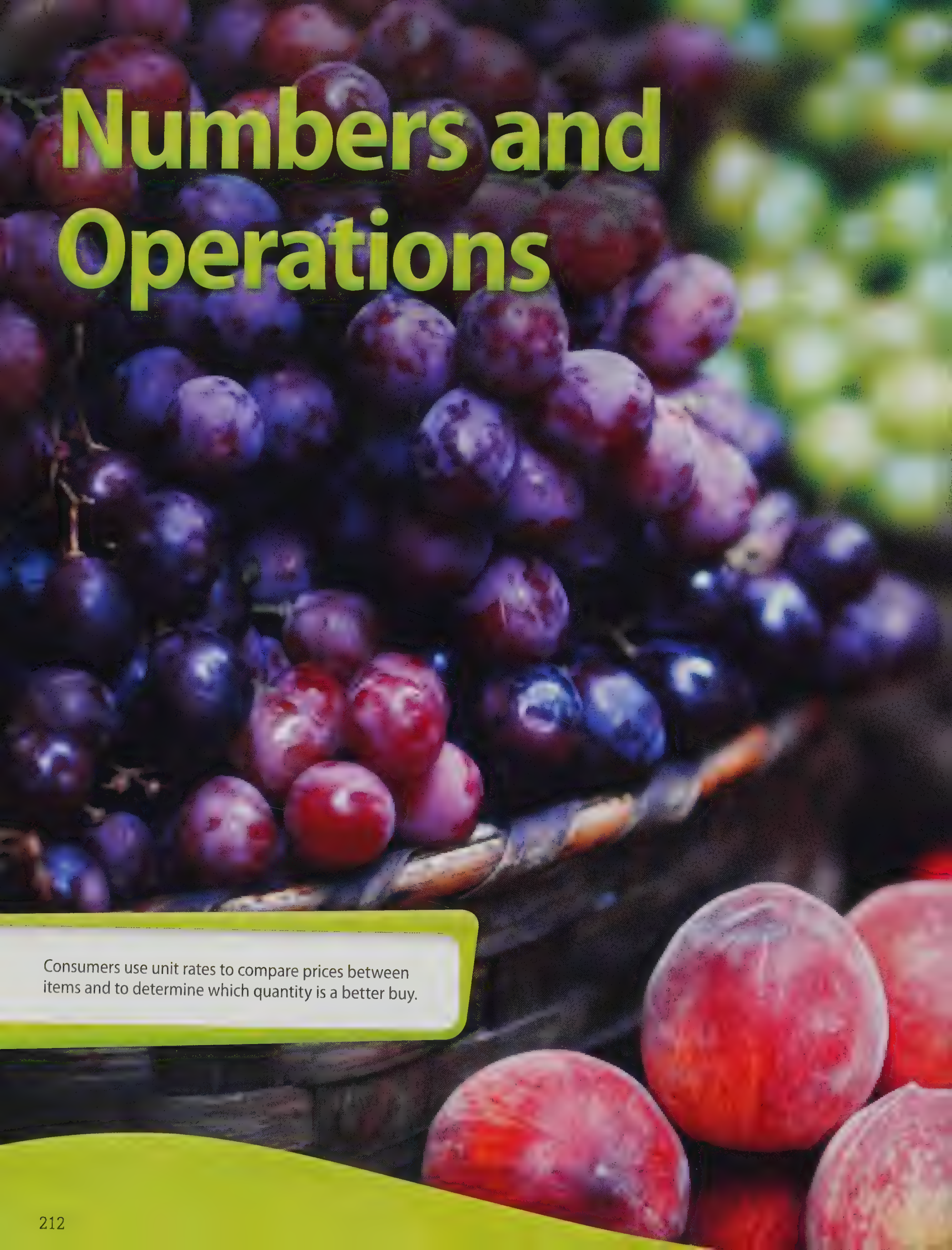
Use a number line to find 3 × 6 = ?

3 jumps of 6 spaces



3 × 6 = 18

Numbers and Operations



Consumers use unit rates to compare prices between items and to determine which quantity is a better buy.

Decimals 214

Number Theory 266

Fractions and Mixed Numbers 300

Ratio and Proportion 362

Percent 378

Estimate with Decimals and Fractions 408

Patterns 430

Problem Solving 446

Place Value to Thousandths

Need More HELP

For help with whole number place value, go to *Place Value to Billions* on pages 40–41.

Understanding Decimal Place Values

Decimal numbers represent values between whole numbers. A decimal point (dot) to the right of the ones place shows where the decimal values begin. You can represent the decimal places on a place-value chart, as shown below. Just as with whole numbers, each decimal place has a value that is *ten times the value of the place to its right* and *one tenth the value of the place to its left.*

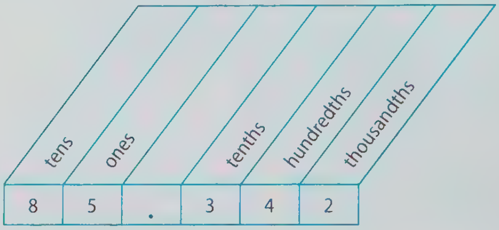
EXAMPLE 1

The place value chart shows the number 85.342.

a. Name the place of the digit 2 in the number 85.342. Then tell its value.
The digit 2 is in the thousandths place.
Its value is *two thousandths.*

b. Name the place of the digit 4 in the number 85.342. Then tell its value.
The digit 4 is in the hundredths place.
Its value is *four hundredths.*

Place-Value Chart



EXAMPLE 2

There are two 6's in the number 5.668. What is the relationship between their values?

The value of the 6 on the left is *six tenths.*

0.6 is 10 × the value of 0.06.

The value of the 6 on the right is *six hundredths.*

0.06 is $\frac{1}{10}$ the value of 0.6.

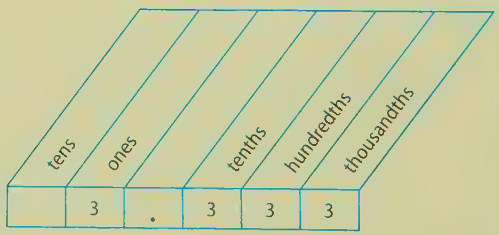
Got to KNOW!

Place Value

Each place has a value that is 10 × the value of the place to its right and $\frac{1}{10}$ the value of the place to its left. In the number shown, the digit 3 has a different value in each place. The values of the places in 3.333 from left to right are:

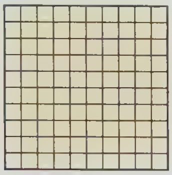
3 ones, 3 tenths, 3 hundredths, 3 thousandths

$3 \qquad \frac{3}{10} \qquad \frac{3}{100} \qquad \frac{3}{1{,}000}$

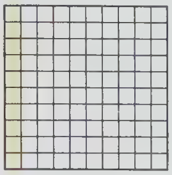


Modeling Decimals

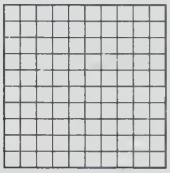
You can use a ten by ten grid to model decimal numbers to hundredths. The whole grid represents 1 whole. Each small square represents one hundredth of the whole. Each row or column of ten small squares represents one tenth of the whole.



1.0



0.1



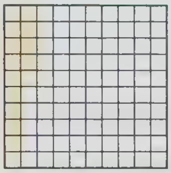
0.01

EXAMPLE 3

Use a ten by ten grid to model 0.15.

0.15 is fifteen hundredths.

Shade 15 squares to model 0.15.



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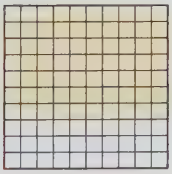
EXAMPLE 4

Use a ten by ten grid to model 0.7.

0.7 is seven tenths.

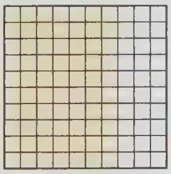
METHOD 1

Shade 7 rows to model 0.7.



METHOD 2

Shade 7 columns to model 0.7.



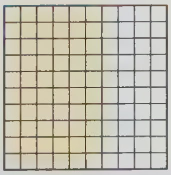
EXAMPLE 5

Complete: 6 tenths = ? hundredths

STEP 1 Model 0.6 on a ten by ten grid.

STEP 2 Count the number of hundredths.

6 tenths = 60 hundredths



Comparing and Ordering Decimals

Comparing Decimals

You can use a number line or place value to compare decimals. The numbers on a number line are ordered from least to greatest. As you move right along the number line, the numbers increase in value. To compare two decimals, use the symbols shown in the *Got to Know!* box.

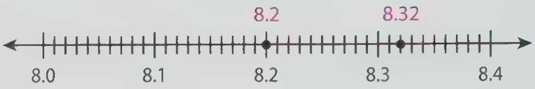
Need More HELP

Each comparison can be written in two ways. If $a > b$, then $b < a$.

EXAMPLE 1

Use $<$, $>$, or $=$ to compare 8.32 and 8.2.

Graph 8.32 and 8.2 on a number line.



8.32 is to the right of 8.2 on the number line, so its value is greater: $8.32 > 8.2$.
You can also say that $8.2 < 8.32$.

EXAMPLE 2

Use $<$, $>$, or $=$ to compare 0.124 and 0.14.

Use place values and compare the digits.

Align the decimal points.

0.124
0.14

You may find it helpful to add a zero to 0.14 so both numbers have the same number of decimal places.

0.124
0.140

Compare the digits in the decimal places from left to right until the digits are different.

Tenths place: Both the digits are 1.
Hundredths place: $4 > 2$

Since 4 hundredths is greater than 2 hundredths, $0.14 > 0.124$.
You can also say that $0.124 < 0.14$.

Got to KNOW!

Comparison Symbols

Symbol	Meaning
$>$	greater than
$<$	less than
$=$	equal to

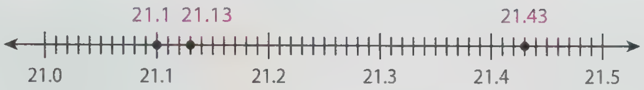
Ordering Decimals

You can use a number line or place value to order decimals. Numbers can be ordered from least to greatest or greatest to least.

EXAMPLE 3

Order from least to greatest: 21.13, 21.43, 21.1

Graph the numbers on a number line.



The numbers are ordered least to greatest as you move left to right along the number line.

Ordered from least to greatest, the numbers are: 21.1, 21.13, 21.43.

Watch Out!

Be sure you understand the scale on the number line. The longer tick marks on these number lines represents tenths. The shorter, unlabeled tick marks represent hundredths.

EXAMPLE 4

The table shows the heights of four students. The height of which student is the least?

Student	Juan	Charlotte	Tomika	Sam
Height (meters)	1.39	1.314	1.309	1.386

Use place values to order the numbers.

Align the decimal points.
Add a zero to 1.39 so all decimals have the same number of decimal places.

1.390
1.314
1.309
1.386

Compare the digits in the decimal places from left to right until the digits are different.

Ones place: All the digits are 1.
Tenths place: All the digits are 3.
Hundredths place: $0 < 1 < 8 < 9$

$1.3\underline{0}9 < 1.3\underline{1}4 < 1.3\underline{8}6 < 1.3\underline{9}$

Tomika's height is the least.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Got to KNOW!

Equivalent Decimals

Equivalent decimals have the same value. Adding a zero to the right of the last decimal place does not change the value of a decimal number.

$4.53 = 4.530$

Adding Decimals

Modeling Decimal Addition

You can use ten by ten grids to model the addition of decimal numbers. The value of each small square in the grid is one hundredth.

SEARCH

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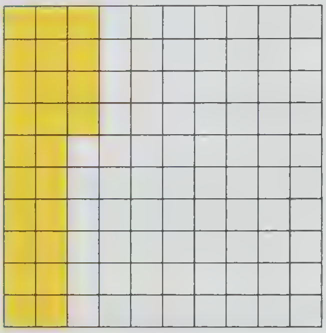
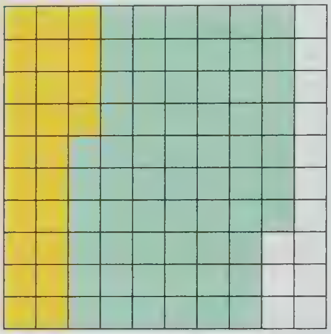
EXAMPLE 1

Use a grid to model 0.24 + 0.63.

Shade 24 hundredths of the grid.



Shade 63 more hundredths.



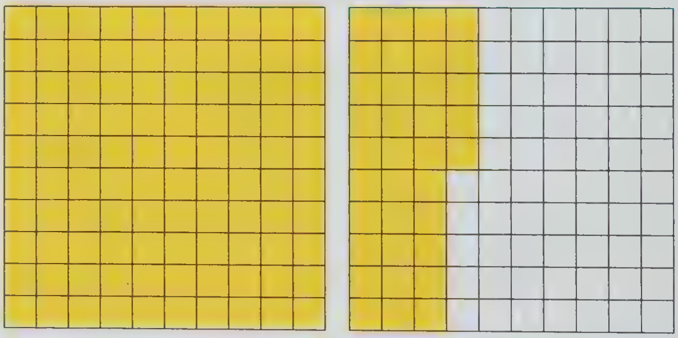
There are 87 hundredths shaded.

0.24 + 0.63 = 0.87

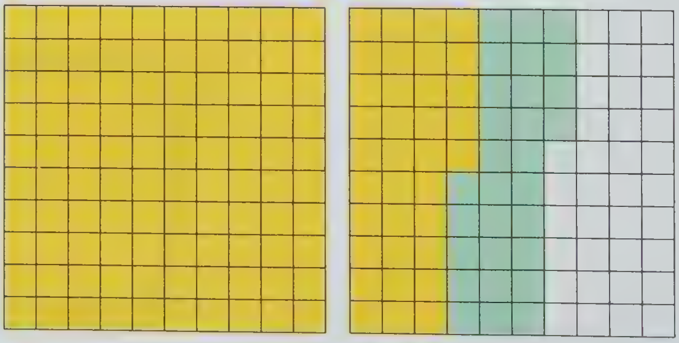
EXAMPLE 2

Use a grid to model 1.35 + 0.29.

Shade 1 whole grid and 35 hundredths of another grid.



Shade 29 more hundredths.



There are 164 hundredths shaded.

1.35 + 0.29 = 1.64

Adding Decimals on a Place-Value Chart

You can also add decimals in a vertical format on a place-value chart. Line up the decimal points in the numbers. The decimal point in the sum will be directly below the decimal points in the addends.

EXAMPLE 3

Add. 11.5 + 5.2

Write the problem vertically on a place-value chart.

	tens	ones	tenths
	1	1.	5
+		5.	2

Add from right to left, as you add whole numbers.

Bring the decimal point straight down into the answer.

	tens	ones	tenths
	1	1.	5
+		5.	2
	1	6.	7

11.5 + 5.2 = 16.7

Try It This Way

You can use mental math to add 11.5 + 5.2.

First, add the whole number parts. Think: 11 + 5 is 16.

Then add the decimal parts. Think: 0.5 + 0.2 is 0.7.

Combine the sums: 16.7

Got to KNOW!

Adding Decimals

When adding decimals vertically, you must align (line up) the decimal points. This keeps each digit in its proper place. The decimal point in the sum is directly below the decimal points in the addends.

$$\begin{array}{r} 11.5 \\ +\ 5.2 \\ \hline 16.7 \end{array}$$

Adding Decimals with Regrouping

When you **regroup**, you organize a value in a different way and rename it. While its name changes, its value remains the same. Suppose you have the sum of 14 tenths. Regroup the tenths as 1 one and 4 tenths. Then place a small 1 above a column of addends to the left to represent the regrouped digits.

Need More HELP

The term *carrying* means the same thing as regrouping. You "carry" a part of the renamed value into the column to its left.

For help with regrouping, go to *Adding Whole Numbers* in *Foundations of Mathematics* (page 82).

EXAMPLE 4

Add. 8.67 + 0.42

Write the problem in vertical format.

Add from right to left, and regroup as needed.

Rename 10 tenths as 1 one and 0 tenths.

Add the whole numbers 8 and 1.

Bring the decimal point straight down into the answer.

	tens	ones	tenths	hundredths
		^{1}8.	6	7
+		0.	4	2
		9.	0	9

8.67 + 0.42 = 9.09

EXAMPLE 5

Marla bought a DVD for C$15.95 and a bottle of water for C$1.25. How much did she spend in all?

Write the problem in vertical format.

Add from right to left, and regroup as needed.

Rename 10 hundredths as 1 tenth and 0 hundredths.

	tens	ones	tenths	hundredths
	1	5.	^{1}9	5
+		1.	2	5
				0

Continue adding.

Rename 12 tenths as 1 one and 2 tenths.

Bring the decimal point straight down into the answer.

	tens	ones	tenths	hundredths
	1	^{1}5.	^{1}9	5
+		1.	2	5
	1	7.	2	0

Marla spent C$17.20 in all.

SEARCH

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MONEY

NOTE: C$ denotes Canadian dollars.

Adding Decimals with More Than Two Addends

Use a place-value chart. Write the problem in vertical form, being careful to line up the decimal points.

EXAMPLE 6

Add. 43.32 + 4.1 + 1.275

Write the problem in vertical format.

Add from right to left.

Bring the decimal point straight down into the answer.

	tens	ones	tenths	hundredths	thousandths
	4	3.	3	2	
		4.	1		
+		1.	2	7	5
	4	8.	6	9	5

43.32 + 4.1 + 1.275 = 48.695

EXAMPLE 7

Add. 11.037 + 7.843 + 10.46

Write the problem in vertical format.

Add from right to left, and regroup as needed.

Rename the 10 thousandths as 1 hundredth and zero thousandths.

Rename the 14 hundredths as 1 tenth and 4 hundredths.

	T	O	T	H	T
	1	1.	^{1}0	^{1}3	7
		7.	8	4	3
+	1	0.	4	6	
				4	0

Continue adding.

Rename the 13 tenths as 1 one and 3 tenths.

Add the whole numbers.

Bring the decimal point straight down into the answer.

	T	O	T	H	T
	1	^{1}1.	^{1}0	^{1}3	7
		7.	8	4	3
+	1	0.	4	6	
	2	9.	3	4	0

11.037 + 7.843 + 10.46 = 29.34

Try It This Way

For another visual representation of the numbers, add placeholder zeros so each number has two whole number places and three decimal places.

43.320
04.100
01.275

This aligns the decimal points but does not change the value of the numbers.

Need More HELP

In Example 7, you do not need to include the zero in the thousandths place when you write the final sum. However, if this were a scientific measurement you would include the zero to make the number more precise. The number 29.340 is precise to thousandths, while 29.34 is precise to hundredths.

Dividing Decimals by Whole Numbers

Need More HELP

The *dividend* is the number to be divided. The *divisor* is the number you divide by. The *quotient* is the answer.

Modeling Decimal Division

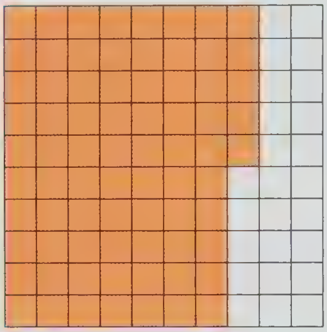
You can use a ten by ten grid to model the division of a decimal number by a whole number. Look at Example 1 and Example 2. Notice the following.

- The quotient has the same number of decimal places as the dividend.
- The quotient is less than the dividend.

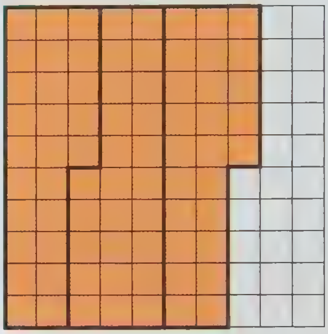
EXAMPLE 1

Divide 0.75 by 3.

Shade 75 hundredths.



Divide 75 hundredths into 3 equal groups.



Each group contains 25 hundredths.

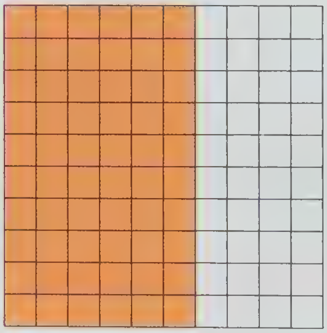
0.75 ÷ 3 = 0.25

EXAMPLE 2

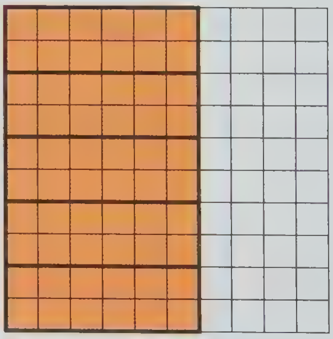
Divide 0.6 ÷ 5.

0.6 = 0.60

Shade 60 hundredths.



Divide 60 hundredths into 5 equal groups.



Each group contains 12 hundredths.

0.6 ÷ 5 = 0.12

Got to KNOW!

Dividing a Decimal by a Whole Number

When a decimal number is divided by a whole number, the quotient is always less than the dividend.

0.60 ÷ 5 = 0.12

12 hundredths is less than 60 hundredths.

Dividing Decimals by Whole Numbers

To divide a decimal by a whole number, write the problem in long division form and divide as with whole numbers. Place the decimal point in the quotient directly above the decimal point in the dividend.

Need More HELP

For help with dividing whole numbers, go to *Divide Whole Numbers* in *Foundations of Mathematics* (p. 100).

EXAMPLE 3

Divide 4.68 ÷ 3.

Write the problem in long division form.

$$3\overline{)4.68}$$

Divide as you do for whole numbers. Multiply, subtract, and bring down the next digit in each step.

Place the decimal point in the quotient directly above the decimal point in the dividend.

$$\begin{array}{r} 1.56 \\ 3\overline{)4.68} \\ \underline{3} \\ 1\,6 \\ \underline{1\,5} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

4.68 ÷ 3 = 1.56

EXAMPLE 4

Mr. Lee bought 18 window shades. The total cost was $179.10. How much did each window shade cost?

Divide the total cost by the number of window shades. 179.10 ÷ 18

Write the problem in long division form.

$$18\overline{)179.10}$$

Divide as you do for whole numbers. Multiply, subtract, and bring down the next digit in each step.

Place the decimal point in the quotient directly above the decimal point in the dividend.

$$\begin{array}{r} 9.95 \\ 18\overline{)179.10} \\ \underline{162} \\ 17\,1 \\ \underline{16\,2} \\ 90 \\ \underline{90} \\ 0 \end{array}$$

Each window shade cost $9.95.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Word Problems

Solving Decimal Word Problems

Often, there is more than one way to solve a word problem. Find the method that works best for you. You may find it helpful to follow the steps shown in *Got To Know!* below. The examples show an application of the steps.

Need More **HELP**

For help solving word problems, go to *Problem Solving* on pages 448–469. You will find a four-step problem-solving process and strategies for problem solving.

The *Check* in Example 1 uses the strategy *Use Logical Reasoning* on page 460.

EXAMPLE 1

Sarah has a watermelon that has a mass of 7.3 kilograms. Water makes up 6.716 kilograms of the mass. How many kilograms of the watermelon's mass are not water?

READ Read the problem carefully. Identify what you know and what you need to find out, or need to know.
Know: Total mass is 7.3 kg, and 6.716 kg of the mass is water.
Need to Know: the part of the watermelon's mass that is not made up of water

PLAN Think about how you can find the information you need to know. You could subtract the mass that is water from the entire mass of the watermelon.

SOLVE Write the problem in vertical form. Add zeros for placeholders and subtract.

$$\begin{array}{rl} 7.300\text{ kg} & \text{total mass of watermelon} \\ \underline{-\ 6.716\text{ kg}} & \text{mass that is water} \\ ?\ \ \text{kg} & \text{mass that is not water} \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}.\overset{12}{\cancel{3}}\overset{\overset{9}{\cancel{1}}}{\cancel{0}}\overset{1}{0} \\ -\ 6.716 \\ \hline 0.584\text{ kg is not water} \end{array}$$

CHECK Did I answer the question? Is my answer reasonable?
6.716 kg + 1 kg = 7.716 kg, which is greater than the total mass of the watermelon. Therefore, the mass of the watermelon that is not water must be less than 1 kg. The answer of 0.584 kg is a reasonable answer.

0.584 kg of the watermelon's mass is not water.

If you state your answer in terms of the question, you will know you answered the correct question. The question in Example 1 asks, "How many kilograms of the watermelon's mass are not water?" The answer states "0.584 kg of the watermelon's mass is not water."

Got to KNOW!

Problem-Solving Process

To solve a word problem, you can use this process.

1. **Read** the problem carefully. Identify what you know and what you need to find out.
2. **Plan** how to find the solution.
3. **Solve** the problem using your plan.
4. **Check** your work and ask if your answer makes sense.

EXAMPLE 2

Jamal bought two T-shirts and a DVD. The T-shirts cost $15.50 each and the DVD cost $14.99. How much did Jamal spend?

READ Know: 2 T-shirts cost $15.50 each, 1 DVD cost $14.99.
Need to Know: Total cost of the 3 items

PLAN There are two ways to solve this problem.

METHOD 1

Multiply to find the total of the T-shirts.
Then add the cost of the DVD to the product.

METHOD 2

Add the cost of the 3 items.

SOLVE

METHOD 1

$$\begin{array}{r} {}^{1\,1} \\ \$15.50 \\ \times\quad 2 \\ \hline \$31.00 \end{array} \qquad \begin{array}{r} \$31.00 \\ +\ \$14.99 \\ \hline \$45.99 \end{array}$$

METHOD 2

$$\begin{array}{r} {}^{1\,1} \\ \$15.50 \\ \$15.50 \\ +\ \$14.99 \\ \hline \$45.99 \end{array}$$

CHECK You can estimate each price and then find the sum.
$15.50 is about $15. $15.50 is about $15. $14.99 is about $15.
Use mental math: 15 × 3 = 45
The calculated answer of $45.99 is reasonable.

Jamal spent $45.99 on the 3 items.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

For help with estimating to check, go to *Estimating to Predict or Check* on page 410.

EXAMPLE 3

Carolina bought 3.62 pounds of grapes. Each pound of grapes costs C$1.49. How much did Carolina spend?

READ Know: 3.62 lb at C$1.49 per lb
Need to Know: Total cost

PLAN Multiply to find how much Carolina spent.

SOLVE

$$\begin{array}{rl} 3.62 & \text{pounds of grapes} \\ \times\ 1.49 & \text{for 1 pound} \\ \hline 3258 & \\ 1448 & \\ 362 & \\ \hline 5.3938 & \text{rounds to C\$5.39} \end{array}$$

CHECK $1.5 \times 4 = 6$ and $1.5 \times 3 = 4.5$
$6 > 5.39 > 4$
C$5.39 is a reasonable answer.

Carolina spent C$5.39 for the grapes.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Solving Decimal Word Problems

EXAMPLE 4

Marla is making sandwiches. She has 4 pounds of turkey. Each sandwich uses 0.25 pounds of turkey. What is the greatest number of sandwiches Marla can make?

READ Know: 4 lb of turkey, 0.25 lb per sandwich
Need to Know: Number of sandwiches Marla can make

PLAN Divide the number of pounds of turkey by the amount needed for sandwich.

SOLVE Divide. Move the decimal point to the right two places.

$$\begin{array}{r} 16. \\ 0.25.\overline{)4.00.} \\ -25 \\ \hline 150 \\ -150 \\ \hline 0 \end{array}$$

CHECK Use logical reasoning and mental math.
Think: $16 \div 4 = 4$
If she can make 16 sandwiches per 4 lb turkey, then she can make 4 sandwiches with 1 lb of turkey.
$1 \text{ lb} \div 4 = \frac{1}{4} \text{ lb} = 0.25 \text{ lb}$
The answer is reasonable.

The greatest number of sandwiches Marla can make is 16.

EXAMPLE 5

Sam bought three lemons for $0.20 each and four oranges for $0.65 each. He gave the clerk a $10.00 bill. How much change did he receive?

READ Know: 3 lemons at $0.20 each, 4 oranges at $0.65 each, gave clerk $10
Need to Know: Amount of change received

PLAN Multiply twice and then add to find total cost of the items. Subtract to find the change.

SOLVE Perform the operations in the plan.

Cost of lemons

$$\begin{array}{ll} \$0.20 & \text{per lemon} \\ \times \quad 3 & \text{lemons} \\ \hline \$0.60 & \end{array}$$

Cost of oranges

$$\begin{array}{ll} \$0.65 & \text{per orange} \\ \times \quad 4 & \text{oranges} \\ \hline \$2.60 & \end{array}$$

Total cost

$$\begin{array}{r} \$0.60 \\ +\ \$2.60 \\ \hline \$3.20 \end{array}$$

Find the change.

$$\begin{array}{r} \$10.00 \\ -\ \$\ \ 3.20 \\ \hline \$\ \ 6.80 \end{array}$$

CHECK Review your multiplication. To estimate the change, think: $10 - 3 = 7$, so $6.80 in change is reasonable.

Sam received $6.80 in change.

EXAMPLE 6

Feng is building a fence around his garden. He needs a total of 46.28 meters of fencing. He has one piece of fencing that is 24.65 meters in length and another piece that is 21.75 meters in length. Does Feng have enough fencing for his garden?

READ Know: Feng has 24.65 m and 21.75 m of fencing.
Need to Know: Is this length equal to or greater than 46.28 m?

PLAN Add to find the total amount of fencing Feng has. Compare the sum to what he needs.

SOLVE Perform the operations in the plan.

Add.

$$\begin{array}{r} {}^{1}\ {}^{1} \\ 24.65 \\ +\ 21.75 \\ \hline 46.40 \end{array}$$

Compare.

46.40 m > 46.28 m
has > needs

The amount of fencing Feng has is greater than the amount he needs.

Feng has 46.40 meters of fencing.

CHECK Rounding shows that 24 + 21 = 45 and 25 + 22 = 47. This tells you that your answer is reasonable, but for this problem you need a precise comparison. Use a calculator to check your addition, or subtract 46.40 − 21.75 to see if the difference is 24.65.

Feng has enough fencing for his garden.

Watch Out !

When a problem asks if some quantity is enough, rounding is not the best way to check the answer.

EXAMPLE 7

An orange juice carton is labeled: 1.92 liters, 8 servings. How many milliliters of juice are in a serving?

READ Know: 1.92 L of juice will make 8 servings.
Need to Know: Number of mL in each serving

PLAN Divide the amount of juice by the number of servings. Convert the answer to mL.

SOLVE Perform the operations in the plan.

Divide.

$$\begin{array}{r} .24 \\ 8\overline{)1.92} \\ -16 \\ \hline 32 \\ -32 \\ \hline 0 \end{array}$$

Convert liters to milliliters.

1 liter = 1,000 milliliters

Multiply 0.24 by 1,000 by moving the decimal point three places to the right.

0.24 L = 240 mL

There are 0.24 liters per serving.

CHECK Use compatible numbers. Divide: 1.6 ÷ 8 = 0. 2 L per serving. Express 0.2 L as 200 mL. 200 mL is close to 240 mL. The answer is reasonable.

There are 240 milliliters of juice in each serving.

Need More HELP ?

For help with compatible numbers, go to *Estimating Decimal Sums and Differences* on page 414.

Rational Numbers and Number Sets

Need More HELP

Sometimes the *set of rational numbers* is defined as numbers that can be expressed as the *ratio* of two integers.

Sets of Numbers

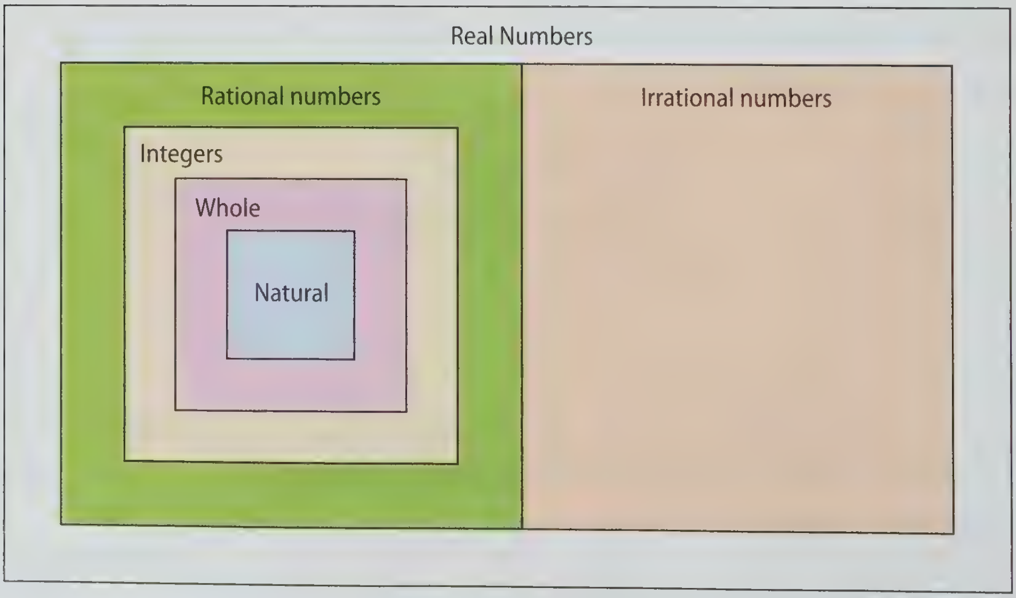
A **set** is a collection of objects. You can represent sets in different ways. Here are two examples that name the same set:

- the set of all the colors on a traffic light
- $C = \{\text{red, yellow, green}\}$

Mathematicians work with the following sets of numbers.

Set	Symbol	Examples
Real numbers All of the numbers that can be represented by a point on a number line. This set contains the set of rational numbers and the set of irrational numbers.	R	$0, 5, -8, \frac{3}{7}, \pi, \sqrt{2},$ 1.0100100010001,... $-\sqrt{137}$
Rational numbers Numbers that can be written as the quotient of two integers, $\frac{n}{d}$, where $d \neq 0$.	Q	$0, 3, \frac{1}{2}, \frac{3}{4}, 3.2, -\frac{7}{8}, -57,$ $-0.0001, -6\frac{2}{3}$, 1,000,0001
Integers Whole numbers and their opposites.	Z	$\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$
Whole numbers The set of natural numbers and 0 (the number of members in an empty set).	W	$0, 1, 2, 3, \ldots$
Natural numbers (counting numbers): The numbers with which we count. The number of members in a nonempty set.	N	$1, 2, 3, \ldots$
Irrational numbers Numbers that cannot be written as the quotient of two integers. Their decimal form is nonrepeating and nonterminating.	No symbol	$\sqrt{2}, \pi$ (3.14159...), $\sqrt{7}$, 2.718281828...

The Venn diagram shows the relationships among these sets of numbers. Each set of numbers contains all of the sets below it. For example, the set of integers contains the set of whole numbers and the set of natural numbers.



Use the table and the Venn diagram on the facing page to identify the sets to which each number belongs.

EXAMPLE 1

To which sets of numbers does $\frac{1}{5}$ belong?

The number $\frac{1}{5}$ belongs to the following sets.

Set	**Why does it belong?**
Rational numbers	The number $\frac{1}{5}$ is the quotient of two integers. $(1 \div 5 = 0.2)$
Real numbers	The set of real numbers contains the set of rational numbers.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 2

To which sets of numbers does 0.4 belong?

The number 0.4 belongs to the following sets.

Set	**Why does it belong?**
Rational numbers	$0.4 = \frac{4}{10}$, so it is the quotient of two integers.
Real numbers	The set of real numbers contains the set of rational numbers.

EXAMPLE 3

To which sets of numbers does 0 belong?

The number 0 belongs to the following sets.

Set	**Why does it belong?**
Whole numbers	The set of whole numbers contains the natural numbers and 0.
Integers	The set of integers contains the set of whole numbers and their opposites, and zero.
Rational numbers	Zero can be written as the ratio of two integers: $\frac{0}{5}$
Real numbers	The set of real numbers contains the set of rational numbers.

Writing Mixed Numbers

You can use what you know about fraction and decimal equivalents to change the form of a mixed number.

EXAMPLE 10

Write $3\frac{4}{5}$ as a mixed decimal number.

Whole number part: 3.0

Fractional part: Express the fraction as an equivalent decimal.

METHOD 1

To express the fraction as a decimal, divide 4 by 5.

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ \underline{40} \\ 0 \end{array}$$

METHOD 2

Express $\frac{4}{5}$ as an equivalent fraction with a denominator of 10.

$$\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

Express the fraction as 0.8.

$$3\frac{4}{5} = 3.8$$

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 11

Write 7.425 as a mixed number.

Whole number part: 7

Decimal part: Express the decimal as an equivalent fraction.

Identify the least decimal place value. The least decimal place is the thousandths. $0.42\underline{5}$

Use 1,000 as the denominator. $\frac{425}{1{,}000}$

Express the fraction in simplest form. $\frac{425 \div 25}{1{,}000 \div 25} = \frac{17}{40}$

$$7.425 = 7\frac{17}{40}$$

Need More HELP

Make sure you find the correct decimal place. Remember that the places to the right of the decimal point are, in order:

- tenths
- hundredths
- thousandths
- ten-thousandths

Percents

A **percent** is a ratio of some quantity to 100. Percents are usually written with a percent sign (%) instead of as a fraction. You can express a percent as an equivalent fraction or decimal.

Need More **HELP**

For help with percents, go to *Meaning of Percents* on page 380.

EXAMPLE 12

Write 37% as a fraction.

Write the percent as a fraction with a denominator of 100. $37\% = \frac{37}{100}$

$37\% = \frac{37}{100}$

EXAMPLE 13

Write 40% as a fraction in simplest form.

Write the percent as a fraction with a denominator of 100. $40\% = \frac{40}{100} = \frac{40 \div 10}{100 \div 10} = \frac{4}{10}$

$40\% = \frac{4}{10}$

EXAMPLE 14

Write 0.75 as a percent.

Write the decimal as a fraction. $0.75 = \frac{75}{100}$

Express the fraction as a percent. $\frac{75}{100} = 75\%$

$0.75 = 75\%$

EXAMPLE 15

Write $\frac{1}{2}$ as a percent.

Multiply the numerator and denominator by 50 so that the denominator is equal to 100. $\frac{1 \times 50}{2 \times 50} = \frac{50}{100}$

Write the fraction as a percent. $\frac{50}{100} = 50\%$

$\frac{1}{2} = 50\%$

Improper Fractions and Mixed Numbers

Understanding Mixed Numbers and Improper Fractions

A **proper fraction** has a numerator that is less than its denominator. Examples: $\frac{1}{2}, \frac{3}{4}, \frac{5}{9}$

An **improper fraction** has a numerator that is greater than or equal to its denominator. Examples: $\frac{7}{2}, \frac{15}{7}, \frac{10}{10}$

A **mixed number** contains a whole number and a proper fraction. Mixed numbers name the sum of a whole and a part of a whole. Examples: $1\frac{1}{3}, 2\frac{3}{8}, 4\frac{1}{6}$

SEARCH

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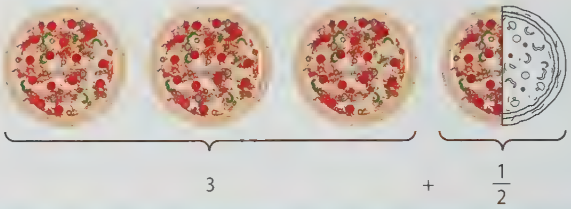
EXAMPLE 1

Liam and his friends ate three and one-half pizzas. Write a mixed number to represent the number of pizzas eaten.

Use a model to help you.

Three whole pizzas and one-half of another pizza are shaded.

Liam and his friends ate $3\frac{1}{2}$ pizzas.



Expressing Mixed Numbers as Improper Fractions

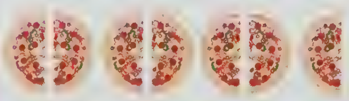
Sometimes it is necessary to express a mixed number as an improper fraction.

EXAMPLE 2

Write $3\frac{1}{2}$ as an improper fraction.

METHOD 1

Use a diagram to model $3\frac{1}{2}$.



There are seven half-pizzas shown in the diagram.

METHOD 2

Think: There are 2 halves in 1 pizza, so there are $2 \times 3 = 6$ halves in 3 pizzas: $\frac{6}{2}$

Add this fraction to one-half.

$$\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

METHOD 3

Multiply the whole number by the denominator.

Then add the numerator.

$$3\frac{1}{2} = \frac{(3 \times 2) + 1}{2}$$
$$= \frac{7}{2}$$

The mixed number $3\frac{1}{2}$ written as an improper fraction is $\frac{7}{2}$.

Expressing Improper Fractions as Mixed Numbers

To change an improper fraction to a mixed number, follow these steps.

- Divide the numerator by the denominator.
- Use the quotient as the whole number part of the mixed number.
- Use the remainder as the numerator of the fraction in the mixed number.
- Keep the original denominator that you used as the divisor.

EXAMPLE 3

Express each improper fraction as a mixed number.

a. $\frac{5}{3}$

The quotient 1 is the whole number part of the mixed number.

The remainder 2 is the numerator of the proper fraction.

The improper fraction $\frac{5}{3}$ written as a mixed number is $1\frac{2}{3}$.

$$\begin{array}{r} 1 \\ 3\overline{)\,5} \\ -3 \\ \hline 2 \end{array} \longrightarrow 1\frac{2}{3}$$

b. $\frac{17}{5}$

The quotient 3 is the whole number part of the mixed number.

The remainder 2 is the numerator of the proper fraction.

The improper fraction $\frac{17}{5}$ written as a mixed number is $3\frac{2}{5}$.

$$\begin{array}{r} 3 \\ 5\overline{)\,17} \\ -15 \\ \hline 2 \end{array} \longrightarrow 3\frac{2}{5}$$

c. $\frac{109}{4}$

The quotient 27 is the whole number part of the mixed number.

The remainder 1 is the numerator of the proper fraction.

The improper fraction $\frac{109}{4}$ written as a mixed number is $27\frac{1}{4}$.

$$\begin{array}{r} 27 \\ 4\overline{)\,109} \\ -8 \\ \hline 29 \\ -28 \\ \hline 1 \end{array} \longrightarrow 27\frac{1}{4}$$

Need More HELP

For help with division, go to *Dividing Whole Numbers* in *Foundations of Mathematics* (p. 100).

Try It This Way

For Example 3a, take 5 counters and divide them into groups of 3. You can make 1 group of 3. There are 2 counters left over that represent $\frac{2}{3}$ of the whole.

Try this method for Example 3(b), starting with 17 counters and making groups of 5.

Got to KNOW!

Renaming Mixed Numbers and Improper Fractions

Mixed number → Improper fraction

- Multiply the whole number by the denominator.
- Then add the numerator.

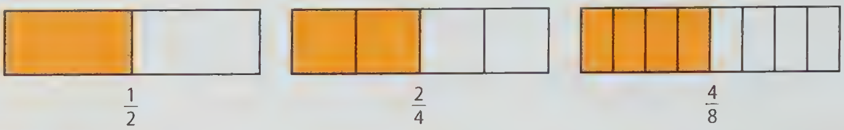
Improper fraction → Mixed number

- Divide the numerator by the denominator.
- Use the quotient as the whole number.
- Use the remainder as the numerator.

Equivalent Fractions

Modeling Equivalent Fractions

Equivalent fractions are fractions that name the same value, or amount. You can model equivalent fractions using fraction strips or a number line. The models below use equivalent fractions to name the same quantity: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

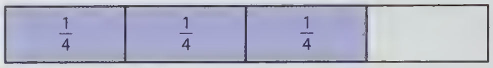


$\frac{1}{2}$
$\frac{2}{4}$
$\frac{4}{8}$

EXAMPLE 1

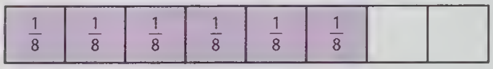
Use fraction strips to write two fractions that are equivalent to $\frac{3}{4}$.

STEP 1 Model $\frac{3}{4}$ using fourths strips.



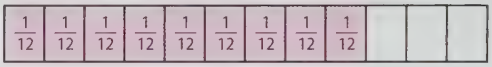
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$

STEP 2 It takes 6 eighths to make a strip of equal length.



$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$

STEP 3 It takes 9 twelfths to make a strip of equal length.



$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$
$\frac{1}{12}$

The fractions $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{9}{12}$ are equivalent fractions.

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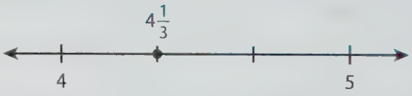
To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 2

Use a number line to write a fraction equivalent to $4\frac{1}{3}$.

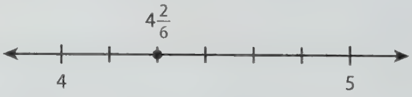
Divide the distance between 4 and 5 into thirds.

Graph $4\frac{1}{3}$ on a number line.



Divide the distance between 4 and 5 into sixths.

Graph $4\frac{2}{6}$ on a number line.



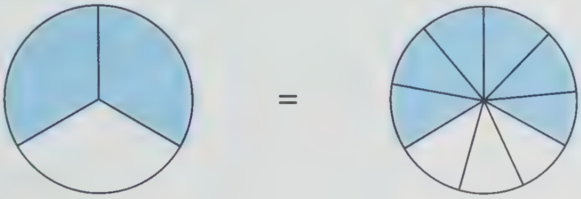
The fractions $4\frac{1}{3}$ and $4\frac{2}{6}$ name the same point on a number line. They are equivalent fractions.

Naming Equivalent Fractions

EXAMPLE 3

Write the equivalent fractions shown in the model.

a.

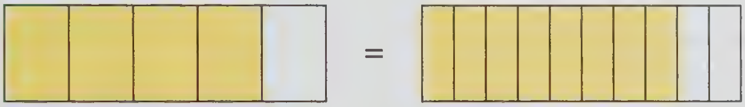


The circle on the left is divided into thirds. Two-thirds are shaded.

The circle on the right is divided into ninths. Six-ninths are shaded.

So, $\frac{2}{3} = \frac{6}{9}$.

b.



The rectangle on the left is divided into fifths. Four-fifths are shaded.

The rectangle on the right is divided into tenths. Eight-tenths are shaded.

So, $\frac{4}{5} = \frac{8}{10}$.

Writing Equivalent Fractions

If you multiply or divide the numerator and the denominator of a fraction by the same non-zero number, the result is an equivalent fraction.

EXAMPLE 4

Write four fractions that are equivalent to $\frac{2}{5}$.

$\frac{2 \times 2}{5 \times 2} = \frac{4}{10}$ $\frac{2 \times 3}{5 \times 3} = \frac{6}{15}$ $\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$ $\frac{2 \times 5}{5 \times 5} = \frac{10}{25}$

The fractions $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$. They are equivalent fractions.

Try It This Way

In Example 4, the numerators are multiples of 2, and the denominators are multiples of 5.

You may find it easier to skip count to find equivalent fractions: $\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}$

Each pair of multiples forms a fraction equivalent to two-fifths.

Got to KNOW!

Multiplying to Find Equivalent Fractions

Multiplication by 1 does not change the value of a number.

Multiplying the numerator and denominator of a fraction by the same number is the same as multiplying the fraction by 1.

Writing Equivalent Fractions

EXAMPLE 5

Write two equivalent fractions for each given fraction.

a. $\frac{5}{8}$

Multiply the numerator and denominator by the same number.

$$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

The fractions $\frac{10}{16}$ and $\frac{15}{24}$ are equivalent to $\frac{5}{8}$.

b. $\frac{15}{25}$

Multiply the numerator and denominator by the same number.

$$\frac{15}{25} = \frac{15 \times 4}{25 \times 4} = \frac{60}{100}$$

Divide the numerator and denominator by the same number.

$$\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$$

The fractions $\frac{60}{100}$ and $\frac{3}{5}$ are equivalent to $\frac{15}{25}$.

EXAMPLE 6

What denominator would make these fractions equivalent? $\frac{5}{7} = \frac{50}{?}$

Think: What number multiplied by 5 is equal to 50?

$5 \times 10 = 50$

$$\frac{5}{7} = \frac{5 \times 10}{7 \times 10} = \frac{50}{70}$$

Multiply the denominator, 7, by the same number.

So, $\frac{5}{7} = \frac{50}{70}$. The denominator 70 makes the fractions equivalent.

Try It This Way

When finding a missing numerator or denominator, as in Example 6, setting up the problem first will help you visualize it.

Since $50 > 5$, you must multiply.

$$\frac{5 \times ?}{7 \times ?} = \frac{50}{?}$$

In the problem below, $4 < 16$, so you must divide.

$$\frac{12 \div ?}{16 \div ?} = \frac{?}{4}$$

Got to KNOW!

Dividing to Find Equivalent Fractions

Dividing the numerator and denominator of a fraction by the same number is the same as dividing by 1.

$$\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$$

If you cannot divide the numerator and denominator of a fraction by the same number, the fraction is in *simplest form*.

$\frac{11}{25}$ 11 and 25 have no common factors.

Simplest Form of a Fraction

A fraction is in **simplest form** when the numerator and denominator have no common factor other than one. To write a fraction in simplest form, divide the numerator and denominator repeatedly by their common factors or once by their greatest common factor.

EXAMPLE 7

Write $\frac{24}{30}$ in simplest form.

METHOD 1

Find the GCF of 18 and 24. 18: 1, 2, 3, 6, 9, 18

List factors of 18 and 24. 24: 1, 2, 3, 4, 6, 8, 12, 24

The GCF is 6.

Divide the numerator and denominator by 6. $\frac{24}{30} = \frac{24 \div 6}{30 \div 6} = \frac{4}{5}$

The numbers 4 and 5 have no common factors other than 1. The fraction is in simplest form.

The fraction $\frac{24}{30}$ written in simplest form is $\frac{4}{5}$.

METHOD 2

Find a common factor of 24 and 30. Both numbers are divisible by 2. $\frac{24}{30} = \frac{24 \div 2}{30 \div 2} = \frac{12}{15}$

Divide the numerator and denominator by 2.

The numbers 12 and 15 share a common factor, 3.

Divide the numerator and denominator by 3. $\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$

The numbers 4 and 5 have no common factors other than 1. The fraction is in simplest form.

The fraction $\frac{24}{30}$ written in simplest form is $\frac{4}{5}$.

Need More HELP

For help with finding the GCF, go to *Factors* on page 286.

EXAMPLE 8

Write $2\frac{14}{21}$ in simplest form.

Find the simplest form of the fraction part of the mixed number.

The GCF of 14 and 21 is 7, so $\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$.

The mixed number $2\frac{14}{21}$ written in simplest form is $2\frac{2}{3}$.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Adding Fractions with Like Denominators

Modeling Adding Fractions with Like Denominators

You can model adding fractions with like denominators using fraction strips or diagrams.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

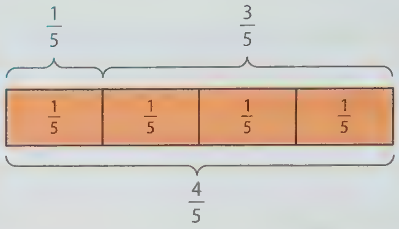
EXAMPLE 1

Use fraction strips to model $\frac{1}{5} + \frac{3}{5}$.

Model $\frac{1}{5}$ plus $\frac{3}{5}$.

Count the number of fifths.
There are 4 fifths.

$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$



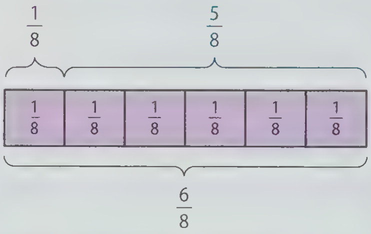
EXAMPLE 2

Use fraction strips to model $\frac{1}{8} + \frac{5}{8}$.

STEP 1 Model $\frac{1}{8}$ plus $\frac{5}{8}$.

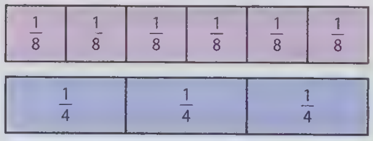
Count the number of eighths.
There are 6 eighths.

$\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$



STEP 2 Simplify.

$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$



$\frac{1}{8} + \frac{5}{8} = \frac{3}{4}$

Need More HELP

For help with simplifying fractions, go to *Simplest Form of a Fraction* on page 311.

EXAMPLE 3

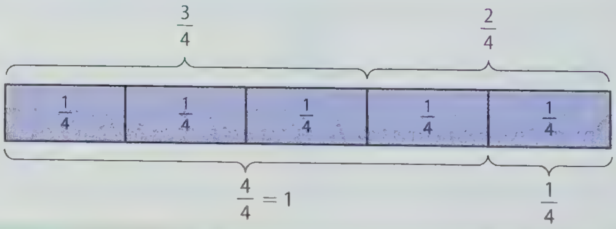
Use fraction strips to model $\frac{3}{4} + \frac{2}{4}$.

Model $\frac{3}{4}$ plus $\frac{2}{4}$.

There are 5 fourths.

Simplify: $\frac{5}{4} = \frac{4}{4} + \frac{1}{4}$, or $1\frac{1}{4}$

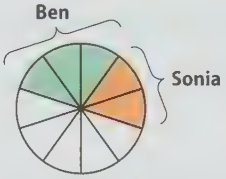
$\frac{3}{4} + \frac{2}{4} = 1\frac{1}{4}$



EXAMPLE 4

A round pizza is divided into ten equal slices. Ben eats 3 slices, or $\frac{3}{10}$ of the pizza. Sonia eats 2 slices, or $\frac{2}{10}$ of the pizza. What fraction represents the part of the pizza that was eaten?

STEP 1 Draw a diagram to represent the situation. Shade the part of the pizza that was eaten.



STEP 2 Count the number of slices eaten. 5.

$\frac{3}{10} + \frac{2}{10} = \frac{5}{10}$

STEP 3 Write the sum in simplest form.
Divide the numerator and denominator by 5: $\frac{5 \div 5}{10 \div 5} = \frac{1}{2}$

Ben and Sonia ate one-half of the pizza.

Adding Fractions

Like fractions are fractions with the same denominator. To add like fractions, add the numerators and put the sum over the common denominator.

EXAMPLE 5

Add. $\frac{2}{7} + \frac{3}{7}$

The fractions are like fractions. Add the numerators: $2 + 3 = 5$

$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

Got to KNOW!

Adding Fractions with Like Denominators

- Add the numerators. $\frac{5}{12} + \frac{3}{12}$
- Write the sum over their common denominator. $= \frac{8}{12}$
- Simplify if possible. $= \frac{8 \div 4}{12 \div 4}$

$= \frac{2}{3}$

Adding Fractions

EXAMPLE 6

Add. $\frac{1}{9} + \frac{5}{9} + \frac{2}{9}$

Add the numerators of the like fractions. $1 + 5 + 2 = 8$

Put the sum over the denominator. $\frac{8}{9}$

$\frac{1}{9} + \frac{5}{9} + \frac{2}{9} = \frac{8}{9}$

Need More HELP

Remember, the *numerator* is above the fraction bar. It is the number of parts being considered. The *denominator* is below the fraction bar. It is the total number of equal parts.

EXAMPLE 7

Add. $\frac{11}{12} + \frac{7}{12}$

Add the numerators of the like fractions. $11 + 7 = 18$

Put the sum over the denominator. $\frac{18}{12}$

The sum is an improper fraction. Write the improper fraction as a mixed number. $= \frac{12}{12} + \frac{6}{12} = 1\frac{6}{12}$

Simplify the fraction. Divide the numerator and denominator by their GCF, 6. $\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$

$\frac{11}{12} + \frac{7}{12} = 1\frac{1}{2}$

EXAMPLE 8

Add. $\frac{8}{15} + \frac{4}{15} + \frac{7}{15}$

Add the numerators of the like fractions. $8 + 4 + 7 = 19$

Put the sum over the denominator. $\frac{19}{15}$

The sum is an improper fraction. Write the improper fraction as a mixed number.

METHOD 1

$$\frac{19}{15} = \frac{15}{15} + \frac{4}{15}$$

$$= 1\frac{4}{15}$$

METHOD 2

Divide the numerator by the denominator.

$$\begin{array}{r} 1 \\ 15\overline{)19} \\ \underline{15} \\ 4 \end{array} \qquad 1\text{R}4 = 1\frac{4}{15}$$

Adding Mixed Numbers

To add mixed numbers, add the whole number parts and the fraction parts separately.

EXAMPLE 9

Add. $2\frac{2}{15} + 5\frac{11}{15}$

Write the problem in vertical form.

Add the whole number parts.

Add the numerators of the like fractions.

$$\begin{array}{r} 2\frac{2}{15} \\ +5\frac{11}{15} \\ \hline 7\frac{13}{15} \end{array}$$

$2\frac{2}{15} + 5\frac{11}{15} = 7\frac{13}{15}$

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 10

Farrah bought $1\frac{1}{4}$ pounds of Swiss cheese, $\frac{3}{4}$ pound of American cheese, and $2\frac{1}{4}$ pounds of provolone cheese. How much cheese did she buy in all?

Add the amounts of cheese.

Write the problem in vertical form.

Add the whole number parts.

Add the numerators of the like fractions.

$$\begin{array}{r} 1\frac{1}{4} \\ \frac{3}{4} \\ +2\frac{1}{4} \\ \hline 3\frac{5}{4} \end{array}$$

Simplify the improper fraction in the sum.

$$\begin{aligned} 3\frac{5}{4} &= 3 + \frac{4}{4} + \frac{1}{4} \\ &= 3 + 1 + \frac{1}{4} \\ &= 4\frac{1}{4} \end{aligned}$$

Farrah bought $4\frac{1}{4}$ pounds of cheese in all.

Got to KNOW!

Adding Mixed Numbers

- Add the whole number parts.
- Add the fraction parts.
- Simplify the sum if necessary.

Adding Fractions with Unlike Denominators

Modeling Addition with Unlike Fractions

Sometimes you need to add fractions with different denominators, or **unlike fractions**. These fractions do not have a common denominator so we say they have unlike denominators. You can use fraction strips to model adding fractions with unlike denominators.

Watch Out!

When you line up the equivalent fraction strips, make certain that they align with the left edge of the strips above them.

EXAMPLE 1

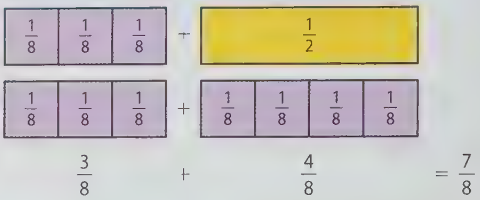
Use fraction strips to model $\frac{3}{8} + \frac{1}{2}$.

METHOD 1

Model the addition with fraction strips.

Use eighths fraction strips to rename $\frac{1}{2}$.

Count the number of eighths. $\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$



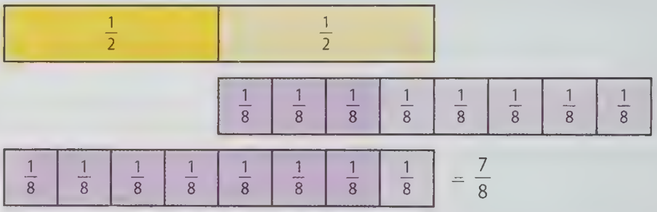
1/8 1/8 1/8 + 1/2
1/8 1/8 1/8 + 1/8 1/8 1/8 1/8
3/8 + 4/8 = 7/8

METHOD 2

Model the addition without folding fraction strips.

Model the sum using eighths. strips Count the number of eighths.

$\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$



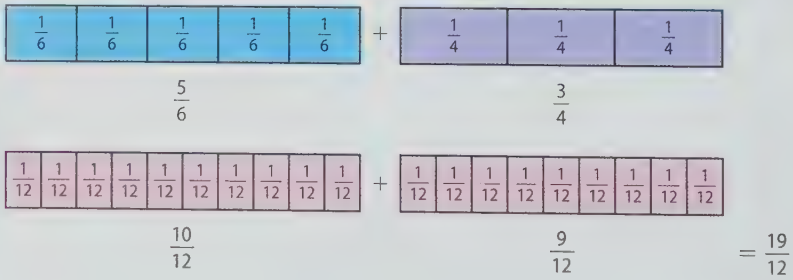
1/2 1/2
1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8
1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8 = 7/8

EXAMPLE 2

Use fraction strips to model $\frac{5}{6} + \frac{3}{4}$.

Model the addition with fraction strips.

Use twelfths fraction strips to rename both fractions.



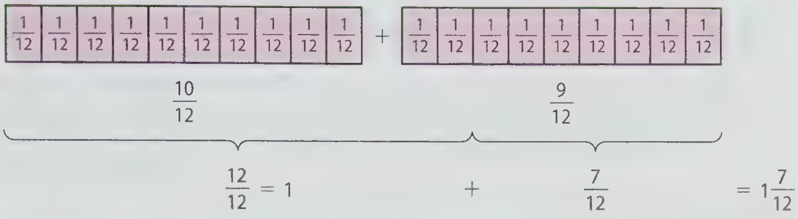
1/6 1/6 1/6 1/6 1/6 + 1/4 1/4 1/4
5/6
3/4
1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 + 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12
10/12
9/12
= 19/12

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Simplify the improper fraction.

Write the sum as a mixed number.

$\frac{5}{6}+\frac{3}{4}=1\frac{7}{12}$



Using Equivalent Fractions to Add

Other ways to add unlike fractions include finding the least common denominator (LCD) of the fractions and drawing a diagram.

EXAMPLE 3

Add. $\frac{2}{5}+\frac{3}{10}$

METHOD 1

Write equivalent fractions for $\frac{2}{5}$ and $\frac{3}{10}$ using their least common denominator (LCD).

The LCD of two fractions is its LCM (least common multiple).

Multiples of 5: 5, 10, …
Multiples of 10: 10, 20, …

Find the LCM of 5 and 10.

LCM = 10

Rename $\frac{2}{5}$ using the denominator 10.

$\frac{2}{5}=\frac{2\times 2}{5\times 2}=\frac{4}{10}$

Now the fractions are like.

Add the numerators and put the sum over the denominator.

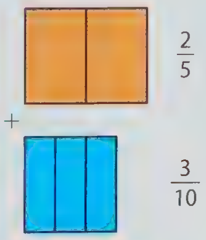
$\frac{4}{10}+\frac{3}{10}=\frac{4+3}{10}=\frac{7}{10}$

$\frac{2}{5}+\frac{3}{10}=\frac{7}{10}$

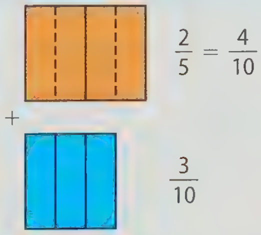
METHOD 2

Use a diagram.

STEP 1 Draw a diagram to model the problem.



STEP 2 Change the model for $\frac{2}{5}$ into tenths.



STEP 3 Count the sections. 7

$\frac{2}{5}+\frac{3}{10}=\frac{7}{10}$

Need More HELP

For help with least common multiples and least common denominators, go to *Multiples* on page 288.

Word Problems

Solving Fraction and Mixed Number Word Problems

There is more than one way to solve many word problems. Find the method that works best for you. You may find it helpful to follow the steps in the *Got to Know!* box on the facing page. Examples 1 and 2 show a formal application of the steps.

EXAMPLE 1

Simon checked out a book from the library. He read $\frac{1}{5}$ of the book on Tuesday, $\frac{1}{3}$ of the book on Wednesday, and $\frac{1}{6}$ of the book on Thursday. How much of the book has Simon read in all?

READ Read the problem carefully. Identify what you know and what you need to find out.

Know: He read $\frac{1}{5}$, $\frac{1}{3}$, and $\frac{1}{6}$ of the book.

Need to Know: The fraction of the book he read altogether.

PLAN Think about how you can find the information you need to know. The phrase "in all" usually means addition. You could add the three fractions.

SOLVE Write the problem in vertical form. Find the common denominator and add.

The LCD of 5, 3, and 6 is 30. $\frac{1}{5} = \frac{6}{30}$

Write equivalent fractions using the LCD. $\frac{1}{3} = \frac{10}{30}$

Add the numerators. $+\frac{1}{6} = \frac{5}{30}$

Simplify the sum. $\frac{21}{30} = \frac{7}{10}$

CHECK Did I answer the question? Is my answer is reasonable?

Simon has not read the entire book, so the answer must be less than 1. The fractions $\frac{1}{5}$ and $\frac{1}{6}$ are both a little less than $\frac{1}{4}$, so they represent not quite $\frac{1}{2}$ of the book. The fraction $\frac{1}{3}$ is a little greater than $\frac{1}{4}$. So Simon has read a little over half of the book. Since $\frac{1}{2} = \frac{5}{10}$, then $\frac{7}{10}$ is a reasonable answer.

Simon has read $\frac{7}{10}$ of the book.

Need More HELP

For help solving word problems, go to *Problem Solving* starting on p. 446. You will find a four-step problem solving process and strategies for problem solving.

The Check in Example 1 uses the strategy *Use Logical Reasoning* found on page 460.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 2

Alejandro and Heather made cardboard decorations. Alejandro made a decoration $2\frac{3}{4}$ inches long. Heather made a decoration $1\frac{1}{4}$ inches long. How much longer was Alejandro's decoration than Heather's?

READ Know: Alejandro's decoration was $2\frac{3}{4}$ inches. Heather's was $1\frac{1}{4}$ inches.

Need to Know: The difference in the lengths of the decorations.

PLAN There are two ways to solve the problem.

METHOD 1

Subtract the whole number parts of the mixed numbers.

Then subtract the fractions.

METHOD 2

Write the mixed numbers as improper fractions.

Then subtract.

SOLVE

METHOD 1

$$\begin{array}{r} 2\frac{3}{4} \\ -1\frac{1}{4} \\ \hline 1\frac{2}{4} = 1\frac{1}{2} \end{array}$$

METHOD 2

$$\begin{array}{r} 2\frac{3}{4} = \frac{11}{4} \\ -1\frac{1}{4} = \frac{5}{4} \\ \hline \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2} \end{array}$$

CHECK Estimate each length. Then find an estimated difference.

$2\frac{3}{4}$ is about 3, and $1\frac{1}{4}$ is about 1. Subtract: $3 - 1 = 2$. Since $1\frac{1}{4}$ is actually a little more than 1, the answer will be less than 2, so $1\frac{1}{2}$ is a reasonable answer.

Alejandro's decoration was $1\frac{1}{2}$ inches longer than Heather's.

Need More HELP

For help with estimating, go to *Estimating to Predict or Check* on page 410.

Got to KNOW!

Problem Solving Process

To solve a word problem, you can use this process:

1. **Read** the problem carefully. Identify what you know and what you need to find out.
2. **Plan** how to find the solution.
3. **Solve** the problem using your plan.
4. **Check** your work, and ask if your answer makes sense.

Solving Fraction and Mixed Number Word Problems

Watch Out!

In Example 3, the problem asks how many apples are red, so your answer should be a whole number.

For the check, you could confirm that 10 is two thirds of 15: $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

EXAMPLE 3

A bowl of apples contains 15 apples. Two-thirds of the apples are red. How many apples are red?

READ Know: 15 apples in all and $\frac{2}{3}$ are red.

Need to Know: How many are red?

PLAN Multiply to find $\frac{2}{3}$ of 15 apples.

SOLVE Multiply.

$$\frac{2}{3} \times 15 = \frac{2}{3} \times \frac{15}{1}$$

$$= \frac{2}{\underset{1}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{1}$$

$$= 10$$

CHECK Use logical reasoning and mental math.

If you arranged 15 apples into 3 equal groups, there would be 5 in each group. Two-thirds of the apples would be 2 × 5 = 10. The answer is reasonable.

Ten of the apples are red.

EXAMPLE 4

A pizza maker has $4\frac{1}{2}$ cups of cheese to divide equally among 3 pizzas. How many cups of cheese will be on each pizza?

READ Know: $4\frac{1}{2}$ cups of cheese; 3 pizzas; same amount of cheese on each pizza.

Need to Know: Amount of cheese on each pizza.

PLAN Divide to find the amount of cheese on each pizza.

SOLVE Divide.

$$4\frac{1}{2} \div 3 = \frac{9}{2} \div \frac{3}{1}$$

$$= \frac{\overset{3}{\cancel{9}}}{2} \times \frac{1}{\underset{1}{\cancel{3}}}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

CHECK Add to check your answer.

$$1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2}$$

$$= 3\frac{3}{2}$$

$$= 3 + 1\frac{1}{2}$$

$= 4\frac{1}{2}$, the original amount of cheese

There are $1\frac{1}{2}$ cups of cheese on each pizza.

EXAMPLE 5

A box contains 24 muffins. One-sixth of the muffins are banana muffins. One-half of the banana muffins contain nuts. How many muffins are banana muffins with nuts?

READ Know: 24 muffins, $\frac{1}{6}$ are banana, $\frac{1}{2}$ of the banana muffins contain nuts.

Need to Know: How many are banana muffins with nuts?

PLAN **STEP 1** Multiply to find the number of banana muffins.

STEP 2 Multiply the number of banana muffins by $\frac{1}{2}$.

SOLVE **STEP 1** $\frac{\overset{4}{\cancel{24}}}{1} \times \frac{1}{\underset{1}{\cancel{6}}} = 4$ **STEP 2** $\frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{1} = 2$

CHECK Use another method to check. Multiply $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ to find the fraction of the muffins that are banana nut muffins. Then multiply $\frac{1}{12} \times 24 = 2$ muffins are banana with nuts. ✔

Two of the muffins are banana muffins with nuts.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 6

Rajat walked $3\frac{3}{4}$ miles. Marian walked $\frac{2}{5}$ as far as Rajat. How many more miles did Rajat walk than Marian?

READ Know: Rajat walked $3\frac{3}{4}$ miles. Marian walked $\frac{2}{5}$ of the distance Rajat walked.

Need to Know: How much farther Rajat walked.

PLAN

STEP 1 Multiply to find the distance Marian walked.

STEP 2 Subtract this distance from the distance Rajat walked.

SOLVE

STEP 1

$$\frac{2}{5} \text{ of } 3\frac{3}{4} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} = \frac{3}{2} = 1\frac{1}{2} \text{ miles}$$

STEP 2

$$3\frac{3}{4} - 1\frac{1}{2} = \frac{15}{4} - \frac{3}{2} = \frac{15}{4} - \frac{6}{4} = \frac{9}{4} = 2\frac{1}{4} \text{ miles}$$

CHECK Round and estimate to check. Rajat walked about 4 miles. Two-fifths is about one-half. So Marian walked about one-half of 4, or 2 miles. Since $4 - 2 = 2$, Rajat walked about 2 miles more than Marian. The answer is reasonable.

Rajat walked $2\frac{1}{4}$ miles more than Marian.

Measurement

Architects convert perimeter and area measurements to make scale drawings and models of the buildings that they are designing.

U.S. Customary System 476
Metric System 498
Perimeter and Area 522
Surface Area and Volume 556
Scientific Notation 586

Measuring U.S. Customary Length

Units of Length in the U.S. Customary System

Length is the distance between two points. In the U.S. customary measurement system, the most commonly used units of length are *inch, foot, yard,* and *mile.* Inches, feet, and yards are often measured using a ruler, yardstick, or tape measure. On most rulers each inch is divided evenly into smaller units.

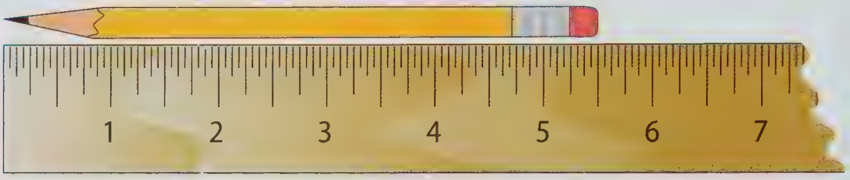
An **odometer** is an instrument used to measure distances in miles. Today most cars are equipped with a trip meter that can be reset. Drivers use it to record the distance in miles or kilometers (see page 500 for metric measurements) traveled during a trip.

Need More HELP ?

Each inch on the ruler is divided into fractional parts including half of an inch, quarter of an inch, eighth of an inch, and sixteenth of an inch.

EXAMPLE 1

Find the length of the pencil in inches.



STEP 1 Align the pencil with the beginning of the scale on the ruler. The scale on the ruler does not show zero. Place the pencil so one end is exactly even with the beginning of the scale.

STEP 2 Count the number of whole inches along its length. The pencil measures 5 whole inches.

STEP 3 Count the additional fractional parts of an inch along its length. The pencil measures an additional one-half inch.

The length of the pencil is $5\frac{1}{2}$ inches.

Got to KNOW!

U.S. Customary Units of Length							Unit Abbreviations
1 inch	=	$\frac{1}{12}$ foot	=	$\frac{1}{36}$ yard			in.
12 inches	=	**1 foot**	=	$\frac{1}{3}$ yard			ft
36 inches	=	3 feet	=	**1 yard**			yd
		5,280 feet	=	1,760 yards	=	**1 mile**	mi

Renaming Units of Length

Sometimes lengths are expressed using different units. Often it is easier to work with the lengths if they are expressed using the same unit.

EXAMPLE 2

Which distance is longer, 48 inches or 5 feet?

Use one of the methods below to find the number of inches in 5 feet. Then compare the number of inches to 48 inches.

METHOD 1

Use a chart that renames the units.

feet	1	2	3	4	5
inches	12	24	36	48	60

5 ft = 60 in.

$60 > 48$

METHOD 2

Multiply to change the larger unit to smaller units.

Think: 5 ft = ? in.

1 ft = 12 in.

Multiply: 5 ft × 12 in. per ft = 60 in.

$60 > 48$

CHECK Division undoes multiplication. Divide to find the number of feet in 60 inches.

60 in ÷ 12 in. per ft = 5 ft

60 in. = 5 ft ✓

$60 > 48$ so 5 feet is longer than 48 inches.

Need More HELP

When you write a unit in a different form, you rename the unit. The directions below mean the same thing.

Rename 5 feet as inches.

Express 5 feet as inches.

Convert 5 feet to inches.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Got to KNOW!

Rules for Renaming Units of Length

Larger unit → smaller unit	Smaller unit → larger unit
You need more of the smaller unit.	You need fewer of the larger unit.
Multiply.	Divide.
20 yd = ? feet	72 in. = ? yd
20 yd × 3 ft per yd = 60 ft	72 in. ÷ 36 in. per yd = 2 yd

Measuring Capacity in the Metric System

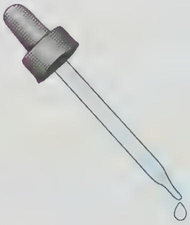
Ways to
REMEMBER

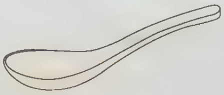
To help you remember metric capacity, think about how bottled water and sodas are packaged. They are usually available in 1-, 2-, or 3-liter bottles.

Units of Capacity

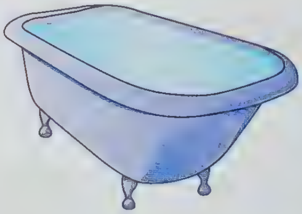
Capacity is the measure of the maximum amount of a liquid (or some other substance) a container can hold. The basic unit of capacity in the metric system is the **liter**.

The most common metric units of capacity are the *milliliter* and *liter.* However, other units include *centiliter* and *kiloliter.* All units of capacity in the metric system are related through powers of ten.


a milliliter is about 10 drops

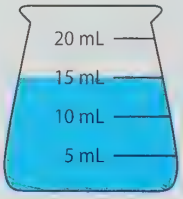

centiliter

liter


the capacity of 5 bathtubs is about 1 kiloliter

EXAMPLE 1

Ben has a beaker with water in it.



a. How many milliliters of water are in the beaker?

The water is level with the mark for 15 mL.

There are 15 mL of water in the beaker.

b. How many liters of water are in the beaker?

Convert milliliters to liters.

1,000 milliliters = 1 liter

15 mL ÷ 1,000 L per mL = 0.015 L

There is 0.015 L of water in the beaker.

Got to KNOW!

Metric Units of Capacity (Abbreviations)

1 milliliter (mL) = 0.1 centiliter = 0.001 liter

10 milliliters = **1 centiliter (cL)** = 0.01 liter

1,000 milliliters = 100 centiliters = **1 liter (L)** = 0.001 kiloliter

1,000 liters = **1 kiloliter (kL)**

Renaming Units of Capacity

Sometimes it is easier to work with different capacities if the same types of units are used.

EXAMPLE 2

Carmen bought a 3-liter bottle of lemonade for her party. Her cups are measured in milliliters. How many milliliters of lemonade does she have?

METHOD 1

A chart can be used to help rename the units.

Liters	1	2	3	4	5
Milliliters	1,000	2,000	3,000	4,000	5,000

3 L = 3,000 milliliters

Carmen has 3,000 milliliters of lemonade.

METHOD 2

Multiplication can be used to change larger units to smaller units.

Think: 3 L = ? mL 1 L = 1,000 mL

Multiply. 3 L × 1,000 mL per L = 3,000 mL

Carmen has 3,000 milliliters of lemonade.

CHECK Division can be used to change smaller units to larger units.

How many liters are in 3,000 mL?

Think: 3,000 mL = ? L 1,000 mL = 1 L

Divide. 3,000 mL ÷ 1,000 mL per L = 3 L

3,000 mL = 3 L ✓

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

When multiplying or dividing by 1,000, you move the decimal point 3 places to the right or 3 places to the left.

Example:

3.000 × 1,000 = 3,000

2,735.0 ÷ 1,000 = 2.735

Got to KNOW!

Renaming Metric Units of Capacity

Larger unit → smaller unit

You need more units. Multiply.

Kiloliters	**→ Liters**
8 kL	× 1,000 = 8,000 L
Centiliters	**→ Milliliters**
8 cL	× 10 = 80 mL

Smaller unit → larger unit

You need fewer units. Divide.

Centiliters	**→ Liters**
800 cL	÷ 100 = 8 L
Milliliters	→ Liters
8 mL	÷ 1,000 = 0.008 L

Converting U.S. Customary and Metric

Need More HELP

Remember, when writing a conversion factor, use the unit that is being changed as the denominator.

For more help with conversion factors, go to *Dimensional Analysis* on page 492.

Converting Between U.S. Customary and Metric Measures

Measurements can be expressed in the U.S. customary system or in the metric system. You need to use a *conversion factor* to convert between the two systems. In these conversions, it does not matter if you are going from small units to large or large units to small. The conversion factor allows for the difference in unit size.

However, due to differences in the two systems, all conversions between the U.S. customary and metric measures give approximate measures.

One meter is approximately equal to (≈) 1.09 yards. The conversion factor for changing meters to yards is $\frac{1.09 \text{ yd}}{1 \text{ m}}$.

When you multiply by the conversion factor, the unit that you are converting from cancels out. The unit you are converting to remains.

$$2\ \cancel{\text{m}} \times \frac{1.09 \text{ yd}}{1\ \cancel{\text{m}}} \approx 2 \times 1.09 \text{ yd} = 2.18 \text{ yd}$$

$$2 \text{ m} \approx 2.18 \text{ yd}$$

EXAMPLE 1

8 miles = ? kilometers

STEP 1 Write the conversion factor. Use 1 mile ≈ 1.61 kilometers.

Conversion factor: $\frac{1.61 \text{ km}}{1 \text{ mi}}$

STEP 2 Use the conversion factor to convert miles to kilometers.

$$\frac{8\ \cancel{\text{mi}}}{1} \times \frac{1.61 \text{ km}}{1\ \cancel{\text{mi}}} \approx 12.88 \text{ km}$$

8 miles ≈ 12.88 kilometers

Got to KNOW!

U.S. Customary and Metric Equivalents—Length

U.S. Customary to Metric	Metric to U.S. Customary
1 in. ≈ 2.54 cm	1 mm ≈ 0.04 in.
1 ft ≈ 30.48 cm	1 cm ≈ 0.39 in.
1 yd ≈ 91.44 cm	1 m ≈ 3.28 ft ≈ 1.09 yd
1 mi ≈ 1.61 km	1 km ≈ 0.62 mi

Converting Capacity

To convert units of capacity, first write a conversion factor. Then multiply the units you want to change by the conversion factor.

EXAMPLE 2

7 liters = ? fluid ounces

STEP 1 Write the conversion factor. Use 1 liter ≈ 33.8 fluid ounces.

Conversion factor: $\frac{33.8\text{ fl oz}}{1\text{ L}}$

STEP 2 Use the conversion factor to convert liters to fluid ounces.

$$\frac{7\cancel{\text{L}}}{1} \times \frac{33.8\text{ fl oz}}{1\cancel{\text{L}}} \approx 7 \times 33.8\text{ fl oz} \approx 236.6\text{ fl oz}$$

7 liters ≈ 236.6 fluid ounces

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 3

12 cups = ? liters

STEP 1 Write the conversion factor. Use 1 cup ≈ 236.6 mL.

Conversion factor: $\frac{236.6\text{ mL}}{1\text{ c}}$

STEP 2 Use the conversion factor to convert cups to milliliters.

$$\frac{12\cancel{\text{c}}}{1} \times \frac{236.6\text{ mL}}{1\cancel{\text{c}}} \approx 12 \times 236.6 \approx 2{,}839.2\text{ mL}$$

STEP 3 Convert 2,839.2 milliliters to liters. Use 1 L ≈ 1,000 mL.

2,839.2 mL ÷ 1,000 mL per L ≈ 2.8391 L

12 cups ≈ 2.8392 liters

Watch Out!

In Example 3, Step 3, you do not need to find a conversion factor because 1,000 is a power of 10. Divide 2,839.2 by 1,000 by moving the decimal point three places to the left.

Got to KNOW!

U.S. Customary and Metric Equivalents—Capacity

U.S. Customary to Metric	Metric to U.S. Customary
1 fl oz ≈ 29.6 mL	1 mL ≈ 0.03 fl oz
1 cup ≈ 236.6 mL	1 cL = 0.34 fl oz
1 pt ≈ 473.2 mL	1 L ≈ 33.8 fl oz ≈ 1.06 qt
1 qt ≈ 0.95 L	1 kL ≈ 264.2 gal
1 gal ≈ 3.79 L	

Converting Mass and Weight

To convert units of weight and mass, first write a conversion factor. Then multiply the units you want to change by the conversion factor.

EXAMPLE 4

A doctor needs to calculate how many milligrams of a certain medicine to give to a patient. The dosage is calculated based on the mass of the person taking it. If the patient weighs 125 lb, what is the patient's mass in kilograms?

STEP 1 Write the conversion factor. Use 1 pound ≈ 0.45 kilograms.

Conversion factor: $\frac{0.45\text{ kg}}{1\text{ lb}}$

STEP 2 Use the conversion factor to convert pounds to kilograms.

$$\frac{125\,\cancel{\text{lb}}}{1} \times \frac{0.45\text{ kg}}{1\,\cancel{\text{lb}}} \approx 125 \times 0.45\text{ kg} \approx 56.25\text{ kg}$$

The patient's mass is approximately equal to 56.25 kilograms.

EXAMPLE 5

A boulder has a mass of 2.5 metric tons. How many pounds does the boulder weigh?

STEP 1 Write the conversion factor. Use 1 metric ton ≈ 2,205 pounds.

Conversion factor: $\frac{2{,}205\text{ lb}}{1\text{t}}$

STEP 2 Use the conversion factor to convert metric tons to pounds.

$$\frac{2.5\,\cancel{\text{t}}}{1} \times \frac{2{,}205\text{ lb}}{1\,\cancel{\text{t}}} \approx 2.5 \times 2{,}205\text{ lb} \approx 5{,}512.5\text{ lb}$$

The boulder weighs approximately 5,512.5 pounds.

Got to KNOW!

U.S. Customary and Metric Equivalents—Weight and Mass

U.S. Customary to Metric	Metric to U.S. Customary
1 oz ≈ 28.35 g	1 mg ≈ 0.000035 oz
1 lb ≈ 0.45 kg	1 g ≈ 0.0353 oz
1 short ton ≈ 907.2 kg ≈ 0.91 metric ton	1 kg ≈ 35.27 oz ≈ 2.2 b
	1 metric ton ≈ 2,205 lb ≈ 1.1 short ton

Solving Problems with Unit Conversions

Remember, when solving problems that require a conversion between U.S. customary and metric systems, the answer is almost always an approximate measure. You may need to round the estimate up or down to answer the question.

EXAMPLE 6

Anita needs to buy enough energy drink to fill a 5-gallon cooler completely. The brand of energy drink that she buys is sold only in 1-liter bottles. How many bottles of energy drink should she buy?

Before you can answer this question, you need to know the number of liters in 5 gallons.

STEP 1 Write a conversion factor for gallons to liters. Use 1 gallon $\approx$ 3.79 liters.

Conversion factor: $\frac{3.79\text{ L}}{1\text{ gal}}$

STEP 2 Use the conversion factor to convert 5 gallons to liters.

$$\frac{5\ \cancel{\text{gal}}}{1} \times \frac{3.79\text{ L}}{1\ \cancel{\text{gal}}} \approx 5 \times 3.79\text{ L} \approx 18.95\text{ L}$$

$5\text{ gal} \approx 18.95\text{ L}$

STEP 3 Compare the quantities.

18 1-L bottles $<$ 18.95 L $<$ 19 1-L bottles

Since the capacity of the cooler is 18.95 L, Anita should buy 19 bottles if she wants to fill the cooler completely.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 7

Hector has 23 meters of rope. He needs 76 feet of rope to mark off a section of a parking lot. Does he have enough rope?

STEP 1 Write a conversion factor for meters to feet. Use 1 meter $\approx$ 3.28 feet.

Conversion factor: $\frac{3.28\text{ ft}}{1\text{ m}}$

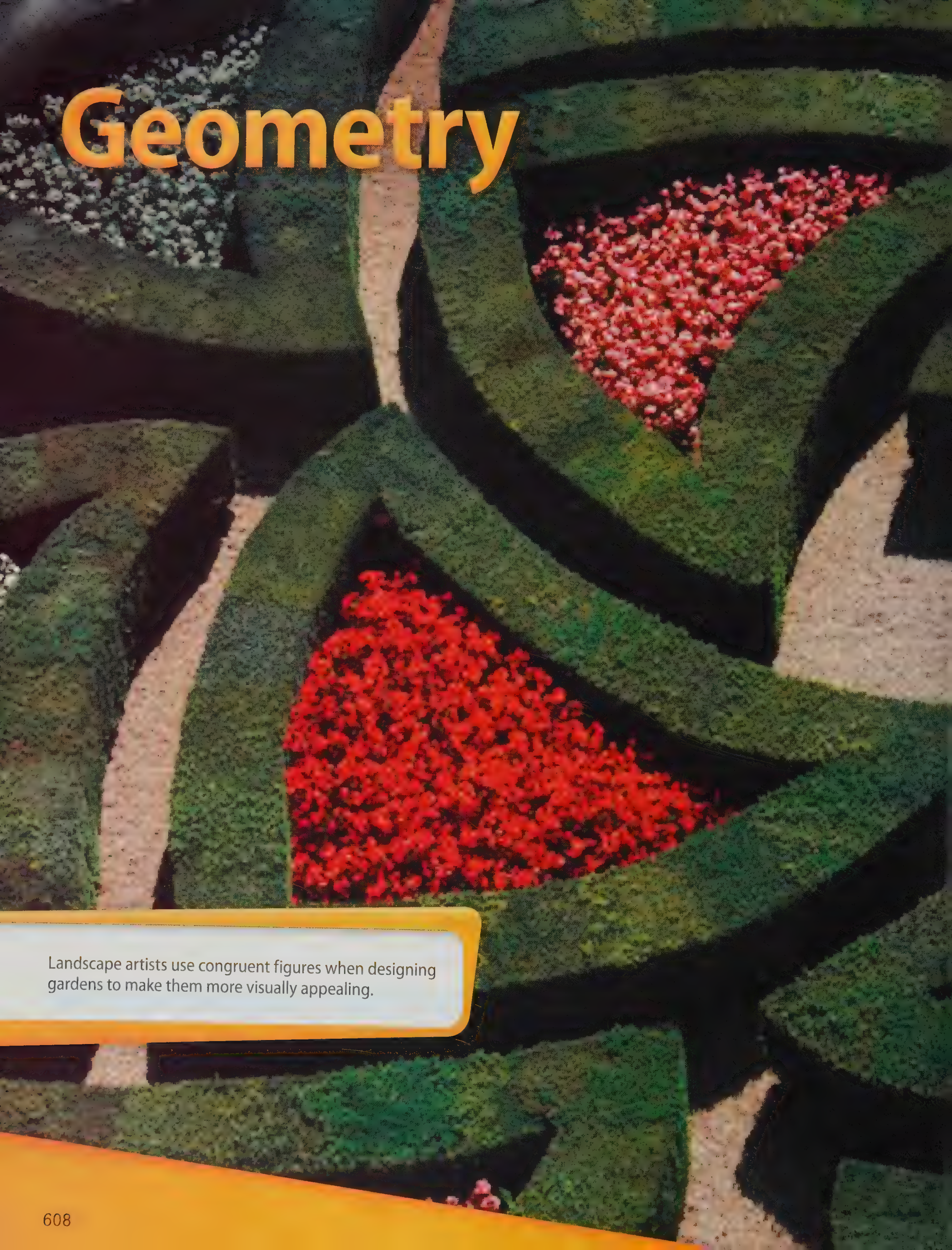
STEP 2 Use the conversion factor to convert meters to feet.

$$\frac{23\ \cancel{\text{m}}}{1} \times \frac{3.28\text{ ft}}{1\ \cancel{\text{m}}} \approx 75.44\text{ ft}$$

The length of Hector's rope is 75.44 feet.

Since 75.44 feet $<$ 76 feet, Hector does not have enough rope.

Geometry



Landscape artists use congruent figures when designing gardens to make them more visually appealing.

Reasoning and Proof 610

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Deductive Reasoning

Using the Law of Detachment

People often use examples to support a proposal and show that it is true in specific cases. But in order to show that a proposal is *always* true, you must prove it. When you use properties, rules, definitions, theorems, given facts, and the laws of logic to arrive at a conclusion, you are using **deductive reasoning**.

One law of deductive reasoning is the **Law of Detachment**.

Law of Detachment
If a conditional statement is true and its hypothesis is true, then the conclusion is true.

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EXAMPLE 1

Assume the given statements are true. Use the Law of Detachment to draw a valid conclusion.

Given: **If Mark mows at least three lawns this weekend, then he can afford to go on the school trip.**
Mark mows four lawns this weekend.

Identify the hypothesis.	Mark mows at least three lawns this weekend.
Identify the conclusion.	He can afford to go on the school trip.

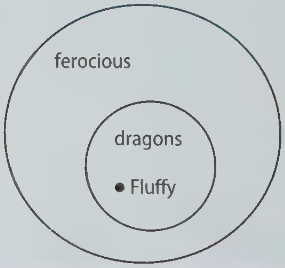
Because the conditional and its hypothesis are both true, the Law of Detachment says the conclusion must be true. You can conclude that Mark can afford to go on the school trip.

You can also use a Venn diagram to help you reason logically.

EXAMPLE 2

Assume the following statements are true. What can you conclude?

All dragons are ferocious.
Fluffy is a dragon.



All the points in the "dragons" circle are inside the "ferocious" circle. Since Fluffy is in the "dragons" circle, Fluffy is also in the "ferocious" circle. You can conclude that Fluffy is ferocious.

Using the Law of Syllogism

Another important law of deductive reasoning is the **Law of Syllogism**.

Law of Syllogism

If $p \longrightarrow q$ is true and $q \longrightarrow r$ is true, then $p \longrightarrow r$ must be true.

EXAMPLE 3

The given statements are true. Use the Law of Syllogism to draw a valid conclusion.

Given: If the sum of the digits in a number is divisible by 9, then the number is divisible by 9.
If a number is divisible by 9, then the number is divisible by 3.

STEP 1 Assign p, q, and r as follows:

p: The sum of the digits in a number is divisible by 9.
q: The number is divisible by 9.
r: The number is divisible by 3.

STEP 2 Rewrite the given statements symbolically.

$p \longrightarrow q$
$q \longrightarrow r$

Now it is easy to see that q is the conclusion of the first conditional as well as the hypothesis of the second conditional. You can use the Law of Syllogism to conclude that $p \longrightarrow r$: If the sum of the digits in a number is divisible by 9, then the number is divisible by 3.

EXAMPLE 4

Use the Law of Detachment and the Law of Syllogism to draw a conclusion from the given statements.

Given: (1) If the perimeter of a square is 20 cm, then one side is 5 cm long.
(2) If one side of a square is 5 cm long, then the area is 25 cm^2.
(3) The perimeter of a square is 20 cm.

METHOD 1

Use the Law of Detachment first.

You can use statements (1) and (3) and the Law of Detachment to conclude that one side of the square is 5 cm long.

Then you can use this result, statement (2), and the Law of Detachment to conclude that the area of the square is 25 cm^2.

METHOD 2

Use the Law of Syllogism first.

You can use statements (1) and (2) and the Law of Syllogism to conclude that if the perimeter of a square is 20 cm, then the area of the square is 25 cm^2.

Now you can use this conditional, statement (3), and the Law of Detachment to conclude that the area of the square is 25 cm^2.

GOT TO KNOW!

Law of Detachment

If $p \longrightarrow q$ is true and p is true, then q is true.

Law of Syllogism

If $p \longrightarrow q$ is true and $q \longrightarrow r$ is true, then $p \longrightarrow r$ is true.

Inductive Reasoning

Patterns and Conjectures

When you use **inductive reasoning** you make a conclusion, also called a **conjecture**, based on observations. This process often involves finding a pattern. Unlike deductive reasoning, you cannot use inductive reasoning to prove something. The conjecture that you make as a result of inductive reasoning may or may not be true.

EXAMPLE 1

Describe each pattern. Use the pattern to guess the next term in the pattern.

a. February, April, June, . . .

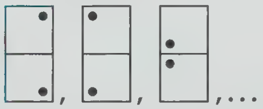
This is a list of every other month beginning with February. The next term is August.

b. 1, 4, 9, 16, . . .

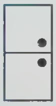
$1 = 1^2$ $4 = 2^2$ $9 = 3^2$ $16 = 4^2$

The numbers are the squares of consecutive integers. The next term is 5^2, or 25.

c.



In each figure, the dot in the top half moves counterclockwise to the next corner, and the dot in the bottom half moves clockwise. The next term is:



Not every conjecture made using inductive reasoning is true. You can prove a conjecture false by finding just one counterexample.

EXAMPLE 2

Dolores looks at the equations below, sees a pattern, and makes the conjecture that every multiple of 5 ends in 5. Prove that this conjecture is false.

$1(5) = 5$ $3(5) = 15$ $5(5) = 25$ $7(5) = 35$

A counterexample is the number 20. It is a multiple of 5 and does not end in 5.

GOT TO KNOW!

Types of Reasoning

Deductive Reasoning	Inductive Reasoning
• Uses properties, rules, definitions, theorems, given facts, and the laws of logic to arrive at a conclusion. • The conclusion must be true if the hypotheses are true.	• Uses observations and patterns. • The conclusion is not necessarily true.

EXAMPLE 3

Use inductive reasoning to form a conjecture.

a. The sum of an even integer and an odd integer

STEP 1 Find examples.

$2 + 3 = 5$ $\quad 6 + 1 = 7$ $\quad 4 + 3 = 7$ $\quad 8 + 5 = 13$

$4 + 5 = 9$ $\quad 8 + 3 = 11$ $\quad 10 + 5 = 15$ $\quad 2 + 7 = 9$

STEP 2 Look for a pattern.

All the sums are odd.

STEP 3 Form a conjecture.

The sum of an even integer and an odd integer is an odd integer.

b. The product of a positive integer and a negative integer

STEP 1 Find examples.

$2(-3) = -6$ $\quad 2(-4) = -8$ $\quad 2(-5) = -10$

$3(-3) = -9$ $\quad 3(-5) = -15$ $\quad 3(-6) = -18$

$4(-10) = -40$ $\quad 4(-15) = -60$ $\quad 4(-17) = -68$

STEP 2 Look for a pattern.

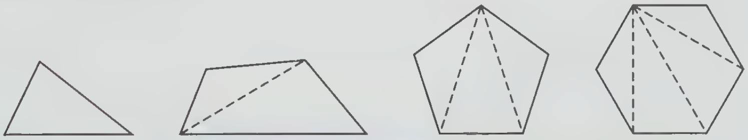
All the products are negative.

STEP 3 Form a conjecture.

The product of a positive integer and a negative integer is a negative integer.

c. The number of sides of a polygon and the number of triangles formed by drawing all the diagonals from one vertex of the polygon

STEP 1 Find examples.



STEP 2 Make a table and look for a pattern.

Number of Sides	3	4	5	6
Number of Triangles	1	2	3	4

Each number in the second row is two less than the corresponding number in the first row.

STEP 3 Form a conjecture.

The number of triangles formed when you draw all the diagonals from one vertex of a polygon is two less than the number of sides.

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Need More HELP

Remember: A vertex is the point where two sides of a polygon intersect.

EXAMPLE 4

Make a conjecture about the sum of the first *n* positive even integers.

Find the sum for $n = 3$. $\quad 2 + 4 + 6 = 12 = 3(4)$

Find the sum for $n = 4$. $\quad 2 + 4 + 6 + 8 = 20 = 4(5)$

Find the sum for $n = 5$. $\quad 2 + 4 + 6 + 8 + 10 = 30 = 5(6)$

Find the sum for $n = 6$. $\quad 2 + 4 + 6 + 8 + 10 + 12 = 42 = 6(7)$

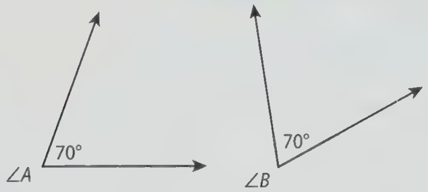
In each case, the sum equals the product of n and $n + 1$.

Conjecture: The sum of the first n positive even integers is $n(n + 1)$.

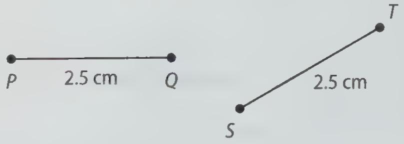
Properties of Congruence

Congruence

Congruent (≅) figures have the exact same shape and size. **Congruent segments** have equal lengths. **Congruent** angles have equal measures. Corresponding sides and corresponding angles of congruent polygons are congruent.



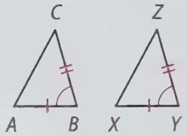
$m\angle A = m\angle B$ if and only if $\angle A \cong \angle B$



$PQ = ST$ if and only if $\overline{PQ} \cong \overline{ST}$

You can mark segments and angles alike to indicate that they are congruent. The markings in the figures at the right show that:

$\angle B \cong \angle Y$ $\quad \overline{AB} \cong \overline{XY}$ $\quad \overline{BC} \cong \overline{YZ}$



EXAMPLE 1

Complete the congruence statements for the given segments and angles.

$AB = 3$	$m\angle 1 = 35°$	$CD = 6$	$m\angle 3 = 35°$	$HI = 5$
$m\angle 2 = 50°$	$DE = 5$	$m\angle 4 = 40°$	$JK = 3$	$m\angle 5 = 50°$

a. $\overline{AB} \cong \underline{?}$

$AB = JK$, so $\overline{AB} \cong \overline{JK}$.

b. $\angle 5 \cong \underline{?}$

$m\angle 5 = m\angle 2$, so $\angle 5 \cong \angle 2$.

c. $\overline{HI} \cong \underline{?}$

$HI = DE$, so $\overline{HI} \cong \overline{DE}$.

GOT TO KNOW!

Congruence Properties

Property	Segments	Angles
Reflexive	$\overline{AB} \cong \overline{AB}$	$\angle X \cong \angle X$
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.	If $\angle X \cong \angle Y$, then $\angle Y \cong \angle X$.
Transitive	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.	If $\angle X \cong \angle Y$ and $\angle Y \cong \angle Z$, then $\angle X \cong \angle Z$.

The properties of congruence in geometry are very much like the properties of equality for real numbers. Though they may seem simple, you will find them extremely useful in proofs.

EXAMPLE 2

Name the property of equality or congruence that justifies each statement.

a. **If $\angle CDE \cong \angle GHI$, then $\angle GHI \cong \angle CDE$.**

Symmetric Property of Congruence

b. **If $m\angle P = m\angle Q$ and $m\angle Q = 120°$, then $m\angle P = 120°$.**

Transitive Property of Equality

c. **$\overline{MN} \cong \overline{NM}$**

Reflexive Property of Congruence

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EXAMPLE 3

Given: $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$, $m\angle 1 = (3x + 5)°$, and $m\angle 3 = 65°$. Find the value of x.

STEP 1 Use the Transitive Property.

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

STEP 2 Use the definition of congruent angles.

If $\angle 1 \cong \angle 3$, then $m\angle 1 = m\angle 3$.

STEP 3 Substitute for $m\angle 1$ and $m\angle 3$.

$3x + 5 = 65$

STEP 4 Solve for x.

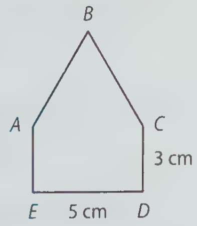
$$\begin{aligned} 3x + 5 &= 65 \\ 3x &= 60 \\ x &= 20 \end{aligned}$$

Need More HELP

For help with solving equations, go to *Two-Step Equations* in *Algebra* (Book 2, p. 1464).

EXAMPLE 4

Find the perimeter of polygon *ABCDE*.



$\overline{AB} \cong \overline{BC}$ $\overline{BC} \cong \overline{ED}$ $\overline{AE} \cong \overline{CD}$

$\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{ED}$, so $\overline{AB} \cong \overline{ED}$ and $AB = ED = 5$.

$\overline{BC} \cong \overline{ED}$, so $BC = ED = 5$.

$\overline{AE} \cong \overline{CD}$, so $AE = CD = 3$.

$$\begin{aligned} \text{perimeter} &= AB + BC + CD + ED + AE \\ &= 5 + 5 + 3 + 5 + 3 \\ &= 21 \text{ cm} \end{aligned}$$

The perimeter of polygon *ABCDE* is 21 cm.

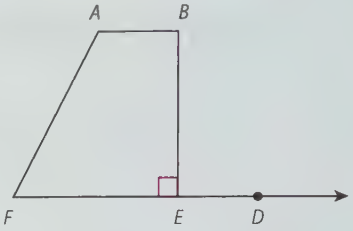
Proving Angles Congruent

Classifying Angles

You can classify angles by their measures. The measure of a **right angle** is 90°. The measure of a **straight angle** is 180°. The measure of an **acute angle** is greater than 0° and less than 90°. The measure of an **obtuse angle** is greater than 90° and less than 180°.

EXAMPLE 1

State whether each angle appears to be acute, right, obtuse, or straight.



a. $\angle FEB$

Look for the right angle symbol. $\angle FEB$ is a right angle.

b. $\angle F$

$\angle F$ appears to be acute.

c. $\angle A$

$\angle A$ appears to be obtuse.

d. $\angle FED$

Points D, E, and F are collinear, so $\angle FED$ is a straight angle.

Ways to REMEMBER

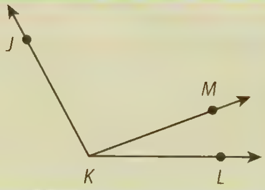
To remember what an acute angle looks like, think of the capital letter *A*, as in *Acute*. The angle at the top of the letter A is acute.

Angle Addition Postulate

A **postulate** is a statement that is accepted as true without being proven. Postulates are one of the basic tools used in proofs. Here is a basic postulate about angles.

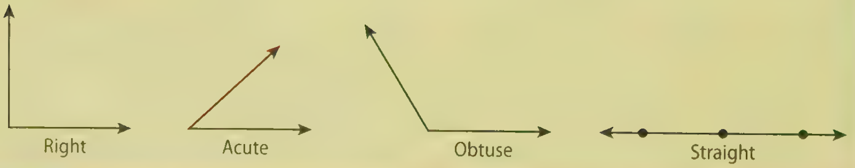
Angle Addition Postulate

If point M is in the interior of $\angle JKL$, then $m\angle JKM + m\angle MKL = m\angle JKL$.



GOT TO KNOW!

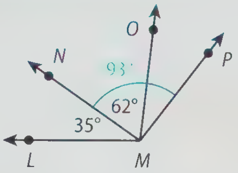
Types of Angles



EXAMPLE 2

Find each angle measure.

a. $m\angle LMO$



Use the Angle Addition Postulate. $m\angle LMO = m\angle LMN + m\angle NMO$

Substitute the given angle measures. $= 35° + 62°$

$= 97°$

$m\angle LMO = 97°$

b. $m\angle OMP$

Use the Angle Addition Postulate. $m\angle NMP = m\angle NMO + m\angle OMP$

Substitute the given angle measures. $93° = 62° + m\angle OMP$

Subtract 62 from both sides. $31° = m\angle OMP$

$m\angle OMP = 31°$

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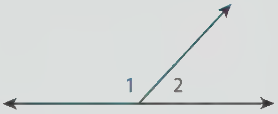
Identifying Angle Pairs

Two angles are **complementary** if the sum of their measures is 90°. Two angles are **supplementary** if the sum of their measures is 180°. Two angles that have the same vertex and share a side but have no common interior points are called **adjacent angles**. If two angles are adjacent and their exterior sides lie on a straight line, then the angles form a **linear pair**.



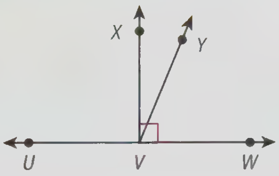
∠1 and ∠2 form a linear pair.

Watch Out!

Complementary angles and supplementary angles do not have to be adjacent angles.

EXAMPLE 3

Use the diagram to name the angle pairs.



a. complementary angles

∠XVY and ∠YVW

b. supplementary angles

∠UVX and ∠XVW, ∠UVY and ∠YVW

c. adjacent angles

∠UVX and ∠XVW, ∠UVY and ∠YVW, ∠UVX and ∠XVY

d. linear pairs

∠UVX and ∠XVW, ∠UVY and ∠YVW

Theorems about Angle Pairs

A **theorem** is a statement that can be proven through deductive reasoning. This theorem states that a linear pair of angles is always supplementary.

Linear Pair Theorem
If two angles form a linear pair, then they are supplementary.

When you prove a theorem, you must justify every statement with a definition, property, postulate, or previously proved theorem.

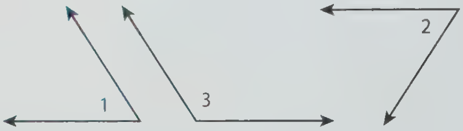
A **paragraph proof** uses complete sentences to explain and justify each step, eventually reaching the desired conclusion.

EXAMPLE 4

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Write a paragraph proof.

Given: $\angle 1$ is supplementary to $\angle 3$.
$\angle 2$ is supplementary to $\angle 3$.
Prove: $\angle 1 \cong \angle 2$



$\angle 1$ and $\angle 3$ are supplementary.

By definition, $m\angle 1 + m\angle 3 = 180°$. Similarly, $m\angle 2 + m\angle 3 = 180°$.

Substituting, you get $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$. The Subtraction Property of Equality lets you subtract $m\angle 3$ from both sides.

So, $m\angle 1 = m\angle 2$ and $\angle 1 \cong \angle 2$.

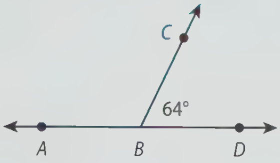
This proves the Congruent Supplements Theorem. The proof of the Congruent Complements Theorem is similar.

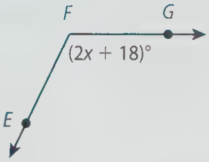
GOT TO KNOW!

Congruent Supplements Theorem
If two angles are supplementary to the same angle (or to congruent angles), then the angles are congruent.
Congruent Complements Theorem
If two angles are complementary to the same angle (or to congruent angles), then the angles are congruent.

EXAMPLE 5

$\angle CBD$ and $\angle EFG$ are supplementary. Find the value of x.





Use the Linear Pair Theorem and the Congruent Supplements Theorem.

$\angle ABC$ and $\angle CBD$ form a linear pair. $m\angle ABC + m\angle CBD = 180$

Substitute 64 for $m\angle CBD$. $m\angle ABC + 64 = 180$

$m\angle ABC = 116°$

Supplements of the same angle are congruent. $m\angle ABC = m\angle EFG$

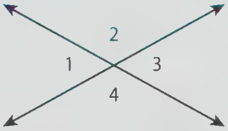
Substitute. $116 = 2x + 18$

Solve the equation for x. $98 = 2x$

$x = 49$

Vertical Angles

Vertical angles are the pairs of non-adjacent angles formed when two lines intersect.



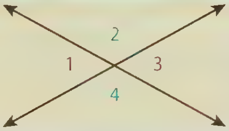
$\angle 1$ and $\angle 3$ are vertical angles.

$\angle 2$ and $\angle 4$ are vertical angles.

Vertical angles have a very special property.

Vertical Angles Theorem

Vertical angles are congruent.
$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

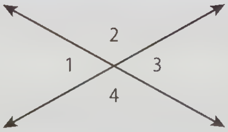


Watch Out!

Vertical angles are not always in a vertical position relative to each other. They are always opposite each other across their common *vertex*.

Proof of the Vertical Angle Theorem

Given: $\angle 1$ and $\angle 3$ are vertical angles.
Prove: $\angle 1 \cong \angle 3$



$\angle 1$ and $\angle 2$ form a linear pair. By the Linear Pair Theorem, $\angle 1$ and $\angle 2$ are supplementary. Similarly, $\angle 2$ and $\angle 3$ are supplementary. Since $\angle 1$ and $\angle 3$ are both supplementary to $\angle 2$, $\angle 1 \cong \angle 3$ by the Congruent Supplements Theorem.

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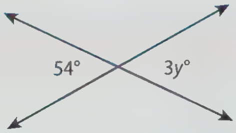
Using the Vertical Angles Theorem

EXAMPLE 6

Find the value of *y*.

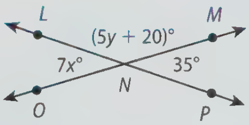


Vertical angles are congruent. $3y = 54$

Divide both sides by 3. $y = 18$

EXAMPLE 7

Find the values of *x* and *y*.



Vertical angles are congruent. $m\angle LNO = m\angle MNP$

Substitute. $7x = 35$

Divide both sides by 7. $x = 5$

Linear pairs are supplementary. $m\angle LNM + m\angle MNP = 180°$

Substitute. $(5y + 20) + 35 = 180$

Simplify. $5y + 55 = 180$

Subtract 55 from both sides. $5y = 125$

Divide both sides by 5. $y = 25$

So, $x = 5$ and $y = 25$.

Right Angles

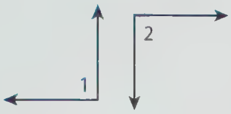
You already know that the measure of a right angle is 90° by definition. Here are two theorems that tell you more about right angles.

Right Angle Congruence Theorem
All right angles are congruent.
Theorem
If two angles are congruent and supplementary, then each angle is a right angle.

EXAMPLE 8

Prove the Right Angle Congruence Theorem.

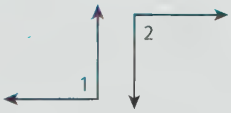
Given: $\angle 1$ and $\angle 2$ are right angles.
Prove: $\angle 1 \cong \angle 2$



By definition of a right angle, $m\angle 1 = 90°$ and $m\angle 2 = 90°$.
By the Transitive Property of Equality, $m\angle 1 = m\angle 2$, or $\angle 1 \cong \angle 2$.

EXAMPLE 9

Given: $\angle 1$ and $\angle 2$ are congruent and supplementary.
Prove: $\angle 1$ and $\angle 2$ are right angles.

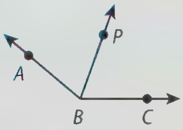


By definition of supplementary angles, $m\angle 1 + m\angle 2 = 180°$.
By definition of congruent angles, $m\angle 1 = m\angle 2$.
Using substitution and algebra, $m\angle 1 + m\angle 1 = 180°$.

$$2(m\angle 1) = 180°$$
$$m\angle 1 = 90°$$

Then, by definition of congruent angles, $m\angle 2 = 90°$ since $\angle 1 \cong \angle 2$. So $\angle 1$ and $\angle 2$ are both right angles by definition of a right angle.

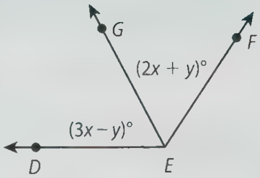
An **angle bisector** is a ray that divides an angle into two congruent coplanar angles.



$\overrightarrow{BP}$ bisects $\angle ABC$.
$\angle ABP \cong \angle CBP$
$m\angle ABP = m\angle CBP = \frac{1}{2}m\angle ABC$.

EXAMPLE 10

$\overrightarrow{EG}$ bisects $\angle DEF$ and $m\angle DEF = 150°$. Find the values of x and y.



Write and solve two equations in two variables.

Use the Angle Addition Postulate.	$m\angle DEG + m\angle GEF = m\angle DEF$
Substitute.	$(3x - y) + (2x + y) = 150$
Simplify.	$5x = 150$
Divide both sides by 5.	$x = 30$
Use the definition of an angle bisector.	$m\angle DEG = m\angle GEF$
Substitute.	$3x - y = 2x + y$
Substitute 30 for x.	$3(30) - y = 2(30) + y$
Simplify.	$90 - y = 60 + y$
Solve for y.	$30 = 2y$
	$y = 15$

So, $x = 30$ and $y = 15$.

Need More HELP ?

For a review on solving systems of equations, go to *Solving a System by Substitution* in *Algebra* (Book 2, p. 1568).

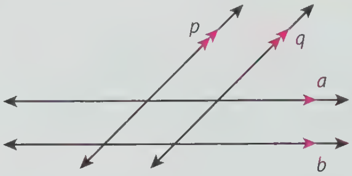
Theorems About Parallel Lines

Watch Out!

Two segments that do not intersect are not necessarily parallel. To be parallel, they must be parts of parallel lines.

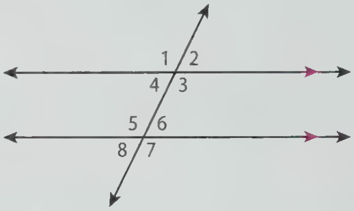
Properties of Parallel Lines

Lines that do not intersect are either *parallel* or *skew*. Coplanar lines that do not intersect are **parallel**. Non-coplanar lines that do not intersect are **skew**. Diagrams use solid red arrowheads to denote lines that are parallel.



Line a is parallel to line b ($a \parallel b$) and line p is parallel to line q.

A **transversal** is a line that intersects two or more coplanar lines at different points. When a transversal intersects parallel lines, the pairs of angles formed have special names.



Corresponding angles lie in corresponding positions on the same side of a transversal.

$\angle 1$ and $\angle 5$ $\angle 4$ and $\angle 8$ $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$

Alternate interior angles lie between the parallel lines on opposite sides of a transversal.

$\angle 4$ and $\angle 6$ $\angle 3$ and $\angle 5$

Same-side interior angles lie between the parallel lines on the same side of a transversal.

$\angle 3$ and $\angle 6$ $\angle 4$ and $\angle 5$

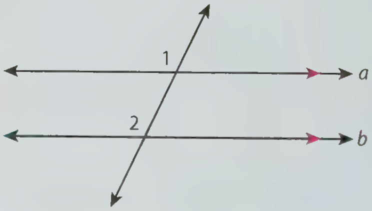
Alternate exterior angles lie outside the parallel lines on opposite sides of a transversal.

$\angle 1$ and $\angle 7$ $\angle 2$ and $\angle 8$

Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then corresponding angles are congruent.

EXAMPLE 1

Find the value of x if $m\angle 1 = (10x + 16)°$ and $m\angle 2 = (13x - 8)°$.



The diagram shows that $a \parallel b$.

Corresponding angles are congruent. $13x - 8 = 10x + 16$

Subtract $10x$ from both sides. $3x - 8 = 16$

Add 8 to both sides. $3x = 24$

Divide both sides by 3. $x = 8$

GOT TO KNOW!

Parallel Postulate

Given a line and a point not on the line, there is exactly one line through the point parallel to the line.

You can use the Corresponding Angles Postulate to prove other theorems about pairs of angles formed when a transversal cuts parallel lines.

Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

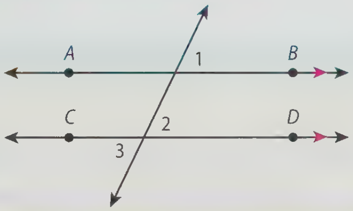
Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

In a **two-column proof** you organize the steps of a proof and their justifications in separate columns. The first step generally states the given information. Here is a two-column proof of the Alternate Exterior Angles theorem.

EXAMPLE 2

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	1. Given
2. $\angle 1 \cong \angle 2$	2. If two $\parallel$ lines are cut by a transversal, then corr. $\angle$s are $\cong$.
3. $\angle 2 \cong \angle 3$	3. Vertical angles are $\cong$.
4. $\angle 1 \cong \angle 3$	4. Transitive Property of $\cong$

SEARCH

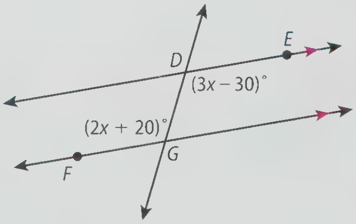
To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

The proof of the Alternate Interior Angles theorem is very similar.

EXAMPLE 3

Find $m\angle GDE$.

Alternate interior $\angle$s are $\cong$.	$3x - 30 = 2x + 20$
Add 30 to both sides.	$3x = 2x + 50$
Subtract $2x$ from both sides.	$x = 50$
Substitute 50 for x.	$m\angle GDE = (3x - 30)°$
	$= 3(50) - 30$
	$= 120$



$m\angle GDE = 120°$

Same-Side Interior Angles

Every pair of angles formed by parallel lines and a transversal is either congruent or supplementary. The next theorem involves pairs of angles that lie on the same side of a transversal and between two parallel lines. The proof of this theorem is in Example 4.

Same-Side Interior Angles Theorem

If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

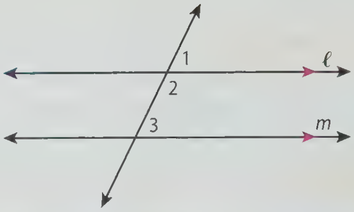
Watch Out!

When a transversal intersects two parallel lines, all the acute angles are congruent and all the obtuse angles are congruent. Every acute angle is supplementary to every obtuse angle.

EXAMPLE 4

Prove the Same-Side Interior Angles Theorem.

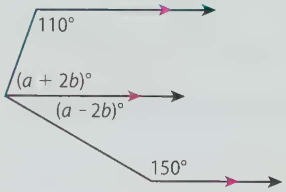
Given: $\ell \parallel m$
Prove: $\angle 2$ **and** $\angle 3$ **are supplementary.**



Statements	Reasons
1. $\ell \parallel m$	1. Given
2. $m\angle 1 + m\angle 2 = 180°$	2. If two angles form a linear pair, then they are supplementary.
3. $m\angle 1 = m\angle 3$	3. If two $\parallel$ lines are cut by a transversal, then corr. $\angle$s are $\cong$.
4. $m\angle 3 + m\angle 2 = 180°$	4. Substitution
5. $\angle 2$ and $\angle 3$ are supplementary.	5. Definition of supplementary angles

EXAMPLE 5

Find the values of *a* and *b*.



$$a + 2b + 110 = 180$$
$$a + 2b = 70 \quad \textbf{(1)}$$

$$a - 2b + 150 = 180$$
$$a - 2b = 30 \quad \textbf{(2)}$$

METHOD 1

Add Equations **(1)** and **(2)**.

$$\begin{array}{rcl} a + 2b & = & 70 \\ \underline{a - 2b} & \underline{=} & \underline{30} \\ 2a & = & 100\text{, so } a = 50 \end{array}$$

Substitute into Equation **(1)**.

$$50 + 2b = 70$$

$$2b = 20\text{, so } b = 10$$

METHOD 2

Solve Equation **(2)** for *a*, then substitute into Equation **(1)**.

$$a = 30 + 2b$$
$$(30 + 2b) + 2b = 70 \quad \textbf{(1)}$$
$$4b = 40\text{, so } b = 10$$
$$a + 2b = 70 \quad \textbf{(1)}$$
$$a + 2(10) = 70\text{, so } a = 50$$

Need More HELP

For help with solving systems of equations, go to *Solving a System by Substitution* and *Solving a System by Addition or Subtraction* in *Algebra* (Book 2, pp. 1568, 1572).

Sometimes the converse of a conditional is true. In particular, the converse of the Corresponding Angles Postulate is true.

Converse of the Corresponding Angles Postulate

If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

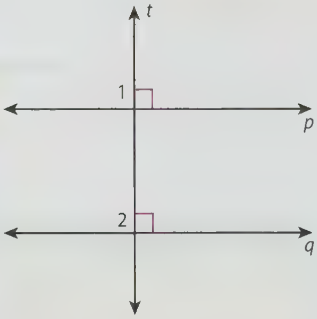
You can use this postulate to prove the following theorem.

Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel.

EXAMPLE 6

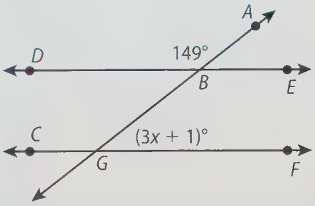
Given: Lines p and q are coplanar, $p \perp t$, and $q \perp t$.
Prove: $p \parallel q$



Statements	Reasons
1. $p \perp t$	1. Given
2. $\angle 1$ is a right angle.	2. $\perp$ lines form right $\perp$s.
3. $q \perp t$	3. Given
4. $\angle 2$ is a right angle.	4. $\perp$ lines form right $\angle$s.
5. $\angle 1 \cong \angle 2$	5. Right $\angle$s are $\cong$.
6. $p \parallel q$	6. If two lines are cut by a trans. and corr. $\angle$s are $\cong$, then the lines are $\parallel$.

EXAMPLE 7

Find the value of x that makes $\overleftrightarrow{DE} \parallel \overleftrightarrow{CF}$.



$\overleftrightarrow{DE} \parallel \overleftrightarrow{CF}$ if $\angle ABE \cong \angle BGF$.

STEP 1 Find $m\angle ABE$.

$\angle ABD$ and $\angle ABE$ form a linear pair, so they are supplementary.

$$m\angle ABD + m\angle ABE = 180^\circ$$
$$149^\circ + m\angle ABE = 180^\circ$$
$$m\angle ABE = 31^\circ$$

STEP 2 Write an equation about $m\angle ABE$ and $m\angle BGF$.

$\overleftrightarrow{DE} \parallel \overleftrightarrow{CF}$ if $\angle ABE \cong \angle BGF$, so $m\angle ABE = m\angle BGF$.

$$31^\circ = (3x + 1)^\circ$$
$$3x = 30$$
$$x = 10$$

Watch Out!

Is this theorem still true if you remove the words "in a plane"? No, the lines could be perpendicular, but they could also be skew.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Proving Lines Parallel

The converses of three theorems introduced earlier in this lesson are also true.

Converse of the Alternate Interior Angles Theorem

If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Converse of the Alternate Exterior Angles Theorem

If two lines are cut by a transversal and alternate exterior angles are congruent, then the lines are parallel.

Converse of the Same-Side Interior Angles Theorem

If two lines are cut by a transversal and same-side interior angles are supplementary, then the lines are parallel.

EXAMPLE 8

SEARCH

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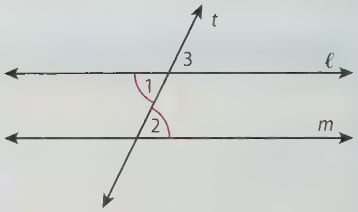
Prove the Converse of the Alternate Interior Angles Theorem.

Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$



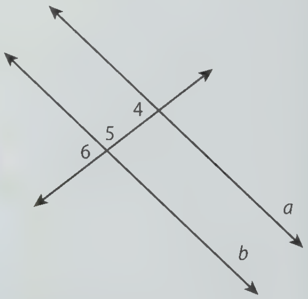
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1 \cong \angle 3$	2. Vertical $\angle$s are $\cong$.
3. $\angle 2 \cong \angle 3$	3. Substitution
4. $\ell \parallel m$	4. If two lines are cut by a trans. and corr. $\angle$s are $\cong$, the lines are $\parallel$.

The proof of the Converse of the Alternate Exterior Angles Theorem is very similar.

EXAMPLE 9

Prove the Converse of the Same-Side Interior Angles Theorem.

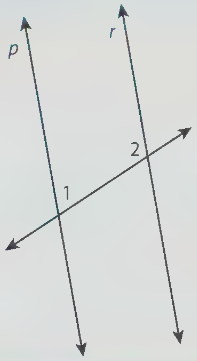
Given: $\angle 4$ **and** $\angle 5$ **are supplementary.**
Prove: $a \parallel b$



Statements	Reasons
1. $\angle 4$ and $\angle 5$ are supplementary.	1. Given
2. $\angle 5$ and $\angle 6$ are supplementary.	2. If two angles form a linear pair, then they are supplementary.
3. $\angle 4 \cong \angle 6$	3. Two $\angle$s supplementary to the same $\angle$ are $\cong$.
4. $a \parallel b$	4. If two lines are cut by a trans. and corr. $\angle$s are $\cong$, the lines are $\parallel$.

EXAMPLE 10

In the figure, $m\angle 1 = (5x + 14)°$ and $m\angle 2 = (8x - 3)°$. Find the value of x that makes $p \parallel r$.

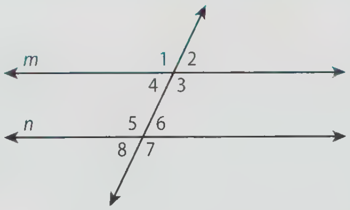


$\angle 1$ and $\angle 2$ are same-side interior angles.

$p \parallel r$ if $\angle 1$ and $\angle 2$ are supplementary.	$m\angle 1 + m\angle 2 = 180°$
Substitute.	$5x + 14 + 8x - 3 = 180$
Simplify.	$13x + 11 = 180$
Subtract 11 from both sides.	$13x = 169$
Divide both sides by 13.	$x = 13$

EXAMPLE 11

In the figure, $m\angle 2 = (6x - 9)°$ and $m\angle 8 = (-2x + 39)°$. Find the value of x that makes $m \parallel n$.



$\angle 2$ and $\angle 8$ are alternate exterior angles.

$m \parallel n$ if $\angle 2$ and $\angle 8$ are congruent.	$m\angle 2 = m\angle 8$
Substitute.	$6x - 9 = -2x + 39$
Add $2x$ to both sides.	$8x - 9 = 39$
Add 9 to both sides.	$8x = 48$
Divide both sides by 8.	$x = 6$

GOT TO KNOW!

Ways to Prove Lines Parallel

Show that:

- CORRESPONDING ANGLES are CONGRUENT.
- ALTERNATE INTERIOR ANGLES are CONGRUENT.
- ALTERNATE EXTERIOR ANGLES are CONGRUENT.
- SAME-SIDE INTERIOR ANGLES are SUPPLEMENTARY.
- Two lines are coplanar and are perpendicular to the same line.

Quadrilaterals

What Came Before?

- Classifying quadrilaterals
- Theorems about parallel lines and congruence

What's This About?

- The properties of parallelograms
- The study of special quadrilaterals
- Proving figures to be parallelograms and other special quadrilaterals

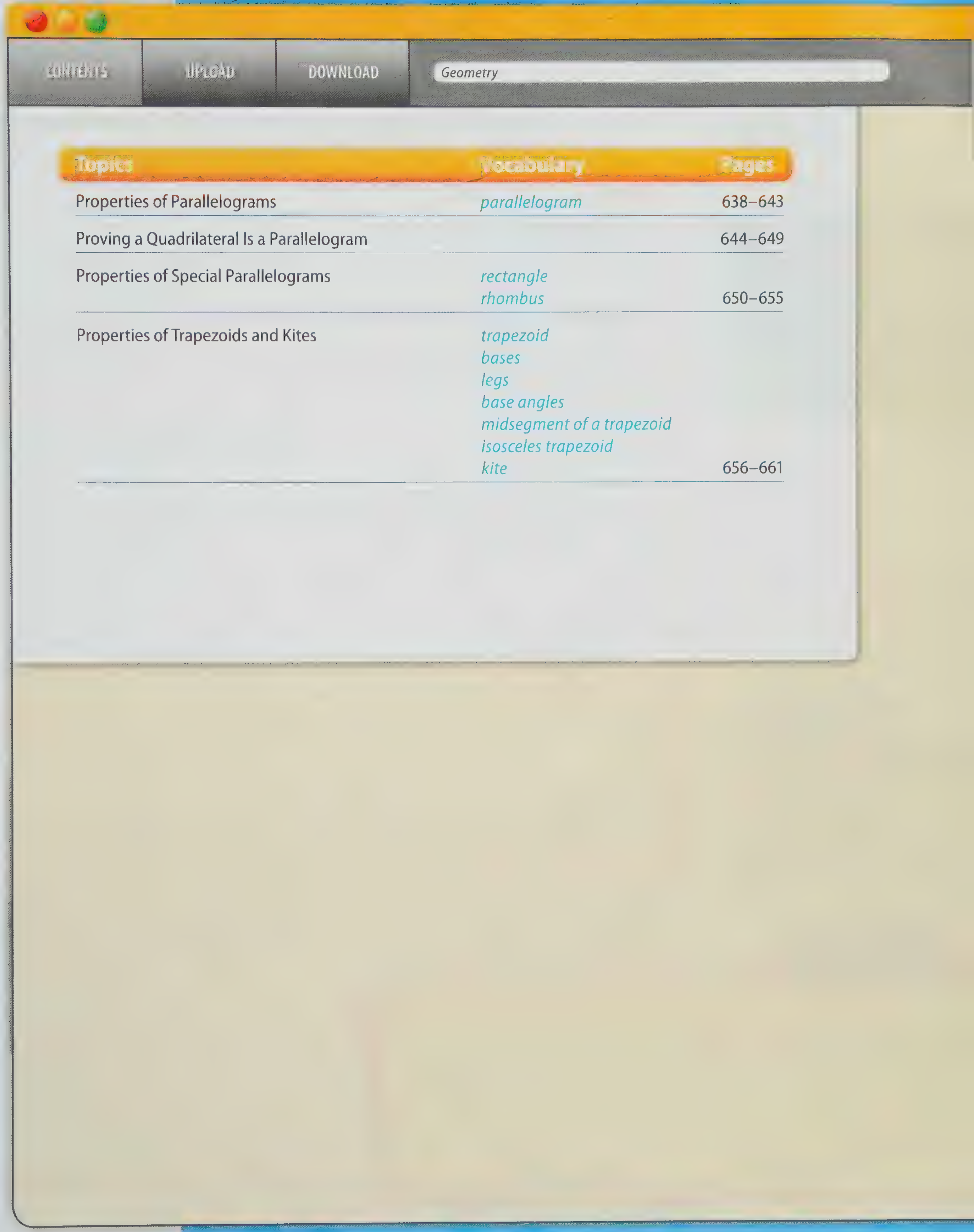
Practical Apps

- The Romans used Golden Rectangles to build the Parthenon.
- Artists use the properties of quadrilaterals in sectioning their paintings.

Just For FUN!

Q: Why is the obtuse angle always upset?

A: Because he's never right.



Topics	Vocabulary	Pages
Properties of Parallelograms	*parallelogram*	638–643
Proving a Quadrilateral Is a Parallelogram		644–649
Properties of Special Parallelograms	*rectangle* *rhombus*	650–655
Properties of Trapezoids and Kites	*trapezoid* *bases* *legs* *base angles* *midsegment of a trapezoid* *isosceles trapezoid* *kite*	656–661

Properties of Parallelograms

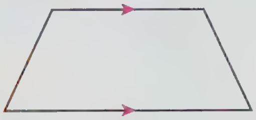
Opposite Sides of a Parallelogram

You know that a quadrilateral is a polygon with four sides. A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.

EXAMPLE 1

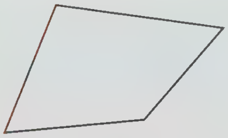
Is the polygon a parallelogram? Explain.

a.



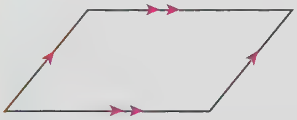
This quadrilateral has only one pair of parallel opposite sides, not two. It is not a parallelogram.

b.



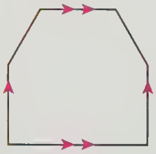
This quadrilateral does not have any parallel sides. It is not a parallelogram.

c.



This quadrilateral has two pairs of parallel opposite sides. It is a parallelogram.

d.



This polygon has two pairs of parallel opposite sides, but it is not a quadrilateral, so it is not a parallelogram.

Need More HELP

To review parallelograms, go to *Classifying Quadrilaterals* in *Foundations of Mathematics* (p. 158).

SEARCH

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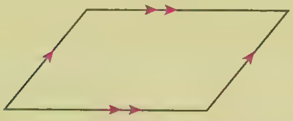
Several other properties follow from the definition of a parallelogram. One of these is that opposite sides are congruent.

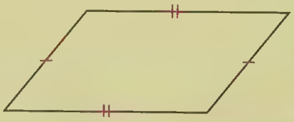
GOT TO KNOW!

Sides of a Parallelogram

Opposite sides are parallel.



Opposite sides are congruent.

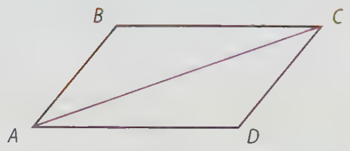


EXAMPLE 2

Prove that opposite angles of a parallelogram are congruent.

Given: $ABCD$ is a parallelogram. $\overline{AC}$ is a diagonal.

Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$.



By definition of a parallelogram, $\overline{AB} \parallel \overline{CD}$. If two parallel lines are cut by a transversal, then alternate interior angles are congruent. So $\angle BAC \cong \angle DCA$ and $\angle BCA \cong \angle DAC$.

$AC = AC$ by the Reflexive Property, so $\triangle ABC \cong \triangle CDA$ by Angle-Side-Angle (ASA).

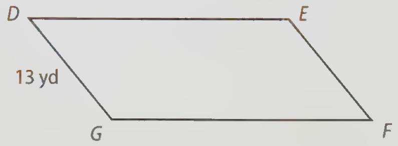
Since corresponding parts of congruent triangles are congruent, it follows that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$.

Try It This Way

Draw a parallelogram. Cut along a diagonal. Match up the triangles. Which sides of the parallelogram are congruent? Which angles are congruent?

EXAMPLE 3

In parallelogram $DEFG$, $\overline{DE}$ is twice as long as $\overline{GD}$. Find the perimeter of the parallelogram.



$\overline{EF} \cong \overline{GD}$, so $\overline{EF}$ is 13 yards long.

$\overline{DE}$ is twice the length of $\overline{GD}$, so $\overline{DE}$ is $2 \times 13 = 26$ yards long.

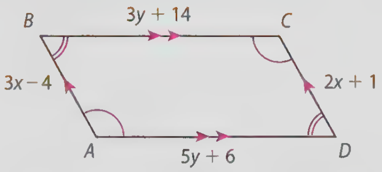
$\overline{GF} \cong \overline{DE}$, so $\overline{GF}$ is 26 yards long.

$$\begin{aligned} \text{perimeter} &= DE + EF + FG + GD \\ &= 26 + 13 + 26 + 13 \\ &= 78 \end{aligned}$$

The perimeter is 78 yd.

EXAMPLE 4

***ABCD* is a parallelogram. Find its perimeter.**



STEP 1 Write two equations. Use the fact that the opposite sides are congruent.

$3x - 4 = 2x + 1$ and $3y + 14 = 5y + 6$

STEP 2 Solve both equations.

$x = 5$ and $y = 4$

STEP 3 Substitute to find the lengths of the sides.

$AB = 3x - 4 = 3(5) - 4 = 11$

$CD = 2x + 1 = 2(5) + 1 = 11$

$BC = 3y + 14 = 3(4) + 14 = 26$

$AD = 5y + 6 = 5(4) + 6 = 26$

STEP 4 Find the perimeter.

$$\begin{aligned} \text{perimeter} &= AB + BC + CD + AD \\ &= 11 + 26 + 11 + 26 = 74 \end{aligned}$$

The perimeter of parallelogram *ABCD* is 74 units.

Need More HELP

For a review of finding the perimeter, go to *Perimeter* in *Measurement* (p. 524).

Angles of a Parallelogram

The angles in a parallelogram have two important properties. The first is that opposite angles are congruent.

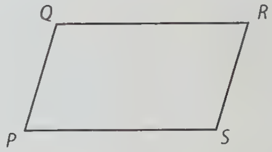
EXAMPLE 5

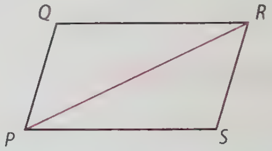
Prove that opposite angles of a parallelogram are congruent.

Given: *PQRS* is a parallelogram.

Prove: $\angle P \cong \angle R$ and $\angle Q \cong \angle S$

In Example 2 you saw that when you drew the diagonal of a parallelogram two congruent triangles were formed. If you draw $\overline{PR}$, you can prove that $\triangle PQR \cong \triangle RSP$ and therefore $\angle Q \cong \angle S$ by CPCTC. Similarly, if you draw $\overline{QS}$, you can prove that $\angle P \cong \angle R$.





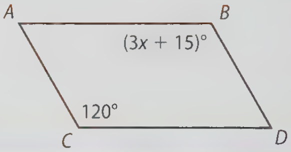
EXAMPLE 6

***ABCD* is a parallelogram. Find the value of *x*.**

Opposite angles are congruent, so $3x + 15 = 120$.

Solve for x. $\quad 3x = 105$

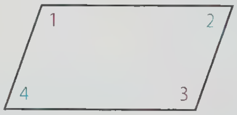
$x = 35$



Watch Out!

Do not confuse opposite angles with consecutive angles, which are angles that share a side.

The second property of angles in a parallelogram is that consecutive angles are supplementary. This makes sense when you realize that consecutive angles are same-side interior angles between parallel lines.

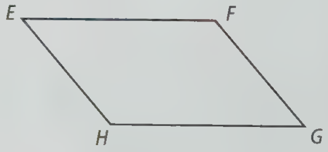


supplementary pairs: $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 4$ and $\angle 1$

EXAMPLE 7

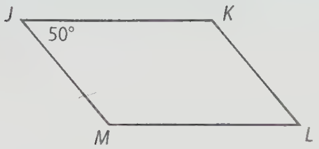
***EFGH* is a parallelogram. What can you conclude?**

Opposite sides are parallel.	$\overline{EF} \parallel \overline{GH}$ and $\overline{EH} \parallel \overline{FG}$
Opposite sides are ≅.	$\overline{EF} \cong \overline{GH}$ and $\overline{EH} \cong \overline{FG}$
Opposite angles are ≅.	$\angle E \cong \angle G$ and $\angle F \cong \angle H$
Consecutive angles are supplementary.	$\angle E$ and $\angle F$ are supplementary. $\angle F$ and $\angle G$ are supplementary.
$\angle G$ and $\angle H$ are supplementary.	$\angle H$ and $\angle E$ are supplementary.



EXAMPLE 8

Find the missing angle measures in parallelogram *JKLM*.



METHOD 1

$\angle J$ and $\angle K$ are supplementary, so $50° + m\angle K = 180°$.

$$m\angle K = 130°$$

$\angle K$ and $\angle L$ are supplementary, so $130° + m\angle L = 180°$.

$$m\angle L = 50°$$

$\angle L$ and $\angle M$ are supplementary, so $50° + m\angle M = 180°$.

$$m\angle M = 130°$$

METHOD 2

Opposite $\angle$s are congruent.	$\angle L \cong \angle J$, so $m\angle L = 50°$.
Consecutive $\angle$s are supplementary.	$\angle K$ and $\angle L$ are supplementary, so $m\angle K + 50° = 180°$. $m\angle K = 130°$
Opposite $\angle$s are congruent.	$\angle K \cong \angle M$, so $m\angle M = 130°$.

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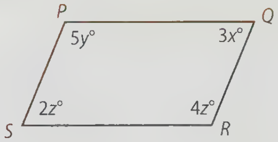
EXAMPLE 9

Find the values of *x*, *y*, and *z* in parallelogram *PQRS*.



STEP 1 Write an equation that uses like terms only.

Consecutive $\angle$s are supplementary. $4z + 2z = 180$

STEP 2 Solve the equation.

Combine like terms. $6z = 180$

Divide both sides by 6. $z = 30$

STEP 3 Write equations to find the values of the other variables.

Opposite angles are congruent. $3x = 2z$ $5y = 4z$

STEP 4 Substitute for z and simplify.

$3x = 2(30)$ $5y = 4(30)$

$3x = 60$ $5y = 120$

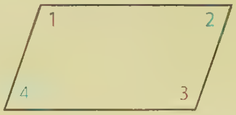
STEP 5 Solve for x and y.

Divide by 3, and divide by 5. $x = 20$ $y = 24$

So, $x = 20$, $y = 24$, and $z = 30$.

GOT TO KNOW!

Angles in a Parallelogram



Opposite angles are congruent.

$\angle 1 \cong \angle 3$

$\angle 2 \cong \angle 4$

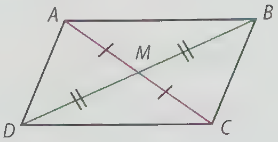
Consecutive angles are supplementary.

$\angle 1$ and $\angle 2$ $\angle 3$ and $\angle 4$

$\angle 2$ and $\angle 3$ $\angle 4$ and $\angle 1$

Diagonals of a Parallelogram

The diagonals of a parallelogram bisect each other.

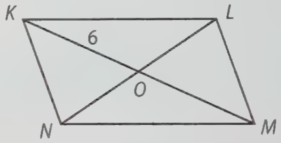


$\overline{AM} \cong \overline{CM}$ $\overline{BM} \cong \overline{DM}$

Sometimes you can use this property to find unknown lengths in a parallelogram.

EXAMPLE 10

In parallelogram *KLMN*, *KO* = 6 and *LN* = 20. Find *OM*, *KM*, *NO*, and *LO*.



The diagonals of a parallelogram bisect each other, so $KO = OM = \frac{1}{2}KM$. Therefore, $OM = 6$ and $KM = 12$.

Similarly, $NO = LO = \frac{1}{2}LN$. Since $LN = 20$, $NO = 10$ and $LO = 10$.

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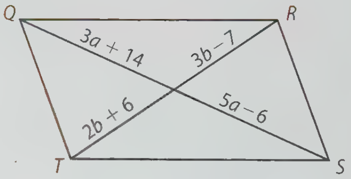
EXAMPLE 11

***QRST* is a parallelogram. Find *QS* and *RT*.**



STEP 1 Use the fact that the diagonals of a parallelogram bisect each other to write two equations. Then solve the equations.

(1) $5a - 6 = 3a + 14$
$2a = 20$
$a = 10$

(2) $3b - 7 = 2b + 6$
$b = 13$

STEP 2 Substitute for a and b.

$$\begin{aligned} QS &= (5a - 6) + (3a + 14) \\ &= 5(10) - 6 + 3(10) + 14 \\ &= 88 \end{aligned}$$

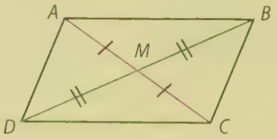
$$\begin{aligned} RT &= (3b - 7) + (2b + 6) \\ &= 3(13) - 7 + 2(13) + 6 \\ &= 64 \end{aligned}$$

$QS = 88$, $RT = 64$

GOT TO KNOW!

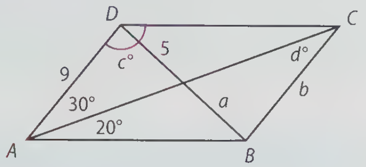
Diagonals of a Parallelogram

The diagonals of a parallelogram bisect each other.



EXAMPLE 12

ABCD is a parallelogram. Find the values of a, b, c, and d.



Diagonals bisect each other. $a = 5$

Opposite sides are congruent. $b = 9$

Consecutive ∠s are supplementary. $(30° + 20°) + c° = 180°$

$c = 130$

Alternate interior ∠s are congruent. $\angle ACB \cong \angle DAC$, so $d = 30$.

So, $a = 5$, $b = 9$, $c = 130$, and $d = 30$.

Watch Out !

Read carefully what is given so you know how to answer.

In the figure, the label for the angle measure is $c°$. The value of c is only a number, 130, because the degree symbol is given.

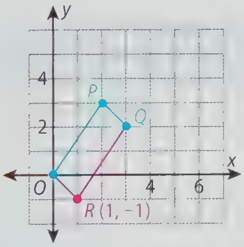
If the figure showed just c for the angle measure, your answer would be 130°.

EXAMPLE 13

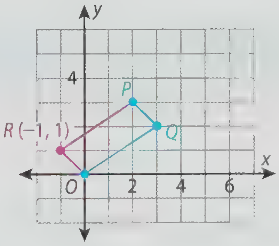
A parallelogram has vertices at $O(0, 0)$, $P(2, 3)$, $Q(3, 2)$, and R. Find three possible sets of coordinates for vertex R.

There are three cases to consider. Draw a sketch for each possibility. Then use slope and the fact that opposite sides are parallel to locate R.

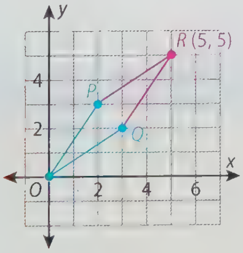
Case 1 Consider parallelogram *OPQR*.



Case 2 Consider parallelogram *OQPR*.



Case 3 Consider parallelogram *OQRP*.



The three possible sets of coordinates for vertex R are $(1, -1)$, $(-1, 1)$, and $(5, 5)$.

Need More HELP ?

For help with slope, go to *Rate of Change and Slope* in *Algebra* (Book 2, p. 1536).

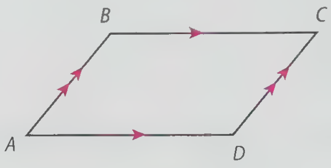
Proving a Quadrilateral Is a Parallelogram

Using Sides

There are three ways to use the sides of a quadrilateral to prove that the quadrilateral is a parallelogram. The first uses the definition of a parallelogram.

If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.

ABCD is a parallelogram.



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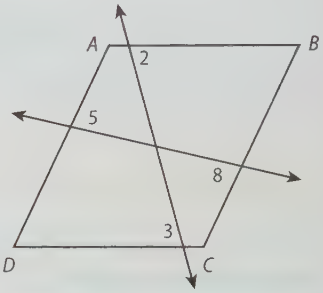
EXAMPLE 1

Given: $\angle 2 \cong \angle 3$ and $\angle 5 \cong \angle 8$
Prove: *ABCD* is a parallelogram.

$\angle 2$ and $\angle 3$ are congruent alternate interior angles formed by the transversal that cuts $\overline{AB}$ and $\overline{CD}$, so $\overline{AB} \parallel \overline{CD}$. $\angle 5$ and $\angle 8$ are congruent alternate interior angles formed by the transversal that cuts $\overline{AD}$ and $\overline{BC}$, so $\overline{AD} \parallel \overline{BC}$. Both pairs of opposite sides are parallel, so *ABCD* is a parallelogram by definition of a parallelogram.



GOT TO KNOW!

Proving Triangles Congruent

Triangles are congruent if:

SSS: three sides of one triangle are congruent to the corresponding three sides of another triangle.

SAS: two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle.

ASA: two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle.

CPCTC: Corresponding Parts of Congruent Triangles are Congruent.

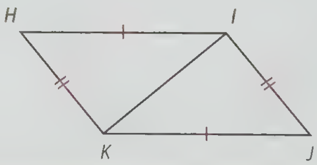
The second way to prove that a quadrilateral is a parallelogram is to show that both pairs of opposite sides are congruent.

Theorem
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

PROOF

Given: $\overline{HI} \cong \overline{JK}$ and $\overline{HK} \cong \overline{JI}$

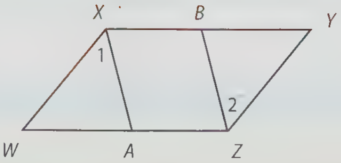
Prove: *HIJK* is a parallelogram.



Statements	Reasons
1. $\overline{HI} \cong \overline{JK}$, $\overline{HK} \cong \overline{JI}$	1. Given
2. $\overline{KI} \cong \overline{IK}$	2. Reflexive property of congruency
3. $\triangle IHK \cong \triangle KJI$	3. SSS
4. $\angle HIK \cong \angle JKI$, $\angle HKI \cong \angle JIK$	4. CPCTC
5. $\overline{HI} \parallel \overline{JK}$, $\overline{HK} \parallel \overline{JI}$	5. If two lines are cut by a transversal and alternate interior $\angle$s are $\cong$, then the lines are $\parallel$.
6. *HIJK* is a parallelogram.	6. Definition of a parallelogram

EXAMPLE 2

Given: *WXYZ* is a parallelogram, $\angle 1 \cong \angle 2$.
Prove: *AXBZ* is a parallelogram.



Statements	Reasons
1. *WXYZ* is a parallelogram; $\angle 1 \cong \angle 2$	1. Given
2. $\angle W \cong \angle Y$	2. Opposite $\angle$s of a parallelogram are $\cong$.
3. $WX = YZ$	3. Opposite sides of a parallelogram are $\cong$.
4. $\triangle WXA \cong \triangle YZB$	4. ASA
5. $AX = BZ$	5. CPCTC
6. $WZ = XY$	6. Opposite sides of a parallelogram are $\cong$.
7. $WA = YB$	7. CPCTC
8. $AZ = XB$	8. Subtraction property of $=$
9. *AXBZ* is a parallelogram.	9. If both pairs of opposite sides of a quadrilateral are $\cong$, then the quadrilateral is a parallelogram.

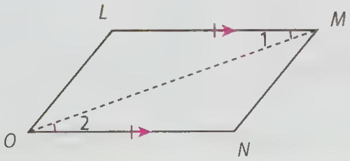
The third method requires information about only one pair of sides.

Theorem
If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

PROOF

Given: $\overline{LM} \cong \overline{NO}$ and $\overline{LM} \parallel \overline{NO}$

Prove: *LMNO* is a parallelogram.



Statements	Reasons
1. $\overline{LM} \cong \overline{NO}$, and $\overline{LM} \parallel \overline{NO}$	1. Given
2. $\angle 1 \cong \angle 2$	2. If two $\parallel$ lines are cut by a transversal, then alternate interior $\angle$s are $\cong$.
3. $\overline{OM} \cong \overline{MO}$	3. Reflexive property of $\cong$
4. $\triangle LMO \cong \triangle NOM$	4. SAS
5. $\overline{LO} \cong \overline{NM}$	5. CPCTC
6. *LMNO* is a parallelogram.	6. If both pairs of opposite sides of a quadrilateral are $\cong$, the quadrilateral is a parallelogram.

Need More HELP

For help with proving triangles congruent, go to *Using SSS and SAS* and *Using ASA and AAS* on pages 768 and 772.

Watch Out!

Is this theorem still true if you take out the word *both*? No.

GOT TO KNOW!

Using Sides to Prove a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel.
- Show that both pairs of opposite sides are congruent.
- Show that two sides are both congruent and parallel.

Two More Ways

Showing that both pairs of opposite angles in a quadrilateral are congruent is sufficient to prove that a quadrilateral is a parallelogram.

Theorem
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

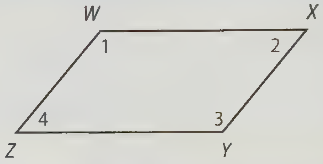
Need More HELP

Remember!
The sum of the measures of the angles of a quadrilateral is 360°.

PROOF

Given: $\angle 1 \cong \angle 3$ **and** $\angle 2 \cong \angle 4$

Prove: ***WXYZ*** **is a parallelogram.**

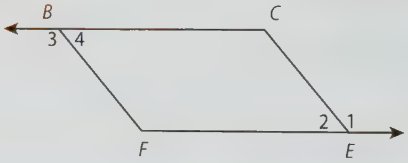


Statements	Reasons
1. $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$	1. Given
2. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$	2. Sum of measures of $\angle$s of a quad. is 360°.
3. $m\angle 1 + m\angle 2 + m\angle 1 + m\angle 2 = 360°$	3. Substitution
4. $2(m\angle 1 + m\angle 2) = 360°$	4. Distributive property
5. $m\angle 1 + m\angle 2 = 180°$	5. Division property of =
6. $m\angle 1 + m\angle 4 = 180°$	6. Substitution
7. $\overline{WX} \parallel \overline{ZY}, \overline{WZ} \parallel \overline{XY}$	7. If two lines are cut by a transversal so that same-side int. $\angle$s are supp., then the lines are $\parallel$.
8. *WXYZ* is a parallelogram.	8. Definition of a parallelogram

EXAMPLE 3

Given: $\angle 1 \cong \angle 3, \angle F \cong \angle C$

Prove: ***BCEF*** **is a parallelogram.**



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Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary; $\angle 3$ and $\angle 4$ are supplementary.	1. If two angles form a linear pair, then they are supplementary.
2. $\angle 1 \cong \angle 3$	2. Given
3. $\angle 2 \cong \angle 4$	3. Supplements of $\cong$ $\angle$s are $\cong$.
4. $\angle F \cong \angle C$	4. Given
5. *BCEF* is a parallelogram.	5. If both pairs of opposite $\angle$s of a quadrilateral are $\cong$, then the quadrilateral is a parallelogram.

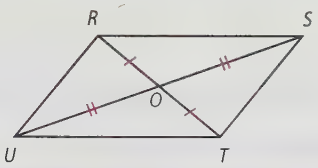
If you can show the diagonals of a quadrilateral bisect each other, then you can prove the quadrilateral is a parallelogram.

Theorem
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

PROOF

Given: $\overline{RT}$ **and** $\overline{SU}$ **bisect each other.**

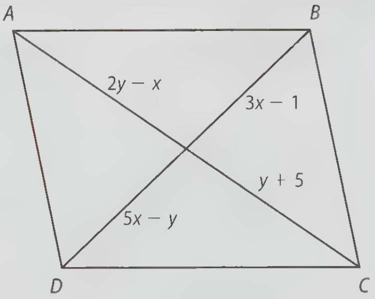
Prove: ***RSTU*** **is a parallelogram.**



Statements	Reasons
1. $\overline{RT}$ and $\overline{SU}$ bisect each other.	1. Given
2. $\overline{RO} \cong \overline{TO}, \overline{UO} \cong \overline{SO}$	2. Definition of bisect
3. $\angle ROS \cong \angle TOU$	3. Vertical $\angle$s are $\cong$.
4. $\triangle ROS \cong \triangle TOU$	4. SAS
5. $\angle RSO \cong \angle TUO, RS = TU$	5. CPCTC
6. $\overline{RS} \parallel \overline{TU}$	6. If two lines are cut by a transversal so that alternate interior $\angle$s are $\cong$, then the lines are $\parallel$.
7. *RSTU* is a parallelogram.	7. If two sides of a quadrilateral are both $\cong$ and $\parallel$, then the quadrilateral is a parallelogram.

EXAMPLE 4

Find the values of *x* and *y* for which *ABCD* must be a parallelogram.



For *ABCD* to be a parallelogram, the diagonals must bisect each other. So,

$2y - x = y + 5$ and $5x - y = 3x - 1$

$y = x + 5$ $\qquad 2x = y - 1$

Substitute $x + 5$ for y. $\qquad 2x = (x + 5) - 1$

$x = 4$

Substitute 4 for x in first equation.

$2y - x = y + 5$

$2y - 4 = y + 5$

$y = 9$

$x = 4$ and $y = 9$

Try It This Way

Before you read the proof of this theorem, write your own proof. There is more than one way to prove this theorem.

GOT TO KNOW!

Ways to Prove a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel.
- Show that both pairs of opposite sides are congruent.
- Show that two sides are both congruent and parallel.
- Show that opposite angles are congruent.
- Show that the diagonals bisect each other.

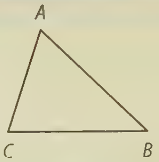
Triangle Inequalities

Inequalities in One Triangle

There are two inequalities that relate the angles and the sides of a triangle.

Theorem

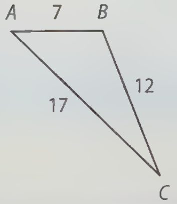
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



If $AB > AC$, then $m\angle C > m\angle B$.

EXAMPLE 1

Order the angles from smallest to largest.



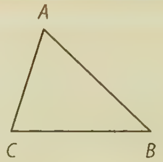
The smallest angle, $\angle C$, is opposite the shortest side, $\overline{AB}$.

The largest angle, $\angle B$, is opposite the longest side, $\overline{AC}$.

From smallest to largest, the angles are $\angle C$, $\angle A$, $\angle B$.

Theorem

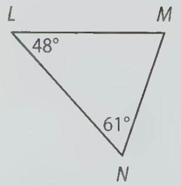
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



If $m\angle A > m\angle C$, then $BC > AB$.

EXAMPLE 2

Name the longest segment in each diagram.

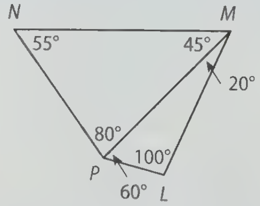


a. **STEP 1** Find $m\angle M$.

$48 + 61 + m\angle M = 180$; $m\angle M = 71°$

STEP 2 Identify the longest side of the triangle.

The longest side lies opposite the largest angle. $\angle M$ is the largest angle, so $\overline{LN}$ is the longest side.



b. **STEP 1** Order the sides in $\triangle MNP$.

$MN > MP > NP$

STEP 2 Order the sides in $\triangle PLM$.

$MP > LM > PL$

STEP 3 Compare the inequalities.

The longest side of $\triangle PLM$ is $\overline{MP}$, but $MN > MP$, so $\overline{MN}$ is the longest segment.

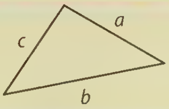
Watch Out!

When a figure is made up of more than one triangle, the longest segment in the figure may not be opposite the largest angle.

This inequality involves the lengths of the three sides of a triangle.

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.



$a + b > c$
$b + c > a$
$a + c > b$

Try It This Way

Here is another way to state the **Triangle Inequality Theorem:**

The shortest distance between two points is along a straight line.

EXAMPLE 3

Determine whether each list could represent side lengths of a triangle.

a. 5, 8, 15

$5 + 8 = 13, 13 \not> 15$

The sum of two side lengths is not greater than the third, so these cannot be the sides of a triangle.

b. 11, 12, 19

$11 + 12 = 23$ $\quad 11 + 19 = 30$ $\quad 12 + 19 = 31$

$23 > 19$ $\quad 30 > 12$ $\quad 31 > 11$

The sum of the lengths of any two sides is always greater than the third, so these can be the sides of a triangle.

c. 18, 12, 6

$12 + 6 = 18$

The sum of two side lengths is equal to but not greater than the third, so these cannot be the sides of a triangle.

EXAMPLE 4

Francisco is building a triangular frame. Two sides are 8 in. and 12 in. long. What are possible lengths for the third side?

Use the Triangle Inequality Theorem to write three inequalities.

$x + 8 > 12 \rightarrow x > 4$

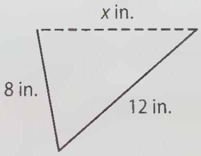
$x + 12 > 8 \rightarrow x > -4$

This is always true because x represents a positive length.

$8 + 12 > x \rightarrow 20 > x$

Combine the inequalities: $4 < x < 20$

The third side must be longer than 4 in. and shorter than 20 in.



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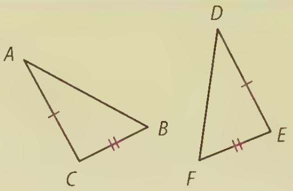
Inequalities in Two Triangles

There are also inequalities that relate the sides and angles of two triangles to each other.

Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, but their included angles are not congruent, then the side opposite the larger included angle is longer than the side opposite the smaller included angle.

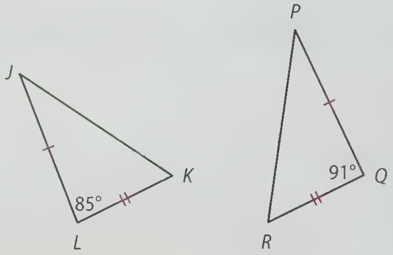


If $m\angle C > m\angle E$, then $AB > DF$.

EXAMPLE 5

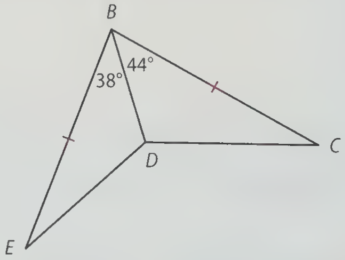
a. Which side is longer, $\overline{JK}$ or $\overline{PR}$?

The triangles have two pairs of congruent sides, so compare the included angles: $m\angle Q > m\angle L$. $\overline{PR}$ is opposite $\angle Q$, so $\overline{PR}$ is longer than $\overline{JK}$.



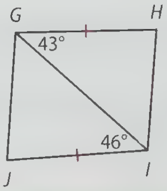
b. Which side is longer, $\overline{DC}$ or $\overline{DE}$?

The triangles have two pairs of congruent sides, so compare the included angles: $m\angle CBD > m\angle EBD$. $\overline{DC}$ is opposite $\angle CBD$, so $\overline{DC}$ is longer than $\overline{DE}$.



c. Which side is shorter, $\overline{JG}$ or $\overline{HI}$?

The triangles have two pairs of congruent sides, so compare the included angles: $m\angle HGI < m\angle GIJ$. $\overline{HI}$ is opposite $\angle HGI$, so $\overline{HI}$ is shorter than $\overline{JG}$.



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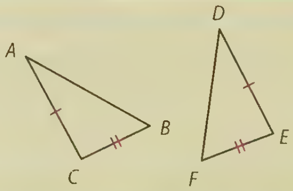
Converse of Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, but the third sides are not congruent, then the angle opposite the longer side is larger than the angle opposite the shorter side.

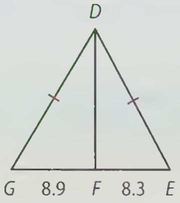


If $AB > DF$, then $m\angle C > m\angle E$.

EXAMPLE 6

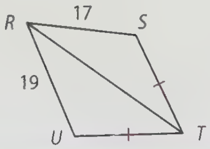
a. Which angle is larger, $\angle EDF$ or $\angle GDF$?

The triangles have two pairs of congruent sides, so compare the non-congruent sides: $GF > EF$. $\angle GDF$ is opposite $\overline{GF}$, so $\angle GDF$ is larger than $\angle EDF$.



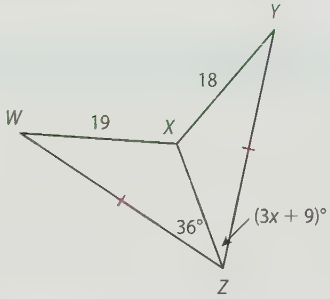
b. Which angle is smaller, $\angle STR$ or $\angle UTR$?

The triangles have two pairs of congruent sides, so compare the non-congruent sides: $SR < UR$. $\angle STR$ is opposite $\overline{SR}$, so $\angle STR$ is smaller than $\angle UTR$.



EXAMPLE 7

Find the range of values for x.



STEP 1 Compare $\angle XZY$ and $\angle XZW$.

The triangles have two pairs of congruent sides, so compare the non-congruent sides: $XY < WX$. $\angle XZY$ is opposite $\overline{XY}$, so $m\angle XZY < m\angle XZW$.

STEP 2 Write and solve an inequality based on your conclusion about the angle measures.

$m\angle XZY < m\angle XZW$

$3x + 9 < 36, x < 9$

STEP 3 $m\angle XZY$ must be positive. Write and solve a second inequality.

$3x + 9 > 0, x > -3$

STEP 4 Combine the inequalities: $-3 < x < 9$

Need More HELP

For help with solving inequalities, go to *Two-Step Inequalities* in *Algebra* (Book 2, p. 1502).

GOT TO KNOW!

Inequalities in One and Two Triangles

In $\triangle ABC$, if $AB > BC$, then $m\angle C > m\angle A$.
If $m\angle C > m\angle A$, then $AB > BC$.

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

If two sides of one triangle are congruent to two sides of another triangle, but their included angles are not congruent, then the side opposite the larger included angle is longer than the side opposite the smaller included angle.

If two sides of one triangle are congruent to two sides of another triangle, but the third sides are not congruent, then the angle opposite the longer side is larger than the angle opposite the shorter side.

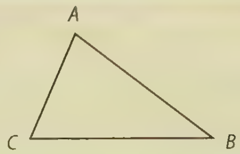
Triangle Angle-Sum Theorem

Angles of a Triangle

Regardless of the triangle, the sum of the angle measures is 180°. You will find this theorem very useful.

Triangle Angle-Sum Theorem

The sum of the angle measures of a triangle is 180°.



$m\angle A + m\angle B + m\angle C = 180°$

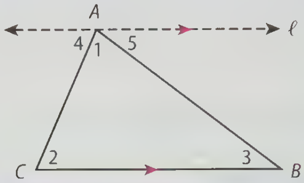
Try It This Way

Draw and cut out a paper triangle. Number the angles. Then tear off the corners and rearrange them to form a straight angle.

PROOF

Given: $\triangle ABC$ **with line** $\ell \parallel \overline{BC}$

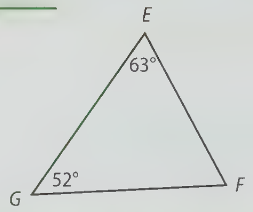
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statements	Reasons
1. $\ell \parallel \overline{BC}$	1. Given
2. $m\angle 4 = m\angle 2$, $m\angle 5 = m\angle 3$	2. If two $\parallel$ lines are cut by a trans., alt. int. $\angle$s are $\cong$.
3. $m\angle 1 + m\angle 4 + m\angle 5 = 180°$	3. Def. straight angle, Angle Addition Postulate
4. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	4. Substitution

EXAMPLE 1

Find $m\angle F$.



$$m\angle E + m\angle F + m\angle G = 180°$$
$$63° + m\angle F + 52° = 180°$$
$$m\angle F = 65°$$

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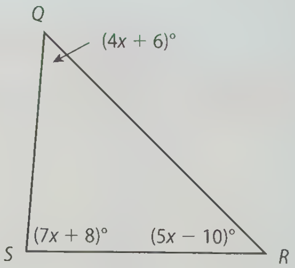
EXAMPLE 2

Find the measure of each angle in $\triangle QRS$.



Triangle Angle-Sum Theorem	$m\angle Q + m\angle R + m\angle S = 180°$
Substitute.	$(4x + 6) + (5x - 10) + (7x + 8) = 180$
Combine like terms.	$16x + 4 = 180$
Solve for x.	$16x = 176$
	$x = 11$
Substitute 11 for x.	$m\angle Q = 4(11) + 6 = 50°$
	$m\angle R = 5(11) - 10 = 45°$
	$m\angle S = 7(11) + 8 = 85°$

A **corollary** is a statement whose proof follows directly from another theorem.

Corollary
The acute angles of a right triangle are complementary.

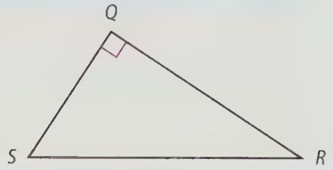
Watch Out!

This is only true for right triangles.

PROOF

Given: $\angle Q$ is a right angle.

Prove: $\angle R$ and $\angle S$ are complementary.



Statements	Reasons
1. $\angle Q$ is a right angle.	1. Given
2. $m\angle Q = 90°$	2. Definition of right $\angle$
3. $m\angle Q + m\angle R + m\angle S = 180°$	3. Sum of measures of $\angle$s in a triangle is 180°.
4. $90° + m\angle R + m\angle S = 180°$	4. Substitution
5. $m\angle R + m\angle S = 90°$	5. Subtraction Property of Equality
6. $\angle R$ and $\angle S$ are complementary.	6. Definition of complementary $\angle$s

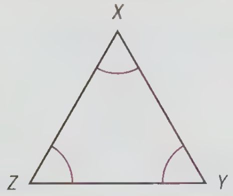
All the angles of an **equiangular triangle** are congruent.

Corollary
The measure of each angle of an equiangular triangle is 60°.

PROOF

Given: $\triangle XYZ$ is equiangular.

Prove: $m\angle X = m\angle Y = m\angle Z = 60°$



Statements	Reasons
1. $\triangle XYZ$ is equiangular.	1. Given
2. $m\angle X = m\angle Y = m\angle Z$	2. Definition of an equiangular triangle
3. $m\angle X + m\angle Y + m\angle Z = 180°$	3. Sum of measures of $\angle$s in a triangle is 180°.
4. $3(m\angle X) = 180°$	4. Substitution
5. $m\angle X = 60°$	5. Division Property of Equality
6. $m\angle X = m\angle Y = m\angle Z = 60°$	6. Substitution

GOT TO KNOW!

Triangle Angle Relationships

The sum of the angle measures of a triangle is 180°.

The acute angles of a right triangle are complementary.

The measure of each angle of an equiangular triangle is 60°.

Pythagorean Theorem

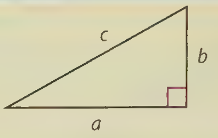
The Pythagorean Theorem

One of the most famous and most widely used theorems of geometry is the Pythagorean Theorem. Pythagoras was a Greek mathematician and philosopher who was born around 540 B.C. and who is credited with the first proof of this theorem.

In a right triangle, the **hypotenuse** is the side opposite the right angle and is the longest side. The **legs** are the two shorter sides that form the right angle.

Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

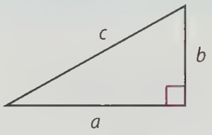


$a^2 + b^2 = c^2$

There are now over 300 proofs of the Pythagorean Theorem. The following proof uses area.

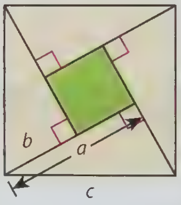
PROOF of Pythagorean Theorem

Given: a right triangle with legs of lengths *a* and *b* and hypotenuse of length *c*



Prove: $a^2 + b^2 = c^2$

The figure shows four congruent right triangles. They determine two squares. Each side of the **larger** square has length c. Each side of the **smaller** square has length $a - b$. The area of the larger square equals the sum of the areas of the four triangles plus the area of the smaller square.



area of larger square		4 × area of triangle		area of smaller square
c^2	$=$	$4 \times \left(\frac{1}{2}ab\right)$	$+$	$(a - b)^2$
c^2	$=$	$2ab$	$+$	$a^2 - 2ab + b^2$

$c^2 = a^2 + b^2$

EXAMPLE 1

Find the length of the hypotenuse of a right triangle with legs of lengths 5 and 12.

Let c = length of the hypotenuse.

$c^2 = a^2 + b^2$

$c^2 = 5^2 + 12^2$

$c^2 = 25 + 144$

$c^2 = 169$

$c = 13$

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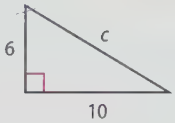
EXAMPLE 2

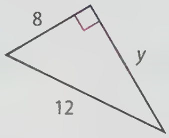
Find the value of the variable. Leave your answer in simplified radical form.

a. value of *c*



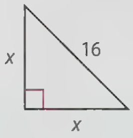
Use the Pythagorean Theorem.	$c^2 = 6^2 + 10^2$
Simplify.	$c^2 = 36 + 100$
Solve.	$c^2 = 136$
	$c = \sqrt{136}$
Simplify the radical.	$c = \sqrt{4 \cdot 34} = \sqrt{4} \cdot \sqrt{34}$
	$c = 2\sqrt{34}$

b. value of *y*



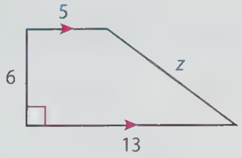
Use the Pythagorean Theorem.	$8^2 + y^2 = 12^2$
Simplify.	$64 + y^2 = 144$
Solve.	$y^2 = 80$
Simplify the radical.	$y = \sqrt{80} = \sqrt{16 \cdot 5}$
	$y = 4\sqrt{5}$

c. value of *x*

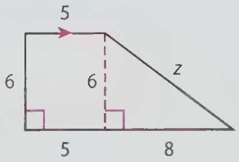


Use the Pythagorean Theorem.	$x^2 + x^2 = 16^2$
Simplify.	$2x^2 = 256$
Solve.	$x^2 = 128$
Simplify the radical.	$x = \sqrt{128} = \sqrt{64 \cdot 2}$
	$x = 8\sqrt{2}$

d. value of *z*



STEP 1 The figure is a trapezoid. Draw an altitude as shown, forming a 6-by-5 rectangle and a right triangle.



STEP 2 Find the lengths of the legs of the right triangle. The legs are 6 and $13 - 5 = 8$.

STEP 3 Use the Pythagorean Theorem.

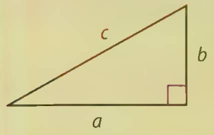
$$6^2 + 8^2 = z^2$$
$$36 + 64 = z^2$$
$$100 = z^2$$
$$z = 10$$

Need More HELP

For help with simplifying radicals, go to *Square Roots* in *Algebra* (Book 2, p. 1400).

GOT TO KNOW!

Pythagorean Theorem

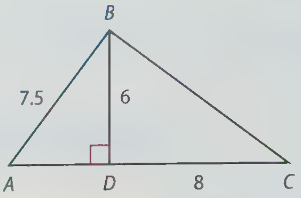


$a^2 + b^2 = c^2$

Applying the Pythagorean Theorem

EXAMPLE 3

Find the perimeter of $\triangle ABC$.



The perimeter of $\triangle ABC = AB + BC + AC$.

STEP 1 Find AC.

Use right $\triangle ADB$ to find AD.

$$6^2 + AD^2 = 7.5^2$$
$$36 + AD^2 = 56.25$$
$$AD^2 = 20.25$$
$$AD = \sqrt{20.25} = 4.5$$

$AC = AD + DC$ $\qquad AC = 4.5 + 8 = 12.5$

STEP 2 Use right $\triangle BDC$ to find BC.

$$BC^2 = 6^2 + 8^2$$
$$BC^2 = 36 + 64 = 100$$
$$BC = \sqrt{100}$$
$$BC = 10$$

STEP 3 Find the perimeter of $\triangle ABC$.

$$\text{perimeter} = AB + BC + AC$$
$$= 7.5 + 10 + 12.5 = 30$$

The perimeter of $\triangle ABC$ is 30 units.

Try It This Way

Use a calculator to find the value of $\sqrt{20.25}$.

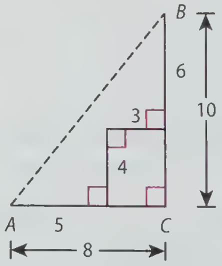
Watch Out !

$\sqrt{36 + 64} \neq \sqrt{36} + \sqrt{64}$

EXAMPLE 4

Dhara rides her bike 5 km east, 4 km north, then 3 km east and 6 km north. How much shorter would her ride be if she rode her bike in a straight line from beginning to end?

STEP 1 Draw a diagram.



STEP 2 Find AB, the distance along a straight line.

$\overline{AB}$ is the hypotenuse of a right triangle with legs 8 and 10, so use the Pythagorean Theorem.

$$AB^2 = 8^2 + 10^2$$
$$AB^2 = 164$$
$$AB = \sqrt{164} \approx 12.8 \text{ km}$$

STEP 3 Find the total distance Dhara rode her bike.

total distance = 5 km + 4 km + 3 km + 6 km = 18 km

STEP 4 Find the difference.

18 km − 12.8 km = 5.2 km

Dhara's ride would be 5.2 km shorter.

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Pythagorean Triples

Three whole numbers that satisfy the Pythagorean Theorem form a **Pythagorean triple**. For example, the numbers 3, 4, 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

Watch Out!

The term *Pythagorean triples* refers only to positive integers, not every group of three numbers that satisfies the Pythagorean Theorem.

EXAMPLE 5

Do the numbers form a Pythagorean triple? Explain.

a. 5, 12, 13

$5^2 + 12^2 \stackrel{?}{=} 13^2$

$25 + 144 = 169$ ✓

Yes, the numbers form a Pythagorean triple. They are whole numbers and they satisfy the Pythagorean Theorem.

b. 0.3, 0.4, 0.5

$0.3^2 + 0.4^2 \stackrel{?}{=} 0.5^2$

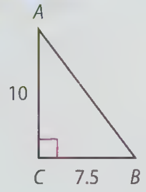
$0.09 + 0.16 = 0.25$ ✓

No, the numbers do not form a Pythagorean triple. They satisfy the Pythagorean Theorem, but they are not whole numbers.

Any multiple of a Pythagorean triple is also a Pythagorean triple. For example, because 5, 12, 13 is a Pythagorean triple, it follows that 10, 24, 26 is also a Pythagorean triple. You can use this fact to simplify some right triangle problems.

EXAMPLE 6

Find *AB*.



METHOD 1

Use the Pythagorean Theorem.

$$\left(7\tfrac{1}{2}\right)^2 + 10^2 = AB^2$$

$$\frac{225}{4} + 100 = AB^2$$

$$\frac{625}{4} = AB^2$$

$$AB = \sqrt{\frac{625}{4}} = \frac{25}{2} = 12\frac{1}{2}$$

METHOD 2

Notice that $7.5 = 3(2.5)$ and $10 = 4(2.5)$.

So, $(7.5, 10, AB)$ belongs to the $(3, 4, 5)$ family.

Therefore, $AB = 5(2.5) = 12.5$.

GOT TO KNOW!

Common Pythagorean Triples and Their Multiples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39		

Classifying Triangles

You use side lengths to classify a triangle as right, acute, or obtuse. You can use the converse of the Pythagorean Theorem to determine if given side lengths form a right triangle.

Converse of the Pythagorean Theorem

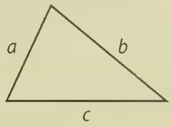
If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

The next theorem is an extension of the Converse of the Pythagorean Theorem.

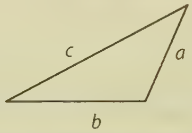
Pythagorean Inequality Theorem

Given: a, b, and c are the side lengths of a triangle with c the length of the longest side

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



EXAMPLE 7

Classify the triangle with the given side lengths. If no triangle is possible, write *not possible*.

a. 5, 8, 9

Compare c^2 to $a^2 + b^2$. Always substitute the greatest number for c.

$9^2\ [\overset{?}{=}]\ 5^2 + 8^2$

$81\ [\overset{?}{=}]\ 25 + 64$

$81 < 89$

The triangle is acute.

b. 9, 12, 17

$17^2\ [\overset{?}{=}]\ 9^2 + 12^2$

$289\ [\overset{?}{=}]\ 81 + 144$

$289 > 225$

The triangle is obtuse.

c. 13, 5, 25

Use the Triangle Inequality Theorem.

$13 + 5 \not> 25$

Not possible

d. 30, 40, 50

$50^2\ [\overset{?}{=}]\ 30^2 + 40^2$

$2500 = 900 + 1600$

The triangle is a right triangle.

Watch Out!

When you compare c^2 to $a^2 + b^2$, always substitute the greatest value for c.

Need More HELP

For a review of the Triangle Inequality Theorem, go to *Triangle Inequalities* in *Triangles and Polygons* on page 664.

EXAMPLE 8

Shauna wants to determine if the table leg she is repairing is perpendicular to the top of the table. She measures from the bottom of the 3.2-ft leg to a point 1.2 ft from the end of the table. The tape measure reads 3.5 ft. What type of angle does the leg form with the table top?

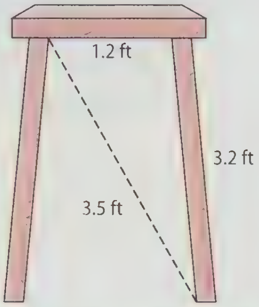
The tape measure, the leg, and the table top form a triangle. The longest side is 3.5 ft.

$3.5^2 [\overset{?}{=}] 3.2^2 + 1.2^2$

$12.25 [\overset{?}{=}] 10.24 + 1.44$

$12.25 > 11.68$

The angle is obtuse.



SEARCH

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EXAMPLE 9

Builders sometimes use a rope with 13 knots spaced at 1-ft intervals to determine whether a wall is perpendicular to the floor. Why does this work?

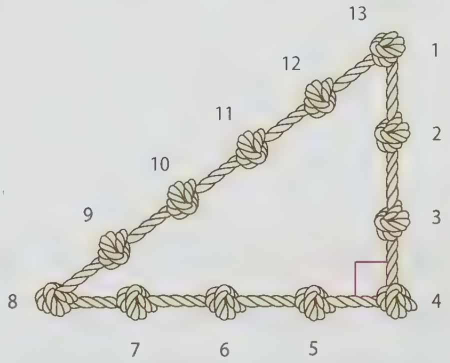
The builder puts a 3-ft length along a wall and a 4-ft length along the floor.

If possible, he or she then connects the first and thirteenth knot.

If the first and last knot meet exactly, then the three lengths form a 3-4-5 triangle.

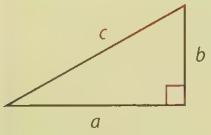
The angle between the 3-ft side and the 4-ft side must be a right angle by the Converse of the Pythagorean Theorem.

If the knots do not meet, or if there is overlap, then the angle is not a right angle.



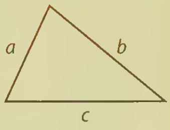
GOT TO KNOW!

Types of Triangles and Their Side Lengths



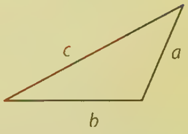
right triangle

$c^2 = a^2 + b^2$



acute triangle

$c^2 < a^2 + b^2$



obtuse triangle

$c^2 > a^2 + b^2$

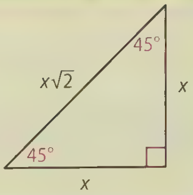
Special Right Triangles

45°–45°–90° Triangles

Base angles of an isosceles triangle are congruent. The acute angles of a right triangle are complementary. Therefore, the base angles of an isosceles right triangle both measure 45°. Another name for an isosceles right triangle is a **45°–45°–90° triangle**.

45°–45°–90° Triangle Theorem

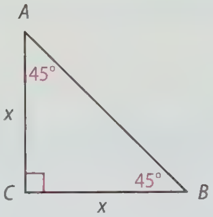
In a 45°–45°–90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.



PROOF

Given: $\triangle ABC$ is a 45°–45°–90° triangle.

Prove: $AB = AC\sqrt{2}$



Let $AC = x$. Then $BC = x$ because the triangle is isosceles. By the Pythagorean Theorem:

$$AB^2 = x^2 + x^2$$

$$AB^2 = 2x^2$$

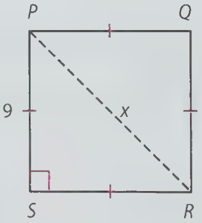
$$AB = \sqrt{2x^2}$$

$$AB = x\sqrt{2}$$

$$AB = AC\sqrt{2}$$

EXAMPLE 1

***PQRS* is a square. Find the value of *x*.**



A diagonal of a square divides it into two congruent 45°–45°–90° triangles.

$$\text{hypotenuse} = \sqrt{2} \cdot \text{leg}$$

$$x = 9\sqrt{2}$$

CHECK Use the Pythagorean Theorem to verify that your answer is correct.

Pythagorean Theorem $PS^2 + SR^2 = PR^2$

Substitute. $9^2 + 9^2 \stackrel{?}{=} (9\sqrt{2})^2$

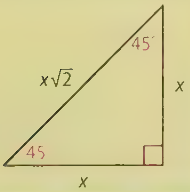
Simplify. $81 + 81 \stackrel{?}{=} (81)(2)$

$162 = 162$ ✓

GOT TO KNOW!

45°–45°–90° Triangle Theorem

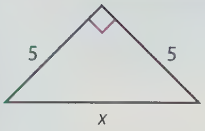
In a 45°–45°–90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.



EXAMPLE 2

Find the value of *x* in each triangle.

a.

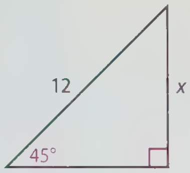


This is a 45°–45°–90° triangle.

hypotenuse = $\sqrt{2} \cdot$ leg

$x = 5\sqrt{2}$

b.



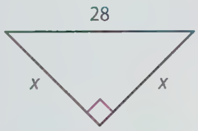
hypotenuse = $\sqrt{2} \cdot$ leg $\qquad 12 = x\sqrt{2}$

Divide both sides by $\sqrt{2}$. $\qquad \frac{12}{\sqrt{2}} = x$

Simplify the radical. $\qquad \frac{12\sqrt{2}}{2} = x$

$x = 6\sqrt{2}$

c.



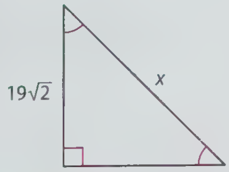
hypotenuse = $\sqrt{2} \cdot$ leg $\qquad 28 = x\sqrt{2}$

Divide both sides by $\sqrt{2}$. $\qquad \frac{28}{\sqrt{2}} = x$

Simplify the radical. $\qquad \frac{28\sqrt{2}}{2} = x$

$x = 14\sqrt{2}$

d.



hypotenuse = $\sqrt{2} \cdot$ leg $\qquad x = (19\sqrt{2})(\sqrt{2})$

Simplify. $\qquad x = 19 \cdot 2$

$x = 38$

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More **HELP**

For help with rationalizing the denominator, go to *Dividing Rational Expressions* in *Advanced Algebra* (Book 2, p. 1962).

30°–60°–90° Triangles

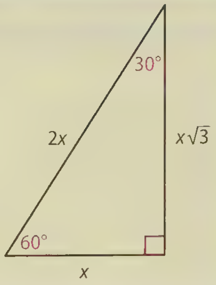
There is a special relationship between the lengths of the sides of a 30°–60°–90° triangle.

Watch Out!

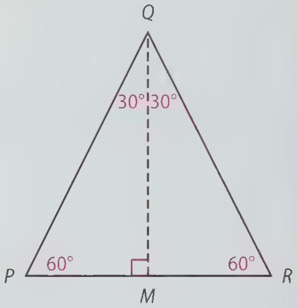
You will use the properties of 45°–45°–90° and 30°–60°–90° triangles often. Memorize them.

30°–60°–90° Triangle Theorem

In a 30°–60°–90° triangle, the hypotenuse is twice the length of the shorter leg. The longer leg is $\sqrt{3}$ times the length of the shorter leg.



The bisector of an angle in an equilateral triangle divides the triangle into two congruent 30°–60°–90° triangles.



PROOF

Given: $\triangle PQR$ is equilateral; $\overline{QM}$ bisects $\angle PQR$.

Prove: $PQ = 2PM$ and $QM = PM\sqrt{3}$

$\overline{QM}$ divides $\triangle PQR$ into two congruent 30°–60°–90° triangles, and $PR = 2PM = PQ$. By the Pythagorean Theorem:

$$PM^2 + QM^2 = PQ^2$$

$$PM^2 + QM^2 = (2PM)^2$$

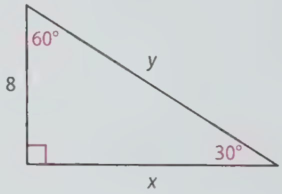
$$PM^2 + QM^2 = 4PM^2$$

$$QM^2 = 3PM^2$$

$$QM = PM\sqrt{3}$$

EXAMPLE 3

Find the values of *x* and *y*.



You are given the length of the shorter leg.

STEP 1 Find *x*, the length of the longer leg.

longer leg = shorter leg • $\sqrt{3}$ $\qquad x = 8\sqrt{3}$

STEP 2 Find *y*, the length of the hypotenuse.

hypotenuse = 2 • shorter leg $\qquad y = 2(8) = 16$

CHECK Use the Pythagorean Theorem to verify that your answers are correct.

Pythagorean Theorem $\qquad 8^2 + x^2 = y^2$

Substitute. $\qquad 8^2 + (8\sqrt{3})^2 \stackrel{?}{=} 16^2$

Simplify. $\qquad 64 + 64(3) \stackrel{?}{=} 256$

$$64 + 192 \stackrel{?}{=} 256$$

$$256 = 256 \checkmark$$

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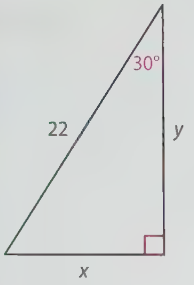
EXAMPLE 4

Find the values of the variables.

a.



You are given the length of the hypotenuse.

STEP 1 Find the length of the shorter leg.

hypotenuse = 2 • shorter leg $22 = 2x$

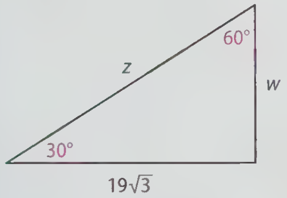
$x = 11$

STEP 2 Find the length of the longer leg.

longer leg = shorter leg • $\sqrt{3}$ $y = 11\sqrt{3}$

$x = 11; y = 11\sqrt{3}$

b.



You are given the length of the longer leg.

STEP 1 Find the length of the shorter leg.

longer leg = shorter leg • $\sqrt{3}$ $19\sqrt{3} = w\sqrt{3}$

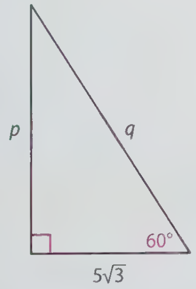
Divide both sides by $\sqrt{3}$. $w = 19$

STEP 2 Find the length of the hypotenuse.

hypotenuse = 2 • shorter leg $z = 2(19) = 38$

$w = 19; z = 38$

c.



You are given the length of the shorter leg.

STEP 1 Find the length of the longer leg.

longer leg = shorter leg • $\sqrt{3}$ $p = 5\sqrt{3} \cdot \sqrt{3}$

Simplify. $p = 5(3) = 15$

STEP 2 Find the length of the hypotenuse.

hypotenuse = 2 • shorter leg $q = 2(5\sqrt{3}) = 10\sqrt{3}$

$p = 15; q = 10\sqrt{3}$

Need More HELP

Remember! The shortest side of a triangle is opposite the smallest angle.

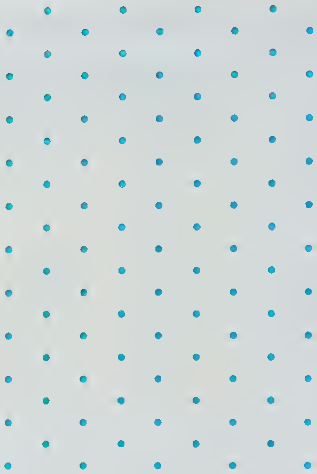
Isometric Drawings

Creating Isometric Drawings

You live in three-dimensional space. This means that every object you encounter, even something as thin as a page in this book, has length, width, and depth. Artists and engineers draw two-dimensional models of three-dimensional objects. Artists use shading and perspective to accomplish this. Geometry uses a few different methods.

The first of these methods is **isometric drawings**. An isometric drawing shows three sides of a figure, using triangular isometric dot paper, also called a *triangular lattice*. The dots are vertices of equilateral triangles. Each 60° angle on the dot paper represents a right angle of a three-dimensional figure.

You can use isometric drawings to represent figures constructed from congruent cubes.



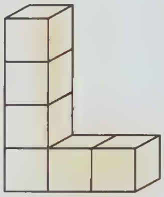
Isometric dot paper

Try It This Way

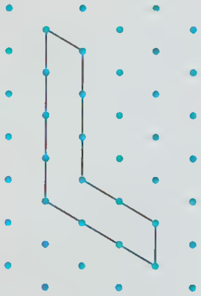
If you are having difficulty visualizing the corner view of a figure constructed from cubes, use actual cubes to build it.

EXAMPLE 1

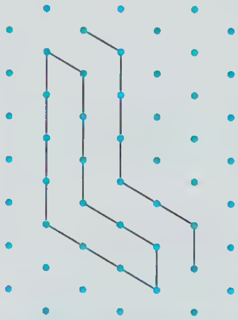
Lou is arranging cubes to form the letters in his name. Use isometric dot paper to draw the letter "L."



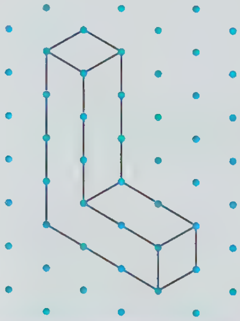
STEP 1 Draw the front.



STEP 2 Draw the back.



STEP 3 Connect corresponding vertices.

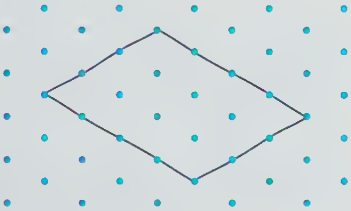


You can use isometric drawings to represent other three-dimensional figures.

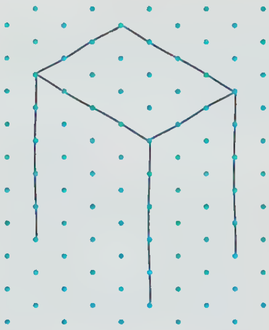
EXAMPLE 2

Use isometric dot paper to draw a rectangular prism 5 units high, 4 units long, and 3 units wide.

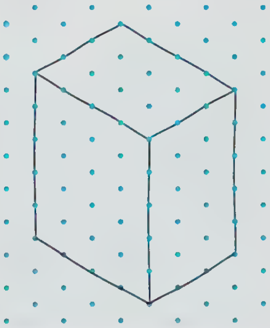
STEP 1 Draw the top of the figure, a 3 × 4 rectangle.



STEP 2 Draw vertical segments from each of the three "front" vertices of the top. Each segment is 5 units long.



STEP 3 Draw the two visible edges of the bottom. Notice that the segments you just drew are parallel to the edges of the top.



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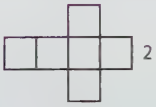
EXAMPLE 3

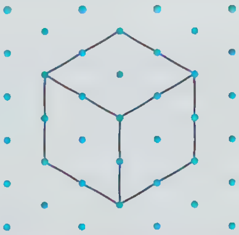
Use isometric dot paper to draw the solid with the given net.



STEP 1 Identify the solid.

The solid is a 2 × 2 × 2 cube.

STEP 2 Draw the cube.



Need More HELP

For a review of nets, go to *Nets* in *Spatial Reasoning*, p. 694.

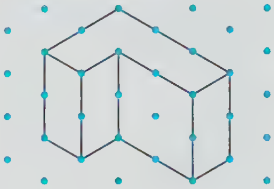
Using Isometric Drawings

Since isometric drawings represent three-dimensional figures, you can use them to determine properties of those figures. One thing to keep in mind when using isometric drawings is that the drawings do not show all the faces. Some faces are hidden from view.

EXAMPLE 4

Determine the numbers of faces, edges, and vertices of each solid.

a.



STEP 1 Count the faces. Five faces are visible in the drawing. Three faces are hidden from view: the back right, the back left, and the bottom.

The figure has 8 faces.

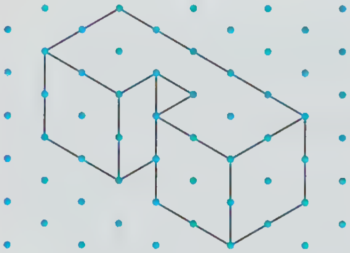
STEP 2 Count the edges. There are 15 visible edges. In addition, there are 3 edges hidden from view: 1 vertical edge where the right back and left back faces intersect, and 2 edges of the bottom base.

The figure has 18 edges.

STEP 3 Count the vertices. There are 11 visible vertices. In addition, there is 1 hidden vertex at the rear of the figure.

The figure has 12 vertices.

b.



STEP 1 Count the faces. Including one face that is partially obscured, there are six visible faces. Four faces are hidden from view: one side of the indentation, the back right, the back left, and the bottom.

The figure has 10 faces.

STEP 2 Count the edges. Including 1 edge that is partially hidden, there are 18 visible edges. In addition, 6 edges are hidden from view: 3 edges in the indentation and 3 edges at the back of the figure.

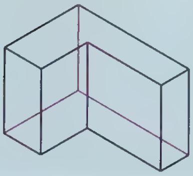
The figure has 24 edges.

STEP 3 Count the vertices. There are 13 visible vertices. In addition, 3 vertices are hidden from view: 2 in the indentation and 1 in the back corner.

The figure has 16 vertices.

Try It This Way

If you are having difficulty visualizing the hidden edges, copy the figure and draw the hidden edges with a colored pencil.



GOT TO KNOW!

Isometric Dot Paper

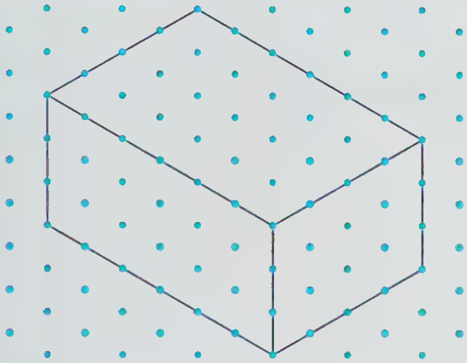
An isometric drawing uses triangular isometric dot paper. The dots are vertices of equilateral triangles. Each 60° angle on the dot paper represents a right angle of a three-dimensional figure.

Isometric drawings can also help you calculate the surface area and volume of geometric solids. In this lesson, you can assume that a segment joining two dots has a length of 1 unit.

EXAMPLE 5

Find the total surface area and volume of each figure.

a.



STEP 1 Find the total surface area (*SA*).

The figure is a rectangular prism. The formula for its surface area is

$SA = 2(\ell w + \ell h + wh)$, where ℓ is the length, w is the width, and h is the height.

Substitute values from the figure.

$SA = 2[(6 \cdot 4) + (6 \cdot 3) + (4 \cdot 3)]$

Simplify.

$= 2(24 + 18 + 12)$

$SA = 108$

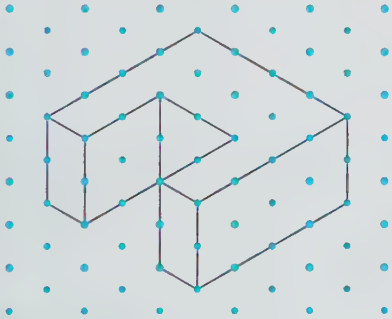
The total surface area is 108 square units.

STEP 2 Find the volume (*V*).

Use the volume formula. $V = \ell \cdot w \cdot h = 6 \cdot 4 \cdot 3 = 72$

The volume is 72 cubic units.

b.



STEP 1 Find the total surface area.

The figure is a 4 × 4 × 2 rectangular prism with a 2 × 2 × 2 rectangular prism removed. The faces of the indentation are congruent to the faces that have been removed from the larger prism, so the formula for its surface area is still:

$SA = 2(\ell w + \ell h + wh)$

$SA = 2[(4 \cdot 4) + (4 \cdot 2) + (4 \cdot 2)] = 2(16 + 8 + 8)$
$= 64$

The total surface area is 64 square units.

STEP 2 Find the volume.

The volume of the figure is the difference between the volume of the 4 × 4 × 2 rectangular prism and the volume of the 2 × 2 × 2 rectangular prism.

$V = (4 \cdot 4 \cdot 2) - (2 \cdot 2 \cdot 2) = 24$

The volume is 24 cubic units.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

For help with finding the volume of a prism, go to *Volumes of Prisms and Cylinders* in *Measurement*, p. 570.

Orthographic Drawings

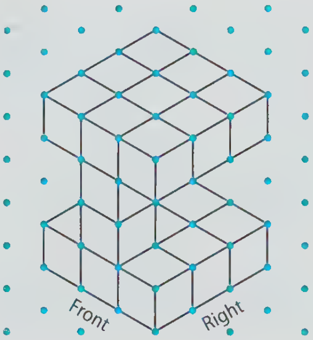
Creating Orthographic Drawings

When you look at an object straight on, your line of sight is perpendicular to the surface of the object. You may see only one side of the object, and not its depth.

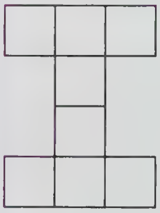
An **orthographic drawing** of a three-dimensional object shows up to six views of the object: the front, back, top, bottom, right, and left sides. A heavy segment indicates a break in the surface.

EXAMPLE 1

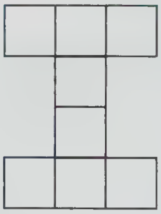
Given the isometric drawing of a figure, make an orthographic drawing.



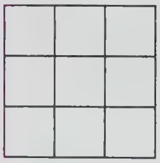
Front View



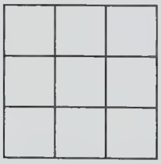
Back View



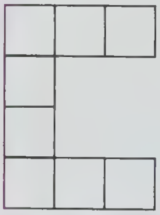
Top View



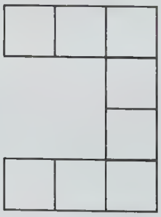
Bottom View



Right-Side View



Left-Side View



GOT TO KNOW!

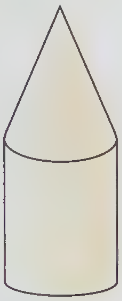
Orthographic Drawings

An orthographic drawing shows up to six views: front, back, right side, left side, top, and bottom.

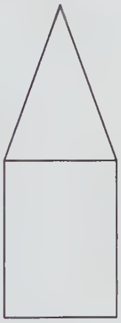
Orthographic drawings are not limited to figures constructed from cubes. You can make an orthographic drawing of any three-dimensional figure, including one with curved surfaces.

EXAMPLE 2

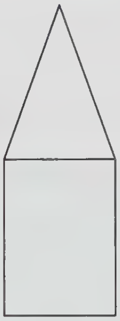
Make an orthographic drawing of the figure below.



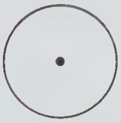
Front View
From the front, the figure looks like a rectangle topped by a triangle.



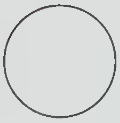
Back View



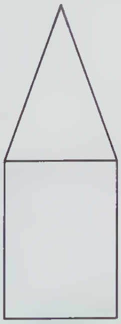
Top View
The point represents the top of the cone.



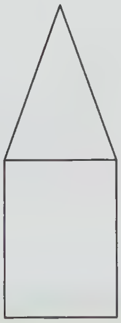
Bottom View



Right-Side View



Left-Side View



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Watch Out!

The bottom view is not the same as the top view. Do not include a point at the center of the circle.

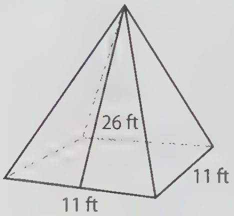
Surface Area of a Pyramid

For a three-dimensional figure, the surface area is the sum of the areas of all of its surfaces. For a pyramid, the surface area is the area of the base plus the lateral area.

EXAMPLE 5

Find the surface area of each pyramid.

a. The figure is a square pyramid.



STEP 1 Find the area of the base.

Write the formula. $A = s^2$

Substitute and simplify. $= 11^2 = 121$

STEP 2 Find the lateral area.

Write the formula. $\text{LA} = \frac{1}{2}Ps$

Substitute values. $= \frac{1}{2}(4 \cdot 11)(26)$

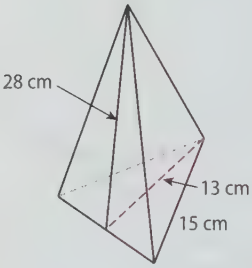
Simplify. $= 572$

STEP 3 Add the area of the base and the lateral area.

$121 + 572 = 693$

The surface area of the pyramid is 693 ft^2.

b. The figure is an equilateral triangular pyramid.



STEP 1 Find the area of the base.

Write the formula. $A = \frac{1}{2}bh$

Substitute values. $= \frac{1}{2}(15)(13) = 97.5$

STEP 2 Find the lateral area.

Write the formula. $\text{LA} = \frac{1}{2}Ps$

Substitute values. $= \frac{1}{2}(3 \cdot 15)(28)$

Simplify. $= 630$

STEP 3 Add the area of the base and the lateral area.

$97.5 + 630 = 727.5$

The surface area of the pyramid is 727.5 cm^2.

GOT TO KNOW!

How Many Bases?

A pyramid has one base. Other solid figures, such as prisms and cylinders, have two bases.

Need More HELP

For help with perimeter, go to *Perimeter* in *Measurement*, p. 524.

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To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

GOT TO KNOW!

Surface Area and Lateral Area

The surface area of any solid figure consists of the area of the base(s) plus the lateral area. The lateral area of any solid *does not* include the area of the base(s).

Cones

A **cone** is a three-dimensional figure with a circular base, a **vertex** not in the same plane as the base, and a curved surface determined by the base and the vertex. The curved surface of a cone is called its **lateral surface**.

The **axis of a cone** is the line segment that has one endpoint at the vertex of the cone and the other endpoint at the center of the base. A **right cone** is a cone whose axis is perpendicular to the base. An **oblique cone** is a cone whose axis is not perpendicular to the base. Any segment from the vertex to a point on the boundary of the base is called an **element** of the cone.

Ways to REMEMBER

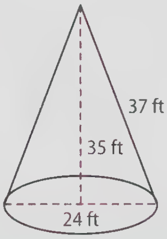
An element of a cone is similar to the slant height of a pyramid. Both segments represent the shortest distance from the vertex to the base.

EXAMPLE 6

Determine whether each cone is a right cone or an oblique cone.

To determine whether a cone is right or oblique, form a triangle using a radius of the base, the axis of the cone, and an element of the cone. Then use the Converse of the Pythagorean Theorem to decide if that triangle is a right triangle.

a. Use the cone at the right.



STEP 1 Find the radius of the base.

Write the formula. $r = \frac{d}{2}$

Substitute values. $= \frac{24}{2} = 12$

STEP 2 Use the Converse of the Pythagorean Theorem.

Write the formula. $a^2 + b^2 = c^2$

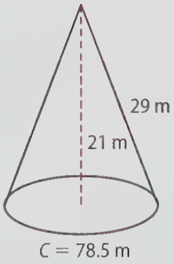
Substitute values. $12^2 + 35^2 \stackrel{?}{=} 37^2$

Find the squares. $144 + 1225 \stackrel{?}{=} 1369$

Add. $1369 = 1369$

The triangle is a right triangle, so the cone is a right cone.

b. Use the cone at the right.



STEP 1 Find the radius of the base. Use 3.14 for π.

Write the formula. $r = \frac{C}{2\pi}$

Substitute values. $\approx \frac{78.5}{2(3.14)}$

Simplify. $= 12.5$

STEP 2 Use the Converse of the Pythagorean Theorem.

Write the formula. $a^2 + b^2 = c^2$

Substitute values. $12.5^2 + 21^2 \stackrel{?}{=} 29^2$

Find the squares. $156.25 + 441 \stackrel{?}{=} 841$

Add. $597.25 \neq 841$

The triangle is not a right triangle, so the cone is an oblique cone.

GOT TO KNOW!

Pythagorean Theorem

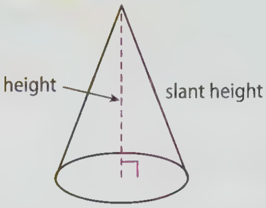
When you use the Pythagorean Theorem, you start with the fact that $a^2 + b^2 = c^2$ and you find one of the missing values a, b, or c.

Converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem states that if the lengths of the sides of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Lateral Area of a Cone

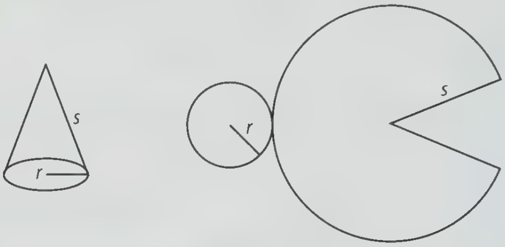
As with pyramids, it is important to make a distinction between the *height of a cone* and the *slant height of a cone*. The **height of a cone** is the perpendicular distance from the vertex to the plane containing the base. For a right cone, the height of the cone is equal to the length of the axis. The **slant height of a right cone** is the length of an element of the cone.



The **lateral area of a cone** is the area of its curved surface. To derive a formula for the lateral area of a cone, start by looking at a right cone and its net.

Need More HELP

For help with sectors of a circle, go to *Arcs and Sectors*, p. 728.



Consider the lateral surface of the cone as it appears in the net above. It is part of a circle, and the radius of that circle is s, the slant height of the cone. The circumference of the partial circle is $2\pi r$ because that is the circumference of the base of the cone.

The lateral area of the cone is the same as the area of the partial circle, and that area is a fraction of the area of the circle whose radius is s. That fraction is the ratio $\frac{2\pi r}{2\pi s}$, which simplifies to $\frac{r}{s}$. So:

$$\text{LA} = \frac{r}{s} \times (\text{area of circle with radius } s) = \frac{r}{s} \times (\pi s^2) = \pi rs$$

Watch Out!

A key step in deriving a formula for the lateral area of a cone uses the fact that if you remove a pie-shaped sector of a circle, you take away a proportional part of the area of that circle.

Therefore, if a cone has a base with radius r and slant height s, a formula for the lateral area of the cone is $\text{LA} = \pi rs$. The total surface area of a cone is the lateral area plus the area of its circular base, so a formula for the total surface area (SA) of a cone is

$$\text{SA} = \pi r^2 + \pi rs$$

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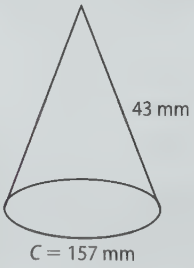
EXAMPLE 7

Find the lateral area of the cone. Use 3.14 for π.



STEP 1 Find the radius of the base.

Write the formula. $r = \frac{C}{2\pi}$

Substitute values and simplify. $\approx \frac{157}{2(3.14)} = \frac{157}{6.28} = 25$

STEP 2 Find the lateral area.

Write the formula. $\text{LA} = \pi rs$

Substitute values from the figure. $\approx (3.14)(25)(43)$

Multiply. $= 3375.5$

The lateral area of the cone is about 3375.5 mm^2.

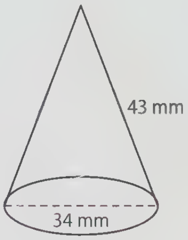
Surface Area of a Cone

The previous page shows a formula for the surface area (SA) of a cone whose base has radius r and whose slant height is s. That formula is $SA = \pi r^2 + \pi rs$.

EXAMPLE 8

Find the total surface area of each cone. Use 3.14 for π.

a. Use the cone at the right.



STEP 1 Find the radius.

Write the formula. $r = \frac{d}{2}$

Substitute and simplify. $= \frac{34}{2} = 17$

STEP 2 Find the area of the base.

Write the formula. $A = \pi r^2$

Substitute values. $\approx (3.14)(17^2)$

Simplify. $\approx (3.14)(289)$

$= 907.46$

STEP 3 Find the lateral area.

Write the formula. $LA = \pi rs$

Substitute values. $\approx (3.14)(17)(43)$

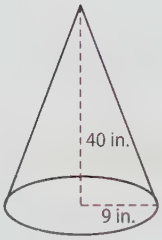
Multiply. $= 2295.34$

STEP 4 Add the area of the base and the lateral area.

$907.46 + 2295.34 = 3202.8$

The total surface area of the cone is about 3202.8 mm^2.

b. Use the cone at the right.



STEP 1 Use the Pythagorean Theorem to find the slant height of the cone.

Write the formula. $a^2 + b^2 = c^2$

Substitute values. $9^2 + 40^2 = c^2$

Find the squares. $81 + 1600 = c^2$

Add. $1681 = c^2$

Find the square roots of both sides. $41 = c$

STEP 2 Find the area of the base.

Write the formula. $A = \pi r^2$

Substitute values. $\approx (3.14)(9^2)$

Simplify. $\approx (3.14)(81) = 254.34$

STEP 3 Find the lateral area.

Write the formula. $LA = \pi rs$

Substitute values. $\approx (3.14)(9)(41)$

Multiply. $= 1158.66$

STEP 4 Add the area of the base and the lateral area.

$254.34 + 1158.66 = 1413$

The total surface area of the cone is about 1413 in.^{2}.

GOT TO KNOW!

Surface Area Formula

In the formula $SA = \pi r^2 + \pi rs$, the term πr^2 is used to calculate the area of the base of the cone. The term πrs is used to calculate the lateral area of the cone.

GOT TO KNOW!

Pyramid and Cone Formulas

For a **regular pyramid** whose base has perimeter P and whose slant height is s,

$LA = \frac{1}{2}Ps$

For a **cone** whose base has radius r and whose slant height is s,

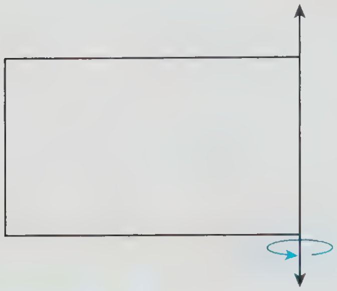
$LA = \pi rs$

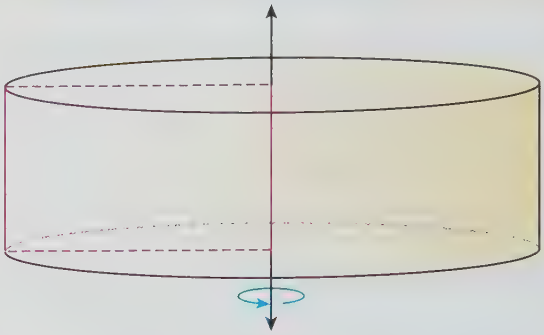
$SA = \pi r^2 + \pi rs$

Solids of Revolution

Creating Solids of Revolution

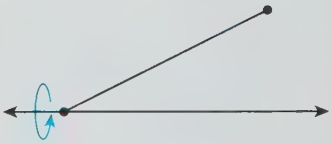
If a figure is revolved about a line, the resulting shape is a **solid of revolution**. The line about which it revolves is the **axis of revolution**. Solids of revolution are figures with curved surfaces, such as cylinders and cones.





EXAMPLE 1

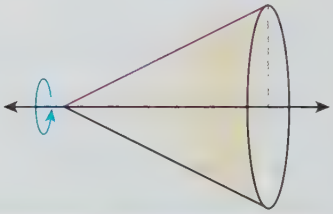
Draw the solid of revolution that results from rotating a line segment as shown.



Draw the rotation around the horizontal line.

The vertex at the origin does not move. To picture the rotation of the circle, think of the line spinning, with the point at the origin staying in place. The figure becomes a cone.

Draw a cone with base radius equal to the distance from the end of the segment to the horizontal line.



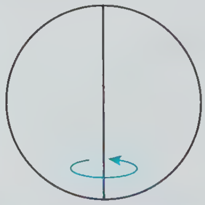
Try It This Way

If you are having difficulty visualizing the solid of revolution formed by rotating a two-dimensional figure, try using a physical object with the same general shape. For example, hold a straw at one end, and rotate it around another straw.

EXAMPLE 2

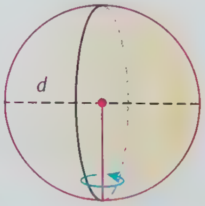
Draw the solid of revolution that results from rotating a circle around its diameter.

STEP 1 Draw the two-dimensional figure described in the direction. There are no directions of the size of the circle, so draw a circle of any size.



STEP 2 Draw the rotation around the diameter.

To picture the rotation of the circle, think of a coin spinning on its edge. The coin appears to be a solid sphere. So, draw a sphere with the same diameter as the circle from Step 1.

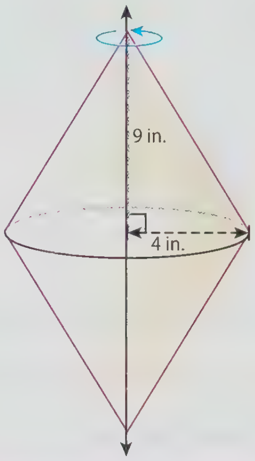


The solid of revolution is a sphere with the same diameter as the circle that formed it.

Solids of revolution have surface area and volume, just like any other three-dimensional figure. Dimensions of the solids can be obtained from information given in the two-dimensional figures forming the solids.

EXAMPLE 3

Rotating a diamond across a line forms two congruent cones. Find the volume of the solid of revolution to the right.



Use the volume formula for a cone.

Multiply by 2 for the two cones. $V = 2 \cdot \frac{1}{3}\pi r^2 h$

Substitute values from the figure. $\approx 2 \cdot \frac{1}{3}(3.14)(4^2)(9)$

Simplify. $\approx 2 \cdot \frac{1}{3}(3.14)(16)(9)$

≈ 301.44

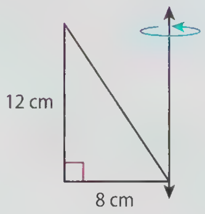
The volume is approximately 301.44 in.^{3}.

Need More HELP

For help with volumes of cylinders and cones, see *Volumes of Prisms and Cylinders* and *Volumes of Pyramids and Cones* in *Measurement* (pp. 570, 574).

EXAMPLE 4

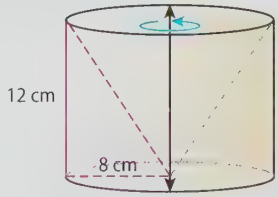
Find the volume of the solid of revolution formed by rotating the figure as indicated. Use 3.14 for π.



STEP 1 Draw the solid of revolution.

The solid of revolution looks like a cylinder with height equal to the length of the long leg of the triangle and radius equal to the length of the short leg.

Missing from the cylinder is an inverted cone with the same height and radius as the cylinder.



STEP 2 Find the volume of the cylinder.

Write the formula. $V = \pi r^2 h$

Substitute values from the figure. $\approx (3.14)(8^2)(12)$

Find the square. $\approx (3.14)(64)(12)$

Multiply. ≈ 2411.52

STEP 3 Find the volume of the cone.

Write the formula. $V = \frac{1}{3}\pi r^2 h$

Substitute values from the figure. $\approx \frac{1}{3}(3.14)(8^2)(12)$

Find the square. $\approx \frac{1}{3}(3.14)(64)(12)$

Multiply. ≈ 803.84

STEP 4 Subtract the volumes.

$$2411.52 - 803.84 = 1607.68$$

The volume of the solid of revolution is approximately 1607.68 cm^3.

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Cross Sections

Describing Cross Sections

Imagine a pre-packaged loaf of bread that you might buy at the grocery store. As a whole, the loaf is a rectangular prism. Each slice of bread, though, is basically a square.

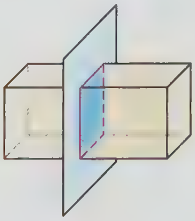
As with a loaf of bread, we can look at "slices," or *cross sections,* of solid figures. A **cross section** is the intersection of a three-dimensional figure and a plane. Since planes are two-dimensional, cross sections are two-dimensional figures.

Watch Out !

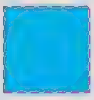
The loaf of bread can also be called a "square prism," because the two bases are squares.

One caution, though, is that sometimes a *square prism* refers to a cube, because all six faces are squares.

If you see the term *square prism,* be sure to check the dimensions to determine how many faces are squares.



plane intersecting a rectangular prism

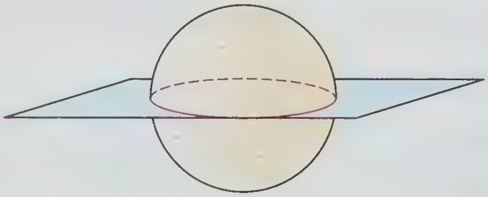


square cross section

EXAMPLE 1

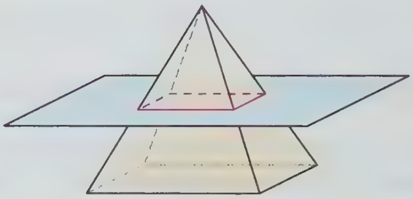
Describe each cross section.

a.



The cross section is a circle.

b.

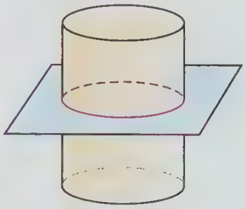


The cross section is a rectangle.

EXAMPLE 2

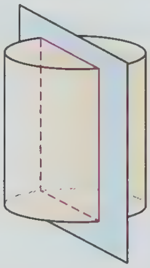
Describe how to cut a cylinder to make each cross section. Draw the figure.

a. circle



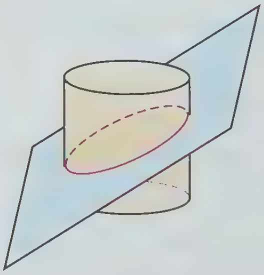
Cut parallel to bases.

b. rectangle



Cut perpendicular to bases.

c. ellipse



Cut diagonally without cutting the bases.

GOT TO KNOW!

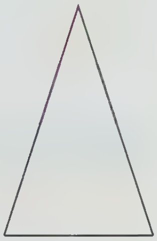
Multiple Cross Sections

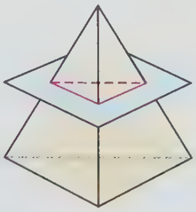
Since there are an infinite number of planes that can intersect a three-dimensional figure, each solid figure has an infinite number of possible cross sections.

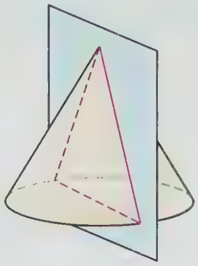
EXAMPLE 3

Name the two-dimensional figure. Describe and draw two solids that can have this figure as a cross section.

The figure is an acute triangle. It can be a cross section of a triangular pyramid or of a cone.







Watch Out!

A triangular prism can also have a triangle as a cross section. For prisms and pyramids, pay close attention to the shape of the base as it will affect the shape of the cross section.

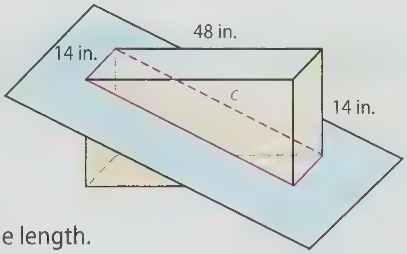
Perimeter and Area of Cross Sections

Cross sections are two-dimensional figures. You can find the perimeter and area of a cross section.

EXAMPLE 4

Find the perimeter and area of the cross section.

The figure is a square prism, and the cross section is a rectangle.



STEP 1 Find the length of the rectangular cross section.

The short edge of the prism, the long edge of the prism, and the long side of the cross section form a right triangle. Use the Pythagorean Theorem to find the length.

Write the formula. $a^2 + b^2 = c^2$

Substitute values from the figure. $14^2 + 48^2 = c^2$

Find the squares. $196 + 2304 = c^2$

Add. $2500 = c^2$

Find the square roots of both sides. $50 = c$

STEP 2 Find the perimeter of the cross section.

Write the formula. $P = 2\ell + 2w$

Substitute 50 for ℓ and 14 for w. $= 2(50) + 2(14)$

Multiply. $= 100 + 28$

Add. $= 128$

STEP 3 Find the area of the cross section.

Write the formula. $A = \ell w$

Substitute 50 for ℓ and 14 for w. $= (50)(14)$

Multiply. $= 700$

The perimeter of the cross section is 128 in. The area of the cross section is 700 in.^{2}.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

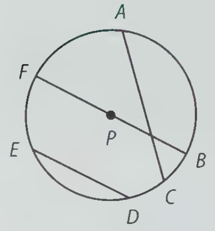
For help with perimeter and area of rectangles, see *Perimeter* and *Areas of Rectangles and Parallelograms* in *Measurement* (pp. 524, 530).

Chords and Arcs

Chords and Central Angles

A **chord** is a segment whose endpoints lie on a circle.
A **diameter** is a chord that contains the center of a circle.

$\overline{AC}$, $\overline{BF}$, and $\overline{DE}$ are chords. $\overline{BF}$ is a diameter.

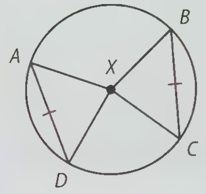


Theorem
In the same circle (or in congruent circles), congruent chords have congruent central angles.

PROOF

Given: $\overline{AD} \cong \overline{BC}$

Prove: $\angle AXD \cong \angle BXC$



Statements	Reasons
1. $\overline{AD} \cong \overline{BC}$	1. Given
2. $\overline{AX} \cong \overline{BX}, \overline{DX} \cong \overline{CX}$	2. All radii of a circle are ≅.
3. $\triangle AXD \cong \triangle BXC$	3. SSS
4. $\angle AXD \cong \angle BXC$	4. CPCTC

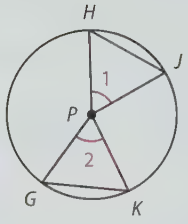
The converse of this theorem is also true.

Theorem
In the same circle (or in congruent circles), congruent central angles have congruent chords.

PROOF

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{GK} \cong \overline{HJ}$

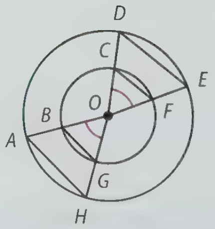


Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $PG = PK = PH = PJ$	2. All radii of a circle are ≅.
3. $\triangle GPK \cong \triangle HPJ$	3. SAS
4. $\overline{GK} \cong \overline{HJ}$	4. CPCTC

EXAMPLE 1

Name the congruent chords.

In congruent circles, congruent central angles have congruent chords, so $\overline{CF} \cong \overline{BG}$ and $\overline{DE} \cong \overline{AH}$.



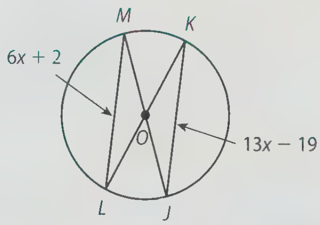
Watch Out !

$\overline{CF}$ and $\overline{DE}$ have congruent central angles, but they are not congruent chords.

EXAMPLE 2

Find LM.

Vertical $\angle$s are $\cong$.	$\angle LOM \cong \angle KOJ$
Congruent central $\angle$s have $\cong$ chords.	$LM = JK$
Substitute.	$6x + 2 = 13x - 19$
Solve.	$21 = 7x$
	$x = 3$
Substitute 3 for x.	$LM = 6(3) + 2$
	$LM = 20$



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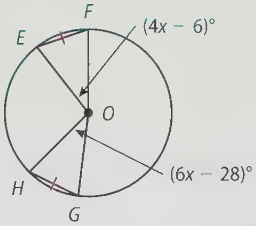
EXAMPLE 3

Find $m\angle EOF$.

Congruent chords have $\cong$ central $\angle$s.	$m\angle EOF = m\angle GOH$
Substitute.	$4x - 6 = 6x - 28$
Solve.	$22 = 2x$
	$x = 11$
Substitute 11 for x.	$m\angle EOF = 4(11) - 6$
	$m\angle EOF = 38°$

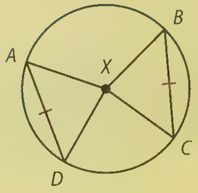


GOT TO KNOW!

Chords and Central Angles

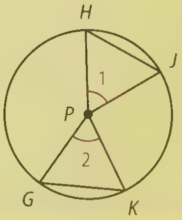
In the same circle (or in congruent circles), congruent chords have congruent central angles.

If $\overline{AD} \cong \overline{BC}$, then $\angle AXD \cong \angle BXC$.



In the same circle (or in congruent circles), congruent central angles have congruent chords.

If $\angle 1 \cong \angle 2$, then $\overline{GK} \cong \overline{HJ}$.



Chords and Arcs

Congruent arcs lie in the same circle (or in congruent circles) and have the same measure. A chord that is not a diameter cuts off a minor arc and a major arc. The minor arc is called the **arc of the chord**.

Watch Out!

The congruent arcs must be in the same circle or in congruent circles.

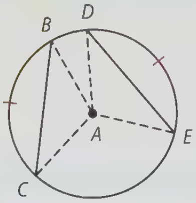
Theorem

In the same circle (or in congruent circles), congruent arcs have congruent chords.

PROOF

Given: $\overset{\frown}{BC} \cong \overset{\frown}{DE}$

Prove: $\overline{BC} \cong \overline{DE}$



Statements	Reasons
1. $\overset{\frown}{BC} \cong \overset{\frown}{DE}$	1. Given
2. $\angle BAC \cong \angle DAE$	2. Congruent arcs have congruent central $\angle$s.
3. $\overline{BC} \cong \overline{DE}$	3. Congruent central $\angle$s have congruent chords.

The converse of this theorem is also true.

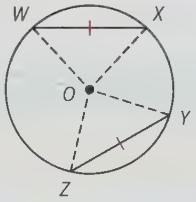
Theorem

In the same circle (or congruent circles), congruent chords have congruent arcs.

PROOF

Given: $\overline{WX} \cong \overline{ZY}$

Prove: $\overset{\frown}{WX} \cong \overset{\frown}{ZY}$

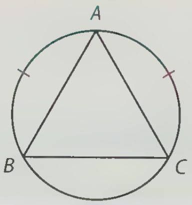


Statements	Reasons
1. $\overline{WX} \cong \overline{ZY}$	1. Given
2. $\angle WOX \cong \angle ZOY$	2. Congruent chords have congruent central $\angle$s.
3. $\overset{\frown}{WX} \cong \overset{\frown}{ZY}$	3. Congruent central $\angle$s have congruent arcs.

EXAMPLE 4

Given: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$

Prove: $\angle B \cong \angle C$



Statements	Reasons
1. $\overset{\frown}{AB} \cong \overset{\frown}{AC}$	1. Given
2. $\overline{AB} \cong \overline{AC}$	2. Congruent arcs have congruent chords.
3. $\angle B \cong \angle C$	3. Base $\angle$s of an isosceles triangle are congruent.

EXAMPLE 5

Find $m\widehat{XY}$.

Congruent chords have ≅ arcs. $m\widehat{XY} = m\widehat{VW}$

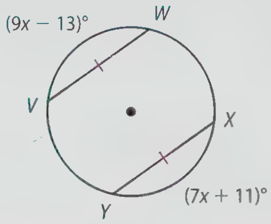
Substitute. $7x + 11 = 9x - 13$

Solve. $24 = 2x$

$x = 12$

Substitute 12 for x. $m\widehat{XY} = 7(12) + 11$

$m\widehat{XY} = 95°$



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EXAMPLE 6

Find GH.

STEP 1 Find $m\widehat{HE}$.

$m\widehat{HE} + m\widehat{GH} + m\widehat{GF} + m\widehat{FE} = 360°$

$m\widehat{HE} + 60 + 110 + 130 = 360$

$m\widehat{HE} = 60°$, so $\widehat{HE} \cong \widehat{GH}$.

STEP 2 Find the value of x.

Congruent arcs have ≅ chords. $GH = EF$

Substitute. $2x + 17 = 10x - 7$

Solve. $24 = 8x$

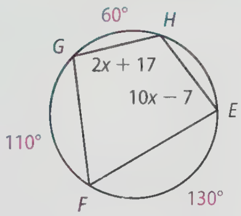
$x = 3$

STEP 3 Find GH.

$GH = 2x + 17$

Substitute 3 for x. $GH = 2(3) + 17$

$GH = 23$



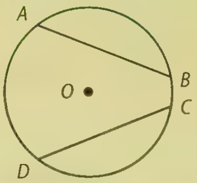
GOT TO KNOW!

Chords and Arcs

In the same circle (or in congruent circles), congruent arcs have congruent chords.

In the same circle (or in congruent circles), congruent chords have congruent arcs.

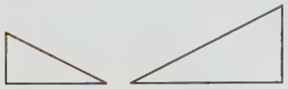
$\widehat{AB} \cong \widehat{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.



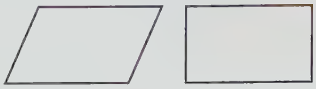
Congruent Figures

Congruence

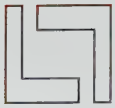
Figures that have the exact same size and shape are **congruent**.



These two figures are the same shape but not the same size. They are not congruent.



These two figures are not the same size and not the same shape. They are not congruent.

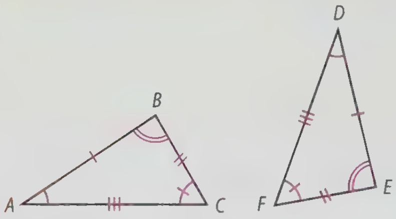


These figures are the same size and shape. They are congruent.

Watch Out !

The way the two figures are positioned does not matter when determining whether they are congruent.

Angles and sides that are in the same position in two different figures are called **corresponding angles** and **corresponding sides**. Two figures are congruent if and only if each pair of corresponding angles and corresponding sides is congruent.

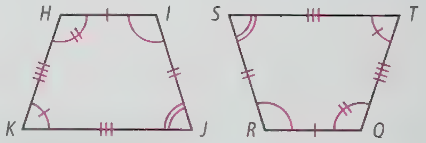


Corresponding Sides	Corresponding Angles
$\overline{AB} \cong \overline{DE}$	$\angle A \cong \angle D$
$\overline{BC} \cong \overline{EF}$	$\angle B \cong \angle E$
$\overline{CA} \cong \overline{FD}$	$\angle C \cong \angle F$

$\triangle ABC \cong \triangle DEF$

Corresponding Sides	Corresponding Angles
$\overline{HI} \cong \overline{QR}$	$\angle H \cong \angle Q$
$\overline{IJ} \cong \overline{RS}$	$\angle I \cong \angle R$
$\overline{JK} \cong \overline{ST}$	$\angle J \cong \angle S$
$\overline{KH} \cong \overline{TQ}$	$\angle K \cong \angle T$

$HIJK \cong QRST$



The order in which the vertices are listed in a congruence statement is very important. It indicates which sides and angles correspond to each other.

EXAMPLE 1

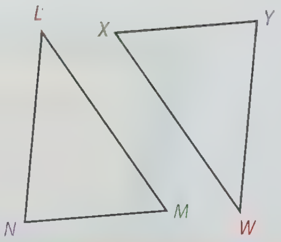
Given $\triangle LMN \cong \triangle WXY$, list all pairs of congruent corresponding parts.

Use the order in which the vertices are named.

$\triangle LMN \cong \triangle WXY$

$\angle L \cong \angle W$, $\angle M \cong \angle X$, and $\angle N \cong \angle Y$

$\overline{LM} \cong \overline{WX}$, $\overline{MN} \cong \overline{XY}$, and $\overline{NL} \cong \overline{YW}$



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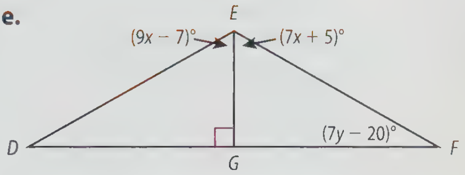
EXAMPLE 2

Given $\triangle DEG \cong \triangle FEG$ and $\angle DGE$ is a right angle, find each value.



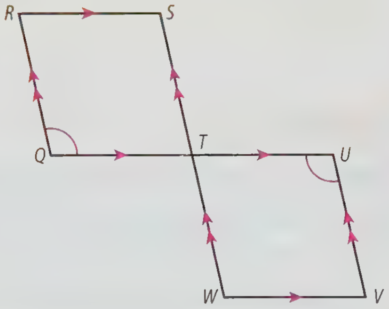
a. $m\angle DEG$

Corresponding ∠s in ≅ △s are ≅.	$\angle DEG \cong \angle FEG$
Definition of ≅ ∠s.	$m\angle DEG = m\angle FEG$
Substitute given values.	$9x - 7 = 7x + 5$
Solve for x.	$2x = 12$
	$x = 6$
Substitute for x.	$m\angle DEG = 9(6) - 7$
	$= 47°$

b. y

Sum of ∠s in a triangle is 180°.	$m\angle DGE + m\angle DEG + m\angle EDG = 180°$
Subtract.	$m\angle EDG = 180° - m\angle DGE - m\angle DEG$
Substitute.	$m\angle EDG = 180° - (90°) - (47°)$
	$= 43°$
Corresponding ∠s in ≅ △s are ≅.	$\angle EFG \cong \angle EDG$
Definition of ≅ ∠s.	$m\angle EFG = m\angle EDG$
Substitute.	$7y - 20 = 43°$
Solve for y.	$7y = 63$
	$y = 9$

EXAMPLE 3



Given: Parallelograms *QRST* and *UVWT*, *T* is the midpoint of $\overline{QU}$ and $\overline{SW}$, $\angle TQR \cong \angle TUV$, and parallel segments as marked

Prove: $QRST \cong UVWT$

Because they are vertical angles, $\angle UTW \cong \angle QTS$. Because opposite angles in a parallelogram are congruent, $\angle UTW \cong \angle UVW$, $\angle QRS \cong \angle QTS$, $\angle TWV \cong \angle TUV$, and $\angle TSR \cong \angle TQR$. $\angle TSR \cong \angle TWV$. $\angle QRS \cong \angle UVW$ by the Transitive Property of Congruence. Therefore, corresponding angles are congruent.

$\overline{TS} \cong \overline{TW}$ and $\overline{QT} \cong \overline{TU}$ by definition of a midpoint. Opposite sides in a parallelogram are congruent, so $\overline{RS} \cong \overline{QT}$, $\overline{TU} \cong \overline{VW}$, $\overline{QR} \cong \overline{TS}$, and $\overline{TW} \cong \overline{UV}$. $\overline{RS} \cong \overline{VW}$ and $\overline{QR} \cong \overline{UV}$ by transitivity. Therefore, corresponding sides are congruent. $QRST \cong UVWT$.

GOT TO KNOW!

Properties of Congruent Figures

Same size

Same shape

Congruent corresponding sides

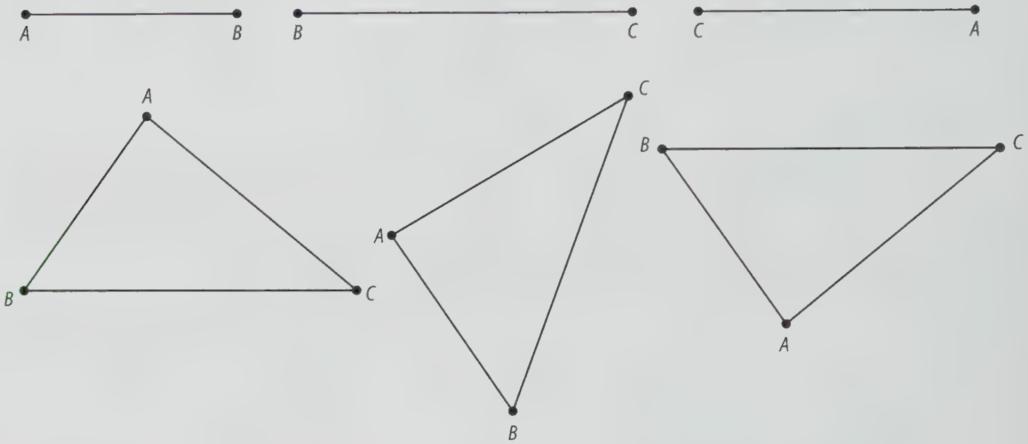
Congruent corresponding angles

Using SSS and SAS

Side-Side-Side Congruence

As you have learned, two triangles are congruent if and only if each pair of corresponding angles is congruent and each pair of corresponding sides is congruent. However, it is not necessary to know all of this to know two triangles are congruent.

The three segments given below can be put together to form a triangle. No matter how you put the segments together, the resulting triangle is the same.



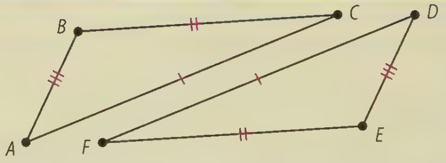
Try It This Way

Put three pencils together to form a triangle. See how many different triangles you can make.

The triangle may be in different positions, but it is still the same triangle. This fact is stated in the following postulate.

Side-Side-Side Congruence Postulate (SSS)

If three sides of one triangle are congruent to the corresponding three sides of another triangle, then the triangles are congruent.



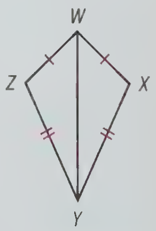
$\triangle ABC \cong \triangle DEF$

EXAMPLE 1

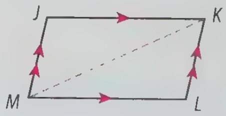
Write and justify a congruence statement that describes each pair of triangles.

a. $\overline{WY} \cong \overline{WY}$ by the Reflexive Property of Congruence. It is given that $\overline{WZ} \cong \overline{WX}$ and $\overline{ZY} \cong \overline{XY}$.

Three pairs of corresponding sides are congruent, so $\triangle WZY \cong \triangle WXY$ by SSS congruence.



b. Opposite sides are parallel, so the quadrilateral is a parallelogram. This means opposite sides are also congruent, so $\overline{JK} \cong \overline{LM}$ and $\overline{JM} \cong \overline{LK}$. $\overline{KM} \cong \overline{MK}$ by the Reflexive Property of Congruence. Three pairs of corresponding sides are congruent, so $\triangle MJK \cong \triangle KLM$ by SSS congruence.



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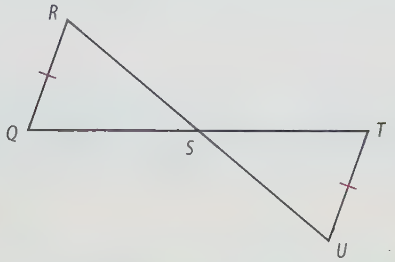
EXAMPLE 2

Given: $\overline{QR} \cong \overline{TU}$, S is the midpoint of $\overline{QT}$ and $\overline{RU}$

Prove: $\triangle QRS \cong \triangle TUS$



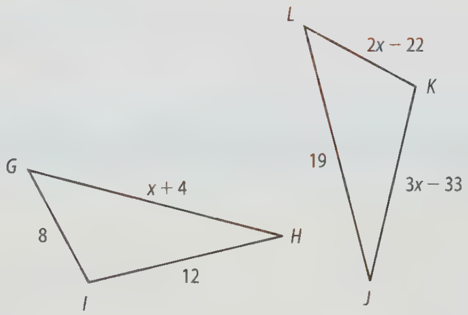
Statements	Reasons
1. $\overline{QR} \cong \overline{TU}$	1. Given
2. S is the midpoint of $\overline{QT}$ and $\overline{RU}$.	2. Given
3. $\overline{SR} \cong \overline{SU}, \overline{SQ} \cong \overline{ST}$	3. Definition of midpoint
4. $\triangle QRS \cong \triangle TUS$	4. SSS congruence

Watch Out !

To say two triangles are congruent, it is not enough to know that there are two corresponding congruent sides (SS). You must know that there are three corresponding congruent sides (SSS).

EXAMPLE 3

Verify that $\triangle GHI \cong \triangle LJK$ given $x = 15$.



$GH = x + 4$
$= (15) + 4$
$= 19$
$= LJ$

So, $\overline{GH} \cong \overline{LJ}$.

$JK = 3x - 33$
$= 3(15) - 33$
$= 45 - 33$
$= 12$
$= HI$

So, $\overline{HI} \cong \overline{JK}$.

$LK = 2x - 22$
$= 2(15) - 22$
$= 30 - 22$
$= 8$
$= GI$

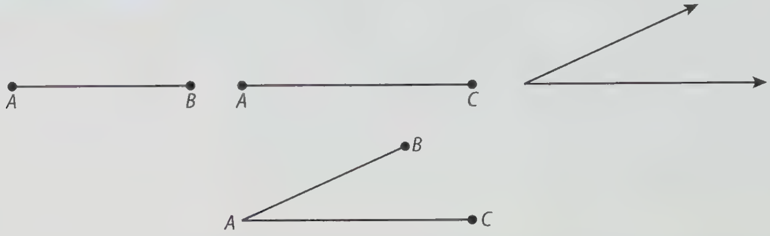
So, $\overline{GI} \cong \overline{LK}$.

So, $\triangle GHI \cong \triangle LJK$ by SSS congruence.

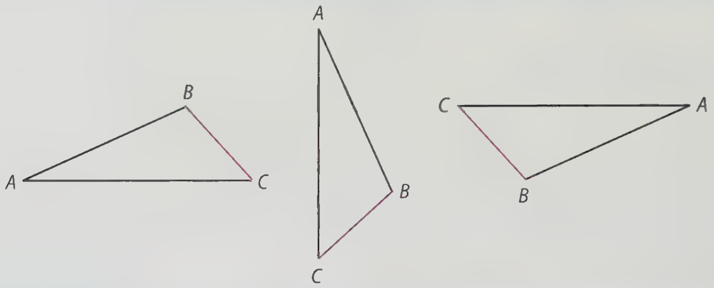
Side-Angle-Side Congruence

Another way to check that two triangles are congruent involves two sides and their *included angle*. An **included angle** is the angle that is formed by two consecutive sides.

The two segments given below can be put together to form the given angle.



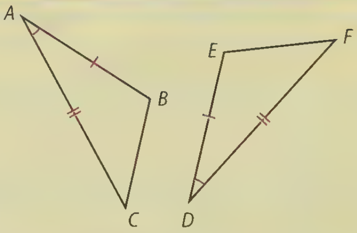
Regardless of the position of $\angle BAC$, to form a triangle, $\overline{BC}$ can have only one length.



Given any two segments and an included angle, it is possible to make only one triangle. The two sides and an included angle determine the size and shape of a triangle. This fact is stated in the following postulate.

Side-Angle-Side Congruence Postulate (SAS)

If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.



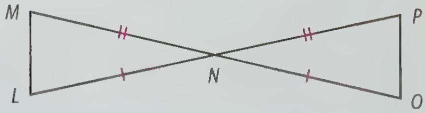
$\triangle ABC \cong \triangle DEF$

EXAMPLE 4

Write and justify a congruence statement that describes the two triangles.

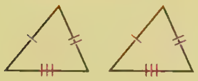
$\angle LNM \cong \angle ONP$ because they are vertical angles.
It is given that $\overline{LN} \cong \overline{ON}$ and $\overline{MN} \cong \overline{PN}$.

Two pairs of corresponding sides and their included angles are congruent, so $\triangle LNM \cong \triangle ONP$ by SAS congruence.

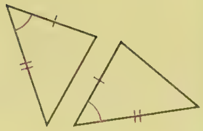


GOT TO KNOW!

Two Ways to Show Triangle Congruence



SSS

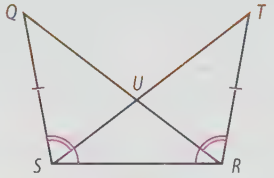


SAS

EXAMPLE 5

Given: $\overline{QS} \cong \overline{TR}$, $\angle QST \cong \angle TRQ$, $\angle TSR \cong \angle QRS$

Prove: $\triangle QRS \cong \triangle TSR$



Statements	Reasons
1. $\overline{QS} \cong \overline{TR}$	1. Given
2. $\angle QST \cong \angle TRQ$, $\angle TSR \cong \angle QRS$	2. Given
3. $m\angle QST = m\angle TRQ$, $m\angle TSR = m\angle QRS$	3. Definition of $\cong$ $\angle$s
4. $m\angle QST + m\angle TSR = m\angle TRQ + m\angle QRS$	4. Addition property of equality
5. $m\angle QSR = m\angle QST + m\angle TSR$, $m\angle TRS = m\angle TRQ + m\angle QRS$	5. Angle addition
6. $m\angle QSR = m\angle TRS$	6. Substitution
7. $\angle QSR \cong \angle TRS$	7. Definition of $\cong$ $\angle$s
8. $\overline{SR} \cong \overline{RS}$	8. Reflexive property of congruence
9. $\triangle QRS \cong \triangle TSR$	9. SAS congruence

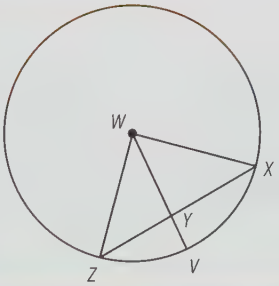
SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 6

Given: $\odot W$, $\overline{VW} \perp \overline{XZ}$

Prove: $\triangle WYZ \cong \triangle WYX$

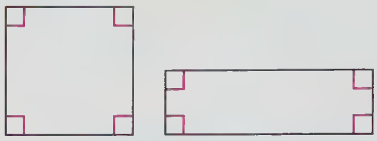


Statements	Reasons
1. $\overline{VW} \perp \overline{XZ}$	1. Given
2. $\overline{VW}$ bisects $\overline{XZ}$	2. If a radius is $\perp$ to a chord, it bisects the chord.
3. $\overline{YZ} \cong \overline{YX}$	3. Definition of bisects
4. $\angle WYZ$ and $\angle WYX$ are right angles	4. Definition of $\perp$
5. $\angle WYZ \cong \angle WYX$	5. All right $\angle$s are $\cong$.
6. $\overline{WY} \cong \overline{WY}$	6. Reflexive property of congruence
7. $\triangle WYZ \cong \triangle WYX$	7. SAS congruence

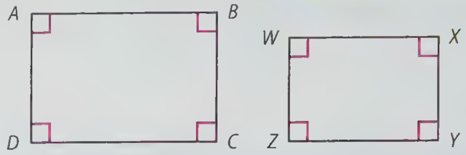
Similar Polygons

Similarity

Figures that have the exact same shape but not necessarily the same size are **similar**.



These figures are not the exact same shape, so they are not similar.

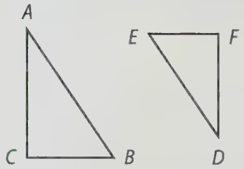


These figures are the exact same shape, so $ABCD \sim WXYZ$ (read $ABCD$ is similar to $WXYZ$).

Need More HELP

For help with proportions, go to *Equivalent Ratios and Proportions* in *Numbers and Operations*, p. 368.

Two figures are similar if and only if each pair of corresponding angles is congruent and each pair of corresponding sides is proportional.



Corresponding Sides

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

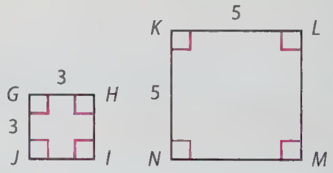
Corresponding Angles

$$\angle A \cong \angle D$$
$$\angle B \cong \angle E$$
$$\angle C \cong \angle F$$

$$\triangle ABC \sim \triangle DEF$$

As with congruence, the order in which the vertices are listed in a similarity statement indicates which sides and angles correspond to each other.

The ratio of the lengths of corresponding sides of similar figures is called the **similarity ratio**. The similarity ratio of $GHIJ$ to $KLMN$ is $\frac{3}{5}$. The similarity ratio of $KLMN$ to $GHIJ$ is $\frac{5}{3}$.



EXAMPLE 1

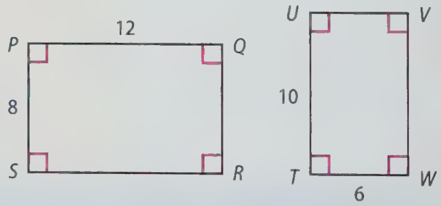
Determine whether the rectangles are similar.

All angles in rectangles are right angles, so the angles are congruent.

Now check that the sides are proportional.

$\frac{PS}{TW} = \frac{8}{6} = \frac{4}{3}$ $\quad \frac{PQ}{TU} = \frac{12}{10} = \frac{6}{5}$ $\quad \frac{4}{3} \neq \frac{6}{5}$

Corresponding sides are not proportional, so *PQRS* is not similar to *TUVW*.



GOT TO KNOW!

Characteristics of Similar Figures

The have the exact same shape.

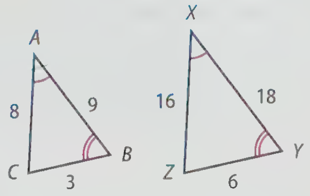
They may or may not be the same size.

Their corresponding angles are congruent.

Their corresponding sides are proportional.

EXAMPLE 2

Determine whether the triangles are similar.



It is given that $\angle A \cong \angle X$ and $\angle B \cong \angle Y$. This means that the third angles in the triangles must be congruent, so $\angle C \cong \angle Z$. So, all the corresponding angles are congruent.

Now check that the sides are proportional.

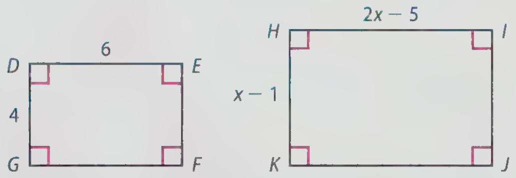
$$\frac{BC}{YZ} = \frac{3}{6} = \frac{1}{2} \qquad \frac{CA}{ZX} = \frac{8}{16} = \frac{1}{2} \qquad \frac{AB}{XY} = \frac{9}{18} = \frac{1}{2}$$

Corresponding sides are proportional with a similarity ratio of $\frac{1}{2}$. So, $\triangle ABC \sim \triangle XYZ$.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 3

Given *DEFG* ~ *HIJK*. Solve for *x*.



Since the rectangles are similar, their side lengths must be proportional.

$$\frac{DG}{HK} = \frac{DE}{HI}$$

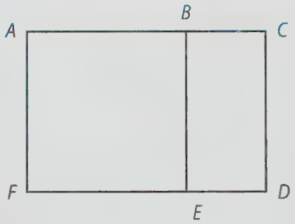
Substitute the given values in the proportion.

$$\frac{4}{x-1} = \frac{6}{2x-5}$$

Cross multiply to find the value for x.

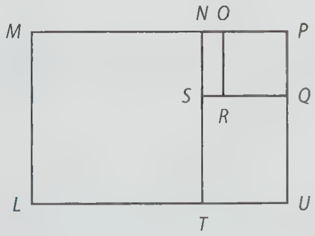
$$4(2x - 5) = 6(x - 1)$$
$$8x - 20 = 6x - 6$$
$$2x = 14$$
$$x = 7$$

A golden rectangle is a rectangle that can be divided into a square and a rectangle so that the rectangles are similar. The new, smaller rectangle is also a golden rectangle. All golden rectangles are similar to each other. In the diagram, *ACDF* ~ *EBCD*.



EXAMPLE 4

***MPUL* is a golden rectangle.**



a. Identify all similar rectangles.

MPUL, *TNPU*, *NPQS*, and *SNOR* are all golden rectangles, so they are all similar to each other.
MPUL ~ *TNPU* ~ *NPQS* ~ *SNOR*

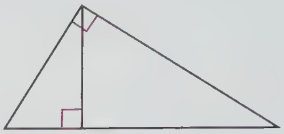
b. Identify all sides that correspond to $\overline{ML}$.

You can tell by looking at the similarity statements that $\overline{TU}$, $\overline{NS}$, and $\overline{SR}$ all correspond to $\overline{ML}$.

Self-Similarity

A figure is considered **self-similar** if it can be divided into parts that are similar to the entire figure. Some self-similar figures you have already learned about are shown below.

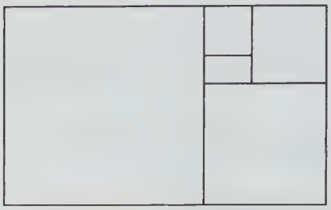
A right triangle with the altitude drawn to the hypotenuse is self-similar.



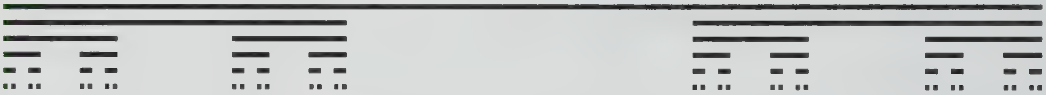
Need More HELP

For more information about golden rectangles, go to *Similar Polygons*, p. 790.

A golden rectangle divided into other golden rectangles is self-similar.

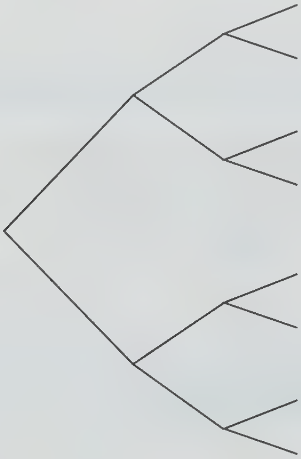


Self-similar figures are formed by a process called **iteration**, that is, repeatedly performing a rule. Below is an example of an iteration, the Cantor set, formed by removing the middle third of a segment.



You can see the same rule is applied to each new part of the figure during each iteration.

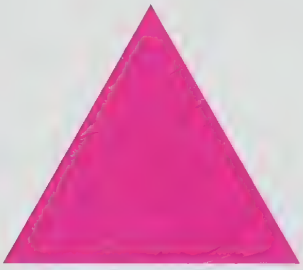
Another example you may be more familiar with is that of a tree diagram.



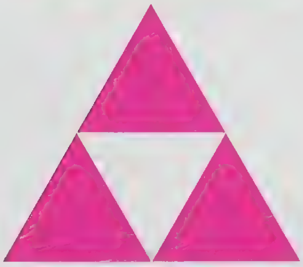
You start with a “V” shape. During each iteration, you add a “V” to the end of each branch.

A **fractal** is a self-similar figure made by iteration. A well-known example is the Sierpinski triangle. To make a Sierpinski triangle, follow the steps below.

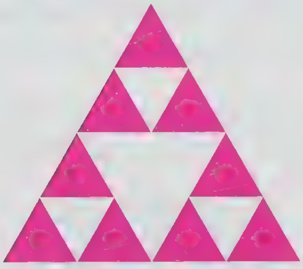
Start with an equilateral triangle.



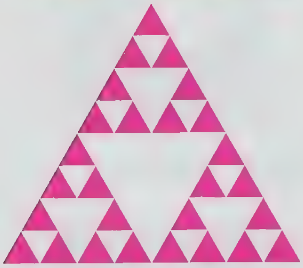
Remove the midsegment triangle. This is the first iteration.



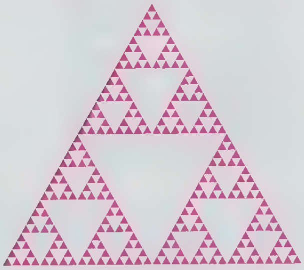
Now remove the midsegment triangle from each of the three newly formed triangles. This is the second iteration.



This is the third iteration.



Repeating the process two more times results in the following figure.



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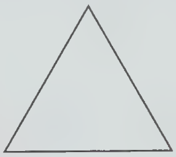
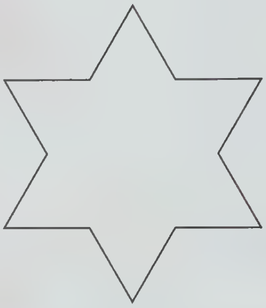
Forming Fractals

The Koch snowflake is another example of a fractal that begins with a triangle. To make a Koch snowflake, follow the steps below.

Start with an equilateral triangle.



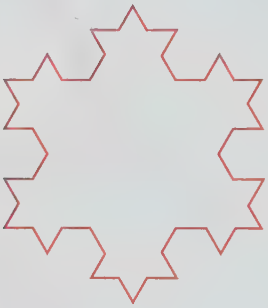
Add an equilateral triangle, one-third the size of the original, to the center of each side.



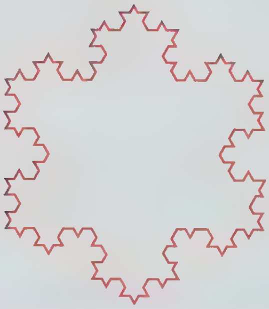
EXAMPLE 1

Draw the next two iterations of the Koch snowflake.

Starting with the first iteration, add a triangle to the center of each side to get the following figure.



Repeat the process to get the next iteration.



GOT TO KNOW!

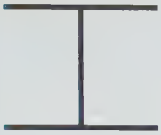
Characteristics of Fractals

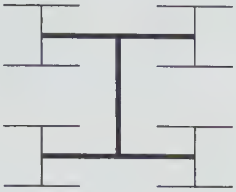
A small part is similar to the whole figure.

They are formed by a repeated process.

EXAMPLE 2

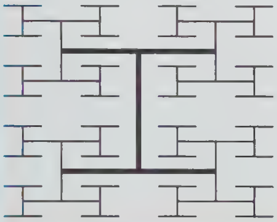
Describe the rule for the fractal given by the following figure and first iteration. Then draw the next two iterations.

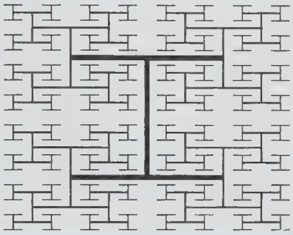




A sideways "H" is placed on each of the four corners at the midpoint of each new H.

This shows the second and third iterations.





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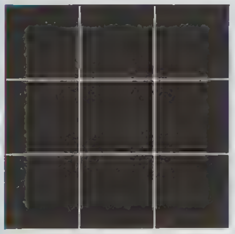
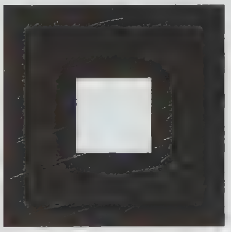
EXAMPLE 3

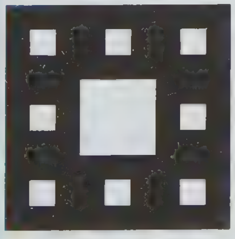
The Sierpinski carpet is a fractal formed by dividing a square into 9 congruent squares, removing the center square, and then repeating this process. Draw the first three iterations.



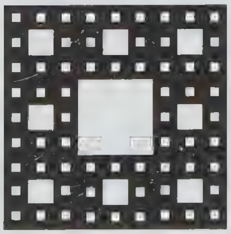
Start with a square.



The first iteration removes 1 center square.



The second iteration removes 8 center squares.



The third iteration removes 64 center squares.

Need More HELP

For the second iteration, do to each of the eight remaining squares what you did to the original square.

Rotations in the Coordinate Plane

When rotating a figure in the coordinate plane, this book assumes that the origin $O(0, 0)$ is the center of rotation. A rotation of 90° or −90° is a **quarter-turn**. A rotation of 180° or −180° is a **half-turn**. In a half-turn, the origin is the midpoint of each segment joining a point and its image. For quarter-turns and half-turns, you can use the rules in the *Got to Know!* box shown below to find the coordinates of the image points.

SEARCH

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EXAMPLE 3

Graph the image of $\triangle ABC$ after a half-turn about the origin.

METHOD 1

Use the fact that the origin O is the midpoint of $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$ to locate A', B', and C'. In the figure, $AO = OA'$, $BO = OB'$, and $CO = OC'$.

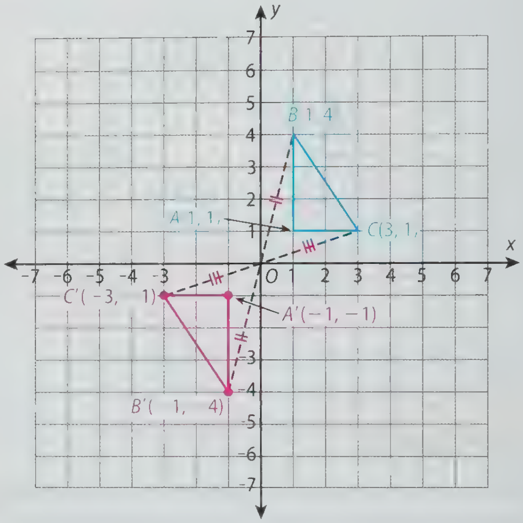
Draw $\triangle A'B'C'$.

METHOD 2

A half-turn is a rotation of 180° about the origin O. Use the rule $R_{O,180°}(x, y) \to (-x, -y)$ to find the coordinates of A', B', and C'.

Pre-image		Image
$A(1, 1)$	→	$A'(-1, -1)$
$B(1, 4)$	→	$B'(-1, -4)$
$C(3, 1)$	→	$C'(-3, -1)$

Draw $\triangle A'B'C'$.

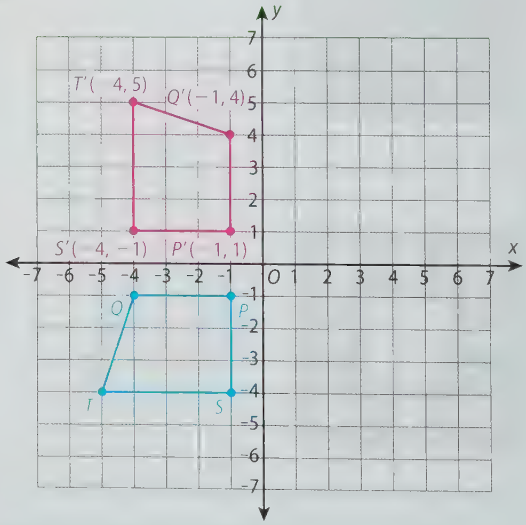


EXAMPLE 4

Graph quadrilateral $PQTS$ with vertices $P(-1, -1)$, $Q(-4, -1)$, $T(-5, -4)$, and $S(-1, -4)$. Find the coordinates of the vertices of the image after a rotation of −90°. Then graph the image.

A rotation of −90° is a clockwise quarter-turn. Use the rule $R_{O,-90°}(x, y) \to (y, -x)$ to find the coordinates of P', Q', T', and S'.

Pre-image		Image
$P(-1, -1)$	→	$P'(-1, 1)$
$Q(-4, -1)$	→	$Q'(-1, 4)$
$T(-5, -4)$	→	$T'(-4, 5)$
$S(-1, -4)$	→	$S'(-4, 1)$



GOT TO KNOW!

Half-turns

$R_{O,180°}(x, y) \to (-x, -y)$

$R_{O,-180°}(x, y) \to (-x, -y)$

Quarter-turns

$R_{O,90°}(x, y) \to (-y, x)$

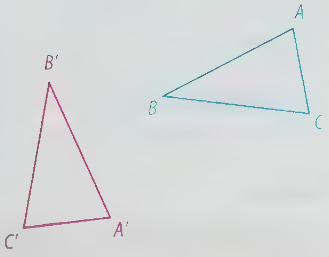
$R_{O,-90°}(x, y) \to (y, -x)$

Finding the Center of Rotation

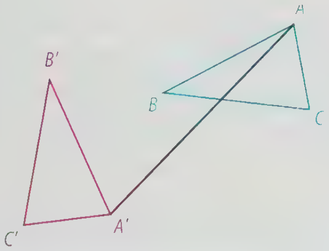
You can use the perpendicular-bisector construction to find the center of a rotation if you are given at least two points and their images.

EXAMPLE 5

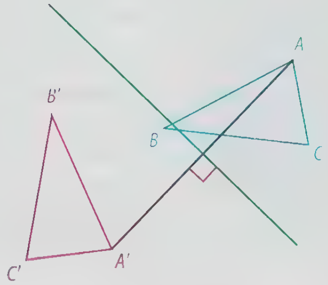
Find the center of rotation for this transformation.



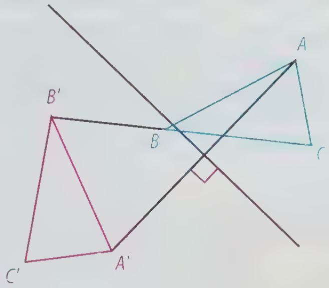
STEP 1 Draw $\overline{AA'}$.



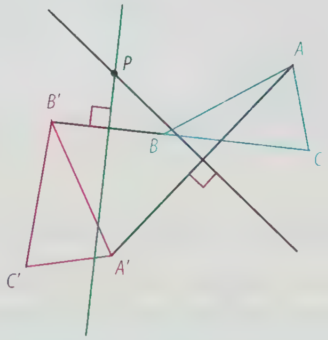
STEP 2 Construct the perpendicular bisector of $\overline{AA'}$.



STEP 3 Draw $\overline{BB'}$.



STEP 4 Construct the perpendicular bisector of $\overline{BB'}$. Label the intersection of the two perpendicular bisectors as point P. P is the center of rotation.



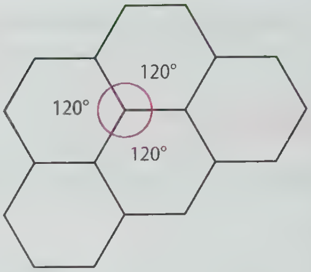
Need More HELP

For help with constructing the perpendicular bisector of a segment, go to *Perpendicular Bisector of a Segment* in *Constructions* (p. 864).

Tessellations

Regular Tessellations

A **tessellation** is an arrangement of shapes that cover the plane with no overlaps and no gaps. A tessellation is said to **tessellate** the plane. A tessellation consisting of congruent copies of a regular polygon is called a **regular tessellation**.



These hexagons form a regular tessellation. The measure of each interior angle of a regular hexagon is 120°. Notice that the sum of the measures of the interior angles at each vertex of the tessellation is 360°.

EXAMPLE 1

Which regular polygons can be used to form a regular tessellation?

STEP 1 Find the measures of the interior angles of regular polygons with 8 or fewer sides.

STEP 2 For each of the regular polygons in the table, divide 360° by the measure of the interior angles.

Number of Sides	Interior Angle	$\frac{360°}{\text{Interior Angle}}$
3	60°	6
4	90°	4
5	108°	$3\frac{1}{3}$
6	120°	3
7	$128\frac{4}{7}°$	2.8
8	135°	$2\frac{2}{3}$

A whole number of copies of the regular polygon must meet at each vertex of a tessellation to form a 360° angle; otherwise there will be a gap or overlap. So 360° divided by the measure of the interior angles of the polygon must be a whole number. The whole number tells how many of that polygon will meet at each vertex.

At least 3 regular polygons must meet at each vertex. If a regular polygon has more than 6 sides, then the measure of its interior angles is greater than 120°. So the sum of the measures at each vertex would be more than 360°. Therefore, the only regular polygons that can be used to form a regular tessellation are those with 3, 4, or 6 sides; in other words, equilateral triangles, squares, or regular hexagons.

Need More HELP

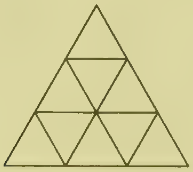
Remember—the formula for finding the measure of the interior angle of a regular polygon with n sides is: $\frac{180(n-2)}{n}$.

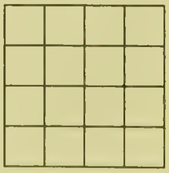
GOT TO KNOW!

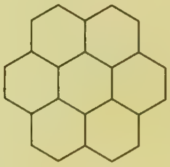
Tessellations

A tessellation is an arrangement of shapes that cover the plane with no overlaps and no gaps.

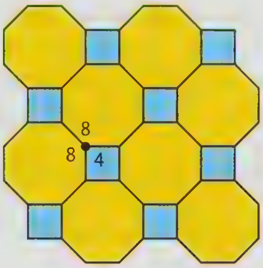
A tessellation of congruent copies of a regular polygon is called a regular tessellation. There are only three polygons that can form regular tessellations: equilateral triangles, squares, and hexagons. These diagrams show the way each type of shape can fit together to cover the whole plane.







Semi-Regular Tessellations



A tessellation is a **semi-regular tessellation** if it consists of more than one type of regular polygon and the arrangement of polygons is the same at each vertex. The tessellation shown here is a semi-regular tessellation.

A vertex of a tessellation can be described by the number of sides of the polygons that meet at the vertex. For example, the vertex arrangement of a tessellation of equilateral triangles, shown on the previous page, is 3-3-3-3-3-3 because there are six equilateral triangles that meet at each vertex. The vertex arrangement of the tessellation shown here is 4-8-8 because a square, a regular octagon, and another regular octagon meet at each vertex. To describe a vertex arrangement, start with the number of sides of the polygon with the fewest sides.

Need More HELP

For help with rotations, go to *Rotations* on pp. 842–845.

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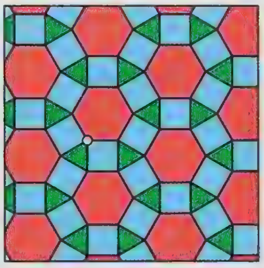
EXAMPLE 2

Describe the vertex arrangement of this semi-regular tessellation.

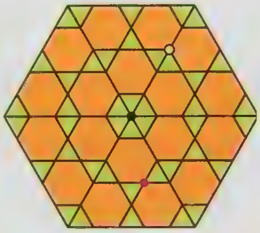


Since you know this is a semi-regular tessellation, you know that the arrangement of polygons is the same at each vertex. The triangle has the fewest sides, so the first number is 3. The next shape is a square, the next is a hexagon, and the last is another square.

So the vertex arrangement is 3-4-6-4.

EXAMPLE 3

Is this tessellation regular, semi-regular, or neither?



STEP 1 Decide whether the tessellation is regular.

All the shapes are regular congruent polygons, but there is more than one type of polygon, so the tessellation is not regular.

STEP 2 Decide whether the tessellation is semi-regular.

The arrangement of polygons is not the same at each vertex. The vertex arrangement at the vertex marked with the white dot is 3-3-6-6. The vertex arrangement at the vertex marked with the black dot is 3-3-3-3-3-3. The vertex arrangement at the vertex marked with the red dot is 3-6-3-6. So this is not a semi-regular tessellation.

This tessellation is neither regular nor semi-regular.

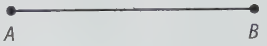
Congruent Segments

Constructing a Congruent Segment

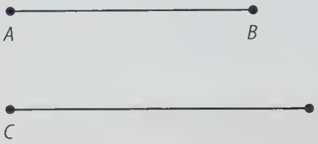
A **construction** is a geometric drawing that is made using only a straightedge and a compass. It is understood that you are also allowed to use a pencil or a pen. Anything that can be used to draw a straight line is a straightedge. When you are making a construction, you are not allowed to measure angles with a protractor or to measure lengths with a ruler.

EXAMPLE 1

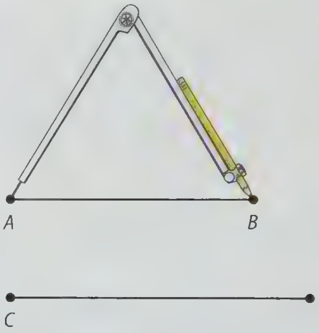
Construct a segment congruent to segment $\overline{AB}$.



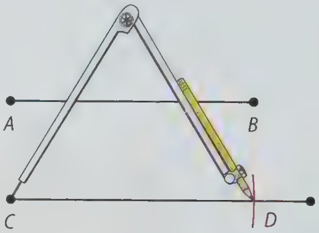
STEP 1 Use a straightedge to draw a segment longer than $\overline{AB}$. Label one of the endpoints of the new segment as C.



STEP 2 Set your compass at the length of $\overline{AB}$.



STEP 3 Keep the compass adjusted to the same radius, and place the compass point on C. Draw an arc intersecting the new segment. Label the point of intersection of the arc and the segment as D. Then $\overline{CD}$ is congruent to $\overline{AB}$.



The distance between the compass point and the pencil point in Steps 2 and 3 is the length of the original segment, so the measure of the original segment and the measure of the constructed segment are the same.

Try It This Way

Draw a segment EF. Measure its length.

Use a ruler to draw another segment the same length as segment EF.

Construct a third segment congruent to segment EF using a straightedge and compass.

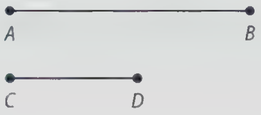
Compare the segment you drew by measuring with a ruler with the segment you drew using a straightedge and compass.

Adding Segments

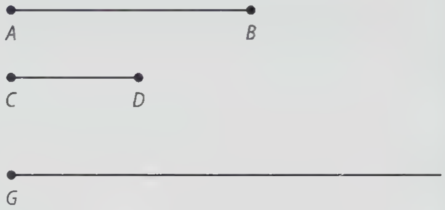
If you know how to construct congruent segments, you can do other constructions.

EXAMPLE 2

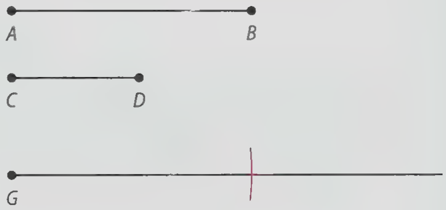
Construct a segment the length of which is equal to the sum of the lengths of $\overline{AB}$ and $\overline{CD}$.



STEP 1 Use a straightedge to draw a segment that is longer than the sum of the lengths of $\overline{AB}$ and $\overline{CD}$. Label one of the endpoints of the segment as G.



STEP 2 Set the compass at the length of $\overline{AB}$. Put the compass point on G. Draw an arc with radius AB intersecting the new segment.

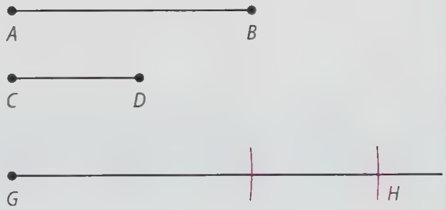


STEP 3 Set the compass radius at the length of $\overline{CD}$.

Place the compass point on the point of intersection of the arc and the segment.

Draw a new arc intersecting the segment as shown.

Label the point where the new arc intersects the segment as H.



The length of $\overline{GH}$ is equal to the sum of the lengths of $\overline{AB}$ and $\overline{CD}$. In symbols, $GH = AB + CD$.

SEARCH

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Congruent Angles

Need More HELP

Remember that a ray is a part of a line that consists of a point and all points on the line that are on one side of that point. That point is called the endpoint of the ray.

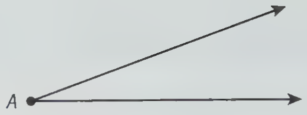
To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Copying an Angle

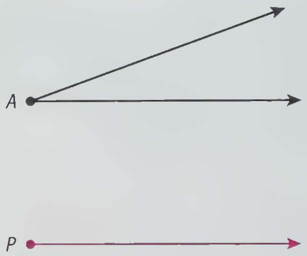
You can copy any angle using a compass and straightedge.

EXAMPLE 1

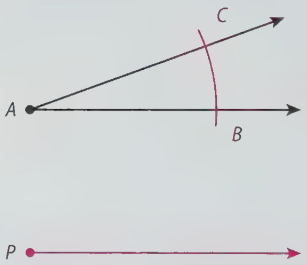
Construct an angle congruent to angle *A*.



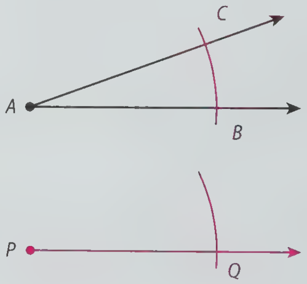
STEP 1 Use a straightedge to draw a ray. Label the endpoint *P*.



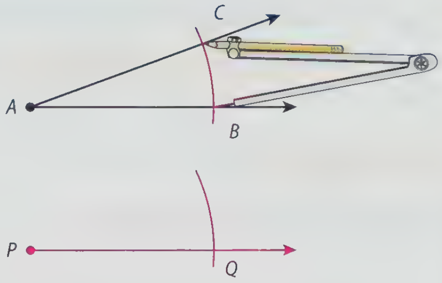
STEP 2 Set the radius of the compass at any convenient length. Place the compass point on *A*. Draw an arc that intersects both rays of the angle. Label the points where the arc crosses the rays as *B* and *C*.



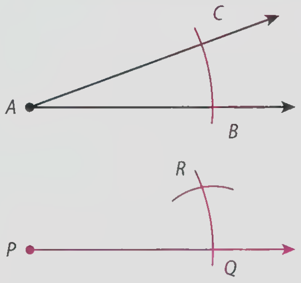
STEP 3 Without changing the compass setting, place the point of the compass on *P* and draw an arc. Label the point where the arc crosses the ray as *Q*.



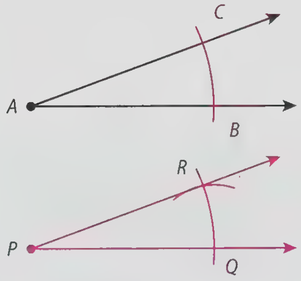
STEP 4 Place the point of the compass on *B*, and set it so the pencil point is on *C*.



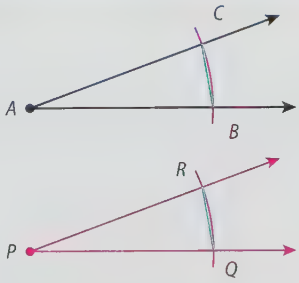
STEP 5 Without changing the compass setting, place the point of the compass on *Q* and draw an arc intersecting the first arc. Label the point where the arcs intersect *R*.



STEP 6 Use a straightedge to draw a ray with endpoint *P* that passes through *R*.



Angle *P* is congruent to angle *A*.



Form triangles by drawing a segment connecting *B* and *C*, and a segment connecting *Q* and *R*. You can prove that angle *P* is congruent to angle *A* by using SSS, the Side-Side-Side Congruence Postulate.

$\overline{AB} \cong \overline{PQ}$ because the radius of the arc with its center at *A* is the same as the radius of the arc with its center at *P*. For a similar reason, $\overline{AC} \cong \overline{PR}$. Because of the way point *R* was obtained, $\overline{BC} \cong \overline{QR}$. Therefore, $\triangle CAB \cong \triangle RPQ$. $\angle A$ and $\angle P$ are corresponding angles. Hence $\angle A \cong \angle P$.

Need More HELP

For help with using the Side-Side-Side Congruence Postulate, see *Using SSS and SAS* on page 768.

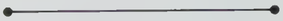
Perpendicular Bisector of a Segment

Constructing a Perpendicular Bisector

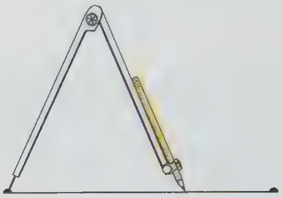
A **perpendicular bisector of a segment** is a line, ray, or segment that is perpendicular to the given segment at its midpoint.

EXAMPLE 1

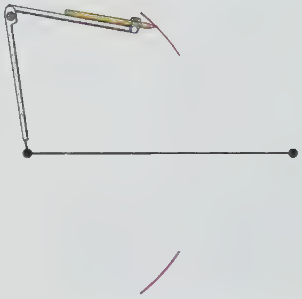
Construct a perpendicular bisector of the segment on the right.



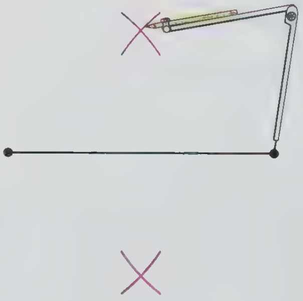
STEP 1 Place the compass on one endpoint of the segment. Open the compass to about two-thirds the length of the segment.



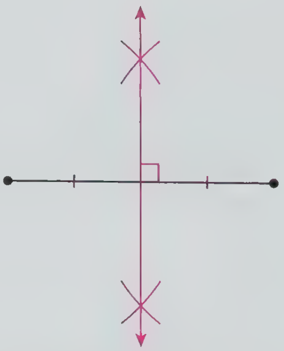
STEP 2 Draw an arc on each side of the segment.



STEP 3 Without changing the compass setting, place the point on the other endpoint of the segment and draw two arcs that intersect the first arcs.



STEP 4 Use a straightedge to draw a line through the two points where the arcs intersect.



The line is perpendicular to the segment and intersects the segment at its midpoint.

Need More HELP

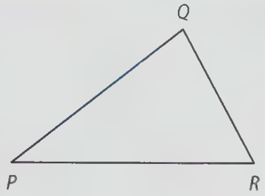
Remember that the midpoint of a segment divides a segment into two congruent parts.

Constructing the Median of a Triangle

A **median of a triangle** is a line segment connecting a vertex of the triangle with the midpoint of the side opposite that vertex. Each triangle has three medians.

EXAMPLE 2

Construct a median of this triangle.

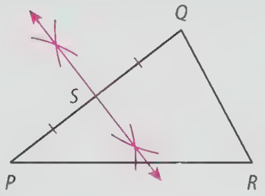


STEP 1 Choose a side of the triangle and construct a perpendicular bisector of that side, following the steps of Example 1.

In this example, a perpendicular bisector of $\overline{PQ}$ is constructed.

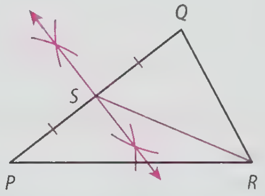
The point where the segment crosses $\overline{PQ}$ is its midpoint.

Label the midpoint S.



STEP 2 Use a straightedge to draw a segment from S to the angle opposite side $\overline{PQ}$, angle R.

Segment RS is a median of triangle PQR.



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Segment RS is one of the three medians of triangle PQR. You can construct the other two medians by following Steps 1–2 for each of the other two sides of the triangle.

The medians of a triangle have a special property, which you will explore in Example 3.

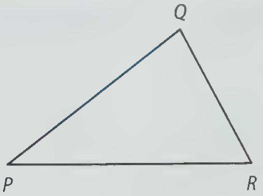
Perpendicular Bisector of a Segment

Constructing the Centroid of a Triangle

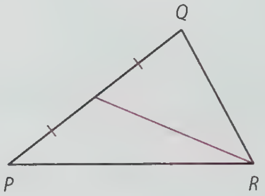
The three medians of a triangle meet at a single point. The point is called the **centroid** of the triangle. The centroid is the balancing point of the triangle.

EXAMPLE 3

Find the centroid of this triangle.

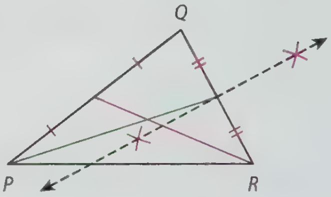


STEP 1 Construct the median to one side of the triangle. Example 2 shows how to construct the median to side $\overline{PQ}$.

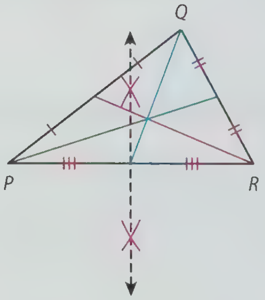


STEP 2 Construct the median to another side of the triangle.

The point where the two medians intersect is the centroid of the triangle. It is not necessary to draw all three medians to find the centroid. Continue with Step 3 to illustrate that all three medians meet at the centroid.



STEP 3 Construct the median to the remaining side of the triangle.

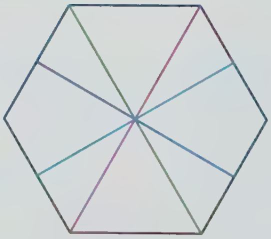


The centroid of the triangle is the point where the medians of the three sides of the triangle meet.

Try It This Way

Draw a triangle on paper using a straightedge. Construct the centroid of the triangle using a compass and straightedge. Cut out the triangle and trace its outline on cardboard or stiff paper. Cut along the outline to make a model of the triangle. Find and mark the point on which the triangular model will balance on the tip of a pencil. Compare the centroid of the triangle with the point on which the triangular model balances.

Three or more lines in the same plane that intersect at the same point are called **concurrent lines**. The point of intersection of the lines is called the **point of concurrency**. The centroid of a triangle is one example of a point of concurrency. Here is another example of concurrent lines. The colored lines meet at the point of concurrency.



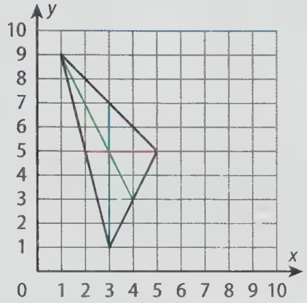
Distance of the Centroid from the Vertices

EXAMPLE 4

The figure shows the medians of the triangle.
For each median, find this ratio:

$$\frac{\textbf{Distance from the vertex to the centroid}}{\textbf{Length of the median}}$$

What conclusion can you draw?



STEP 1 Find the ratio for the blue median.

$$\frac{4\text{ units}}{6\text{ units}} = \frac{2}{3}$$

STEP 2 Find the ratio for the red median.

$$\frac{2\text{ units}}{3\text{ units}} = \frac{2}{3}$$

STEP 3 Find the ratio for the green median.

Use the distance formula. The vertex is at (1, 9), the centroid is at (3, 5), and the midpoint of the opposite side is at (4, 3). The distance from the vertex to the centroid is given by

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-3)^2 + (9-5)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

The length of the median is given by

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-4)^2 + (9-3)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

The ratio for the green median is given by

$$\frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}$$

Therefore, the centroid of the triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.

SEARCH

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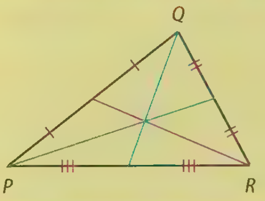
The property that is true for the triangle in Example 4 is true for every triangle: In any triangle the medians intersect at a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side.

GOT TO KNOW!

Location of Centroids

The centroid of any triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.



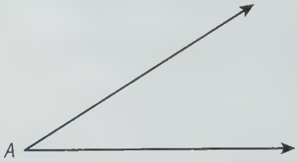
Angle Bisector

Bisecting an Angle

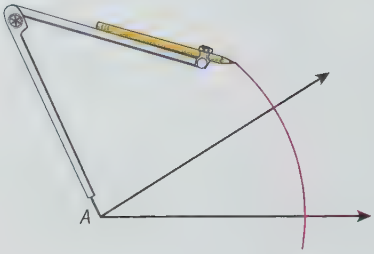
An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent.

EXAMPLE 1

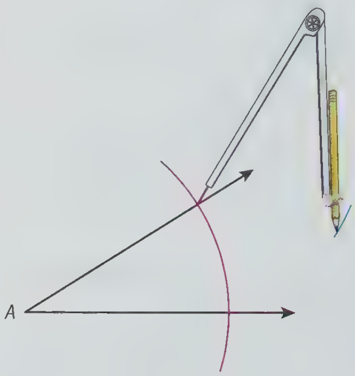
Use a compass and a straightedge to bisect angle *A*.



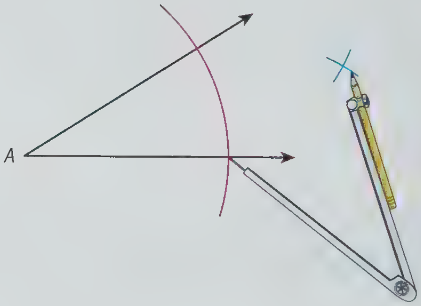
STEP 1 Place the compass point on vertex *A*. Draw an arc that intersects both rays.



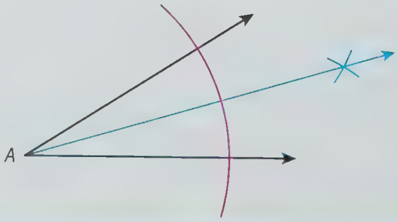
STEP 2 Move the compass point to the intersection of one of the rays and the arc. Draw an arc on the interior of the angle.



STEP 3 Without changing the compass setting, move the point of the compass to the other point of intersection of the other ray and the arc. Draw another arc that intersects the arc that you drew in Step 2.



STEP 4 Use a straightedge to draw a ray from vertex *A* through the point where the two arcs intersect. The ray is the angle bisector of angle *A*.



Try It This Way

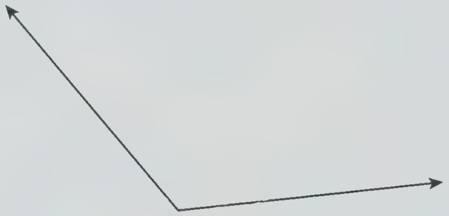
Place a piece of paper over an angle to be bisected. The paper should be thin enough so you can see the angle. Trace the angle. Fold the paper so that the two rays overlap. The fold line will be the angle bisector.

Dividing an Angle into Four Congruent Parts

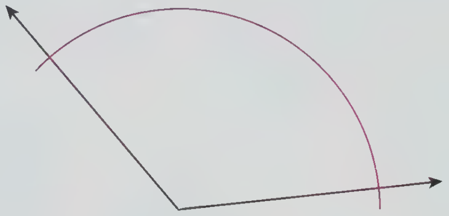
You can divide an angle into four congruent angles by bisecting the angle and then bisecting each of the resulting angles, using the method in Example 1.

EXAMPLE 2

Use a compass and straightedge to divide this angle into four congruent angles.

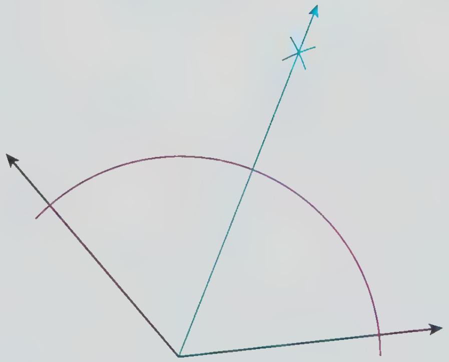


STEP 1 Put the point of the compass on the vertex and draw an arc that crosses both rays.

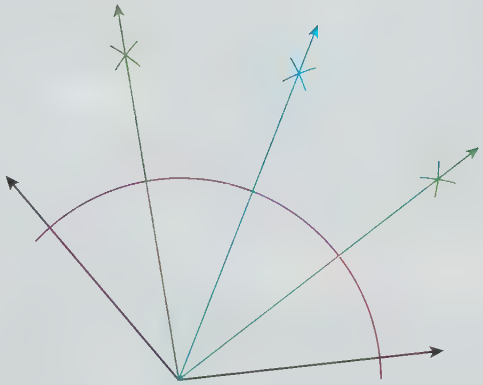


STEP 2 Draw arcs from the intersection of the first arc and each ray to construct an angle bisector as shown in Example 1.

The angle is now divided into two congruent angles.



STEP 3 Bisect each of the two congruent angles using the same process.



The angle is now divided into four congruent angles.

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Trigonometry

Makers of amusement park rides have to meet safety codes in angular velocity that depend on the radius of the ride.

Right Triangle Trigonometry 892

Radian Measure and the Trigonometric Functions 926

Graphs of the Trigonometric Functions 948

Trigonometric Identities 994

Inverse Trigonometric Functions and Trigonometric Equations 1020

Oblique Triangles and Vectors 1046

Polar Coordinates and the Complex Plane 1090

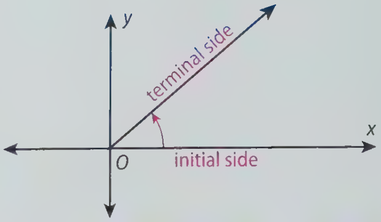
Angles and Degree Measure

Need More HELP

For a review of classifying angles, go to *Classifying Angles* and *Special Pairs of Angles* in *Foundations of Mathematics* (pp. 140, 146).

Angles

A basic figure in trigonometry is the angle. In geometry, you learned that an angle is the union of two rays with a common endpoint. In trigonometry, it is helpful to define an angle differently: An **angle** is formed by rotating a ray around its endpoint. The endpoint is at the origin and is the **vertex** of the angle. The **initial side** of the angle is the side that lies on the x-axis. The other side of the angle is the **terminal side**.



You can classify an angle by its measure. For example, a 90° angle is a **right angle**. An **acute angle** has measure greater than 0° and less than 90°. An **obtuse angle** has measure greater than 90° and less than 180°. If the measure of an angle is 180°, the angle is a **straight angle**.

Pairs of angles also have special names. **Complementary angles** are two angles the sum of whose measures is 90°. **Supplementary angles** are two angles the sum of whose measures is 180°.

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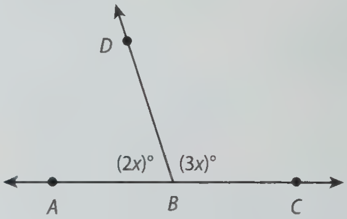
EXAMPLE 1

Find $m\angle ABD$ and $m\angle CBD$. Then classify each angle.



STEP 1 Since $\angle ABC$ is a straight angle, its measure is 180°. Write and solve an equation to find x.

Write the equation. $2x + 3x = 180$

Combine like terms. $5x = 180$

Divide both sides by 5. $x = 36$

STEP 2 Find the measures of the two angles.

$$m\angle ABD = 2x \qquad m\angle CBD = 3x$$
$$= 2(36) \qquad = 3(36)$$
$$= 72° \qquad = 108°$$

STEP 3 Classify the angles by their measures.

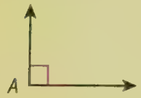
$\angle ABD$ is acute. $\angle CBD$ is obtuse.

GOT TO KNOW!

Classifying Angles and Angle Pairs

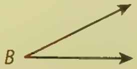
Right angle

$m\angle A = 90°$



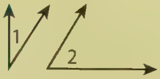
Acute angle

$0° < m\angle B < 90°$



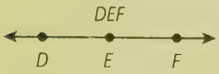
Complementary angles

$m\angle 1 + m\angle 2 = 90°$



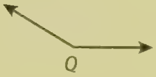
Straight angle

$m\angle DEF = 180°$



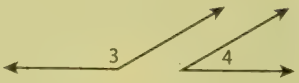
Obtuse angle

$90° < m\angle Q < 180°$



Supplementary angles

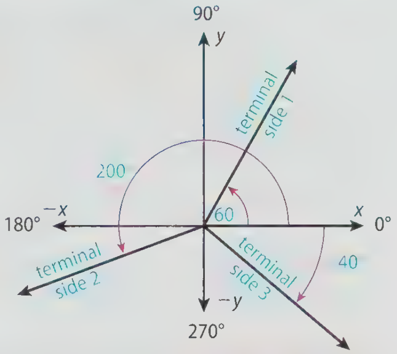
$m\angle 3 + m\angle 4 = 180°$



Angles in Standard Position

An angle is in **standard position** if the vertex is at the origin and the initial side lies on the positive *x*-axis. An angle in standard position is said to lie in the quadrant that contains its terminal side. Remember that the *x*- and *y*-axes divide the coordinate plane into four quadrants; I, II, III, and IV, labeled counterclockwise starting at the top right.

A counterclockwise rotation determines a **positive** angle. If the rotation is clockwise, the angle is **negative**. The figure shows angles of 60°, 200°, and −40°.



If the terminal side of an angle lies on the *x*-axis or the *y*-axis, the angle is a **quadrantal** angle. The measure of a quadrantal angle is always a multiple of 90°. Two angles in standard position that have the same terminal side are **coterminal**. Every angle has infinitely many coterminal angles.

EXAMPLE 2

Find two angles, one positive and one negative, that are coterminal with a 55° angle.

To find a positive angle, add a multiple of 360°.

A positive angle that is coterminal with a 55° angle is 55° + 360° = 415°.

To find a negative angle, subtract a multiple of 360°.

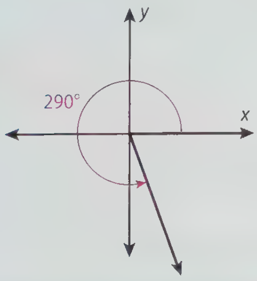
A negative angle that is coterminal with a 55° angle is 55° − 360° = −305°.

EXAMPLE 3

Sketch each angle in standard position.

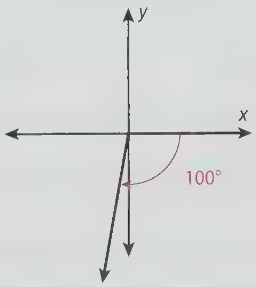
a. 290°

Rotate the ray 290° counterclockwise.



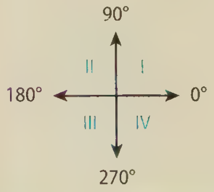
b. −100°

Rotate the ray 100° clockwise.



Ways to REMEMBER

To help remember the degree measures for the quadrants, you may want to sketch a small diagram of a coordinate system.



Computing with Angle Measures

Try It This Way

Graphing calculators and many scientific calculators can convert back and forth between DMS (degrees/minutes/seconds) and decimal degrees. Make sure your calculator is set on degree mode and not radian mode.

Parts of a Degree

When stating an angle measurement, it is important to be precise. Angle measures may be given in **decimal degrees** or in **DMS** (degrees/minutes/seconds) notation. DMS is based on the fact that there are 360 degrees in a circle. Each degree of an angle is divided into 60 **minutes** and each minute is divided into 60 **seconds**. The symbols for minutes and seconds are ' and ", respectively.

minutes: $1° = 60'$ $\quad 1' = \frac{1}{60}°$ $\qquad$ seconds: $1' = 60''$ $\quad 1'' = \frac{1}{60}' = \frac{1}{3600}°$

For example, 20°30' = 20.5°. Both ways of recording angle measures are now used, so you need to be able to convert from one type of measure to the other.

EXAMPLE 1

Convert 35.4° to degrees/minutes/seconds.

STEP 1 Multiply the decimal part of the angle measure by 60 to get minutes.

$0.4(60') = 24'$

STEP 2 Add.

$35° + 24' = 35°24'$

$35.4° = 35°24'$

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EXAMPLE 2

Convert 48°50'22" to decimal degrees. Round to three decimal places.

STEP 1 Write the minutes and seconds as fractions.

$48°50'22'' = 48° + \left(\frac{50}{60}\right)° + \left(\frac{22}{3{,}600}\right)°$

STEP 2 Change the fractions to decimals.

$48°50'22'' \approx 48° + 0.833° + 0.006°$

$48°50'22'' \approx 48.839°$

EXAMPLE 3

Convert 24.683° to degrees/minutes/seconds.

STEP 1 Multiply the decimal part of the angle measure by 60 to get minutes.

$0.683(60') = 40.98'$

STEP 2 Multiply the decimal part of the minutes by 60 to get seconds.

$0.98(60'') = 58.8''$

STEP 3 Add.

$24° + 40' + 58.8''$

$24.683° = 24°40'58.8''$

Computing with Angle Measures

Adding and subtracting angle measures given in degrees, minutes, and seconds may involve renaming.

EXAMPLE 4

Find the sum $47^\circ 35' + 70^\circ 48'$.

STEP 1 Vertically align the angle measures. Then add the degrees and minutes separately.

$$\begin{array}{r} 47^\circ 35' \\ +\ 70^\circ 48' \\ \hline 117^\circ 83' \end{array}$$

STEP 2 Rename 83'. Change 83' to degrees and minutes.

$$\begin{aligned} 83' &= 60' + 23' \\ &= 1^\circ 23' \end{aligned}$$

STEP 3 Add.

$$117^\circ + 1^\circ 23' = 118^\circ 23'$$

$47^\circ 35' + 70^\circ 48' = 118^\circ 23'$

Watch Out !

Be careful not to confuse DMS notation with decimal degrees.

The sum $47^\circ 35' +$ $70^\circ 48'$ is *not* equal to 117.83°.

EXAMPLE 5

Find the difference $180^\circ - 113^\circ 28'$.

STEP 1 Rename 180°.

$$\begin{aligned} 180^\circ &= 179^\circ + 1^\circ \\ &= 179^\circ 60' \end{aligned}$$

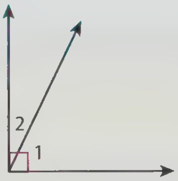
STEP 2 Vertically align the angle measures. Subtract the degrees and minutes separately.

$$\begin{array}{r} 179^\circ 60' \\ -\ 113^\circ 28' \\ \hline 66^\circ 32' \end{array}$$

$180^\circ - 113^\circ 28' = 66^\circ 32'$

EXAMPLE 6

Find $m\angle 2$ if $m\angle 1 = 64^\circ 37' 17''$.



STEP 1 The angles are complementary, so the sum of their measures is 90°. Write an equation.

$$m\angle 1 + m\angle 2 = 90^\circ, \text{ or } m\angle 2 = 90^\circ - m\angle 1$$

STEP 2 Rename 90°: $90^\circ = 89^\circ 59' 60''$

STEP 3 To find $m\angle 2$, subtract $m\angle 1$ from 90°.

$$\begin{array}{r} 89^\circ 59' 60'' \\ -\ 64^\circ 37' 17'' \\ \hline 25^\circ 22' 43'' \end{array}$$

$m\angle 2 = 25^\circ 22' 43''$

Special Triangles and Angles

45°–45°–90° Triangles

Every angle corresponds to a fractional part of a circle. Recall, there are 360 degrees in a circle.

Degrees	30°	45°	60°	90°	120°	135°	180°
Fractional Part of a Circle	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$

Need More HELP

In a right triangle, the hypotenuse is the side opposite the right angle. For more review of the Pythagorean Theorem, go to *Pythagorean Theorem* in *Geometry* (p. 678).

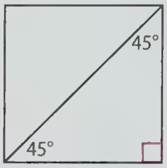
Because angles whose measures are multiples of 30 and 45 appear so frequently in trigonometry, you should know some facts about them. In particular, it is very helpful to know the properties of two special right triangles, the 45°–45°–90° triangle and the 30°–60°–90° triangle.

When you draw a diagonal of a square, you create two congruent 45°–45°–90° triangles, also known as isosceles right triangles. If each leg has length x and the hypotenuse has length c, you can use the Pythagorean Theorem to solve for c in terms of x.

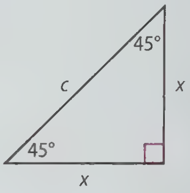
$$x^2 + x^2 = c^2$$

$$2x^2 = c^2$$

$$x\sqrt{2} = c$$



square



45° – 45° – 90° triangle

All isosceles right triangles are similar. In every 45°–45°–90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

EXAMPLE 1

Find the value of *h*. Round your answer to the nearest tenth.

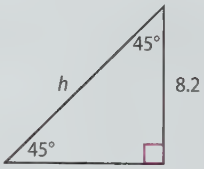
Use the properties of a 45°–45°–90° triangle.

Hypotenuse = $\sqrt{2}$ • leg　　$h = \sqrt{2}(8.2)$

Use a calculator and round to the nearest tenth.　　$h \approx (1.41421)(8.2)$

≈ 11.6

$h \approx 11.6$



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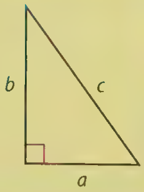
GOT TO KNOW!

The Pythagorean Theorem

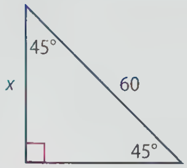
In a right triangle with legs of length a and b and hypotenuse of length c, $a^2 + b^2 = c^2$.



EXAMPLE 2

Find the exact value of x.

Use the properties of a 45°–45°–90° triangle.



Hypotenuse = $\sqrt{2}$ • leg $\quad 60 = x\sqrt{2}$

Divide both sides by $\sqrt{2}$. $\quad x = \frac{60}{\sqrt{2}}$

Rationalize the denominator. $\quad x = \frac{60}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

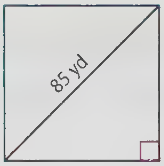
Simplify. $\quad x = 30\sqrt{2}$

Watch Out!

If a problem asks for an "exact answer," leave the square root in your answer and write the square root in simplified form.

EXAMPLE 3

A walking path through a square park is 85 yd long. Find the dimensions of the park.



METHOD 1

Use the Pythagorean Theorem.

Let a = length of each side of the square. $\quad a^2 + a^2 = 85^2$

Simplify. $\quad 2a^2 = 7{,}225$

Divide both sides by 2. $\quad a^2 = 3{,}612.5$

Use a calculator. Round to the nearest tenth. $\quad a \approx 60.1$

METHOD 2

Use the properties of a 45°–45°–90° triangle.

Hypotenuse = $\sqrt{2}$ • leg $\quad 85 = a\sqrt{2}$

Divide both sides by $\sqrt{2}$. $\quad a = \frac{85}{\sqrt{2}}$

Use a calculator. Round to the nearest tenth. $\quad a \approx 60.1$

The park is about 60.1 yd wide and 60.1 yd long.

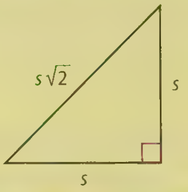
Need More HELP

For a review of rationalizing the denominator, go to *Dividing Radical Expressions* in *Advanced Algebra* (Book 2, p. 1962).

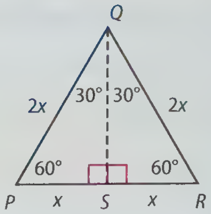
GOT TO KNOW!

45°–45°–90° Triangle Theorem

In a 45°–45°–90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.



30°–60°–90° Triangles



The measure of each angle of an equilateral triangle is 60° and the side lengths are congruent. The bisector of $\angle PQR$ is also the perpendicular bisector of $\overline{PR}$. $\triangle QPS$ is a 30°–60°–90° triangle.

You can use the Pythagorean Theorem to find an expression for QS in terms of x.

Use the Pythagorean Theorem. $PS^2 + QS^2 = PQ^2$

Substitute. $x^2 + QS^2 = (2x)^2$

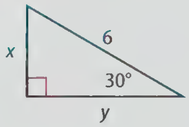
Simplify. $QS^2 = 3x^2$

Solve for QS. $QS = x\sqrt{3}$

All 30°–60°–90° triangles are similar. In every 30°–60°–90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times the shorter leg.

EXAMPLE 4

Find the exact values of *x* and *y*.



STEP 1 Find the value of x.

2 • shorter leg = hypotenuse $2x = 6$

Solve for x. $x = 3$

STEP 2 Find the value of y.

longer leg = $\sqrt{3}$ • shorter leg $y = x\sqrt{3}$

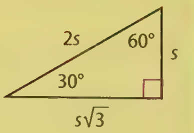
Substitute 3 for x. $y = 3\sqrt{3}$

Watch Out!

The shorter leg in a 30°–60°–90° triangle is half the length of the hypotenuse. It is not half the length of the longer leg.

GOT TO KNOW!

30°–60°–90° Triangle Theorem

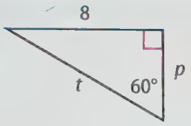


In a 30°–60°–90° triangle,

- The hypotenuse is twice the length of the shorter leg.
- The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

EXAMPLE 5

Find the exact values of *p* and *t*.



STEP 1 Find the length of the shorter leg.

$\sqrt{3}$ • shorter leg = longer leg $\quad \sqrt{3} \cdot p = 8$

Divide both sides by $\sqrt{3}$. $\quad p = \frac{8}{\sqrt{3}}$

Rationalize the denominator. $\quad p = \frac{8\sqrt{3}}{3}$

STEP 2 Find the hypotenuse.

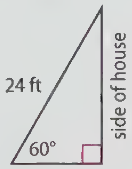
hypotenuse = 2 • shorter leg $\quad t = 2 \cdot \frac{8\sqrt{3}}{3}$

Simplify. $\quad t = \frac{16\sqrt{3}}{3}$

EXAMPLE 6

A 24-ft ladder leans against the side of a house. The ladder makes a 60° angle with the ground. How far up the side of the house does the ladder reach?

STEP 1 Make a sketch.



STEP 2 Find the length of the shorter leg.

Divide the hypotenuse by 2. $\quad \frac{24}{2} = 12$

STEP 3 Find the length of the longer leg.

Multiply the shorter leg by $\sqrt{3}$. $\quad 12\sqrt{3}$

Use a calculator. $\quad 12\sqrt{3} \approx 20.8$

The ladder reaches about 20.8 ft up the side of the house.

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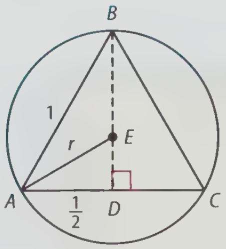
EXAMPLE 7

An equilateral triangle with sides 1 in. long is inscribed in circle *E*. Find the radius of the circle.



Since $\overline{BD}$ bisects $\angle ABC$, $\triangle BAD$ is a 30°–60°–90° triangle.

Since $\overline{BD}$ bisects $\overline{AC}$, $AD = \frac{1}{2}$. Radius EA bisects $\angle BAD$, so $m\angle EAD = 30°$ and $\triangle AED$ is also a 30°–60–90° triangle. Let $EA = r$. Then $ED = \frac{r}{2}$.

Use the Pythagorean Theorem. $\quad \left(\frac{1}{2}\right)^2 + \left(\frac{r}{2}\right)^2 = r^2$

Simplify. $\quad \frac{1}{4} + \frac{r^2}{4} = r^2$

Multiply both sides by 4. $\quad 1 + r^2 = 4r^2$

Solve for *r*. $\quad r = \frac{\sqrt{3}}{3}$

The radius of the circle is $\frac{\sqrt{3}}{3}$.

Need More HELP

For more review of special triangles, go to *Special Right Triangles in Geometry* (p. 684).

Trigonometric Ratios

Sine, Cosine, and Tangent Ratios

The Pythagorean Theorem relates the lengths of the sides of a right triangle. So does *trigonometry*. The word **trigonometry** comes from the Greek language and means "triangle measurement."

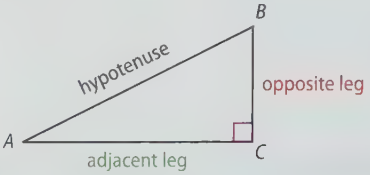
The ratios of side lengths in a right triangle have special names: *sine*, *cosine*, and *tangent ratios*. The definitions are given below.

Definition

sine of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{hypotenuse}}$

cosine of $\angle A = \dfrac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$

tangent of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$



Short Form

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

Ways to REMEMBER

Use SOH CAH TOA to remember the definitions of sine, cosine, and tangent.

SOH stands for "Sine: Opposite over Hypotenuse."

CAH stands for "Cosine: Adjacent over Hypotenuse."

TOA stands for "Tangent: Opposite over Adjacent.

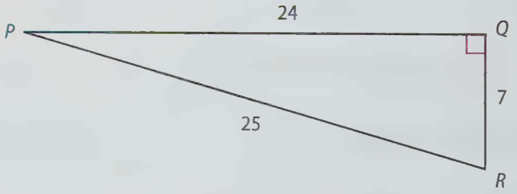
EXAMPLE 1

Write sin *P*, cos *P*, and tan *P* as ratios.

$\sin P = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{7}{25}$

$\cos P = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{24}{25}$

$\tan P = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{7}{24}$



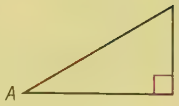
GOT TO KNOW!

Trigonometric Ratios

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

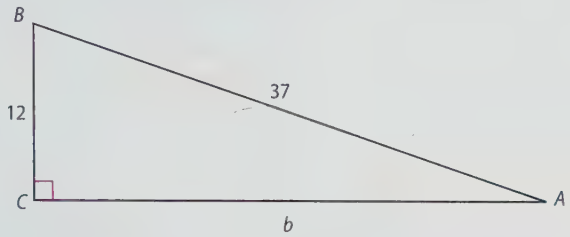
$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$



EXAMPLE 2

Find the sine, cosine, and tangent of $\angle A$.
Round your answers to the nearest hundredth.



STEP 1 Use the Pythagorean Theorem to find the value of b.

Pythagorean Theorem $12^2 + b^2 = 37^2$

Simplify. $144 + b^2 = 1{,}369$

$b^2 = 1{,}225$

$b = 35$

STEP 2 Write the ratios.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{37} \approx 0.32$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{35}{37} \approx 0.95$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{35} \approx 0.34$$

Watch Out!

Be careful not to confuse sine and cosine. Make sure you use the opposite side for sine and the adjacent side for cosine.

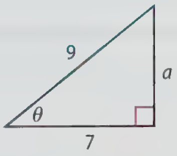
EXAMPLE 3

If $\cos\theta = \frac{7}{9}$, find the values of $\sin\theta$ and $\tan\theta$.

STEP 1 By definition of cosine, you know that

$$\frac{\text{side adjacent to }\theta}{\text{hypotenuse}} = \frac{7}{9}.$$

Draw a right triangle in which the hypotenuse is 9 and the leg adjacent to θ is 7.



STEP 2 Use the Pythagorean Theorem to find the missing side.

Pythagorean Theorem $a^2 + 7^2 = 9^2$

$a^2 = 81 - 49$

$a^2 = 32$

Take the positive square root. $a = \sqrt{32}$

Simplify the radical. $a = \sqrt{16 \cdot 2}$

$a = 4\sqrt{2}$

STEP 3 Write the sine and tangent of θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4\sqrt{2}}{9} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{4\sqrt{2}}{7}$$

SEARCH

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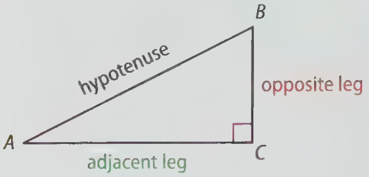
Cosecant, Secant, and Cotangent Ratios

The reciprocals of the sine, cosine, and tangent ratios are the **cosecant**, **secant**, and **cotangent** ratios, abbreviated csc, sec, and cot, respectively.



$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} \qquad \cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} \qquad \tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{leg opposite } \angle A} \qquad \sec A = \frac{\text{hypotenuse}}{\text{leg adjacent to } \angle A} \qquad \cot A = \frac{\text{leg adjacent to } \angle A}{\text{leg opposite } \angle A}$$

Need More HELP

To write the reciprocal of a fraction, mentally FLIP the fraction.

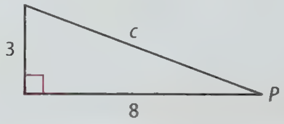
A **trigonometric identity** is an equation that is true for all acceptable values of the variables in the equation. Here are the six **reciprocal identities**. You will use identities in later topics to simplify trigonometric expressions and to solve trigonometric equations.

$$\sin \theta = \frac{1}{\csc \theta} \qquad \cos \theta = \frac{1}{\sec \theta} \qquad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

EXAMPLE 4

Find the values of sin *P*, cos *P*, tan *P*, csc *P*, sec *P*, and cot *P*.



STEP 1 Use the Pythagorean Theorem to find the hypotenuse.

Pythagorean Theorem $3^2 + 8^2 = c^2$

$9 + 64 = c^2$

$c^2 = 73$

Take the positive square root. $c = \sqrt{73}$

STEP 2 Write the ratios in simplified radical form.

$$\sin P = \frac{3}{\sqrt{73}} = \frac{3\sqrt{73}}{73} \qquad \cos P = \frac{8}{\sqrt{73}} = \frac{8\sqrt{73}}{73} \qquad \tan P = \frac{3}{8}$$

$$\csc P = \frac{\sqrt{73}}{3} \qquad \sec P = \frac{\sqrt{73}}{8} \qquad \cot P = \frac{8}{3}$$

GOT TO KNOW!

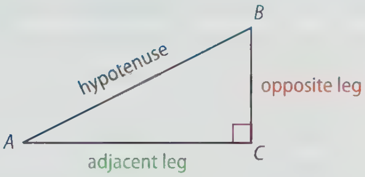
Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta} \qquad \cos \theta = \frac{1}{\sec \theta} \qquad \tan \theta = \frac{1}{\cot \theta}$$

Cofunctions

In every right triangle, the acute angles are complementary. In the figure, $\angle A$ and $\angle B$ are complementary. Notice that $\sin A = \cos B$ and $\sin B = \cos A$. In other words, the sine of an angle equals the cosine of its complement. Sine and cosine are called **cofunctions**. Tangent and cotangent are also cofunctions, as are secant and cosecant.



The following list shows six more identities, the **cofunction identities**.

$\sin\theta = \cos(90° - \theta)$ $\qquad$ $\tan\theta = \cot(90° - \theta)$ $\qquad$ $\sec\theta = \csc(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$ $\qquad$ $\cot\theta = \tan(90° - \theta)$ $\qquad$ $\csc\theta = \sec(90° - \theta)$

EXAMPLE 5

Use a cofunction identity to find the value of x.

a. $\cos 63° = \sin x°$

Use a cofunction identity. $\qquad \cos 63° = \sin(90° - 63°)$

$= \sin 27°$

$x = 27$

b. $\tan 24° = \cot x°$

Use a cofunction identity. $\qquad \tan 24° = \cot(90° - 24°)$

$= \cot 66°$

$x = 66$

[illegible]

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EXAMPLE 6

Find the value of x that satisfies the equation $\cos(x + 8)° = \sin(2x - 14)°$.

Since cosine and sine are cofunctions, you can write an equation showing that the sum of the two expressions is 90.

Write the equation. $\qquad (x + 8) + (2x - 14) = 90$

Combine like terms. $\qquad 3x - 6 = 90$

Add 6 to both sides. $\qquad 3x = 96$

Divide both sides by 3. $\qquad x = 32$

GOT TO KNOW!

Cofunction Identities

$\sin\theta = \cos(90° - \theta)$ $\qquad$ $\tan\theta = \cot(90° - \theta)$ $\qquad$ $\sec\theta = \csc(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$ $\qquad$ $\cot\theta = \tan(90° - \theta)$ $\qquad$ $\csc\theta = \sec(90° - \theta)$

Using a Calculator

You can use the SIN, COS, and TAN keys of your calculator to find the values of the sine, cosine, and tangent ratios, respectively. When you are given the measure of an angle in degrees, make sure your calculator is in degree mode (DEG), and not in radian mode (RAD) or gradian mode (GRA). The degree mode is the default mode on most calculators. If you see RAD or GRA on your calculator screen, you are not in degree mode.

Most trigonometric values (like most square roots) are irrational numbers. The tangent of 45° equals 1 and the cosine of 60° equals 0.5, but these are exceptions. When you use trigonometric ratios, round your answer to a reasonable number of decimal places. If the answer is a rounded value, be sure to use the approximate symbol when you state your answer.

EXAMPLE 1

Use a calculator to evaluate each expression. Round your answer to three decimal places.

a. tan 75°

TAN 7 5 ENTER **3.732050808**

tan 75° ≈ 3.732

b. cos 28.6°

COS 2 8 . 6 ENTER **0.877982975**

cos 28.6° ≈ 0.878

c. sin 110°

SIN 1 1 0 ENTER **0.93969262208**

sin 110° ≈ 0.937

Try It This Way

Your calculator may be able to convert from DMS (degree, minutes, seconds) to decimal degrees. Make sure the calculator is in degree mode first.

EXAMPLE 2

Use a calculator to evaluate sin 49°12′. Round your answer to three decimal places.

STEP 1 Convert 12′ to decimal degrees.

1 2 ÷ 6 0 ENTER **0.2**

STEP 2 Add the result to 49.

+ 4 9 ENTER **49.2**

STEP 3 Use the SIN key.

SIN 4 9 . 2 ENTER **0.756995055**

sin 49.2° ≈ 0.757

Watch Out !

Always check the angle mode on your calculator.

The Reciprocal Ratios

Your calculator has keys for the sine, cosine, and tangent ratios. But, there may be no keys for the other three functions—cosecant, secant, and cotangent. So how do you use a calculator to evaluate these ratios? First of all, remember that these are reciprocal functions.

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

To use a calculator to determine the values for cosecant, secant, and tangent, use the SIN, COS, and TAN keys, respectively, followed by the reciprocal x^{-1} key.

EXAMPLE 3

Use a calculator to evaluate each expression. Round your answer to three decimal places.

a. cot 53°

Use your calculator to evaluate $\frac{1}{\tan 53}$. TAN 5 3) x^{-1} ENTER **0.75355405**

cot 53° ≈ 0.754

b. sec 14.3°

Use your calculator to evaluate $\frac{1}{\cos 14.3°}$. COS 1 4 . 3) x^{-1} ENTER **1.03197499**

sec 14.3° ≈ 1.032

Watch Out!

The order in which you use the keys varies from calculator to calculator. On some calculators you press TAN and then the angle measure. On others, you enter the angle measure first.

EXAMPLE 4

Use a calculator to evaluate csc 52°10′. Round your answer to three decimal places.

STEP 1 Convert 10′ to decimal degrees.

1 0 ÷ 6 0 = **0.166666666**

STEP 2 Add the result to 52.

+ 5 2 = **52.16666667**

STEP 3 Because $\csc \theta = \frac{1}{\sin \theta}$, use the SIN key, followed by the x^{-1} key.

SIN 52.16666667) x^{-1} ENTER **1.266146047**

csc 52°10′ ≈ 1.266

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GOT TO KNOW!

Finding Trigonometric Values with a Calculator

Function	sine	cosine	tangent	cosecant	secant	cotangent
Keys	SIN	COS	TAN	SIN x^{-1}	COS x^{-1}	TAN x^{-1}

Solving Problems with Right Triangles

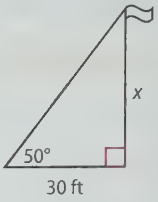
You can use trigonometric ratios to solve many types of problems. To find an unknown length in a right triangle you need only two pieces of information—the lengths of two sides or the measure of one acute angle and the length of one side.

Watch Out !

Copy the drawing carefully and use it to help you solve the problem. If a drawing is not provided, then make one.

EXAMPLE 5

The shadow of a flagpole extends 30 ft from the flagpole. A line from the top of the flagpole to the end of the shadow makes a 50° angle with the ground. Find x, the height of the flagpole.



STEP 1 Write an equation.

$$\tan 50° = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{30}$$

STEP 2 Solve the equation.

Multiply both sides by 30. $30 \cdot \tan 50° = x$

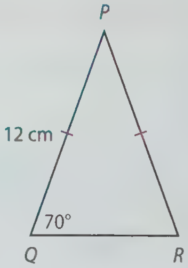
Use a calculator. 3 0 × TAN 5 0 ENTER **35.75260778**

Round your answer to the nearest foot. $x \approx 36$

The flagpole is about 36 ft tall.

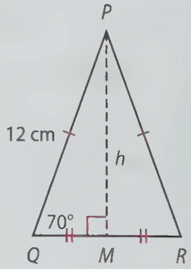
EXAMPLE 6

Triangle PQR is isosceles. The base angles measure 70° and the legs are 12 cm long. Find the area of $\triangle PQR$.



To find the area of a triangle, use the formula $A = \frac{1}{2}bh$, where A is the area, b is the length of the base, and h is the height of the triangle.

STEP 1 Draw the altitude from the vertex P to the base of the triangle. Because the triangle is isosceles, the altitude bisects the base.



STEP 2 Find the height h.

Use a trigonometric ratio. $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

Write an equation. $\sin 70° = \frac{h}{12}$

Solve the equation for h. $h = 12(\sin 70°) \approx 11.276$

STEP 3 Find the length of the base.

The length of the base QR is $2(QM)$.

Use the Pythagorean Theorem to find QM.

Pythagorean Theorem $QM^2 + PM^2 = PQ^2$

Substitute. $QM^2 + 11.276^2 = 12^2$

Solve for QM. $QM \approx 4.105$

$QR = 2(QM) \approx 8.21$

STEP 4 Substitute into the area formula.

$$A = \frac{1}{2}bh = \frac{1}{2}(8.21)(11.276) \approx 46.3$$

The area of $\triangle PQR$ is about 46.3 cm².

Using a Table

Before there were calculators, mathematicians and scientists used tables like this one to compute trigonometric ratios. The earliest known table of trigonometric ratios is part of a Babylonian cuneiform tablet.

θ **Degrees**	θ **Radians**	**sin θ**	**cos θ**	**tan θ**	**cot θ**	**sec θ**	**csc θ**		
6	.1047	.1045	.9945	.1051	9.514	1.006	9.567	1.4661	**84**
7	.1222	.1219	.9925	.1228	8.144	1.008	8.206	1.4486	**83**
8	.1396	.1392	.9903	.1405	7.115	1.010	7.185	1.4312	**82**
9	.1571	.1564	.9877	.1584	6.314	1.012	6.392	1.4137	**81**
10	.1745	.1736	.9848	.1763	5.671	1.015	5.759	1.3963	**80**
11	.1920	.1908	.9816	.1944	5.145	1.019	5.241	1.3788	**79**
12	.2094	.2079	.9781	.2126	4.705	1.022	4.810	1.3614	**78**
		cos θ	**sin θ**	**cot θ**	**tan θ**	**csc θ**	**sec θ**	θ **Radians**	θ **Degrees**

For now, ignore the column labeled *Radians*. You will learn more about radians in *Radian Measure and Trigonometric Functions* starting on page 926.

In the table above, only angles from 6° to 12° and from 78° to 84° are shown. In the complete table, for angles from 0° to 45°, use the angles in the left-hand column and the trigonometric ratios listed at the top. For angles from 45° to 90°, use the angles in the right-hand column and the trigonometric ratios listed at the bottom.

Watch Out!

If you use the angle measures in the right column, be sure to use the column names at the bottom—not the top.

EXAMPLE 7

Use the table to find the value of each trigonometric ratio.

a. sin 6°

Start at the top of the table.
Find the column headed "sin θ."
Go to the row that contains 6 in the "Degrees" column on the left.

$\sin 6° \approx 0.1045$

b. tan 81°

Start at the bottom of the table.
Find the column labeled "tan θ."
Go up to the row that contains 81 in the "Degrees" column on the right.

$\tan 81° \approx 6.314$

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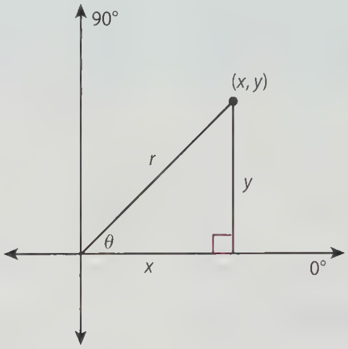
Trigonometric Ratios of Special Angles

0° and 90° Angles

There are some angles that have exact trigonometric values. These appear so often in problem situations that it is a good idea to memorize their values.

To gain understanding of the trigonometric ratios for angles of 0° and 90°, consider a right triangle in the coordinate plane.



Based on this figure, the trigonometric functions can be redefined as follows:

$$\sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r},\ \tan\theta = \frac{y}{x}$$

When θ equals 0°, what happens to x, y, and r? You can see that the value of y reduces to 0 and that the values of r and x will become equal. Next, imagine θ increasing to 90°. Now the value of x reduces to 0 and the values of r and y become equal. These ideas are used in the next example.

Watch Out!

You cannot divide by 0. Division by 0 is undefined.

EXAMPLE 1

Find the six trigonometric ratios for angles of 0° and 90°.

STEP 1 For 0°, use $x = 1$, $y = 0$, and $r = 1$. Write the six ratios.

$\sin 0^\circ = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 0^\circ = \frac{x}{r} = \frac{1}{1} = 1$

$\tan 0^\circ = \frac{y}{x} = \frac{0}{1} = 0$

$\csc 0^\circ = \frac{1}{\sin 0^\circ} = \frac{1}{0}$ undefined

$\sec 0^\circ = \frac{1}{\cos 0^\circ} = \frac{1}{1} = 1$

$\cot 0^\circ = \frac{1}{\tan 0^\circ} = \frac{1}{0}$ undefined

STEP 2 For 90°, use $x = 0$, $y = 1$, and $r = 1$. Write the six ratios.

$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$

$\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$

$\tan 90^\circ = \frac{y}{x} = \frac{1}{0}$ undefined

$\csc 90^\circ = \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1$

$\sec 90^\circ = \frac{1}{\cos 90^\circ} = \frac{1}{0}$ undefined

$\cot 90^\circ = \frac{1}{\tan 90^\circ} = \frac{0}{1} = 0$

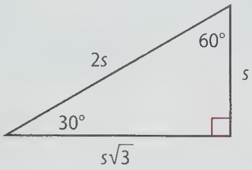
GOT TO KNOW!

Trigonometric Ratios of 0° and 90° Angles

$\sin 0^\circ = 0$	$\cos 0^\circ = 1$	$\tan 0^\circ = 0$	$\csc 0^\circ$ undefined	$\sec 0^\circ = 1$	$\cot 0^\circ$ undefined
$\sin 90^\circ = 1$	$\cos 90^\circ = 0$	$\tan 90^\circ$ undefined	$\csc 90^\circ = 1$	$\sec 90^\circ$ undefined	$\cot 90^\circ = 0$

30° and 60° Angles

In a 30°-60°-90° right triangle, the shorter leg is opposite the 30° angle and the longer leg is opposite the 60° angle. The hypotenuse is twice as long as the shorter leg. The length of the longer leg is $\sqrt{3}$ times the shorter leg.



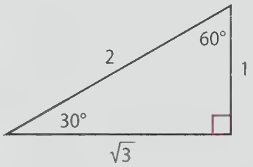
Need More HELP

For a review of 30°-60°-90° triangles, go to *Angles and Degree Measure* on page 894.

EXAMPLE 2

Without using a calculator, find the six trigonometric ratios of a 30° angle.

STEP 1 Draw a 30°-60°-90° right triangle with hypotenuse 2.



STEP 2 Find the sine, cosine, and tangent of 30°.

$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\cos 30° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

STEP 3 Use the reciprocal relationships to find the cosecant, secant, and cotangent of 30°.

$$\csc 30° = \frac{1}{\sin 30°} = \frac{2}{1} = 2$$

$$\sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot 30° = \frac{1}{\tan 30°} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

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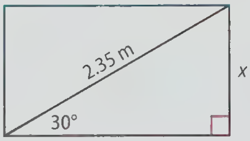
EXAMPLE 3

A board that is 2.35 m long braces a gate in a fence. The brace makes a 30° angle with the bottom of the gate. How tall is the gate? Round your answer to the nearest hundredth.



You are given the hypotenuse in a 30°-60°-90° right triangle. You need to find the measure of the side opposite the 30° angle. Use the sine ratio to find the unknown measure.

Write an equation. $\sin 30° = \frac{x}{2.35}$

Multiply both sides by 2.35. $x = 2.35 \cdot \sin 30°$

Simplify. $x = 2.35 \cdot \frac{1}{2} = 1.175$

The gate is about 1.18 m tall.

Solving Right Triangles

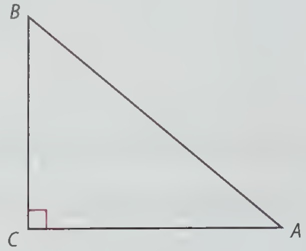
Useful Things to Know

To solve a triangle means to determine the measures of the three sides and the three angles of a given triangle. The following definitions and theorems are often useful when solving a triangle.

Complementary Angles in a Right Triangle

The two acute angles in a right triangle are complementary.

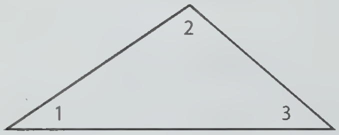
$m\angle A + m\angle B = 90°$



Triangle Angle Sum Theorem

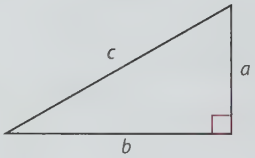
The sum of the angle measures in any triangle is 180°.

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

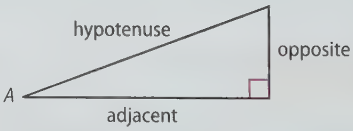


The Pythagorean Theorem

In a right triangle with legs of length a and b and hypotenuse of length c, $a^2 + b^2 = c^2$.



The Trigonometric Ratios



$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

$\csc A = \dfrac{\text{hypotenuse}}{\text{opposite}}$ $\sec A = \dfrac{\text{hypotenuse}}{\text{adjacent}}$ $\cot A = \dfrac{\text{adjacent}}{\text{opposite}}$

Converting DMS Notation to Decimal Degrees

$$x°y'z'' = \left(x + \frac{y}{60} + \frac{z}{3600}\right)°$$

EXAMPLE 1

Tell how you would solve each problem. There may be more than one way.

a. You know two sides in a right triangle. You want to find the length of the third side.

Use the Pythagorean Theorem. Substitute the two given side lengths and solve for the third side.

b. You know one acute angle in a right triangle and want to find the other acute angle.

METHOD 1

Two angles are given. Add their measures and subtract the sum from 180°.

METHOD 2

Subtract the acute angle measure from 90°.

c. You know one side length and one acute angle in a right triangle. You want to find the length of the side opposite the given angle.

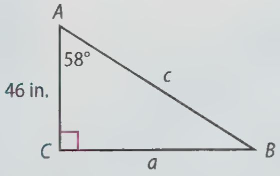
Use the sine or tangent ratio, depending on which side length you are given.

Using One Side and One Angle

When you know the length of a side and the measure of an acute angle in a right triangle, you can use trigonometry to find the lengths of the other sides.

EXAMPLE 2

Find the values of *a* and *c*.



STEP 1 Find the value of a.

Use the tangent ratio: $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

Substitute. $\tan 58° = \frac{a}{46}$

Use a calculator. $a = 46 \cdot \tan 58° \approx 73.6$

STEP 2 Find the value of c.

METHOD 1

Use the cosine ratio.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 58° = \frac{46}{c}$$

$$c = \frac{46}{\cos 58°} \approx 86.8$$

METHOD 2

Use the Pythagorean Theorem.

$$73.6^2 + 46^2 = c^2$$

$$c^2 = 7{,}532.96$$

$$c \approx 86.8$$

$a \approx 73.6$ and $c \approx 86.8$

SEARCH

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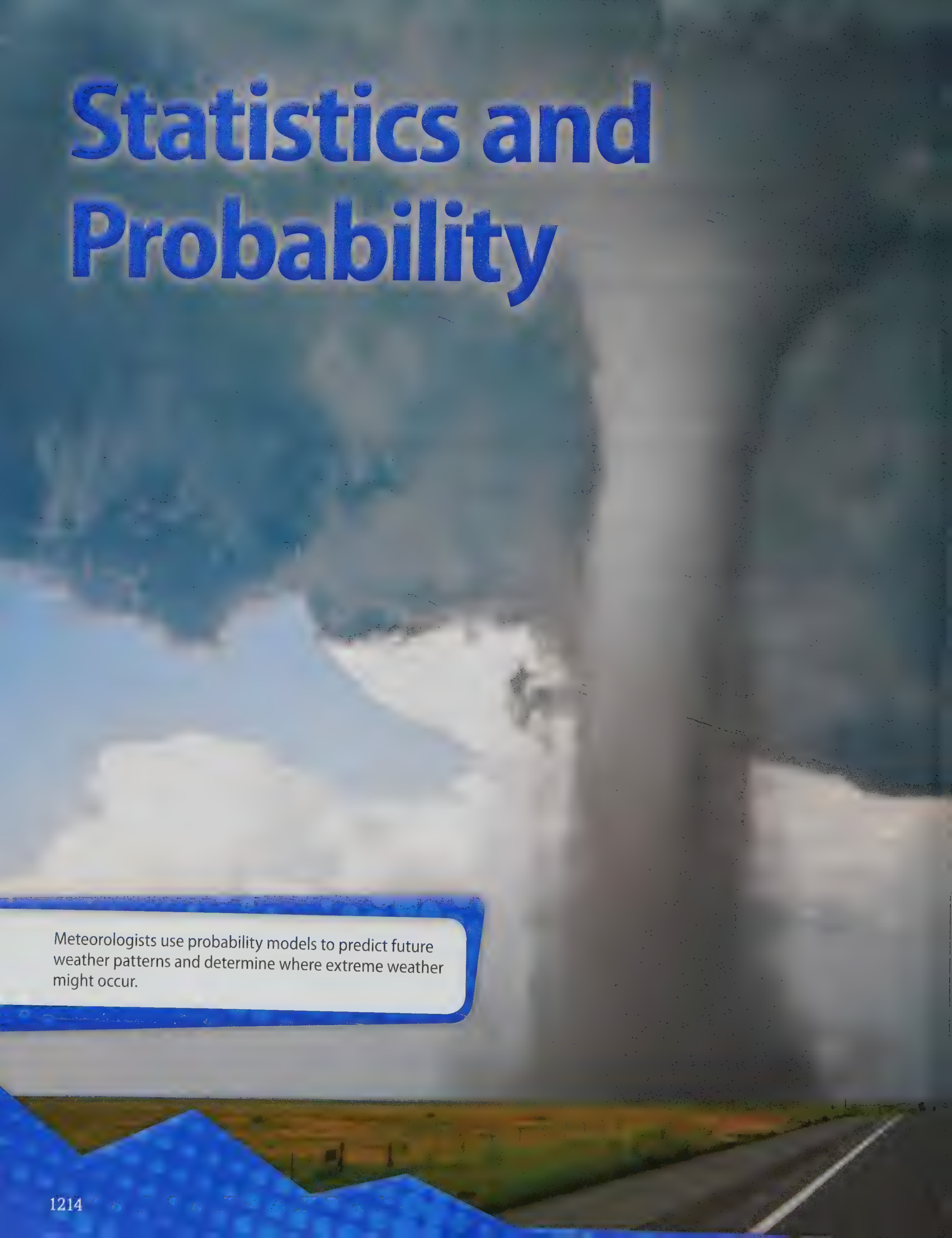
Statistics and Probability



Meteorologists use probability models to predict future weather patterns and determine where extreme weather might occur.

Gathering Data 1216

Theoretical and Experimental Probability . . . 1230

Displaying Data 1258

Describing Data 1292

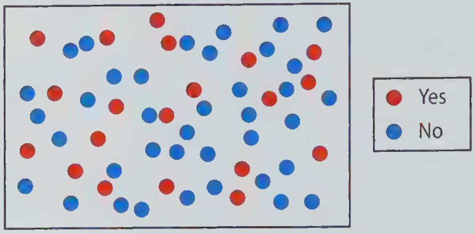
Analyzing and Using Data 1310

Compound Events and Conditional Probability 1356

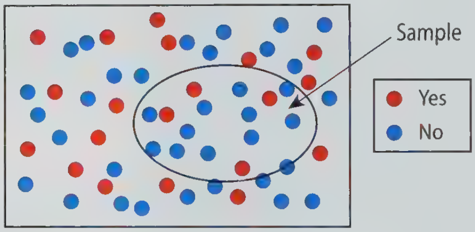
Samples and Populations

Data Sets

A set of **data** is a collection of numbers, measures, or other information. For example, a restaurant owner might record whether or not her customers order dessert. The group of individuals who supply the data is called the **population**. In this example, the restaurant customers are the population.


Would you care for dessert?

Sometimes it is not possible to collect data for every member of a population. Instead, the restaurant owner might look at every fifth receipt, or the receipts from one day only. The selected receipts are a *subgroup* of a population and are called a **sample**. The process of choosing the subgroup is called *sampling*.


Population

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EXAMPLE 1

Identify the sample and population in each situation.

a. A politician asks 50 voters from the registered voter list for their opinions on developing bicycle lanes on city streets.

The population is the voters on the registered voter list.

The sample is the 50 voters who gave their opinions.

b. Radio listeners are invited to log on to a station's website to request songs.

The population is the radio listeners with computer access.

The sample is the listeners who log on to request songs.

Making Predictions

A sample can be used to make predictions about the population or estimates of the size of the population.

Watch Out !

To make a general statement based on information from a sample, the sample must be *random*. To learn more, see *Random Sampling* on page 1224.

EXAMPLE 2

Principal Li wants to know how many of the 1,200 students in her school read the student newspaper. She chooses 90 students at random, and 63 of them say they read the paper. About how many students in the school read the student paper?

Set up a proportion. $\frac{\text{newspaper readers in sample}}{\text{students in sample}} = \frac{\text{newspaper readers in population}}{\text{students in population}}$

Substitute. $\frac{63}{90} = \frac{x}{1{,}200}$

Multiply both sides by 1,200. $1{,}200\left(\frac{63}{90}\right) = \left(\frac{x}{1{,}200}\right)1{,}200$

Simplify. $840 = x$

About 840 students read the student paper.

EXAMPLE 3

At one school, all 476 students participate in a sport. The sports that a random sample of students plays are shown. Estimate how many of the 476 students run track.

Number of Students in Spring Sports	
Ultimate Flying Disc	42
Track	27
Baseball	33

STEP 1 Find the number of students in the sample. $42 + 27 + 33 = 102$

STEP 2 Set up a proportion. $\frac{\text{track runners in sample}}{\text{students in sample}} = \frac{\text{track runners in population}}{\text{students in population}}$

Substitute. $\frac{27}{102} = \frac{x}{476}$

Multiply both sides by 476. $476\left(\frac{27}{102}\right) = \left(\frac{x}{476}\right)476$

Simplify. $126 = x$

About 126 students run track.

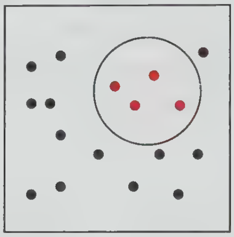
Need More HELP

For help with proportions, go to *Solving Proportions* in *Numbers and Operations* (Book 1, p. 370).

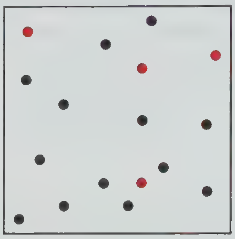
Capture and Recapture

Scientists use a method called "capture/recapture" to estimate the size of a population. The steps in the method are as follows.

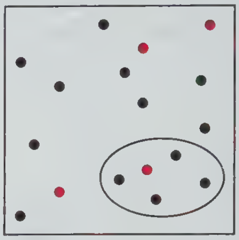
Capture a group from the population. Count and mark the group members.



Return the marked group to the population and wait for it to mix in.



Capture another group. Count how many are marked and unmarked.



After counting the second group, estimate the size of the population by setting up a proportion.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 4

A park superintendent wants to estimate how many squirrels are in a city park. Ten squirrels are captured and tagged. Six weeks later, 20 squirrels are captured and 3 of them have tags. Estimate the number of squirrels in the park.

Set up a proportion. $\frac{\text{tagged squirrels in population}}{\text{total squirrels in population}} = \frac{\text{tagged squirrels captured}}{\text{total squirrels captured}}$

Substitute. $\frac{10}{x} = \frac{3}{20}$

Cross multiply. $10 \cdot 20 = 3 \cdot x$

$200 = 3x$

Divide both sides by 3. $\frac{200}{3} = \frac{3x}{3}$

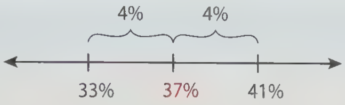
$66.\overline{66} = x$

Round to the nearest number of squirrels. $66.\overline{66} \approx 67$

There are about 67 squirrels in the park.

Margin of Error

Information about a population that is drawn from a sample is an estimate. A report about a survey or poll may include a *margin of error*. For example, it may read, "The company has a satisfaction rating of 37% with a margin of error of plus or minus 4%."



This means it is probable that the percent of satisfied customers is between 33% and 41%. The margin of error is based on the size of the sample and whether or not the sample is a good representation of the population, not on the type of questions asked.

EXAMPLE 5

From a population of 5,000 high school students, a random sample of 850 is selected. The poll shows that 78% are *not* in favor of classes on Saturdays. The margin of error is 3.4%. What is the probable range of students not in favor of Saturday classes?

METHOD 1

STEP 1 Find 78% of 5,000. $0.78 \cdot 5{,}000 = 3{,}900$

STEP 2 Find 3.4% of 5,000. $0.034 \cdot 5{,}000 = 170$

STEP 3 Find the range of students. $3{,}900 - 170 = 3{,}730$
$3{,}900 + 170 = 4{,}070$

There are probably between 3,730 and 4,070 students who do not favor Saturday classes.

METHOD 2

STEP 1 Find the lower percent in the range. $78\% - 3.4\% = 74.6\%$

STEP 2 Find the upper percent in the range. $78\% + 3.4\% = 81.4\%$

STEP 3 Find the range of students. $0.746 \cdot 5{,}000 = 3{,}730$
$0.814 \cdot 5{,}000 = 4{,}070$

There are probably between 3,730 and 4,070 students who do not favor Saturday classes.

Need More HELP ?

For help with percents, go to *Decimals, Fractions, and Percents* in *Numbers and Operations* (Book 1, p. 272).

Got to KNOW!

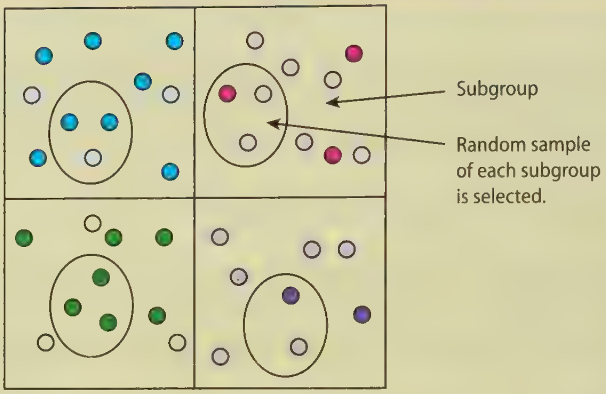
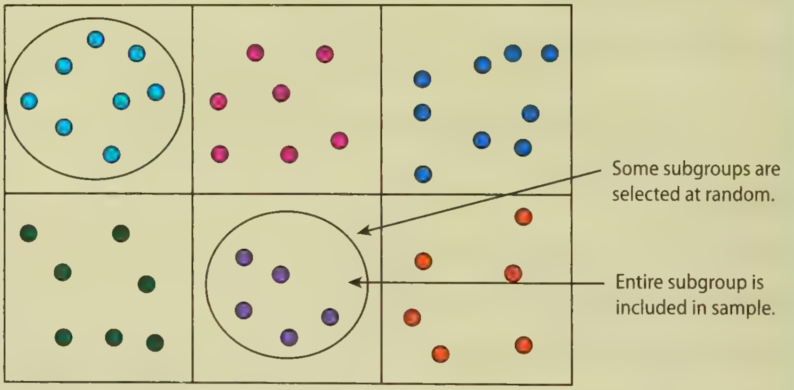
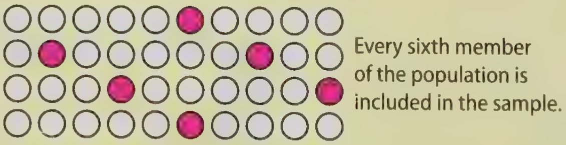
Terms to Know

Data	Collection of numbers, measures, or other information
Population	Group of individuals who supply data
Sample	Any subgroup of the population
Margin of error	An estimate of whether a measure of a sample is a good representative of the measure of the population

Other Sampling Methods

Other Types of Random Samples

A **simple random sample** (SRS) is one in which every member of a population has an equal possibility of being selected. However, there are other types of random samples including *stratified samples, cluster samples,* and *systematic samples.*

Types of Random Samples	
Stratified sample The population is separated into two or more subgroups. Each subgroup is similar in composition. Then a sample is chosen from each subgroup. If the sample is chosen by SRS, then the sampling method is called *stratified random sampling.*	 Subgroup Random sample of each subgroup is selected.
Cluster sample The population is separated into subgroups, which are selected at random. In most cases, every member of the chosen subgroups is included in the sample. (In some sampling situations, not every member of a subgroup is chosen. Instead, members are randomly selected from the selected subgroups.)	 Some subgroups are selected at random. Entire subgroup is included in sample.
Systematic sample The population is ordered in some way. Then members are chosen at regular intervals throughout the population.	 Every sixth member of the population is included in the sample.

Stratified Samples

A stratified sample is a good choice when a population contains subgroups that do not overlap, such as people who live in different ZIP codes. It also ensures that the subgroups in the population are proportionally represented in the sample.

EXAMPLE 1

The state math honors program consists of 132 girls and 108 boys. A math coach wants to choose a representative sample of 12 program members to travel to a tournament out of state. Describe how to use stratified sampling to do this.

STEP 1 The two subgroups are boys and girls. Find the total number of math honors students in the two subgroups: $132 + 108 = 240$

STEP 2 Find the percent of girls and boys in the population.

Girls: $\frac{132}{240} = 55\%$ Boys: $\frac{108}{240} = 45\%$

STEP 3 Find the number of girls in the sample.

$0.55 \cdot 12 = 6.6$ Round 6.6 to 7. Randomly select 7 of the 132 girls.

STEP 4 Find the number of boys in the sample.

$12 - 7 = 5$ Randomly select 5 of the 108 boys.

SEARCH

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Systematic Samples

Systematic samples ensure that the population is evenly sampled. They use a fixed interval period and a random starting point, which makes them easy to work with.

EXAMPLE 2

A community health clinic wants to select a sample of 18 patients from the 144 patients who visit the clinic in one day. Describe how to use systematic sampling to choose this sample.

STEP 1 Number the patients from 1 to 144.

STEP 2 Find the interval period, a ratio of the number of people in the population to the number of people in the sample.

$$\text{Interval period} = \frac{\textit{Number of people in the population}}{\textit{Number of people in the sample}} = \frac{144}{18} = 8$$

STEP 3 Choose the starting point, or first patient in the sample, by randomly selecting a number between 1 and the sample size, 8. For example, roll two number cubes and find their sum. Discard any sum greater than 8. Say you roll 2 and 1. Add: $2 + 1 = 3$. Patient 3 is the first patient in the sample.

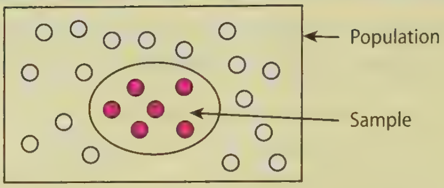
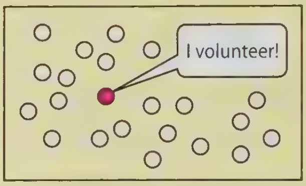
STEP 4 Start with 3 and count on by 8 to find the other patients in the sample.
$3 + 8 = 11$, $11 + 8 = 19$, $19 + 8 = 27$, and so on.
The sample will include patients 3, 11, 19, 27, and so on.

Need More HELP

For help with ratios, go to *Writing and Simplifying Ratios* in *Numbers and Operations* (Book 1, p. 364).

Nonrandom Samples

Suppose you asked a group of your friends, “Who is your favorite author?” You would probably get different answers than if you asked the same question of a random sample of bookstore customers. The sample made up of your friends is called a *convenience sample,* which is one type of nonrandom sample.

Types of Nonrandom Samples	
Convenience sample The members of the population that are easiest to contact or survey are selected for the sample.	
Voluntary-response sample Members of the population volunteer to be in the sample. It is sometimes called a self-selected sample.	

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 3

A nonprofit organization in a state wants to study the recycling programs used in the counties across the state. Researchers randomly select ten counties from all the counties in the state. (This is about one-fourth of the counties.) Then they visit every recycling facility in each of the ten selected counties.

a. What kind of sample did the organization use? Why is it a good choice for this situation?

The organization used a cluster sample. Nonprofit organizations usually do not have a lot of money. A cluster sample is a good choice as it reduces the cost of the study. Instead of visiting a few sites in every county across the state, the researchers can visit every site in a few counties.

b. Using a convenience sample might cost the organization even less. Why might a convenience sample be a poor choice for this study?

A convenience sample would most likely include only nearby recycling programs. If the state is large, this sample may not represent what is happening across the state. Although it could save money, it is likely to be less accurate.

Identifying Types of Samples

EXAMPLE 4

Identify the type of sample in each situation as *convenience, voluntary-response, stratified, cluster, systematic,* or *random*.

a. The owner of a store asks every tenth customer for his or her ZIP code.

This is a systematic sample. Members are chosen at a regular interval.

b. A teacher has each student roll a number cube. Students who roll an even number present their reports on Monday.

This is a random sample because all students are equally likely to roll an even number and need to present their reports on Monday.

c. A student uses a social networking website to ask his friends if they have homework assignments over a holiday weekend.

This is a convenience sample because it is an easy group to ask. It is also a voluntary-response sample because his friends may choose to respond or not respond.

d. A principal selects at random 5 students from each grade in the school.

This is a stratified sample. The different grades are the subgroups. The 5 students from each grade are the random samples of the subgroups.

Need More HELP

Remember that a *sample* is any subgroup of a population. A *random sample* is one in which all members of the population are equally likely to be chosen for the sample.

For help with random samples, go to *Random Sampling* on page 1224.

Got to KNOW!

Advantages and Disadvantages of Different Sampling Methods

Sample Type	Advantage	Disadvantage
Convenience	Easy, quick	Unlikely to represent an entire population.
Voluntary-response	Easy, inexpensive	Likely to be biased. People who have strong opinions or who like to be in surveys are more likely to respond.
Stratified	Subgroups in the population are proportionally represented.	Cannot be used if the population cannot be separated into meaningful subgroups.
Systematic	Simple, ensures that the population will be evenly sampled	Might introduce bias. Suppose a sample is made up of every eighth house on every street. The population has many blocks with only eight houses. The sample might overrepresent corner houses.
Cluster	Less expensive, less time consuming	Less accurate, especially if only a few clusters are selected; may exclude an important subgroup.

Theoretical Probability

What Is Theoretical Probability?

Probability is a way to describe the likelihood that a given event will take place. When probability is calculated by analysis, it is called **theoretical probability**. Theoretical probability tells what you can expect to happen, not what actually happens. Each possible result is an **outcome**. An **event** is any outcome or group of outcomes.

To determine theoretical probability, use the formula below, where *favorable outcomes* are outcomes you want and *total number of outcomes* is the number of all the possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

The probability of any event is 0, 1, or a number between 0 and 1.

A probability may be written as a fraction, a decimal, or a percent.

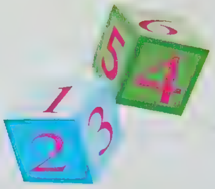
If two outcomes have equal probabilities, the outcomes are **equally likely**.

Need More HELP

Read P(event) as "the probability of an event." There may be a different term in parentheses. For example, P(red) means "the probability of red."

Identifying Outcomes

- Suppose you perform an experiment in which you roll one of the game cubes shown. The given event is rolling the number cube. There are six possible outcomes for this event—rolling a 1, 2, 3, 4, 5, or 6.



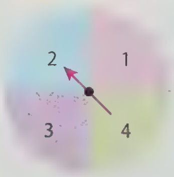
- Suppose the event is tossing a coin. One possible outcome of this experiment is that the coin lands heads up. The other outcome is that the coin lands tails up.

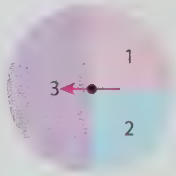
The set of all possible outcomes of an experiment is called the **sample space** for the experiment. The sample space for rolling the game cube is {1, 2, 3, 4, 5, 6}. The sample space for tossing the coin is {heads, tails}.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

- The spinner at the left below is divided into four equal sections. The sample space for using this spinner is {1, 2, 3, 4}. Because the sections are of equal size, the outcomes are all *equally likely*. When you measure the likelihood of an outcome, you are finding the *probability* of the outcome.





- The possible outcomes in the sample space are not always equally likely. The sections of the spinner at the right above are not all the same size. If you spin this spinner, the arrow is more likely to land on 3 than on 1 or 2.

Certain and Impossible Events

A dresser drawer contains 12 T-shirts—3 blue, 5 white, and 4 red. Suppose you select a T-shirt at random and any color is okay. The probability of selecting a blue, white, or red T-shirt is $\frac{12}{12} = 1$. An event with a probability of 1 is called a *certain* event.

Suppose you want to select a gray T-shirt. Since there are no gray T-shirts in the drawer, the probability of selecting a gray T-shirt is $\frac{0}{12} = 0$. An event with a probability of 0 is called an *impossible* event.

EXAMPLE 1

Use the formula for theoretical probability and the definitions of certain and impossible events to find the probability of the following event.

You roll a game cube with sides numbered 1, 2, 3, 4, 5, and 6.

a. Find the probability of rolling a 4.

There are 6 possible outcomes. There is 1 favorable outcome—rolling a 4.

$$P(4) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{6} \approx 0.17 = 17\%$$

b. Find the probability of rolling an odd number.

There are 6 possible outcomes. There are 3 odd numbers—1, 3, and 5.

$$P(\text{odd number}) = \frac{3}{6} = 0.5 = 50\%$$

c. Find the probability of rolling a number less than 5.

There are 6 possible outcomes. There are 4 numbers less than 5—1, 2, 3, and 4.

$$P(\text{number less than 5}) = \frac{4}{6} = \frac{2}{3} \approx 0.67 = 67\%$$

d. Find the probability of rolling a number greater than 6.

None of the numbers are greater than 6, so $P(\text{number} > 6) = \frac{0}{6} = 0$.

Need More HELP

For help converting fractions to decimals and percents, see *Decimals, Fractions, and Percents* in *Numbers and Operations* (Book 1, p. 272).

Got to KNOW!

Probabilities of 0 and 1

The probability of an event that is *certain* is 1.

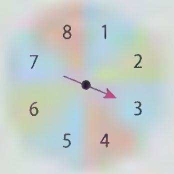
The sum of the probabilities of all possible outcomes of an event is 1.

The probability of an event that is *impossible*, and cannot occur, is 0.

Finding Probability

EXAMPLE 2

Use the spinner to find each probability. The spinner is divided into equal sections.



a. Find the probability of spinning a 3.

There are 8 possible outcomes. One of them is landing on 3.

$P(3) = \frac{1}{8} = 0.125 = 12.5\%$

b. Find the probability of landing on green.

Out of the 8 possible outcomes, 2 of them are landing on green.

$P(\text{green}) = \frac{2}{8} = \frac{1}{4} = 0.25 = 25\%$

c. Find the probability of spinning a number greater than 2 and less than 7.

Out of the 8 possible outcomes, 4 outcomes are between 2 and 7—3, 4, 5, and 6.

$P(\text{number greater than 2 and less than 7}) = \frac{4}{8} = \frac{1}{2} = 0.5 = 50\%$

Got to KNOW!

Probability Formula

The formula for finding probability is:

$P(\text{event}) = \frac{\text{no. of favorable outcomes}}{\text{total number of outcomes}}$

EXAMPLE 3

A drawer contains 6 black socks, 4 blue socks, and 2 white socks. Suppose you take a sock out of the drawer without looking. Are the probabilities of selecting the three colors equally likely? If not, list the probabilities from least to greatest.

STEP 1 Add to find the total number of possible outcomes: $6 + 4 + 2 = 12$.

STEP 2 Find each probability.

$$P(\text{black}) = \frac{\text{number of black socks}}{\text{total number of socks}} = \frac{6}{12} = \frac{1}{2}$$

$$P(\text{blue}) = \frac{\text{number of blue socks}}{\text{total number of socks}} = \frac{4}{12} = \frac{1}{3}$$

$$P(\text{white}) = \frac{\text{number of white socks}}{\text{total number of socks}} = \frac{2}{12} = \frac{1}{6}$$

STEP 3 Compare the outcomes. $\frac{1}{6} < \frac{1}{3} < \frac{1}{2}$

The outcomes are not equally likely. From least to greatest, the probabilities are white sock, blue sock, black sock.

Need More HELP ?

Remember, the sum of all the possible outcomes is equal to 1.

$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} =$

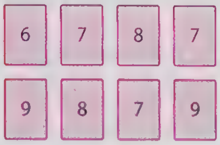
$\frac{3}{6} + \frac{2}{6} + \frac{1}{6} =$

$\frac{6}{6} =$

1

EXAMPLE 4

In a set of number cards, one number card is selected at random.



a. What is the probability of selecting a card with a number less than 10?

All of the cards have numbers less than 10. This event is certain, so its probability is 1.

b. What is the probability of selecting a card with a number divisible by 5?

No number on the cards is divisible by 5. This event is impossible, so its probability is 0.

c. What is the probability of selecting a card whose number is not 8?

METHOD 1

Count the cards whose numbers are not 8. Set up a probability ratio.

$$P(\text{not } 8) = \frac{\text{number of cards that are not 8}}{\text{total number of cards}} = \frac{6}{8} = \frac{3}{4} = 0.75 = 75\%$$

METHOD 2

Find the probability of selecting a card whose number is 8, and subtract the probability from 1.

$$P(8) = \frac{\text{number of cards that are 8}}{\text{total number of cards}} = \frac{2}{8} = \frac{1}{4}$$

$$P(\text{not } 8) = 1 - \frac{1}{4} = \frac{3}{4} = 0.75 = 75\%$$

SEARCH

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Watch Out!

Even though there are four possible numbers one could choose, the total number of cards is eight. Use 8, not 4, as the total number of possible outcomes.

EXAMPLE 5

If a number is chosen at random from the set {3, 6, 9, 12, 15}, what is the probability that the number is a solution of the inequality $6 < x < 15$?

STEP 1 Count the total number of possible outcomes.

There are 5 numbers in the set.

STEP 2 Read the inequality as "x is greater than six and less than fifteen."

STEP 3 There are 2 outcomes that satisfy the inequality.

9 and 12 are both greater than 6 but less than 15.

STEP 4 Write the probability.

$P(\text{number that is solution of the inequality}) = \frac{2}{5}$.

Need More HELP

For help with inequalities, see *Graphing an Inequality as an Interval* in *Algebra* (p. 1492).

Experimental Probability

Conducting Trials for an Experiment

Suppose you toss a coin exactly once. It can land either heads up or tails up, so the theoretical probability that it lands heads up is $\frac{1}{2}$. However, for only one toss, you cannot predict the outcome. Heads and tails are equally likely.

If you toss the coin many times, you can compare the frequency of each result with the total number of times the coin is tossed. The act of tossing the coin many times to find outcomes is an **experiment**. Each toss of the coin is called a **trial** of the experiment.

The tally chart below shows the results of tossing a coin 16 times.

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Heads	✓	✓	✓	✓			✓			✓	✓			✓		✓
Tails					✓	✓		✓	✓			✓	✓		✓	

The **experimental probability** of an event is the ratio of the number of times a favorable outcome occurs to the total number of trials. The coin landed heads up 9 times. In this experiment, the experimental probability of heads is $\frac{9}{16}$.

Look at just the first 2 tosses in the table above. Of these 2 tosses, both landed heads, so $P(\text{heads}) = \frac{2}{2} = 1$.

Now look at just the first 6 tosses—4 heads, 2 tails. For these tosses, $P(\text{heads}) = \frac{4}{6} = \frac{2}{3}$, which is closer to the experimental probability of $\frac{1}{2}$.

In general, as the number of trials of an experiment increases, the experimental probability gets closer and closer to the theoretical probability.

Got to KNOW!

Experimental Probability

$$P(\text{event}) = \frac{\text{number of times the favorable outcome occurs}}{\text{total number of trials}}$$

The experimental probability of an event is also called the *relative frequency* of the event.

Law of Large Numbers

As the number of times an experiment is performed increases, the experimental probability approaches the expected, or theoretical, probability.

EXAMPLE 1

A game cube numbered 1 through 6 is rolled several times. Each number rolled is recorded. The results are shown in the table below.

Result	1	2	3	4	5	6
Number of Occurrences	12	5	8	3	6	6

a. What is the experimental probability of rolling a 1?

Add the total number of occurrences to find the total number of trials:
$12 + 5 + 8 + 3 + 6 + 6 = 40$.
The number of trials with a result of 1 is 12.

$$P(1) = \frac{12}{40} = \frac{3}{10} = 0.3 = 30\%$$

b. What is the experimental probability of rolling an odd number?

The total number of trials is 40. The number of occurrences of an odd number is:
$12 + 8 + 6 = 26$

$$P(\text{odd}) = \frac{26}{40} = \frac{13}{20} = 0.65 = 65\%$$

SEARCH

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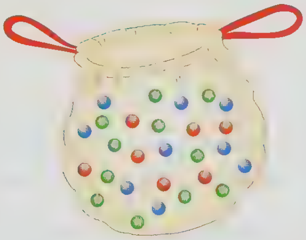
EXAMPLE 2

A bag contains 9 blue marbles, 7 red marbles, and 11 green marbles. A marble is drawn from the bag at random. Its color is recorded, and the marble is put back in the bag. The process is repeated several times. The results are shown in the table below.



Result	Blue	Red	Green
Number of Occurrences	12	7	11

a. What is the experimental probability of selecting a blue marble?

The total number of trials is $12 + 7 + 11 = 30$. A blue marble was drawn 12 times.

$$\text{Experimental: } P(\text{blue}) = \frac{12}{30} = 40\%$$

b. What is the theoretical probability of selecting a blue marble?

The total number of marbles in the bag is $9 + 7 + 11 = 27$. The number of blue marbles in the bag is 9.

$$\text{Theoretical: } P(\text{blue}) = \frac{9}{27} = \frac{1}{3} \approx 33\%$$

c. Was a blue marble chosen more times or fewer times than expected?

More times; 40% is greater than the theoretical probability of 33%.

Need More HELP

For help calculating theoretical probability, go to *Theoretical Probability* on pages 1232–1235.

Observational Results

It is fairly easy to find the theoretical probabilities for an event such as tossing a coin. In other cases, it may be more difficult. In these cases, you can collect information either by performing an experiment or by making observations. Then you can use experimental probability to analyze the results.

EXAMPLE 3

Two game cubes numbered 1 through 6 are rolled several times. The sum of the two numbers rolled—the result—is recorded. The results are shown in the table below.

Result	Number of Occurrences
2	2
3	5
4	3
5	5
6	3
7	12
8	9
9	8
10	0
11	2
12	1

a. Would you expect all the possible outcomes of this experiment to be equally likely? Why or why not?

No. There is only one way to roll a sum of 2 $(1 + 1)$ or a sum of 12 $(6 + 6)$. There are several ways to roll some numbers. For example, you can roll a sum of 7 by rolling 6 and 1, 5 and 2, or 4 and 3.

b. What is the experimental probability of rolling a 3?

Add all numbers of occurrences to find the total number of trials:
$2 + 5 + 3 + 5 + 3 + 12 + 9 + 8 + 2 + 1 = 50$.
The number of trials with a result of 3 is 5.

$$P(3) = \frac{5}{50} = \frac{1}{10} = 0.1 = 10\%$$

c. What is the experimental probability of rolling a number greater than 6?

The total number of trials is 50. The number of trials with a result greater than 6 is $12 + 9 + 8 + 2 + 1 = 32$.

$$P(>6) = \frac{32}{50} = 0.64 = 64\%$$

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Try It This Way

Use your calculator to find the total number of occurrences and to calculate probability.

In Example 3(c), enter 32, press ÷, enter 50, and press =. Your calculator will display 0.64.

EXAMPLE 4

A coin is tossed onto a checkerboard 20 times. The coin may land entirely on a red square, entirely on a black square, or on both colors at the same time. The results of the experiment are shown in the table below.

Result	Number of Occurrences
Red	4
Black	3
Both	13

What is the experimental probability that the coin lands on both red and black?

$$P(\text{both}) = \frac{\text{number of times coin lands on both}}{\text{total number of trials}} = \frac{13}{20} = 0.65 = 65\%$$

Got to KNOW!

Types of Probability

Experimental probability tells the result of an experiment, or something you actually do.

Theoretical probability predicts an expected result. It is a theory, or idea, which may not always show what really happens.

EXAMPLE 5

On her way to school, Rheana needs to cross two intersections that have traffic lights. For 10 days, she records whether each signal shows Walk or Don't Walk when she reaches it. The results are shown in the table.

Day	1	2	3	4	5	6	7	8	9	10
1st light	Walk	Don't Walk	Walk	Walk	Don't Walk	Walk	Don't Walk	Walk	Walk	Don't Walk
2nd light	Don't Walk	Don't Walk	Don't Walk	Don't Walk	Don't Walk	Walk	Don't Walk	Don't Walk	Walk	Don't Walk

a. What is the experimental probability that both traffic signals show Don't Walk on Rheana's way to school?

Both traffic lights signal Don't Walk on 4 days out of 10. The experimental probability is $P(\text{both Don't Walk}) = \frac{4}{10} = \frac{2}{5} = 40\%$

b. What is the experimental probability that at least one traffic light signal shows Don't Walk on Rheana's way to school?

The only days when Rheana did not record a Don't Walk signal were Day 6 and Day 9. On the other 8 days, at least one light signaled Don't Walk. The experimental probability is $P(\text{at least one Don't Walk}) = \frac{8}{10} = \frac{4}{5} = 80\%$

c. Is it more likely that a light signals Walk or Don't Walk when Rheana reaches it?

In part (b) the probability of at least one Don't Walk is 80%, which is greater than 50%. It is more likely that a light will signal "Don't Walk" when Rheana reaches it.

Try It This Way

You could answer Example 5(c) by counting the results in the chart. Don't Walk occurs 12 times, and Walk occurs 8 times. Since $12 > 8$, it is more likely that a light will signal Don't Walk.

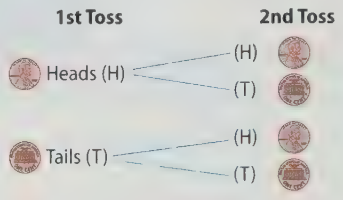
The Counting Principle

Tree Diagrams

When counting the number of outcomes of an event, it is important to make sure that every member of the sample space has been counted. For example, if you toss a coin two times, the possible results are:

Heads-Heads Heads-Tails Tails-Heads Tails-Tails

A **tree diagram** shows all the possible outcomes of one or more events in an organized manner. The arrows or lines on a tree diagram are the "branches" of the tree. The tree diagram below shows the possible outcomes of tossing a coin twice.



Count the number of branches to find the number of different outcomes. On the first toss, the coin lands heads up or tails up. On the second toss, it lands heads up or tails up. There are four possible outcomes: (H, H), (H, T), (T, H), and (T, T).

EXAMPLE 1

The Bagel Palace sells whole-grain bagels and plain bagels. There are 3 choices of toppings: butter, jam, and cream cheese. Use a tree diagram to find the total number of ways a customer can order a bagel with one topping.

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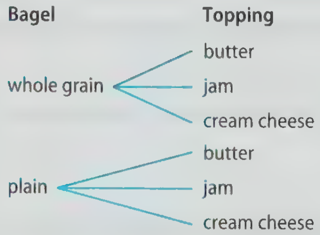
Make a tree diagram. Start with the 2 types of bagels—whole grain and plain. Add branches for the 3 toppings—butter, jam, and cream cheese.



Count the number of branches.

whole grain, butter	whole grain, jam	whole grain, cream cheese
plain, butter	plain, jam	plain, cream cheese

There are 6 different bagel-and-one-topping orders.

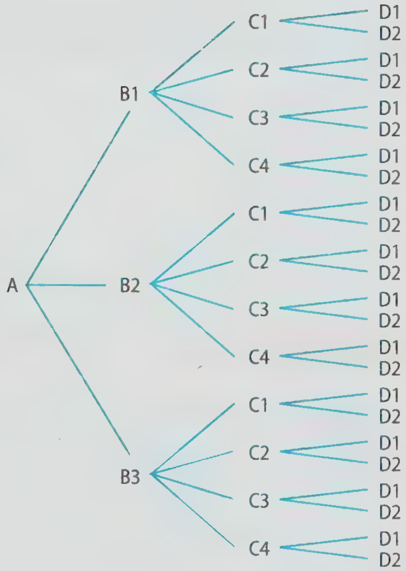
EXAMPLE 2

Jim plans to choose one outfit from these choices.

A. 1 shirt
B. 3 pairs of pants
C. 4 pairs of socks
D. 2 pairs of shoes

Use a tree diagram to find the number of possible outfits that Jim can wear.

METHOD 1



STEP 1 Write A to represent the 1 shirt.

STEP 2 There are 3 choices for pants. Draw 3 branches from A. Label them B1, B2, and B3.

STEP 3 There are 4 choices for socks. Draw 4 branches each from B1, B2, and B3. Label them C1, C2, C3, and C4.

STEP 4 There are 2 choices for shoes. Draw 2 branches each from C1, C2, C3, and C4. Label them D1 and D2.

STEP 5 Count the number of branches on the right side of the tree diagram.

There are 24 branches, so there are 24 possible outfits.

METHOD 2

Multiply the number of possibilities for each choice, or event. This method uses the Fundamental Counting Principle.

The product of the number of choices is equal to the number of outcomes.

1 shirt choice × 3 pants choices × 4 socks choices × 2 shoes choices
$= 1 \times 3 \times 4 \times 2 = 24$ possible outfits

Got to KNOW!

The Fundamental Counting Principle

If one event can occur in m ways and another event can occur in n ways, then there are $m \times n$ ways that both events can occur.

The Fundamental Counting Principle can be extended to any finite number of sets. If there are a outcomes for one event, b outcomes for a second event, c outcomes for a third event, and so on, the number of different ways all these events can occur is $a \times b \times c \times \ldots$.

Using the Fundamental Counting Principle

The Fundamental Counting Principle is a good way to count outcomes of multiple events, especially when the number of choices for each event is very large and it would be difficult to make a tree diagram. To use the Fundamental Counting Principle with a larger number of events, find the product of all the numbers of outcomes for all the events.

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EXAMPLE 3

At the start of a round in one folk dance, 6 boys and 6 girls stand facing each other in two lines. Each boy must shake the hand of each of the girls. If they dance three rounds, how many handshakes is this?

Use the Fundamental Counting Principle to find the total number of outcomes.

There are 6 boys. Each boy shakes hands with 6 girls. There are 3 rounds.

$6 \times 6 \times 3 = 108$

There are 108 handshakes.

EXAMPLE 4

Traviata Pizza offers a choice of 2 bases (tomato sauce or garlic and olive oil), 4 types of cheese, 5 types of meat, and 8 other toppings. How many different types of pizza can a customer order if he or she chooses only one item from each category?

There are 2 bases, 4 cheeses, 5 meats, and 8 other toppings.

Use the Fundamental Counting Principle to find the total number of possible pizzas.

$2 \times 4 \times 5 \times 8 = 320$

There are 320 different ways to order pizza by choosing only one item from each category.

EXAMPLE 5

State license plate numbers often consist of three letters of the alphabet followed by three digits. How many different license plate numbers can be made?

Use the Fundamental Counting Principle to find the total number of outcomes.

There are 26 letter choices (letters *A*–*Z*) for *each* of the first three spaces (events) on the plate number.

There are 10 digit choices (digits 0–9) for *each* of the last three spaces (events) on the plate number.

$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17{,}576{,}000$

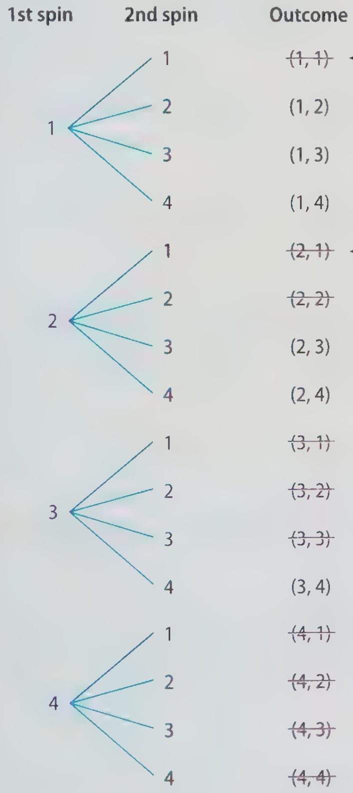
There are 17,576,000 different license plate numbers.

The Counting Principle and Probability

EXAMPLE 6

A spinner has four equal sections labeled 1–4. The pointer on the spinner is spun twice. What is the probability that the result of the first spin is *less than* the result of the second spin?

Make a tree diagram to organize the information you need to answer the question. You are interested only in outcomes in which the first spin is *less than* the second.



The outcome (1,1) is discarded because 1 is not less than 1.

The outcome (2,1) is discarded because 2 > 1.

Of the 16 outcomes, 10 are not counted as favorable because the first spin is not less than the second spin.

6 outcomes show the result of the first spin is less than the result of the second spin.

$P(\text{first spin} < \text{second spin}) = \frac{6}{16} = \frac{3}{8} = 0.375 = 37.5\%$

EXAMPLE 7

Students are invited to make up a three-digit number. Each digit must be greater than 0 and less than 6. If a student's number matches the teacher's number, the student wins a day off from homework. What is the probability of a match?

Use the Fundamental Counting Principle to find the total number of outcomes. There are 5 choices for *each* of the three digits. The number of outcomes is $5 \times 5 \times 5 = 125$.

The probability of winning is $\frac{1}{125} = 0.008 = 0.8\%$.

Watch Out!

In Example 7, when you think of each digit as an event, you will see why you do not multiply 3×5.

1st event: 5 outcomes

2nd event: 5 outcomes

3rd event: 5 outcomes

Multiply the ways in which the three events can occur: $5 \times 5 \times 5$.

Permutations

How Many Different Orders?

You can arrange books on a shelf in alphabetical order or by color or in other ways. You can line people up for a photograph in order of age or height or in some other way. Ordering books on a shelf and lining people up for a photograph in different ways are examples of permutations. A **permutation** is an arrangement of a group, or set, of objects in a particular order.

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EXAMPLE 1

A brother, a sister, a father, a mother, and a grandmother are going to be in a family photograph. In how many ways can these 5 people be arranged in a line?

The number of people available to fill a position decreases each time a person is placed. If the sister is placed at the far left in the photograph, she is no longer available for any other position.

Position	First	Second	Third	Fourth	Fifth
Choices available	5	4	3	2	1

There are 5 people available for the first position, only 4 people available for the second position, and so on. Use the Fundamental Counting Principle to find the total number of ways the people can be arranged.

$5 \times 4 \times 3 \times 2 \times 1 = 120$

There are 120 ways the 5 family members can be arranged in a line.

Watch Out!

Remember that a natural number is an integer that is greater than 0.

The number of different ways the elements in a set can be arranged does not depend on what the elements are. You can arrange 5 different books on a shelf or arrange 5 different digits to form a number in 120 ways, just as you can arrange 5 people in a line for a photograph in 120 different ways.

In Example 1, notice that to find the number of ways that 5 people can be arranged, 5 is multiplied by all the natural numbers that are less than 5. This rule can be generalized to apply to any natural number n members of a set.

There are $n - 1$ numbers remaining after one place is filled, $n - 2$ numbers remaining after the second place is filled, and so on.

Got to KNOW!

Permutations of n objects

To find the total number of arrangements of n objects, use this formula.

$n! = n \cdot (n - 1) \cdot (n - 2) \cdot (n - 3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

Factorials

The product in which the factors are descending natural numbers is called **n factorial**. This product $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$ can be written as $n!$ and is read "n factorial." The product in Example 1, $5 \times 4 \times 3 \times 2 \times 1 = 120$, is 5!. Factorials become large in value quite quickly, as shown in the table below.

Try It This Way

Most scientific calculators have a factorial key, labeled *x*! or *n*!.

Factorials

n	*n*!
0	0! = 1
1	1! = 1
2	2! = 2 × 1 = 2
3	3! = 3 × 2 × 1 = 6
4	4! = 4 × 3 × 2 × 1 = 24
5	5! = 5 × 4 × 3 × 2 × 1 = 120
6	6! = 6 × 5 × 4 × 3 × 2 × 1 = 720
7	7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5,040
8	8! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320
9	9! = 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 362, 880
10	10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 3,628,800

Need More HELP

By definition, 0! = 1. See *Special Cases of Combinations* on page 1251 to find out why.

EXAMPLE 2

A race has 8 runners competing.

a. In how many orders can the runners finish?

There are 8 possibilities for first place. After one runner finishes in first place, there are 7 runners who might finish in second place, 6 who might finish in third place, and so on.

Use the Fundamental Counting Principle to find the total number of orders in which the runners can finish the race.

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$$

b. What is the probability that the runners will finish in the alphabetical order of their names?

Of the 40,320 orders in which the runners can finish, only one of them is the alphabetical order of their names.

$$P(\text{alphabetical order}) = \frac{1}{40{,}320} \approx 0.0000248 = 0.00248\%$$

0.00248% is about a fourth of 1%.

Finding Permutations

In Example 2, you found the number of different orders in which all 8 runners could finish. Suppose, instead, you want to find the number of different orders in which only the first 3 runners can finish.

EXAMPLE 3

A race has 8 runners. Medals will be given to the first-, second-, and third-place runners. In how many different ways can the three medals be awarded?

There are 8 choices for first place. After the first runner has finished, there are 7 choices for second place. After the second runner has finished, there are 6 choices for third place.

Use the Fundamental Counting Principle to find the number of different ways in which the first three spots can be filled.

$8 \times 7 \times 6 = 336$

There are 336 different ways in which medals for first, second, and third place can be awarded.

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You can use this notation to represent the permutations in Example 3.

$${}_{8}P_{3}$$

number of members in the set → 8; 3 ← number from the set that are in the permutation

In general, this is written ${}_{n}P_{r}$.

The product in Example 3 is similar to a factorial, but the multiplication stops after the first 3 numbers. The last 5 factors of 8! are canceled.

- Multiply and divide the product 8 × 7 × 6 by the factors that are canceled.

$$8 \times 7 \times 6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

- Write the numerator and denominator as factorials. $= \frac{8!}{5!}$
- Write 5 as the difference of 8 and 3. $= \frac{8!}{(8-3)!}$

Got to KNOW!

Factorials

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$ (Read $n!$ as "n factorial.")

$1! = 1$ and $0! = 1$

Permutations of *n* objects taken *r* at a time

To find the number of permutations of r items from a group of n items, use ${}_{n}P_{r} = \frac{n!}{(n-r)!}$.

${}_{n}P_{r}$ may also be written as $P(n, r)$.

Using the formula for ${}_nP_r$ is much simpler than deriving it. Remember that permutations involve ordering the elements that are chosen from the larger set.

Try It This Way

Many scientific and graphing calculators have a permutation function, labeled ${}_nP_r$. It may be a second function on a scientific calculator. It may also be on the MATH or PRB menu of a graphing calculator.

EXAMPLE 4

A class president and a class treasurer will be elected from a group of 12 students. In how many different ways can the two offices be filled?

METHOD 1

Use the permutation formula for 12 items taken 2 at a time. There is a difference between being president and being treasurer, so the order of the items is important.

Substitute 12 for n and 2 for r in the formula. $${}_{12}P_2 = \frac{12!}{(12-2)!}$$

Simplify. $$= \frac{12!}{10!}$$

Write 12! as 12 • 11 • 10!. $$= \frac{12 \cdot 11 \cdot 10!}{10!}$$

Simplify. $$= 12 \cdot 11 = 132$$

There are 132 different ways the two officers can be elected from the group of 12 students.

METHOD 2

Use the Fundamental Counting Principle. There are 12 different choices for the president. After the president is selected, there are 11 different choices for the treasurer.

The number of possible outcomes is $12 \cdot 11 = 132$.

In some permutation problems, elements of the set may be repeated. This situation is called *permutation with repetition*.

EXAMPLE 5

A locker combination is made up of three numbers from 0 through 99. How many different locker combinations can there be?

Order is important. For example, the combination 53-12-76 is not the same as 12-53-76.

Use the Fundamental Counting Principle. A number may be repeated in the combination. There are 100 choices for the first number, 100 choices for the second number, and 100 choices for the third number.

The number of possible locker combinations is $100 \cdot 100 \cdot 100 = 1{,}000{,}000$.

Combinations

How Many Choices?

Choosing people from a group to be on a committee is different from choosing officers of a club or awarding prizes for first, second, and third place in a race. For a committee, the order in which the members are chosen is *not* important. On a committee, the pair "Winton and Alice" is the same as the pair "Alice and Winton."

A selection of items, events, or people from a set without regard to the order is a **combination**. Suppose a group consists of 10 items and 6 of them are chosen and placed in a box. If their order in the box doesn't matter, the set of 6 items is a combination. This is an example of 10 items taken 6 at a time.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

EXAMPLE 1

The science department has 8 teachers. Three of the teachers will be chosen to be on a committee. How many different combinations of 3 teachers chosen from the group of 8 teachers are there?

Science Department Teachers			
Ms. Wong	Mr. Riccio	Mrs. Neel	Mr. Costero
Mr. Singh	Ms. Ueda	Mr. Ramirez	Mrs. deVore

If the order of the teachers were important, the number would be ${}_8P_3$.

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$$

However, this number is too high, because it includes groups of teachers that have exactly the same members but in different orders. Since order does not matter in this case, we want to eliminate groups that have the same members as another in a different order.

There are 3!, or 6, ways to arrange 3 teachers in different orders.

To cancel the effect of repeated groups, divide the number of permutations of 8 items taken 3 at a time, 336, by the number of permutations of 3 items, 6.

$$\frac{336}{6} = 56$$

There are 56 different combinations of 8 teachers taken 3 at a time.

Although both combinations and permutations are calculated from a ratio, the result is always a whole number. For example, there cannot be a fractional number of committees formed.

The Combinations Formula

Just as for permutations, there is a formula for combinations. Again, the number of combinations you can choose from a set does not depend on what the elements of the set are. You can choose 3 teachers from a group of 8 in 56 different ways, and you can choose 3 books from a group of 8 in 56 different ways.

You can use this notation to write the combinations in Example 1.

$$\text{number of members in the set} \rightarrow {}_{8}C_{3} \leftarrow \text{number in each combination}$$

In general, this is written ${}_nC_r$ and is read "*n* choose *r*."

As shown in Example 1, the formula for combinations is based on the formula for permutations. Start with the permutation formula, and then divide by *r*! to eliminate the common permutations:

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

EXAMPLE 2

There are 4 CDs that Jeremy wants to buy, but he has enough money for only 2 of them. How many different combinations of 2 of the 4 CDs are possible?

Notice that order is not important, since buying CD 1 and CD 2 is the same as buying CD 2 and CD 1.

Use the combinations formula for *n* items taken *r* at a time with $n = 4$ and $r = 2$.

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

Substitute 4 for *n* and 2 for *r*.

$$ {}_4C_2 = \frac{4!}{2!(4-2)!} $$

Simplify.

$$ = \frac{4!}{2!2!} $$

$$ = \frac{24}{4} $$

$$ = 6 $$

There are 6 different combinations of 2 of the 4 CDs that Jeremy wants.

Try It This Way

Many scientific and graphing calculators have a combination function, labeled ${}_nC_r$. It may be a second function on a scientific calculator or may be on the MATH menu of a graphing calculator.

Got to KNOW!

Combinations

The formula for *n* items taken *r* at a time is ${}_nC_r = \frac{n!}{r!(n-r)!}$.

${}_nC_r$ is read "*n* choose *r*."

${}_nC_r$ may also be written as $\binom{n}{r}$ or $C(n, r)$.

Frequency

Frequency Tables

In a data set, **frequency** is the number of times an event, a number, or a range of events occurs. Frequency is often shown on a frequency table like the one below. A **tally** shows the number of times an event occurs. Count the number of tallies to find the frequency.

Where do you get your news?

Source	Tally	Frequency
Internet	|||	3
Print	||	2
Television	𝍸 ||	7
Radio	||||	4

The frequency table above shows that most people represented on the table get their news by watching television. The frequency of television is 7.

EXAMPLE 1

Victor rolled a 0–6 number cube a number of times. He made a tally each time he rolled a number. Use the table to find the frequency of Victor rolling the number 3 in this experiment.

Victor's Experiment

Number	Tally	Frequency
1	𝍸 |	6
2	||	2
3	𝍸	?
4	|||	3
5	|	1
6	𝍸 ||||	9

STEP 1 Read down the **Number** column to the row for rolling a 3.

STEP 2 Read across to the **Tally** column.

STEP 3 Count the tally marks.

There are five tallies, so the frequency for rolling the number 3 in this experiment is 5.

Try It This Way

Place your fingertip on the word *Number*. Then move your finger down to the 3 row and across to the tally marks. Count the tally marks and write the answer.

EXAMPLE 2

The types of computers preferred by some students in one middle school are shown in the frequency table below. What is the ratio of laptops to desktops?

Which do you prefer?

Computer	Tally	Frequency
Laptop	𝍸 𝍸 𝍸 𝍸 𝍷	21
Desktop	𝍸 𝍸 𝍸 𝍷𝍷𝍷	18
Tablet	𝍸 𝍷𝍷𝍷𝍷	9

STEP 1 Find the number of students who prefer laptops.
The table shows that 21 students prefer laptops.

STEP 2 Find the number of students who prefer desktops.
The table shows that 18 students prefer desktops.

STEP 3 Write the ratio of laptops to desktops.
$\frac{21}{18} = \frac{21 \div 3}{18 \div 3} = \frac{7}{6}$ or 7:6

The ratio of students who prefer laptops to desktops is 7:6.

Need More HELP

For help with ratios, go to *Writing and Simplifying Ratios* in *Numbers and Operations* (Book 1, p. 364).

EXAMPLE 3

The table shows the range of the number of paintings sold by vendors at the crafts fair. How many more vendors sold at least 16 paintings than vendors who sold at most 15 paintings?

Number of Paintings Sold	Frequency
1–5	1
6–10	2
11–15	10
16–20	18
21–25	3

STEP 1 Find the number of vendors who sold at least 16 paintings. $18 + 3 = 21$

STEP 2 Find the number of vendors who sold at most 15 paintings. $1 + 2 + 10 = 13$

STEP 3 Subtract to find the difference. $21 - 13 = 8$

There were 8 more vendors who sold at least 16 paintings than those who sold at most 15 paintings.

SEARCH

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Relative Frequency

In a data set, the **relative frequency** of an event is a ratio of the number of times the event occurs and the total number of events. The relative frequency can be expressed as a fraction, decimal, or percent.

Need More
HELP

For help with writing fractions as decimals or percents, go to *Decimals, Fractions, and Percents* in *Numbers and Operations* (Book 1, p. 272).

EXAMPLE 4

Data for a Saturday night at a restaurant is shown. What is the relative frequency of dinner reservations for a table for 5?

Reservation	Frequency
Table for 2	12
Table for 4	9
Table for 5	8
Table for 6	3

STEP 1 Find the frequency for a reservation for a table for 5. 8

STEP 2 Add to find the total number of reservations.
$12 + 9 + 8 + 3 = 32$

STEP 3 Express the frequencies as a ratio in simplest form.
$\frac{\text{frequency of a table for 5}}{\text{total number of reservations}} = \frac{8}{32}$, which simplifies to $\frac{1}{4}$.

The relative frequency of a reservation for a table for 5 is $\frac{1}{4}$ of the total reservations.

EXAMPLE 5

Elena is keeping track of the Field Day results. Use the frequency table to find what percent of the games Grade 8 won.

Winner	Frequency
Grade 5	4
Grade 6	6
Grade 7	8
Grade 8	6

STEP 1 Find the number of games won by Grade 8. 6

STEP 2 Add to find the total number of games played.
$4 + 6 + 8 + 6 = 24$

STEP 3 Write a ratio and express it as a percent.
$\frac{6}{24} = 0.25$, or 25%

Grade 8 won 25% of the Field Day games.

Got to KNOW!

Terms to Know

Frequency is the number of times an event, a number, or a range of events occurs.

Relative Frequency is a ratio of the number of times an event occurs to the total number of events in question.

Cumulative Frequency is the sum of the frequencies of the data up to a given level.

Cumulative Frequency

In a data set, the **cumulative frequency** is the sum of the frequencies of the data up to a given level. For example, the table below shows the number of cars washed during a one-week period. The cumulative frequency for Fri–Sat is the sum of the frequency for that interval and the frequency of the intervals that come before it.

Days	Frequency	Cumulative Frequency
Mon–Tue	10	10
Wed–Thu	11	10 + 11 = 21
Fri–Sat	23	**10 + 11 + 23 = 44**
Sun	15	10 + 11 + 23 + 15 = 59

EXAMPLE 6

The table shows the number of volcanic eruptions between Year 1 and Year 8. Find the missing cumulative frequencies to complete the table.

Years	Frequency	Cumulative Frequency
Year 1–Year 2	8	8
Year 3–Year 4	17	8 + 17 = 25
Year 5–Year 6	31	?
Year 7–Year 8	29	?

STEP 1 Add to find the cumulative frequency for Year 1–Year 6.
8 + 17 + 31 = 56

STEP 2 Add to find the cumulative frequency for Year 1–Year 8.
8 + 17 + 31 + 29 = 85

STEP 3 Place the equations in the table.

Years	Frequency	Cumulative Frequency
Year 1–Year 2	8	8
Year 3–Year 4	17	8 + 17 = 25
Year 5–Year 6	31	8 + 17 + 31 = 56
Year 7–Year 8	29	8 + 17 + 31 + 29 = 85

SEARCH

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Scatter Plots

Need More HELP

Here are other names used for scatter plots.

- scattergram
- scatter diagram
- scatter graph
- scatter chart

What Is a Scatter Plot?

Paired data are sometimes displayed in a data table like the one below. Such data can also be displayed on a *scatter plot.* A **scatter plot** is a graph that shows data points on a coordinate grid. Each data point represents a pair of values.

EXAMPLE 1

Display the data in the table on a scatter plot.

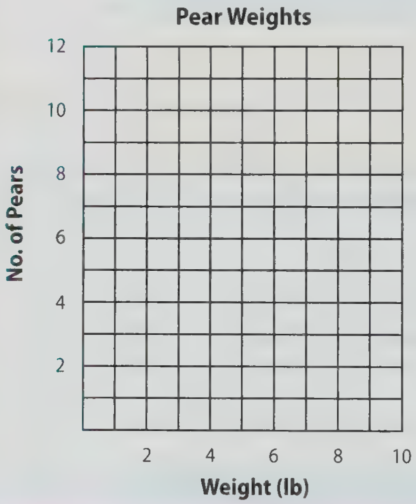
Total Weight	2 lb	2.5 lb	5 lb	6 lb	6.5 lb	7.5 lb	8 lb
No. of Pears	4	5	6	7	8	9	10

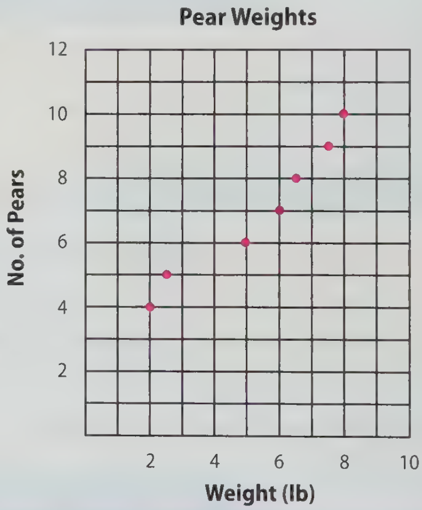
STEP 1 The information in the table tells the total weights of different numbers of pears. The title of the scatter plot could be Pear Weights.

Use the information in the table to label a graph.

STEP 2 Using the paired data, write the coordinates for each data point.
(2, 4) (2.5, 5) (5, 6) (6, 7) (6.5, 8) (7.5, 9) (8, 10)

STEP 3 Plot the data points on the scatter plot.





Need More HELP

For help with plotting points, go to *The Coordinate Plane* in *Algebra* (p. 1442).

Reading a Scatter Plot

You can record the information on a scatter plot in a data table. Use the labels of the x- and y-axes as the row heads for the data table. Use the coordinates of each point as the data in the table.

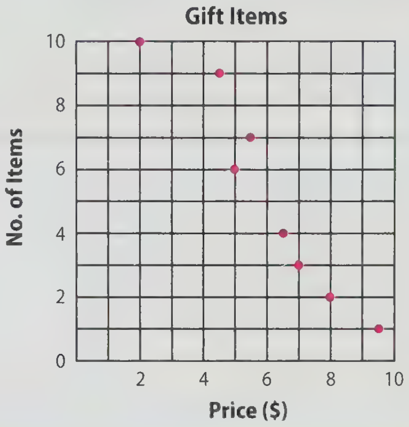
EXAMPLE 2

A booth at a street fair has gift items that cost less than $10. The scatter plot shows the prices of the items and the number of items offered at each price. For example, there are ten items that cost $2.

Use the scatter plot to complete the data table below.

The *x*-axis shows the prices of the items. The *y*-axis shows the number of items available at each price. Find the coordinates for each data point, starting with the one on the left.

- The first data point is at (2, 10). There are 10 items that cost $2.00.
- The second data point is at (4.5, 9). There are 9 items that cost $4.50.
- Continue adding coordinates to the table until all the data points on the graph are shown on the table.



Price	$2.00	$4.50	$5.00	$5.50	$6.50	$7.00	$8.00	$9.50
No. of Items	10	9	6	7	4	3	2	1

Ways to REMEMBER

The first number in an ordered pair represents a data point's position along the horizontal axis, or *x*-axis. The second number represents its position along the vertical axis, or *y*-axis.

To recall this order, remember that *h* comes before *v* in the alphabet. To recall the directions of the axes, think, "*y* points up to the sky."

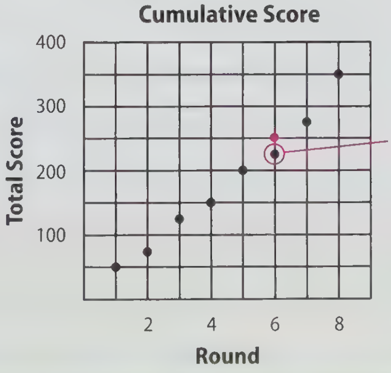
EXAMPLE 3

Team A played 8 rounds of a game and scored a total of 350 points. The data table shows the total points scored by the end of each round. For example, by the end of round 5, Team A had scored a total of 200 points.

Round	1	2	3	4	5	6	7	8
Total Score	50	75	125	150	200	250	275	350

One of the players on the team displayed the data on a scatter plot but made a mistake. Identify the data point on the scatter plot that is plotted incorrectly.

The table shows that the total score after round 6 is 250. The data point is incorrectly placed at (6, 225) on the scatter plot. The red point shows the correct placement.



(6, 225) should be (6, 250).

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Histograms

Need More HELP

Another name for a histogram is a histograph.

For help with frequency, go to *Frequency* on page 1260.

Reading a Histogram

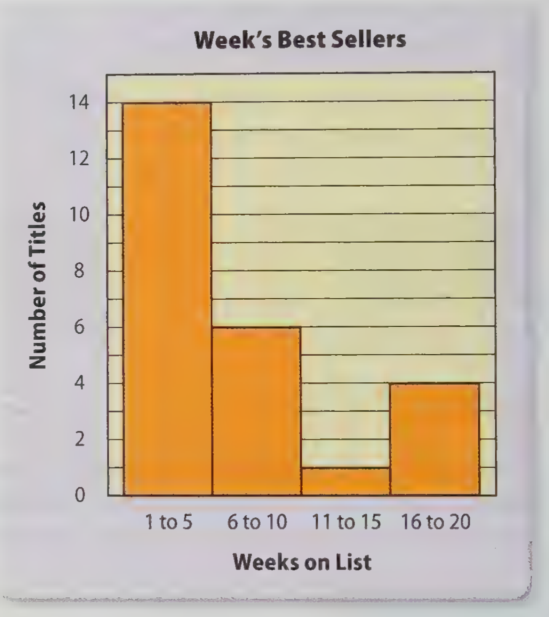
A histogram displays the frequency of a set of data in equal-size intervals, or ranges of data. Because the intervals are equal in size, the bars on a histogram have equal width. There are no spaces between the bars on a histogram because there are no spaces between the intervals.

EXAMPLE 1

Each week the newspaper lists the 25 top-selling fiction books, known as best sellers. The histogram below shows information about this week's best sellers. Use the labels on the histogram to help you understand the data displayed on it.

The *x*-axis label is "Weeks on List." The *x*-axis numbers are intervals, such as 1 to 5. These intervals are ranges of the numbers of weeks that this week's best sellers have been on the list.

The *y*-axis label is "Number of Titles." The scale is 0 to 14. Use this scale to measure the heights of the bars on the histogram. This will tell you how many book titles have been on the best-seller list for a certain period of time (range of weeks).



a. How many of this week's best sellers have been on the list for 5 weeks or less?

The height of the bar for 1 to 5 "Weeks on List" is 14. Fourteen books have been on the list 5 weeks or less.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

b. What time interval represents the least number of books?

The shortest bar is for the interval 11 to 15. Its height is 1. This means that only 1 book has been on the list for 11, 12, 13, 14, or 15 weeks.

c. The sum of the heights of the four bars should be what number? Why?

The sum of the heights should equal 25 because there are 25 books on the best-seller list.

CHECK Add the heights of the bars.

1 to 5		6 to 10		11 to 15		16 to 20	
14	+	6	+	1	+	4	= 25 ✓

Use a Frequency Table to Make a Histogram

EXAMPLE 2

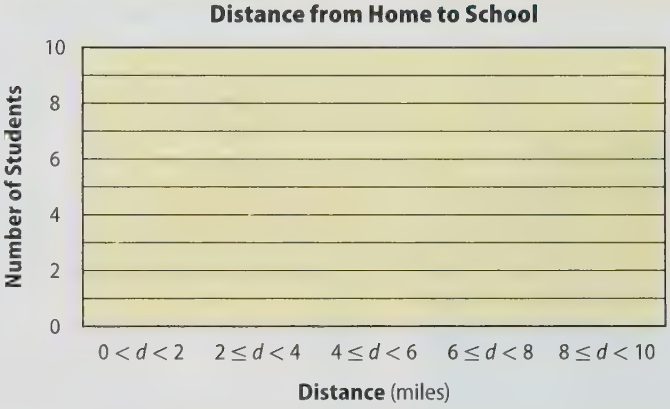
Twenty-seven students estimated the distance from their houses to school. They displayed the estimates on a frequency table.

Distance *d* (miles)	$0 < d < 2$	$2 \le d < 4$	$4 \le d < 6$	$6 \le d < 8$	$8 \le d < 10$
Tally	||	𝍸 ||	𝍸 ||||	𝍸	||||
Frequency	2	7	9	5	4

Display the data on a histogram.

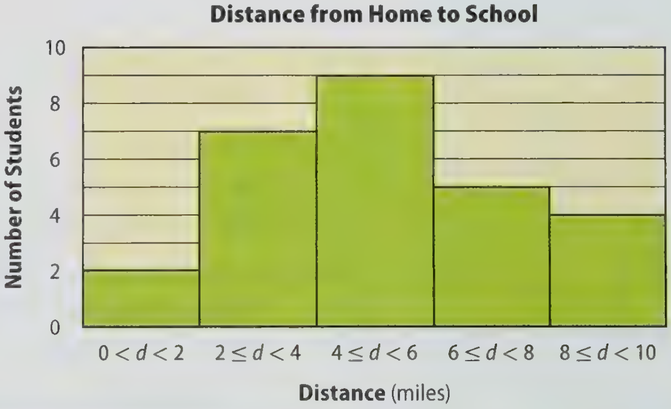
STEP 1 Draw and label the axes of the histogram.

- Let the *x*-axis represent the distance.
- Use the intervals on the table to label the *x*-axis. For example, the first interval shown on the frequency table is $0 < d < 2$.
- Let the *y*-axis represent the number of students.
- The range of the *y*-axis scale should go from zero to the greatest frequency of students, 9.


Distance from Home to School

STEP 2 Add the bars to the histogram.

- Use the information on the frequency table to draw a bar for the frequency of each interval. For example, a frequency of 2 means 2 students.
- Give the histogram a title that describes the data on it. A possible title for this histogram is "Distance from Home to School."


Distance from Home to School

Got to KNOW!

Histograms

The bars on a histogram represent frequency of data in equal-size intervals. Inequality symbols are sometimes used to label the intervals.

Stem-and-Leaf Plots

Making a Stem-and-Leaf Plot

A **stem-and-leaf plot** is a data display that can be used to show how data are distributed. The data are in numerical order in two columns. The left column contains the *stems*, and the right column contains the *leaves*. The last digit in a number is always a leaf and represents the value of the ones place.

Suppose you have the data values 27, 35, 41, 32, 26, 30, 48, and 27. Group the data in three categories by the values in the tens place—2, 3, and 4. Place each leaf in its corresponding category and put the leaves for each stem in numerical order. Write a key that explains what the digits mean.

Stem	Leaf
2	6 7 7
3	0 2 5
4	1 8

Key: 2 | 6 means 26

Need More HELP

Sometimes a stem-and-leaf plot is referred to as a *stemplot* or a *stem and leaf display*.

EXAMPLE 1

Organize the data from the table in a stem-and-leaf plot.

Competition Scores

68	75	75	92	81	88	67
83	72	79	64	70	90	95

STEP 1 Order the data from least to greatest.

64, 67, 68, 70, 72, 75, 75, 79, 81, 83, 88, 90, 92, 95

STEP 2 Determine the numbers for the stems and for the leaves.

The data range from 64 to 95, so the stems will be the tens digits 6, 7, 8, and 9. The leaves will be the ones digits that go with each stem.

STEP 3 Make a two-column plot titled Competition Scores. In the left column, list the stems from least to greatest. In the right column, list the leaves for each stem from least to greatest. Insert a key to tell what the stems and leaves represent.

Competition Scores

Stem	Leaf
6	4 7 8
7	0 2 5 5 9
8	1 3 8
9	0 2 5

Key: 6 | 4 means 64

Try It This Way

To help you see which digits are stems and which are leaves, try this process.

- List the data in a column.
- Draw a horizontal line under each category that will be a stem.
- Write the stems. Use the last digit of every number in the category as the leaves.

64
67 6|4 7 8
68
70
72
75 7|0 2 5 5 9
75
79
81
83 8|1 3 8
88
90
92 9|0 2 5
95

EXAMPLE 2

a. The table shows the batting averages for a baseball team. Organize the data in a stem-and-leaf plot.

Batting Averages

.317	.351	.318	.340	.324	.331	.322	.342	.303
.286	.330	.300	.323	.311	.329	.338	.326	.337

STEP 1 Order the data from least to greatest: .286, .300, .303, .311, .317, .318, .322, .323, .324, .326, .329, .330, .331, .337, .338, .340, .342, .351

STEP 2 Choose the values for the stems. Remember, the leaves are the last digit in each number, so the stems are: .28, .29, .30, .31, .32, .33, .34, and .35.

STEP 3 Write the stems in the left column. Write the leaves for each stem in the right column. Insert a title and a key to explain what the stems and leaves represent.

Batting Averages

Stem	Leaf
.28	6
.29	
.30	0 3
.31	1 7 8
.32	2 3 4 6 9
.33	0 1 7 8
.34	0 2
.35	1

Key: .28 | 6 represents .286

b. Describe what you can tell about the data by looking at the plot.

From the graph, you can see these facts.

- No two players have the same batting average.
- No player has a batting average in the .290's.
- The batting average of .286 is an outlier.
- Of the 18 data points, 12 are in the .311-to-.338 range, so the data cluster in that range.

Watch Out!

Even though the data do not include any batting averages in the .290's, you still need to list .29 as a stem with no leaf. You cannot use 0 as a leaf because that would mean that there is a data value of .290.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Need More HELP

An outlier is a data point that is much less or greater than the rest of the data set. For help with outliers, go to *Outliers* on page 1298.

Median and Range

You can find the median and range of a set of data that is displayed in a stem-and-leaf plot.

EXAMPLE 3

A group hiked part of the Appalachian Trail. The stem-and-leaf plot shows the ages of the hikers in the group.

a. What is the median age of the hikers?

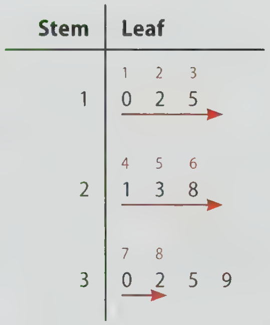
Ages of Hikers

Stem	**Leaf**
1	0 2 5
2	1 3 8
3	0 2 5 9
4	4 7 8
5	2 6

Key: 1 | 0 = 10

The **median** is the middle number in a list. There are 15 numbers, so the eighth leaf represents the median, or middle number, in the data set. Starting at the top of the plot, count to the eighth leaf.

Ages of Hikers



The median age of the hikers is 32.

b. What is the range of the ages of the hikers?

The **range** is the difference between the greatest number in a list and the least. Subtract to find the range: 56 − 10 = 46.

The range of the ages of the hikers is 46 years.

Need More HELP

For more about median and mean, go to *The "Center" of a Data Set* (p. 1294), and for more about range, go to *Effects of Changing Data Values* (p. 1302).

Try It This Way

To find the median of 15 numbers, write the numbers 1–15. Then cross off numbers two at a time, one from the front and one from the end of the list, as shown here.

~~1~~, ~~2~~, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, ~~14~~, ~~15~~

When you have one unpaired number left, it is the median.

Mean and Mode

EXAMPLE 4

The stem-and-leaf plot shows the total number of cars parked at a garage each day for the past 10 days.

Cars Parked
(Past 10 Days)

Stem	Leaf
0	1 2 3 6
1	0 0 3 6
2	0 9

Key: 2 | 9 = 29

a. What was the mean number of cars parked per day at the garage?

The **mean** is the average of the numbers in a list. List each value on the stem-and-leaf plot. Then find the sum.

$1 + 2 + 3 + 6 + 10 + 10 + 13 + 16 + 20 + 29 = 110$ cars

Divide the sum by the number of days: $110 \div 10 = 11$.

The mean number of cars parked per day at the garage was 11.

b. What is the mode of the data set?

The **mode** is the number or numbers that appear most often in a list.

Look at the data in the plot above. Is there a data value that appears more frequently than others? Yes, the stem 1 has two 0 leaves. The data value 10 appears twice.

The mode is 10.

c. Why is the 0 leaf in the 2 stem not included in the mode count?

As modeled in the key, 2|9 represents 29 cars. So 2|0 represents 20 cars. The mode is 1|0, or 10 cars, which appears twice.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Got to KNOW!

Stem-and-Leaf Plots

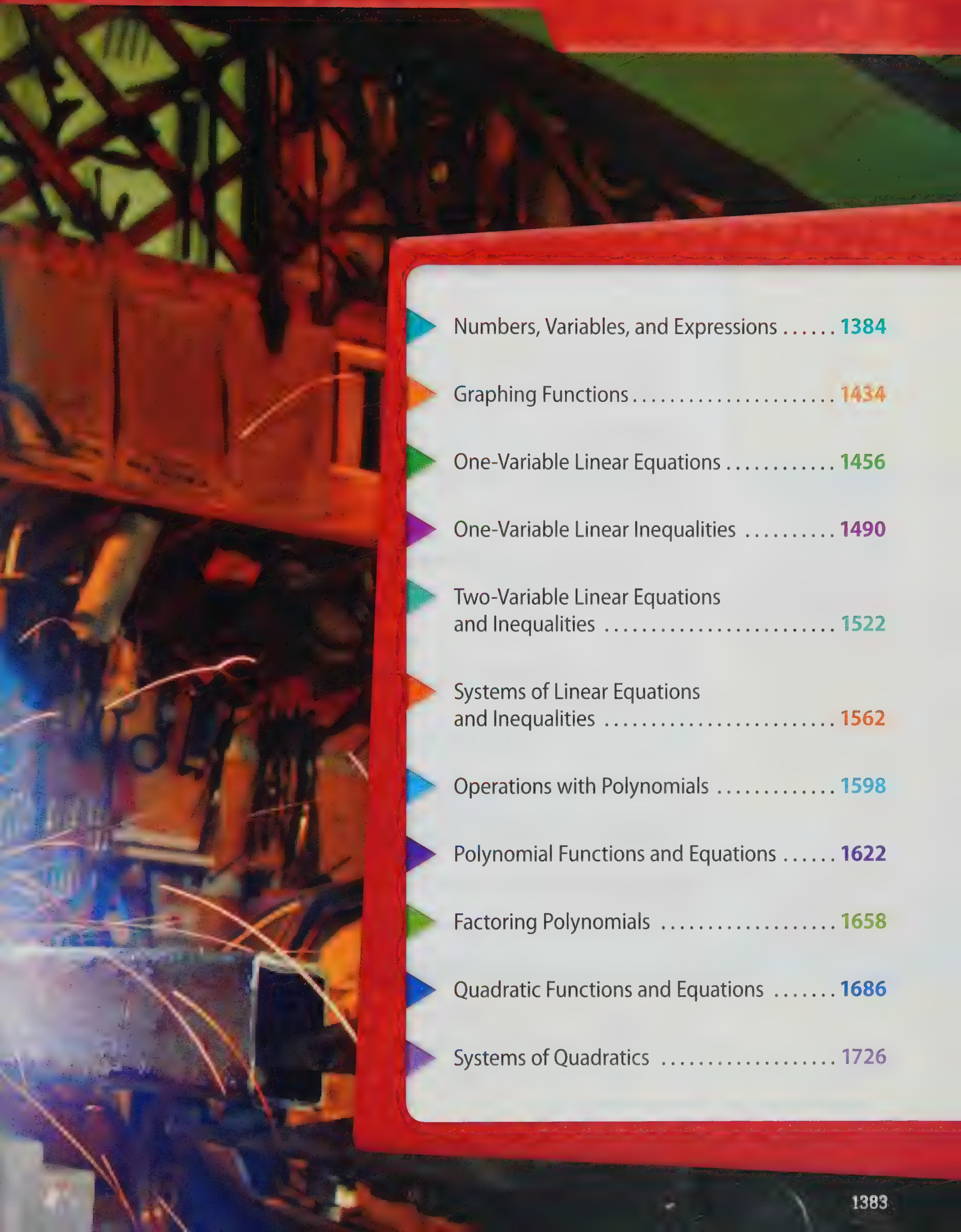
The data in a stem-and-leaf plot are displayed by place value. The ***stems*** may have one or more digits. The ***leaves*** are always the digits with the least place value in the numbers. The **key** tells how to read the plot.

Key: 13 | 2 = 132

The digits to the left of the bar represent 1 hundred and 3 tens. The digit to the right represents 2 ones. The total value of these digits is 132.

Algebra

Welders use linear inequalities to account for the expansion and contraction of metal when it is heated or cooled.



Numbers, Variables, and Expressions 1384

Graphing Functions 1434

One-Variable Linear Equations 1456

One-Variable Linear Inequalities 1490

Two-Variable Linear Equations and Inequalities 1522

Systems of Linear Equations and Inequalities 1562

Operations with Polynomials 1598

Polynomial Functions and Equations 1622

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Quadratic Functions and Equations 1686

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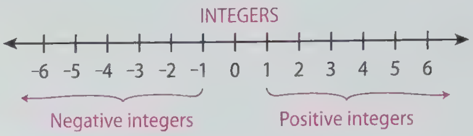
Integers and Absolute Value

Integers

The set of whole numbers, their *opposites,* and zero are **integers**. Integers are often used in life for quantities that can have both positive and negative values, such as temperatures and elevation.



On a number line, **opposites** are numbers that are the same distance from but on opposite sides of zero. For example, on the number line above, -4 and 4 are opposites. The sum of two opposites is zero. Zero is its own opposite.

EXAMPLE 1

Identify the number as an integer or non-integer. Graph the number and its opposite on the number line.

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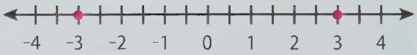
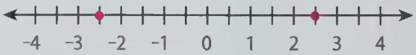
a. 3



The number 3 is a whole number, so it is an integer. Its opposite is -3.

b. $-2\frac{1}{2}$



The number $-2\frac{1}{2}$ is a fraction. It is not an integer.

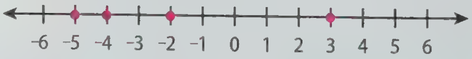
Its opposite is $2\frac{1}{2}$.

Comparing Integers

The numbers on a number line increase in value from left to right. When comparing two integers, the number farther to the right is greater in value.

EXAMPLE 2

Use the number line to compare the integers.



a. 3 and -4

The integer 3 is to the right of -4, so $3 > -4$.

b. -5 and -2

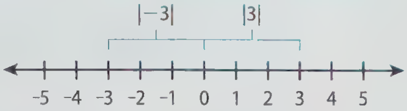
The integer -5 is to the left of -2, so $-5 < -2$.

Finding Absolute Value

The **absolute value** of a number is its distance from zero on a number line. A number and its opposite have the same absolute value. Since absolute value represents a distance, it is always positive.

$|-3| = 3$ The number -3 is 3 units from 0.

$|3| = 3$ The number 3 is also 3 units from 0.



The absolute value of a number greater than zero is the number itself. The absolute value of a number less than zero is the *opposite* of the number.

EXAMPLE 3

Find each absolute value.

a. $|27|$

The absolute value is the number itself, 27.

b. $|-7.06|$

The absolute value is the *opposite* of the number, 7.06.

c. $|0|$

The absolute value is the number itself, 0.

d. $\left|-\frac{4}{5}\right|$

The absolute value is the *opposite* of the number, $\frac{4}{5}$.

Got to KNOW!

Absolute Value Symbol

The symbol for absolute value is a vertical bar on each side of the number.

$|3| = 3$

Read this equation as, "The absolute value of three is three."

Watch Out!

$-x$ looks like a negative number, but $-x$ means the opposite of x. When x is less than 0, $-x$ is ***positive***! For example, if $x = -7$, then $-x = -(-7)$, which is $+7$.

Got to KNOW!

Absolute Value in Variables

You can represent the general form for absolute value using variables.

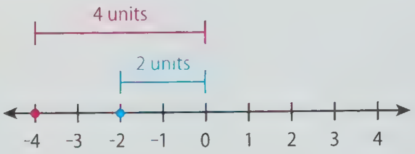
Numbers greater than or equal to 0	For $x \geq 0$, $\lvert x \rvert = x$.	The sign is unchanged.
Numbers less than 0	For $x < 0$, $\lvert x \rvert = -x$.	The sign is changed.

Comparing and Ordering Absolute Value

Just as you compared integers, you can compare absolute values. First find the absolute value of each number. Then compare the numbers. You can use a number line to help visualize the numbers.

When you compare $|-4|$ and $|-2|$, you see that the distance 4 is greater than the distance 2.

$$|-4| > |-2|$$



Watch Out!

Notice that -2 is an integer, not an absolute value. Do not change it to 2.

EXAMPLE 4

Order from least to greatest.

$\left|-\frac{1}{2}\right|, |-7|, |3.5|, -2$

STEP 1 Simplify each expression.

$\left|-\frac{1}{2}\right| = \frac{1}{2}$ $\quad |-7| = 7$ $\quad |3.5| = 3.5$ $\quad -2$

STEP 2 Order the numbers from least to greatest.

$-2, \frac{1}{2}, 3.5, 7$

STEP 3 Rewrite the numbers in their original form.

$-2, \left|-\frac{1}{2}\right|, |3.5|, |-7|$

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EXAMPLE 5

The lake level dropped 9.5 feet last week, but it rose 8.9 feet this week. During which week was the change in the lake level greater?

Consider the lake level drop as a negative number, -9.5.

Find the absolute value of each change.

$|-9.5| = 9.5$ $\quad |8.9| = 8.9.$

Compare the absolute values: $9.5 > 8.9$

The change was greater last week when the lake dropped 9.5 feet.

Simplifying Absolute-Value Expressions

You can simplify expressions that use absolute value. Think of the absolute value symbols as grouping symbols. First simplify what is inside the absolute value symbols. Then simplify any remaining absolute-value expressions and perform any operations following the order of operations.

Need More HELP

Remember the order of operations.

- Parentheses
- Exponents
- Multiply or Divide
- Add or Subtract

EXAMPLE 6

Simplify each absolute value expression.

a. $|5 - 1|$

Subtract inside the symbols. $|5 - 1| = |4|$

Simplify. $= 4$

b. $|2(3)|$

Multiply inside the symbols. $|2(3)| = |6|$

Simplify. $= 6$

c. $|10| + |-10|$

Simplify each absolute value. $|10| + |-10| = 10 + 10$

Add. $= 20$

d. $3 \times |-5|$

Simplify the absolute value. $3 \times |-5| = 3 \times 5$

Multiply. $= 15$

Got to KNOW!

Positive and Negative Numbers as Absolute Values

Every positive number is the absolute value of two numbers, the number itself and the opposite of the number. For example, positive 5 is the absolute value of both 5 and −5.

No negative number is the absolute value of any number!

Adding Integers

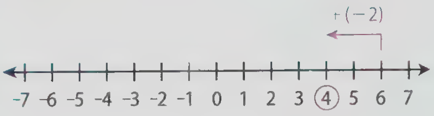
Adding Integers on a Number Line

You can add integers by using a number line. Start at the first value. Move right when you add a positive number, and move left when you add a negative number.

EXAMPLE 1

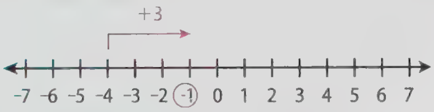
Find each sum.

a. $6 + (-2)$


+(−2)
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

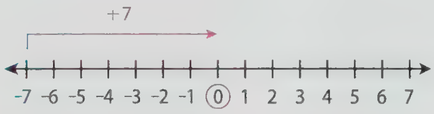
$6 + (-2) = 4$

b. $-4 + 3$


+3
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

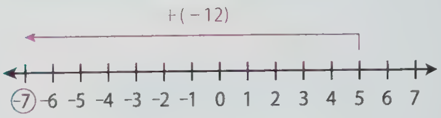
$-4 + 3 = -1$

c. $-7 + 7$


+7
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

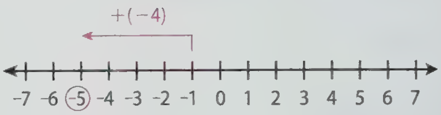
$-7 + 7 = 0$

d. $5 + (-12)$


+(−12)
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

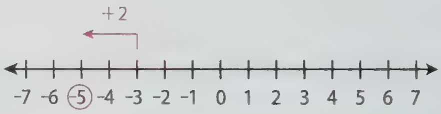
$5 + (-12) = -7$

e. $-1 + (-4)$


+(−4)
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

$-1 + (-4) = -5$

f. $(-3) + (-2)$


+2
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

$(-3) + (-2) = -5$

Got to KNOW!

Adding Opposites

Adding a number and its opposite always gives you zero.

$6 + (-6) = 0$

$a + (-a) = 0$

$-11 + 11 = 0$

$-x + x = 0$

Two-Color Counters

Two-color counters can help you understand integer addition.

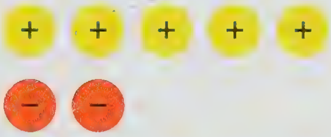
⊕ = 1   ⊖ = −1

Yellow counters represent positive integers.
Red counters represent negative integers.

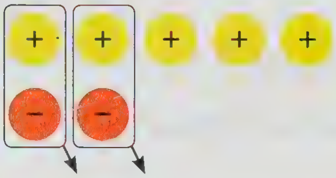
EXAMPLE 2

Use two-color counters to find the sum $5 + (-2)$.

STEP 1 Show 5 counters with a value of $+1$ and 2 counters with a value of -1.



STEP 2 Combine two positive counters and two negative counters to make pairs with a value of zero. Remove the *zero pairs*. The sum is the remaining counters.



$5 + (-2) = 3$

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Using Absolute Value

Absolute value can help you decide whether the sum of two signed integers is positive or negative.

EXAMPLE 3

The overnight, low temperature was −12°F. By noon, the temperature had increased by 9°F. What was the temperature at noon?

The answer is the sum of $-12 + 9$. Notice that the addends have different signs.

STEP 1 Remove the signs and subtract the smaller number from the greater number.

$12 - 9 = 3$

STEP 2 Use the sign of the number with the greater absolute value for the sum.

$|-12| = 12 \qquad |9| = 9$

$12 > 9$, so the sum will have the same sign of -12. The sum of $-12 + 9$ is negative.

$-12 + 9 = -3$

The temperature at noon was −3°F.

Got to KNOW!

Rules for Adding Two Signed Numbers

If the addends have the same sign, add the absolute values and use the sign of the addends.

$5 + 11 = 16 \qquad -5 + (-11) = -16$

If the addends have different signs, subtract their absolute values and use the sign of the addend with the greater absolute value.

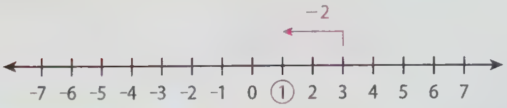
$-7 + 11 = 4 \qquad 7 + (-11) = -4 \qquad -15 + 6 = -9 \qquad 15 + (-6) = 9$

Subtracting Integers

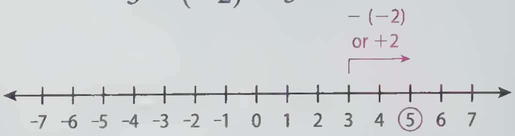
Subtracting Integers on a Number Line

You can subtract integers by using a number line. Start at the first value. Move left when you subtract a positive number and move right when you subtract a negative number.

$3 - 2 = 1$


−2
-7 -6 -5 -4 -3 -2 -1 0 ① 2 3 4 5 6 7

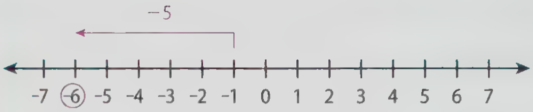
$3 - (-2) = 5$


− (−2)
or +2
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 ⑤ 6 7

EXAMPLE 1

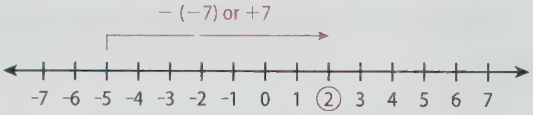
Find each difference.

a. $\mathbf{-1 - 5}$


−5
-7 ⑥ -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

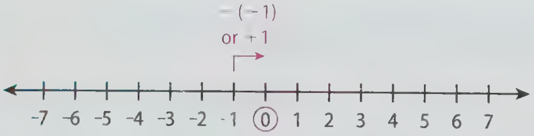
$-1 - 5 = -6$

b. $\mathbf{-5 - (-7)}$


− (−7) or +7
-7 -6 -5 -4 -3 -2 -1 0 1 ② 3 4 5 6 7

$-5 - (-7) = 2$

c. $\mathbf{-1 - (-1)}$


− (−1)
or +1
-7 -6 -5 -4 -3 -2 -1 ⓪ 1 2 3 4 5 6 7

$-1 - (-1) = 0$

Need More HELP

When you subtract a negative number, think of turning left on the number line to subtract, but then walking *backward*.

Got to KNOW!

Subtracting Integers

Subtracting a number is the same as adding its opposite.

$5 - 3 = 2$ $\qquad a - b = a + (-b)$
$5 + (-3) = 2$

To subtract a number, add its opposite. Then use the rules for adding signed numbers.

$4 - 9 = 4 + (-9)$ $\qquad 6 - (-7) = 6 + 7$
$= -5$ $\qquad = 13$

EXAMPLE 2

Find each difference.

a. 8 − (−5)

Add the opposite of −5. $8 - (-5) = 8 + 5$

Simplify. $= 13$

b. −4 − 7

Add the opposite of 7. $-4 - 7 = -4 + (-7)$

Simplify using the rules for adding signed numbers. $= -11$

c. 13 − 22

Add the opposite of 22. $13 - 22 = 13 + (-22)$

Simplify using the rules for adding signed numbers. $= -9$

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When you substract negative fractions or negative decimals, you use the same process as you when you substract negative integers.

EXAMPLE 3

Subtract.

$-\frac{1}{2} - \left(-\frac{2}{3}\right)$

STEP 1 Add the opposite of $-\frac{2}{3}$. $-\frac{1}{2} - \left(-\frac{2}{3}\right) = -\frac{1}{2} + \frac{2}{3}$

STEP 2 Write the fractions using a common denominator. $= -\frac{3}{6} + \frac{4}{6}$

STEP 3 Simplify using the rules for adding signed numbers. $= -\frac{3}{6} + \frac{4}{6} = \frac{1}{6}$

Need More HELP

For help with common denominators, go to *Adding Fractions with Unlike Denominators* in *Numbers and Operations* (Book 1, p. 322).

EXAMPLE 4

Francisco had a bank account balance of $236.85. The bank reverses a previous charge, and −$18.50 is subtracted from the balance. What is Francisco's new balance?

Set up a subtraction problem. $236.85 - (-18.5)$

Add the opposite of −18.5. $236.85 - (-18.5) = 236.85 + 18.5$

Simplify. $= 255.35$

Francisco's new balance is $255.35.

Multiplying and Dividing Integers

Multiplying Integers

Look at the patterns below:

$3 \times 2 = 6$	$-3 \times 2 = -6$
$3 \times 1 = 3$	$-3 \times 1 = -3$
$3 \times 0 = 0$	$-3 \times 0 = 0$
$3 \times (-1) = -3$	$-3 \times (-1) = 3$
$3 \times (-2) = -6$	$-3 \times (-2) = 6$

When you multiply integers, you multiply absolute values. The signs of the factors determine the sign of the product.

Multiplying Integers with the Same Sign

If two integers have the same sign, their product is positive.

Got to KNOW!

Multiplicative Property of 0

For any real number a:
$a \cdot 0 = 0$ and $0 \cdot a = 0$

Multiplicative Property of -1

For any real number a:
$a \cdot (-1) = -a$ and
$(-1) \cdot a = -a$

EXAMPLE 1

Find each product.

a. 7(8)

$|7| = 7$ $|8| = 8$ $7(8) = 56$

So, $7(8) = 56$.

b. $(-1)(-5)$

$|-1| = 1$ $|-5| = 5$ $(1)(5) = 5$

So, $(-1)(-5) = 5$.

Multiplying Integers with Different Signs

If two integers have different signs, their product is negative.

Ways to REMEMBER

Use this sentence to remember how to determine the sign of a product or a quotient of two integers.

Sleeping People Do Nothing.

If both signs are the Same, the result is Positive.

If the signs are Different, the result is Negative.

EXAMPLE 2

Find each product.

a. $12(-9)$

$|12| = 12$ $|-9| = 9$ $12(9) = 108$

So, $12(-9) = -108$.

b. $(-3)(15)$

$|-3| = 3$ $|15| = 15$ $(3)(15) = 45$

So, $(-3)(15) = -45$.

EXAMPLE 3

In one fantasy football league, a team scores −2 points every time its players are charged with a turnover (a fumble or an interception). If a team's players are charged with 9 turnovers, how many points are scored?

To find the total number of points, multiply the number of turnovers by the number of points scored for each turnover.

turnover	times	points for each turnover	equals	total points
9	×	−2	=	−18

The number of points scored is −18.

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Dividing Integers

Use the same rules for the signs when you divide integers. If two integers have the same sign, their quotient is positive. If two integers have different signs, their quotient is negative.

EXAMPLE 4

Find each quotient.

a. $\frac{-40}{-4}$

$|40| = 40$ $|-4| = 4$ $\frac{40}{4} = 10$

The signs are the same. So, $\frac{-40}{-4} = 10$.

b. $-100 \div 5$

$|-100| = 100$ $|5| = 5$ $100 \div 5 = 20$

The signs are different. So, $-100 \div 5 = -20$.

Got to KNOW!

Multiplying or Dividing Integers with the Same Sign

If two numbers have the same sign, their product or quotient is positive.

$(-)(-) = (+)$ $(+)(+) = (+)$ $(-) \div (-) = (+)$ $(+) \div (+) = (+)$

Multiplying or Dividing Integers with Different Signs

If two numbers have different signs, their product or quotient is negative.

$(-)(+) = (-)$ $(+)(-) = (-)$ $(-) \div (+) = (-)$ $(+) \div (-) = (-)$

Exponents

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Exponents and Powers

Sometimes it is helpful to use a shorthand way to write repeated multiplication by the same number. For example, in the following problem, 4 is a factor 3 times.

$4 \times 4 \times 4 = \underline{?}$

A shorter way to write this problem is to use *exponential notation*. **Exponential notation** expresses multiplication as a *power*.

- A **power** is the product written as a base and an exponent.
- The **base** is the number used as a factor.
- The **exponent** tells the number of times the factor is used. The exponent is written as a superscript to the right of the base.

$$\overbrace{4 \times 4 \times 4}^{3 \text{ factors of } 4} \rightarrow \overbrace{\underset{\overset{\uparrow}{\text{Base}}}{4}^{3}}^{\text{Power}} \swarrow \text{Exponent}$$

This power is read as "the third power of four," "four to the third power," or simply, "four to the third."

EXAMPLE 1

Write each repeated multiplication as a power. Evaluate.

a. (2)(2)(2)(2)(2)(2)(2)(2)

There are 8 factors of 2, so the power is 2^8.

Use a calculator: 2 x^y 8 ENTER 256

b. 5 × 5 × 5 × 5 × 5 × 5

There are 6 factors of 5, so the power is 5^6.

Use a calculator: 5 x^y 6 ENTER 15,625

Try It This Way

You may need to use a different key on your calculator, such as y^x or ^.

EXAMPLE 2

Write the power 3^7 as a repeated multiplication.

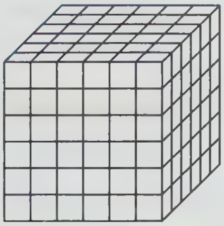
The base is 3, and the exponent is 7, so 3 is used as a factor 7 times.

$3^7 = (3)(3)(3)(3)(3)(3)(3)$

EXAMPLE 3

Find the number of small cubes in the figure.

There are 6 small cubes along each edge, so there are $6^3 = 216$ small cubes in the figure.



Special Exponents

The numbers 0, 1, 2, and 3 have special characteristics when used as exponents.

Exponent of 0: A power with a base $\neq 0$ and an exponent of 0 has a value of 1.

$$a^0 = 1, \text{ when } a \neq 0$$

Exponent of 1: A power with an exponent of 1 has a value equal to the base. As a result, any number equals that number to the first power.

$$a^1 = a$$

Exponent of 2: You know that a base with an exponent of 2 can be read as "(base) to the second power" or "(base) to the second." The power can also be read as "(base) squared."

Exponent of 3: You know that a base with an exponent of 3 can be read as "(base) to the third power" or "(base) to the third." It can also be read as "(base) cubed."

Need More HELP

For help understanding why $2^0 = 1$, see Example 6 on page 1398.

EXAMPLE 4

Simplify.

a. 6^0

$6^0 = 1$

b. 7^1

$7^1 = 7$

c. **4 squared**

4 squared $= 4^2 = 16$

d. **2 cubed**

2 cubed $= 2^3 = 8$

Got to KNOW!

Exponential Notation

$$\underbrace{\text{base}^{\text{exponent}}}_{\text{power}}$$

$\text{Base}^0 = 1$	Base^2 = Base Squared
Base^1 = Base	Base^3 = Base Cubed

Multiplying and Dividing with Exponents

Examine this example of multiplying powers with the same base.

$$5^2 \times 5^4 = (5)(5) \times (5)(5)(5)(5) = (5)(5)(5)(5)(5)(5) = 5^6$$

The **Product of Powers Property** states that to *multiply* powers that have the same base, you add the exponents and keep the same base.

Look at this example of dividing powers with the same base.

$$\frac{3^6}{3^4} = \frac{(3)(3)(3)(3)(3)(3)}{(3)(3)(3)(3)} = \frac{(3)(3)\cancel{(3)}\cancel{(3)}\cancel{(3)}\cancel{(3)}}{\cancel{(3)}\cancel{(3)}\cancel{(3)}\cancel{(3)}} = 3^2$$

The **Quotient of Powers Property** states that to *divide* powers that have the same base, you can *subtract* the exponent of the divisor from the exponent of the dividend and keep the same base.

EXAMPLE 5

Write each product as a power with the same base.

a. $\mathbf{10^{12} \cdot 10^8}$

Use the Product of Powers Property. $10^{12} \cdot 10^8 = 10^{12+8}$

Add the exponents, and keep the base the same. $= 10^{20}$

b. $\mathbf{15 \cdot 15^6}$

Write 15 as 15^1. Use the Product of Powers Property. $15^1 \cdot 15^6 = 15^{1+6}$

Add the exponents, and keep the base the same. $= 15^7$

Watch Out !

You cannot apply the Product of Powers Property to $3^7 \cdot 5^7$, because the bases are different: 3 and 5. The Product of Powers and the Quotient of Powers Properties only apply when the bases are the same.

EXAMPLE 6

Write each quotient as a power with the same base.

a. $\mathbf{\frac{8^5}{8^5}}$

The quotient should equal 1, because the numerator equals the denominator.

Use the Quotient of Powers Property. $\frac{8^5}{8^5} = 8^{5-5}$

Subtract the exponents, and keep the base the same. $= 8^0$

Simplify the exponent 0. $= 1$

b. $\mathbf{16^3 \div 16}$

Write 16 as 16^1. Use the Quotient of Powers Property. $16^3 \div 16^1 = 16^{3-1}$

Subtract the exponents, and keep the base the same. $= 16^2$

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A Power to a Power

Look at the following problem.

$$(4^3)^2 = (4^3)(4^3) = 4^{3+3} = 4^6$$

You can see that $(4^3)^2$ is 4^3 used as a factor twice. According to the Products of Powers Property, the final exponent is found by adding the exponents in each factor together.

If you look again at the initial problem and the final power, you can see that $2 \times 3 = 6$. The **Power of a Power Property** states that to raise a power to a power, you can *multiply* the exponents.

EXAMPLE 7

a. Simplify $(4^2)^3$.

METHOD 1

Use the Product of Powers Property.

Write the cube of 4^2 as a product. $(4^2)^3 = (4^2)(4^2)(4^2)$

Multiply. $= (16)(16)(16) = 4{,}096$

METHOD 2

Use the Power of a Power Property.

Write the base with one exponent. $(4^2)^3 = 4^{2\cdot 3}$

Multiply the exponents. $= 4^6$

Simplify. $= 4{,}096$

b. Simplify $(2^0)^4$.

Use the Power of a Power Property. $(2^0)^4 = 2^{0\times 4}$

Multiply the exponents and simplify. $= 2^0 = 1$

Watch Out!

These rules used to solve problems with the same base apply only to multiplication and division. They cannot be used in addition and subtraction problems involving powers.

Got to KNOW!

Rules for Exponents

For $a \neq 0$, and whole numbers m and n, the following properties are true.

Product of Powers Property

To multiply powers with the same base, use the same base and add the exponents.

$$a^m \times a^n = a^{m+n}$$

Quotient of Powers Property

To divide powers with the same base, use the same base and subtract the exponent of the divisor from the exponent of the dividend.

$$\frac{a^m}{a^n} = a^{m-n}$$

Power of a Power Property

To raise a power to a power, multiply the exponents.

$$(a^m)^n = a^{m\cdot n}$$

Square Roots

Try It This Way

Making a table of perfect squares can help you find integer square roots.

Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144

Squares and Square Roots of Integers

A number that is multiplied by itself, or squared, to form a product is a **square root** of that product. For example, $5^2 = 25$, so 5 is a square root of 25. Every positive number has two square roots.

$25 = 5^2$, so one square root of 25 is 5.

$25 = (-5)^2$, so another square root of 25 is -5.

The positive square root of the number is the **principal square root**. It is indicated with the symbol $\sqrt{}$, which is called a radical sign. The negative square root is represented by $-\sqrt{}$. The symbol $\pm\sqrt{}$ represents both square roots.

The number under the radical sign is called the **radicand**. If the square root of the radicand is an integer, the radicand is called a **perfect square**.

EXAMPLE 1

Simplify, if possible.

a. $\pm\sqrt{81}$

$9^2 = 81$, and $(-9)^2 = 81$, so $\pm\sqrt{81} = 9$ and -9

b. $-\sqrt{49}$

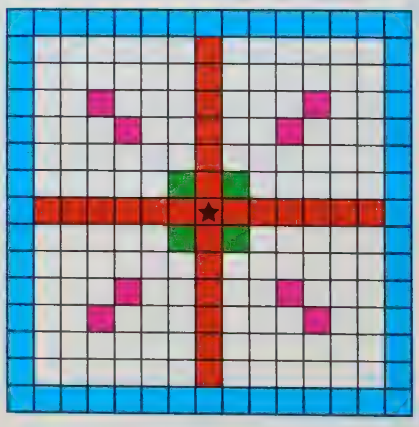
$7^2 = 49$, so $\sqrt{49} = 7$, and $-\sqrt{49} = -7$

c. $\sqrt{-16}$

There is no real solution. No square of a real number is negative.

EXAMPLE 2

There are 225 mini-squares on the game board. How many mini-squares do not touch an edge of the board?



METHOD 1

There are 225 mini-squares, so the number of mini-squares along each edge is $\sqrt{225} = 15$.

There are 56 mini-squares along the edges.

The number of mini-squares that do not touch an edge of the board is $225 - 56 = 169$ mini-squares.

METHOD 2

The area of the board containing mini-squares that do not touch an edge of the board is 13 by 13.

$13^2 = 169$, so 169 mini-squares do not touch an edge of the board.

Squares and Square Roots of Other Real Numbers

Decimals and fractions also have square roots. Some square roots are not whole numbers, but they can be expressed as the quotient of two integers.

$2.5^2 = 6.25$, so $\sqrt{6.25} = 2.5$ $\quad$ $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, so $\sqrt{\frac{1}{4}} = \frac{1}{2}$

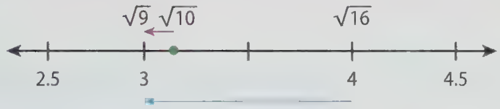
Many square roots, such as $\sqrt{2}$ and $\sqrt{3}$, cannot be expressed as a quotient of two integers. You can approximate square roots that do not have exact values.

EXAMPLE 3

A square patio has an area of 10 m², so each side is $\sqrt{10}$ m long. Approximate the length of a side to the nearest hundredth of a meter.

METHOD 1

Because $\sqrt{9} = 3$ and $\sqrt{16} = 4$, $\sqrt{10}$ must be between 3 and 4. You can estimate the value between 3 and 4 as shown below.



$$3 + \frac{10 - 9}{16 - 9} = 3\frac{1}{7}$$

The decimal value of this square root estimate is 3.142857…

Rounded to the nearest hundredth, the length of a side is 3.14 m.

METHOD 2

Use a calculator. **√** 10 = 3.16227766

Rounded to the nearest hundredth, the length of a side is 3.16 m.

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Simplifying Radical Terms

You can simplify a radical term by factoring out any factors of the radicand that are perfect squares.	$\sqrt{250} = \sqrt{25 \cdot 10}$
Find the square root of any perfect square.	$\sqrt{25} = 5$
Write the expression as the product of the square root and the remaining radical term.	$\sqrt{250} = \sqrt{25 \cdot 10} = 5\sqrt{10}$

EXAMPLE 4

Simplify, if possible.

a. $\sqrt{242}$

$$\begin{aligned}\sqrt{242} &= \sqrt{121 \cdot 2} \\ &= \sqrt{121} \cdot \sqrt{2} = 11\sqrt{2}\end{aligned}$$

b. $\sqrt{65}$

This term is already simplified. No factors of 65 are perfect squares.

Commutative and Associative Properties

The Commutative Properties

You know you can *add* numbers in any order, and the order does not affect the sum.

$5 + 9 = 14$ and $9 + 5 = 14$, so $5 + 9 = 9 + 5$.

You also know you can *multiply* numbers in any order, and the order does not affect the product.

$5 \cdot 9 = 45$ and $9 \cdot 5 = 45$, so $5 \cdot 9 = 9 \cdot 5$.

These relationships are expressed in the following properties.

Commutative Property of Addition
For any real numbers a and b, $a + b = b + a$.
Commutative Property of Multiplication
For any real numbers a and b, $ab = ba$.

Expressions such as $x + 7$ and $7 + x$, which have equal values for any value of x, are called **equivalent expressions**.

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EXAMPLE 1

Find the sum $6 + \frac{1}{8} + 4 + \frac{7}{8}$.

Use the Commutative Property of Addition to rewrite the expression.	$6 + 4 + \frac{1}{8} + \frac{7}{8}$
Add the whole numbers. Then add the two fractions.	$10 + 1$
Find the sum.	$10 + 1 = 11$

EXAMPLE 2

Use a commutative property to write an equivalent expression.

a. $2w$

An equivalent expression using the Commutative Property of Multiplication is $w \cdot 2$.

b. $pq + 8$

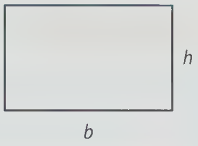
An equivalent expression using the Commutative Property of Multiplication is $qp + 8$.

An equivalent expression using the Commutative Property of Addition is $8 + pq$.

An equivalent expression using both commutative properties is $8 + qp$.

EXAMPLE 3

The area of a rectangle is given by $A = bh$, where b is the base and h is the height. What is another way to write the formula?



Use the Commutative Property of Multiplication.

$A = hb$

EXAMPLE 4

An auditorium contains 34 rows of seats, with 22 seats in each row. The number of seats can be found by multiplying the number of rows by the number of seats in each row, or by multiplying the number of seats in each row by the number of rows. Which property does this represent?

These are two orders for multiplication, 34(22) and 22(34), so this problem represents the Commutative Property of Multiplication.

The Associative Properties

Because you can add numbers in any order, you can also regroup addends without affecting the sum.

$(79 + 38) + 62$	$79 + (38 + 62)$
$117 + 62$	$79 + 100$
179	179

And because you can multiply numbers in any order, you can also regroup factors without affecting the product.

$(8.1 \cdot 3.2) \cdot 12.5$	$8.1 \cdot (3.2 \cdot 12.5)$
$25.92 \cdot 12.5$	$8.1 \cdot 40$
324	324

These relationships are expressed in the following properties.

Associative Property of Addition
For any real numbers a, b, and c, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication
For any real numbers a, b, and c, $(ab)c = a(bc)$.

Expressions such as $(6 + 7) + 5$ and $6 + (7 + 5)$ are equivalent expressions, as was true for expressions with equal values by the commutative properties.

Watch Out !

There are no associative and commutative properties of subtraction and division. You can use addition properties with a subtraction problem if you think of subtraction as being addition of a negative number.

$12 - 5 = 12 + (-5)$

To use multiplication properties with a division problem, you must remember that dividing is the same thing as multiplying the dividend by the reciprocal of the divisor.

$\frac{12}{5} = 12 \cdot \frac{1}{5}$.

Using the Properties

EXAMPLE 5

Use an associative property to write an equivalent expression.

a. $(2x + 17) + 33$

An equivalent expression using the Associative Property of Addition is $2x + (17 + 33)$.

b. $(7p)(p) + 21$

An equivalent expression using the Associative Property of Multiplication is $7(p \cdot p) + 21$.

Ways to REMEMBER

To help you remember which properties are associative and which are commutative, think about what the terms mean.

When you *commute* to work or school, you move from one place to another.

When you *associate* with people in class, you form a group.

You can use the commutative and associative properties together with mental math to solve problems quickly.

EXAMPLE 6

Find the sum or product.

a. $97 + 89 + 103 + 111$

Look for pairs of compatible numbers.

Use the Commutative Property of Addition.	$97 + 103 + 89 + 111$
Use the Associative Property of Addition.	$= (97 + 103) + (89 + 111)$
Add each group of numbers. Then find the sum.	$= 200 + 200 = 400$

b. $3\sqrt{6} \cdot 5\sqrt{24}$

Use the Commutative Property of Multiplication.	$3 \cdot 5 \cdot \sqrt{6} \cdot \sqrt{24}$
Use the Associative Property of Multiplication.	$= (3 \cdot 5) \cdot \left(\sqrt{6} \cdot \sqrt{24}\right)$
Multiply each group of numbers.	$= 15 \cdot \sqrt{144}$
Simplify the radical. Then find the product.	$= 15 \cdot 12 = 180$

c. $2^5 \cdot 0.25x \cdot 2^2$

Use the Commutative Property of Multiplication.	$0.25 \cdot 2^5 \cdot 2^2 \cdot x$
Use the Associative Property of Multiplication.	$= 0.25 \cdot (2^5 \cdot 2^2) \cdot x$
Multiply the powers.	$= 0.25 \cdot 2^7 \cdot x$
Simplify the power.	$= 0.25 \cdot 128 \cdot x$
Use the Associative Property of Multiplication to find the product.	$= (0.25 \cdot 128) \cdot x = 32x$

Need More HELP

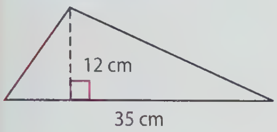
To multiply two square roots, you multiply the numbers under the radical signs.

For every number $a \geq 0$ and $b \geq 0$,

$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

EXAMPLE 7

Find the area of the triangle.



The formula for the area of a triangle is $A = \frac{1}{2}bh$.

METHOD 1

Use the Associative Property of Multiplication to group the fraction with one variable. $A = \left(\frac{1}{2}b\right)h$

Substitute the values for the variables. $= \left(\frac{1}{2} \cdot 35\right)12$

Multiply the grouped factors. Then multiply this product by the other factor. $= 17.5 \cdot 12$
$= 210$

METHOD 2

Use the Associative Property of Multiplication to group the variables. $A = \frac{1}{2}(bh)$

Substitute the values for the variables. $= \frac{1}{2}(35 \cdot 12)$

Multiply the grouped factors. Then multiply this product by the other factor. $= \frac{1}{2}(420)$
$= 210$

The area of the triangle is 210 cm^2.

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Got to KNOW!

The Commutative and Associative Properties

For any real numbers a, b, and c:

Operation	Commutative Property	Associative Property
Addition	$a + b = b + a$	$(a + b) + c = a + (b + c)$
Multiplication	$ab = ba$	$(ab)c = a(bc)$

Distributive Property

Using the Distributive Property to Simplify

The number 27 is actually the sum of 20 and 7. When you multiply a number by 27, you are actually first multiplying the number by 7. Next, you multiply the number by 20, and then, you add these two products.

	12
	× 27
the product of 12 and 7	84
the product of 12 and 20	+240
the sum of these products	324

Distributing one factor over two addends of another factor is called the Distributive Property. This property is sometimes called the Distributive Property of Multiplication over Addition.

The Distributive Property states that for any real numbers *a*, *b*, and *c*:

$$a(b + c) = ab + ac$$

The Distributive Property can help you do mental math.

EXAMPLE 1

Use the Distributive Property to rewrite 14(1,002). Then simplify.

Rewrite 1,002 as 1,000 + 2.	$14(1{,}002) = 14(1{,}000 + 2)$
Use the Distributive Property.	$= 14(1{,}000) + 14(2)$
Multiply.	$= 14{,}000 + 28$
Add.	$= 14{,}028$

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The Distributive Property also works for multiplication over subtraction.

For any real numbers a, b, and c, $a(b - c) = ab - ac$.

EXAMPLE 2

Use the Distributive Property to rewrite 8(99). Then simplify.

Rewrite 99 as 100 − 1.	$8(99) = 8(100 - 1)$
Use the Distributive Property.	$= 8(100) - 8(1)$
Multiply.	$= 800 - 8$
Subtract.	$= 792$

Use the Distributive Property if you see an easier calculation that is equivalent.

EXAMPLE 3

Simplify 2(53) + 2(47).

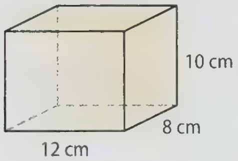
The numbers 53 and 47 are compatible for addition.

Use the *Distributive Property* to rewrite the expression.	$2(53) + 2(47) = 2(53 + 47)$
Add.	$= 2(100)$
Multiply.	$= 200$

The Distributive Property can be extended to apply to more than two terms.

EXAMPLE 4

The surface area of a rectangular prism is given by $SA = 2wh + 2\ell w + 2\ell h$, where w is the width, ℓ is the length, and h is the height. What is the surface area of this rectangular prism?



METHOD 1

Substitute into the formula.

$$\begin{aligned} SA &= 2wh + 2\ell w + 2\ell h \\ &= (2 \cdot 8 \cdot 10) + (2 \cdot 12 \cdot 8) + (2 \cdot 12 \cdot 10) \\ &= 160 + 192 + 240 \\ &= 592 \text{ cm}^2 \end{aligned}$$

METHOD 2

Each term has a factor of 2. Use the Distributive Property to rewrite the formula.

$$\begin{aligned} SA &= 2(wh + \ell w + \ell h) \\ &= 2(8 \cdot 10 + 12 \cdot 8 + 12 \cdot 10) \\ &= 2(80 + 96 + 120) \\ &= 2(296) \\ &= 592 \text{ cm}^2 \end{aligned}$$

Got to KNOW!

The Distributive Property

The Distributive Property of Addition over Multiplication	
For any real numbers a, b, and c, $a(b + c) = ab + ac$ $(b + c)a = ba + ca$	$4(5 + 3) = 4(5) + 4(3)$ $(5 + 3)4 = (5)4 + (3)4$
The Distributive Property of Subtraction over Multiplication	
For any real numbers a, b, and c, $a(b - c) = ab - ac$ $(b - c)a = ba - ca$	$4(5 - 3) = 4(5) - 4(3)$ $(5 - 3)4 = (5)4 - (3)4$

Variable Expressions

One important use of the Distributive Property is in writing equivalent expressions. When the Distributive Property is used to factor out the common factors of two or more *terms*, the resulting expression is equivalent to the original expression. The **terms** of an expression are the parts that are added or subtracted.

$$\underbrace{2x}_{} + \underbrace{y}_{} - \underbrace{6}_{}$$

terms

In the expression above, which terms have a common factor? The terms $2x$ and 6 have 2 as a common factor.

Use the Commutative Property to change the order of the terms.	$2x - 6 + y$
Rewrite 6 as 2(3).	$2x - 2(3) + y$
Use the Distributive Property to write an equivalent expression.	$2(x - 3) + y$

Ways to REMEMBER

One way to remember the distributive property is to think of *distribute,* which means to give something out to members of a group.

EXAMPLE 5

Use the Distributive Property to write an equivalent expression.

a. $-2(y - 5)$

Use the Distributive Property.	$-2(y) + (-2)(-5)$
Multiply.	$-2y + 10$

The expressions $-2(y - 5)$ and $-2y + 10$ are equivalent.

b. $4r + 8$

Write 8 as 4(2).	$4r + 4(2)$
Use the Distributive Property.	$4(r + 2)$

The expressions $4r + 8$ and $4(r + 2)$ are equivalent.

The Distributive Property can also be used to simplify expressions containing *like terms.* **Like terms** contain the same variables raised to the same powers. **Unlike terms** differ in either a variable or the power of a variable. The following table provides several examples of like terms and unlike terms.

Identifying Like Terms

Terms	Variable(s)	Power(s)	Like terms?
$3m$ and $6m$	m	1	Yes
$4x$ and $4y$	x and y	1	No
$3a$ and $4a^2$	a	1 and 2	No

EXAMPLE 6

Use the Distributive Property to simplify, if possible.

a. $11y - y$

These terms are like terms. y is equivalent to $1y$.

Use the Distributive Property to factor out y.	$11y - y = y(11 - 1)$
Add.	$= y(10)$
Use the Commutative Property.	$= 10y$

b. $9x + 4y$

These terms are not like terms. The variables are different.

This expression cannot be simplified.

c. $5n + 7n^2$

These terms are not like terms. The variables are the same, but the powers differ.

This expression cannot be simplified.

Need More HELP

Remember that the variable y has a coefficient of 1 and can be written as $1y$.

EXAMPLE 7

Simplify.

a. $3(m - 2) + 6m$

Use the Distributive Property to multiply.	$3(m - 2) + 6m = 3m - 6 + 6m$
Use the Commutative Property.	$= 3m + 6m - 6$
Use the Distributive Property again.	$= m(3 + 6) - 6$
Subtract.	$= m(9) - 6$
Use the Commutative Property again.	$= 9m - 6$

b. $16 - 2(k - 2)$

Use the Distributive Property to multiply.	$16 - 2(k - 2) = 16 - 2k - 2(-2)$
Multiply.	$= 16 - 2k + 4$
Use the Commutative Property.	$= 16 + 4 - 2k$
Add.	$= 20 - 2k$

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The Identity and Inverse Properties

The Additive Identity Property

When 0 is added to any number, the result is that number, so 0 is called the **additive identity**.

$3 + 0 = 3 \qquad 0 + 6 = 6 \qquad -8 + 0 = -8 \qquad 0 + \left(-\frac{1}{2}\right) = -\frac{1}{2}$

For any real number *a*:

$a + 0 = a$ and $0 + a = a$

The Multiplicative Identity Property

When any number is multiplied by 1, the result is that number, so 1 is called the **multiplicative identity**.

$7 \cdot 1 = 7 \qquad 1 \cdot 2.5 = 2.5 \qquad -25 \cdot 1 = -25 \qquad 1 \cdot \frac{2}{3} = \frac{2}{3}$

For any real number *a*:

$a \cdot 1 = a$ and $1 \cdot a = a$

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EXAMPLE 1

Simplify each expression.

a. $75 + 98 - 75$

Use the Commutative Property.	$75 + 98 - 75 = 98 + 75 - 75$
Use the Associative Property.	$= 98 + (75 - 75)$
Subtract.	$= 98 + 0$
Use the Additive Identity Property.	$= 98$

b. $5 + h - 5$

Use the Commutative Property.	$5 + h - 5 = h + 5 - 5$
Use the Associative Property.	$= h + (5 - 5)$
Subtract.	$= h + 0$
Use the Additive Identity Property.	$= h$

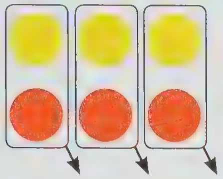
c. $7p - 6p$

Use the Distributive Property.	$7p - 6p = (7 - 6)p$
Subtract.	$= 1p$
Use the Multiplicative Identity Property.	$= p$

The Additive Inverse Property

When any number and its opposite are added, the result is the additive identity, 0. As a result, the opposite of a number is also called its **additive inverse**.

$$3 + (-3) = 0$$
$$-3 + 3 = 0$$



A number and its additive inverse are additive inverses of each other.

For any real number a, there is exactly one additive inverse $-a$, for which

$a + (-a) = 0$ and $-a + a = 0$

Watch Out!

The number -3 is a negative number, negative 3. It can also be read as "the opposite of 3."

The number $-a$ is "the opposite of a." The sign of $-a$ might be positive, or it might be negative or zero. It depends on the sign of a.

EXAMPLE 2

Find the additive inverse of each number.

a. 0.875

Find the opposite of 0.875. -0.875

The additive inverse of 0.875 is -0.875.

b. $-\frac{4}{11}$

Find the opposite of $-\frac{4}{11}$. $\frac{4}{11}$

The additive inverse of $-\frac{4}{11}$ is $\frac{4}{11}$.

c. 0

Find the opposite of 0. 0

The additive inverse of 0 is 0.

d. $5x$

Find the opposite of $5x$. $-5x$

The additive inverse of $5x$ is $-5x$.

EXAMPLE 3

Kadija babysits for her neighbor. In October, she earned \$25 one week and \$45 the next week. She then spent \$30 on a present before earning \$35 the third week. The last week of October, she earned \$30. She then spent \$25 on clothes. How much money did she have at the end of the month?

The money she earns is positive money for her. What she spends is negative. Because $-30 + 30 = 0$ and $25 - 25 = 0$, just add \$45 and \$35. She has \$80 at the end of the month.

The Multiplicative Inverse Property

When any nonzero number is multiplied by its reciprocal, the result is the multiplicative identity, 1. Thus, the reciprocal of any number is also called the **multiplicative inverse** of the number.

A number and its multiplicative inverse are multiplicative inverses of each other.

For any nonzero real number a, there is exactly one multiplicative inverse $\frac{1}{a}$, for which

$$a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1$$

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EXAMPLE 4

Find the multiplicative inverse of each number.

a. **7**

Find the reciprocal of 7. $\frac{1}{7}$

The multiplicative inverse of 7 is $\frac{1}{7}$.

b. $\frac{1}{9}$

Find the reciprocal of $\frac{1}{9}$. $\frac{1}{\frac{1}{9}} = 1 \div \frac{1}{9} = 1 \times \frac{9}{1} = 9$

The multiplicative inverse of $\frac{1}{9}$ is 9.

c. **−4**

Find the reciprocal of −4. $\frac{1}{-4} = -\frac{1}{4}$

The multiplicative inverse of −4 is $-\frac{1}{4}$.

d. **0**

Zero has no multiplicative inverse because division by zero is undefined.

EXAMPLE 5

Find the multiplicative inverse of 2.5.

METHOD 1

Write 2.5 as $\frac{25}{10}$. The multiplicative inverse of $\frac{25}{10}$ is $\frac{10}{25}$. Divide 10 by 25.

$$\begin{array}{r} 0.4 \\ 25\overline{)10.0} \end{array}$$

METHOD 2

Use a calculator. 2.5 x^{-1} = 0.4

EXAMPLE 6

There are about $4\frac{1}{3}$ weeks in the average month. To convert a monthly salary to a weekly salary, multiply the monthly salary by the multiplicative inverse of $4\frac{1}{3}$. Find the weekly salary for a monthly salary of $4,500 a month.

Write $4\frac{1}{3}$ as an improper fraction.

$$4\frac{1}{3} = \frac{4 \cdot 3 + 1}{3} = \frac{13}{3}$$

Find the multiplicative inverse of $\frac{13}{3}$.

$$\frac{1}{\frac{13}{3}} = \frac{3}{13}$$

Multiply $4,500 by $\frac{3}{13}$.

$$\frac{3}{13}(4{,}500) \approx 1{,}038.461538$$

The weekly salary is $1,038.46.

EXAMPLE 7

Pears are on sale at the local grocery for 3 lb for $3.29. To find the cost of a certain weight of pears, multiply the number of pounds by the multiplicative inverse of $\frac{3}{3.29}$. Find the price of 5 lb of pears.

Write $\frac{3}{3.29}$ to include no decimals.

$$\frac{3}{3.29} = \frac{300}{329}$$

Find the multiplicative inverse of $\frac{300}{329}$.

$$\frac{1}{\frac{300}{329}} = \frac{329}{300}$$

Multiply 5 lb by $\frac{329}{300}$.

$$5\left(\frac{329}{300}\right) \approx 5.4833$$

Five pounds of pears cost $5.48.

Need More HELP

The identity properties are sometimes known as the Addition Property of 0 and the Multiplication Property of 1.

The inverse properties were once called the Axiom of Additive Inverses and the Axiom of Multiplicative Inverses.

Got to KNOW!

Identity and Inverse Properties

For any real number a, the following properties apply:

Addition

Identity	$a + 0 = a$	and	$0 + a = a$
Inverse	$a + (-a) = 0$	and	$-a + a = 0$

Multiplication

Identity	$a \cdot 1 = a$	and	$1 \cdot a = a$
Inverse	$a \cdot \frac{1}{a} = 1$	and	$\frac{1}{a} \cdot a = 1\ (a \neq 0)$

Order of Operations

Order of Operations

The expression $17 - 3 \times 4$ contains no grouping symbols. Without a specific order for performing the operations, the expression could be simplified in more than one way. Here are two ways someone might simplify it.

$$\begin{aligned} &\underbrace{17 - 3} \times 4 \\ &\quad 14 \quad \times 4 \\ &\quad 56 \end{aligned} \qquad\qquad \begin{aligned} &17 - \underbrace{3 \times 4} \\ &17 - \quad 12 \\ &5 \end{aligned}$$

To prevent confusion when simplifying expressions with addition, subtraction, multiplication, and division and no grouping symbols, always use the following order:

- Multiply and divide from left to right.
- Add and subtract from left to right.

You don't need to do all multiplications before all divisions, or all additions before subtractions. You can multiply and divide as you get to each operation.

- Divide and multiply from left to right.
- Subtract and add from left to right.

EXAMPLE 1

Simplify each expression.

a. $17 - 3 \times 4$

STEP 1 Multiply.
$$17 - 3 \times 4 = 17 - \underbrace{3 \times 4}_{}$$
$$= 17 - 12$$

STEP 2 Subtract.
$$= 5$$

b. $100 + 50 \div 10 - 5 \times 2$

STEP 1 Divide and multiply from left to right.
$$100 + \underbrace{50 \div 10} - \underbrace{5 \times 2}$$
$$= 100 + 5 - 10$$

STEP 2 Add and subtract from left to right.
$$= 95$$

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EXAMPLE 2

Lashonda uses the calculator application on her computer to evaluate the expression shown. Was the expression calculated correctly?

44 [−] 12 [÷] 4 [+] 20 [×] 5 [=] 140

Use order of operations to check.
$$44 - 12 \div 4 + 20 \times 5$$

Divide and multiply from left to right first.
$$= 44 - 3 + 100$$

Subtract and add from left to right.
$$= 141$$

No, the correct answer is 141. The calculator evaluated all operations from left to right.

Order of Operations with Exponents

Exponents represent repeated multiplication. If an expression contains exponents, simplify any powers before performing any of the four basic operations.

- Simplify powers from left to right.
- Multiply and divide from left to right.
- Add and subtract from left to right.

EXAMPLE 3

Simplify each expression.

a. $54 - 3^3 \times 2$

Simplify the power first. $54 - 3^3 \times 2 = 54 - 3^3 \times 2$

$= 54 - 27 \times 2$

Multiply. $= 54 - 54$

Subtract. $= 0$

b. $(-7) + 2^7 \div 4 - 5^2$

Simplify the powers from left to right first. $(-7) + 2^7 \div 4 - 5^2$

$= (-7) + 128 \div 4 - 25$

Divide. $= (-7) + 32 - 25$

Add and subtract from left to right. $= 0$

Need More HELP

For help with exponents, go to pages 1396–1399.

Order of Operations with Parentheses and Brackets

Grouping symbols such as parentheses and brackets are used to make sure that calculations are done in a specific order. Simplify expressions inside grouping symbols first.

- Perform operations inside parentheses.
- Simplify powers.
- Multiply and divide from left to right.
- Add and subtract from left to right.

This is the expression from the beginning of the lesson, with parentheses added.

$$\underbrace{(17 - 3)}_{14} \times 4$$

$$56$$

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EXAMPLE 4

Simplify each expression.

a. $164 - (10 + 5) \times 2^3$

Perform the operation in parentheses first.	$164 - (10 + 5) \times 2^3$
	$= 164 - 15 \times 2^3$
Simplify the power.	$= 164 - 15 \times 8$
Multiply.	$= 164 - 120$
Subtract.	$= 44$

b. $10 + 16^{(4-3)} \div (24 \div 3)^2$

Perform the operations in parentheses first.	$10 + 16^{(4-3)} \div (24 \div 3)^2$
	$= 10 + 16^{(1)} \div (8)^2$
Simplify the powers.	$= 10 + 16 \div 64$
Divide.	$= 10 + \frac{1}{4}$
Add.	$= 10\frac{1}{4}$

c. $22 + 3^{(1+3)} - (3 \times 2^2)^2$

Perform the operations in parentheses first.

Simplify the powers within parentheses.	$22 + 3^{(1+3)} - (3 \times 2^2)^2$
	$= 22 + 3^{(1+3)} - (3 \times 4)^2$
Simplify within the parentheses.	$= 22 + 3^4 - (12)^2$
Simplify the powers.	$= 22 + 81 - 144$
Add and subtract.	$= -41$

EXAMPLE 5

Felicia had \$220. She bought shoes for \$47.97, jeans for \$34.99, four pairs of socks at \$6.33 a pair, and two T-shirts at \$12.50 each. Then she gave half of what she had left to her mom. Evaluate the expression below to find the amount she gave her mom.

$[220 - (47.97 + 34.99 + 4 \cdot 6.33 + 2 \cdot 12.5)] \div 2$

STEP 1 Multiply inside parentheses.	$[220 - (47.97 + 34.99 + 25.32 + 25)] \div 2$
STEP 2 Add inside parentheses.	$= [220 - (133.28)] \div 2$
STEP 3 You can remove the parentheses.	$= [220 - 133.28] \div 2$
STEP 4 Subtract.	$= [86.72] \div 2$
STEP 5 Divide.	$= 43.36$

Felicia gave her mom \$43.36.

Need More HELP

When an expression has both parentheses and brackets or has nested parentheses, always start with the innermost pair first.

Order of Operations with Other Grouping Symbols

Division bars and square-root symbols are also grouping symbols.

$$\frac{\text{numerator}}{\text{denominator}} \qquad \sqrt{\text{expression}}$$

EXAMPLE 6

Simplify each expression.

a. $\frac{2^2+45}{8(11)-3^4}$

Perform the operations in the numerator and in the denominator first.

Simplify the powers. $\frac{2^2+45}{8(11)-3^4} = \frac{4+45}{8(11)-81}$

Multiply. $= \frac{4+45}{88-81}$

Add and subtract. $= \frac{49}{7}$

Divide. $= 7$

b. $10^2\sqrt{8+4 \cdot 2}$

Perform the operations under the radical symbol first.

Multiply under the radical. $10^2\sqrt{8+4 \cdot 2} = 10^2\sqrt{8+8}$

Add under the radical. $= 10^2\sqrt{16}$

Simplify the square root. $= 10^2(4)$

Simplify the power. $= 100(4)$

Multiply. $= 400$

Ways to REMEMBER

In the U.S., use the sentence **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally to remember the correct order of operations.

P Parentheses
E Exponents
M Multiply
D Divide
A Add
S Subtract

In Canada, brackets, not parentheses, are used. A helpful sentence to remember is **B**ig **E**lephants **D**estroy **M**ice **A**nd **S**nails.

Got to KNOW!

Order of Operations

P Perform operations inside Parentheses.
E Simplify Exponents.
MD Multiply and Divide from left to right.
AS Add and Subtract from left to right.

Translating English into Algebra

Identifying Key Words

To solve word problems algebraically, it is first necessary to change words and phrases into mathematical expressions. You can associate certain words with mathematical operations.

The table below shows some common phrases used in problem solving and their algebraic translations.

Words	Symbols	
a number	x	
a number plus 5 a number increased by 5 5 more than a number the sum of 5 and a number	$x + 5$	+
a number minus 16 a number decreased by 16 a number less 16 16 less than a number the difference of a number and 16	$x - 16$	−
7 times a number a number multiplied by 7 the product of 7 and a number	$7 \cdot x$	×
a number divided by 2 the quotient of a number and 2 the ratio of a number and 2	$\frac{x}{2}$	÷

Translating from Symbols to Words

EXAMPLE 1

Write two possible translations for each expression.

a. $n + 50$

a number n increased by 50 — the sum of n and 50

b. $q \div 8$

a number q divided by 8 — the quotient of q and 8

c. $12 - w$

a number w subtracted from 12 — the difference of 12 and a number w

d. $14y$

14 times y — the product of 14 and y

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An algebraic expression might contain more than one operation. Care should be taken to translate accurately.

EXAMPLE 2

Write two possible translations for each expression.

a. $3x + 10$

3 times a number x, plus 10; the sum of 3 times a number x and 10

b. $3(x + 10)$

3 multiplied by the sum of x and 10; the product of 3 and the sum x plus 10

Translating from Words to Symbols

When you translate from words to symbols, look for words that tell you what operations to perform.

combine, add **Add** extra, more, expand	take away, reduce by **Subtract** discount, deduct, remove	by a factor of, times as many **Multiply** for each	per, for each **Divide** into equal groups

Watch Out!

Other English expressions can also indicate operations, but you need to read the problem to decide the operation. "Separate into *equal groups*" indicates division. "There were 10 *equal groups* of 5" indicates multiplication.

EXAMPLE 3

Write each phrase as an algebraic expression.

a. a number *r* split into 3 equal parts

Split into equal parts means division. $r \div 3$

b. 17 less than a number *c*

Less than means subtraction. $c - 17$

EXAMPLE 4

a. Carol had *d* dollars in her account, and she added $100. Write an expression for the amount in her account now.

STEP 1 Look for a word to indicate the operation. *Added* indicates addition.

STEP 2 Write the expression. The variable d represents the amount Carol had in her account. An expression for the amount in her account now is $d + 100$.

b. Mr. Jackson took *t* minutes to drive to school. The time it took him to drive home was twice that long. Write an expression for the time it took him to drive home.

STEP 1 Look for a word to indicate the operation. *Twice* indicates multiplication.

STEP 2 Write the expression. The variable t represents the time it took him to drive to school. An expression for the time it took him to drive home is $2t$.

Functions

Graphs of Relations

You can represent a relation by a set of ordered pairs, a table, a rule or algebraic expression, or a graph.

[illegible]

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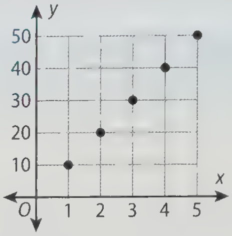
EXAMPLE 1

Look at the relation shown in the graph.



a. What are the ordered pairs?

(1, 10), (2, 20), (3, 30), (4, 40), (5, 50)

b. What are the domain and range?

Domain: {1, 2, 3, 4, 5}

Range: {10, 20, 30, 40, 50}

c. What is a rule relating the variables?

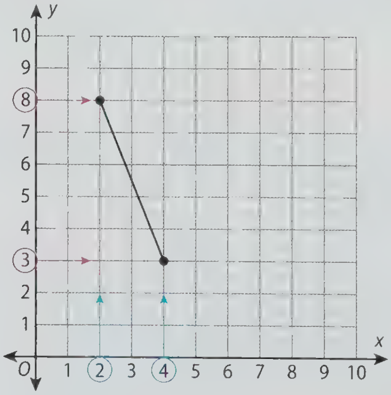
Each y-value is 10 times the corresponding x-value.

When the value of y depends on the value of x, x is the **independent variable** and y is the **dependent variable**.

A graph of a relation can include line segments. In that case, you cannot describe the domain or range by listing ordered pairs, but you can use inequality symbols.

EXAMPLE 2

What are the domain and range of the graph?

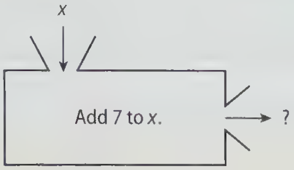


The domain is all x-values from 2 through 4. Domain: $\{2 \leq x \leq 4\}$

The range is all y-values from 3 through 8. Range: $\{3 \leq y \leq 8\}$

Identifying Functions

Suppose you have a machine like the one below.



When you put a number, the input, into the machine, the machine adds 7 to the number. The output from the machine is the sum of your number and 7. If you put in 5, the output is 12. If you put in 100, the output is 107.

If you input 5, the only possible output is 12. Because any given input can have only one possible output, we call this a *function machine*.

A **function** is a relation that assigns each input *exactly one* output.

EXAMPLE 3

Is y a function of x?

a. **(2, 5), (4, 9), (7, 10), (4, 11)**

No, when the input is $x = 4$, there are 2 different possible y-values.

b. **(2, 1), (5, 1), (8, 7), (10, 12)**

Yes, there are no x-values that are paired with two different y-values.

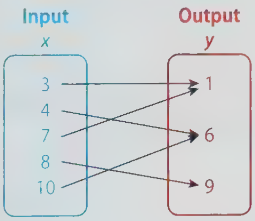
Watch Out!

It's still a function if different inputs generate the same output, such as (2, 1) and (5, 1).

EXAMPLE 4

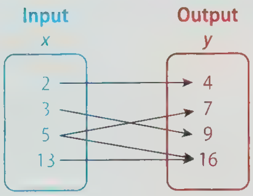
Does each mapping diagram show a function?

a.



No input has more than one output. The relation is a function.

b.



The input 5 has more than one output value, 7 and 16.

The relation is *not* a function.

Determining if a Relation is a Function

EXAMPLE 5

What are the domain and range of each relation? Is the relation a function?

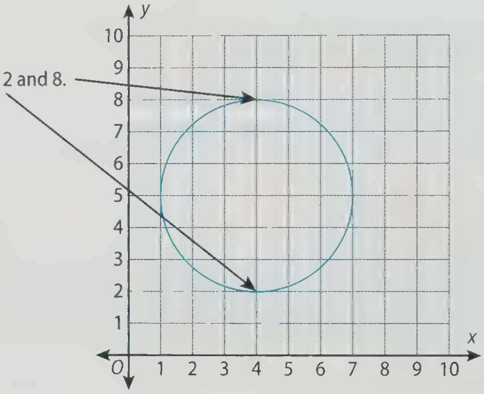
a. **Diamond Prices**

Weight (carats)	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$
Price ($)	3,500	6,500	6,800	7,500	7,500

The domain is $\left\{\frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}\right\}$. The range is {3,500, 6,500, 6,800, 7,500}.

Even though the last two columns are the same, each domain value is paired with exactly one range value. The relation is a function.

b.



The input $x = 4$ has two outputs, 2 and 8.

Each value of x between 1 and 7 is associated with 2 y-values. This relation is *not* a function.

When the domain of a relation has an infinite number of values, you can still make a table of values to see whether you can prove that the relation is not a function.

EXAMPLE 6

Examine the table for the relation $y^2 = x$. Is y a function of x?

When the value of x is 4, y has two values, -2 and 2.

Also, when the value of x is 1, y has two values, -1 and 1.

Either one of these conditions is proof that the relation is *not* a function.

x	y
$(-2)^2 = 4$	-2
$(-1)^2 = 1$	-1
$0^2 = 0$	0
$1^2 = 1$	1
$2^2 = 4$	2

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Try It This Way

To square a negative number like -2 on a scientific calculator, first press the (−) key.

Then type the number.

-2

Then, press the x^2 key.

4

EXAMPLE 7

If *y* is a function of *x*, and the domain is {0, 3, 5, 8}, what value of *x* must be shown in the red box?

x	*y*
0	2
5	4
3	6
🟥	4
8	10

If the choice is 0, when $x = 0$, there will be two *y*-values, 2 and 4.

If the choice is 5, when $x = 5$, there will be only one *y*-value, 4. ✓

If the choice is 3, when $x = 3$, there will be two *y*-values, 6 and 4.

If the choice is 8, when $x = 8$, there will be two *y*-values, 4 and 10.

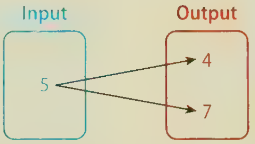
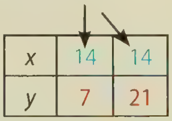
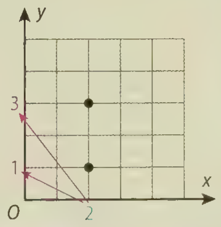
A function can have only one *y*-value for each *x*-value.

So the only possible value in the red box is 5 if *y* is a function of *x*.

Got to KNOW

When a Relation is Not a Function

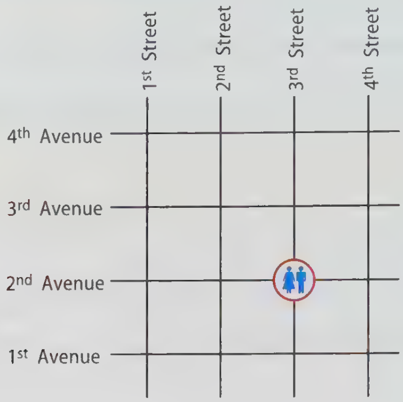
To determine whether or not a relation is a function, look for an input that has at least two different outputs. That shows the relation is *not* a function.

In a set of ordered pairs (*x*, *y*), look for the same value of *x* with two different *y*-values.	{(10, 3), (10, 7), …}
In a mapping diagram, look for two arrows from an input that go to two different numbers in the output.	
In a table of ordered pairs (*x*, *y*), look for the same *x*-value with two different *y*-values.	
In a graph, look for two points, one above the other, with the same *x*-value but two different *y*-values.	

The Coordinate Plane

How Can You Use the Coordinate Plane?

Many cities and towns are set up as a grid of numbered streets and avenues.



Need More HELP

For help in understanding ordered pairs, go to *Relations* on pages 1436–1437.

If you ask a friend to meet you at the intersection shown on the map, you might say, "Meet me at the corner of 2nd Avenue and 3rd Street."

The coordinate plane is like a city map. You can use it to visualize points that represent two related variables. The **coordinate plane** is formed by two number lines that intersect at right angles. The horizontal line is the **x-axis** and the vertical line is the **y-axis**. The point where the two axes intersect is called the **origin**.

Every point on the coordinate grid is identified by an *ordered pair* (x, y).

x-coordinate →
The number of units to the right from the origin

(2, 4)

← **y-coordinate**
The number of units up from the origin

Finding Coordinates

To determine the location of a point on the graph, move down from the point to read the x-coordinate first. Then move left from the point to read the y-coordinate.

EXAMPLE 1

Find the coordinates of each point on the graph.

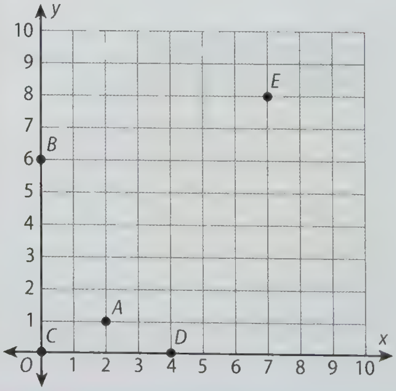
A is located at (2, 1).

B is located at (0, 6).

C is located at the origin (0, 0).

D is located at (4, 0).

E is located at (7, 8).



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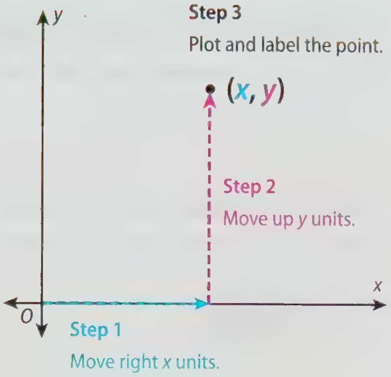
Plotting Points

To plot a point (x, y) on the graph, follow these steps.

- Start at the origin.
- Move right x units.
- Then move up y units.
- Plot and label the point.



Step 3
Plot and label the point.

Step 2
Move up *y* units.

Step 1
Move right *x* units.

Ways to REMEMBER

If you can't recall which variable is up, think of the word *yup*. Yes, *y* is up.

EXAMPLE 2

Plot each point on the coordinate plane.

a. ***F* at (2, 5)**

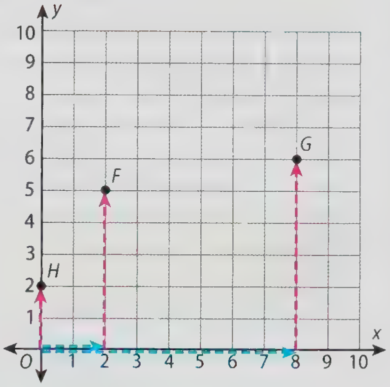
Move right 2 units. Then move up 5 units. Label the point *F*.

b. ***G* at (8, 6)**

Move right 8 units. Then move up 6 units. Label the point *G*.

c. ***H* at (0, 2)**

Move right 0 units. Then move up 2 units. Label the point *H*.



EXAMPLE 3

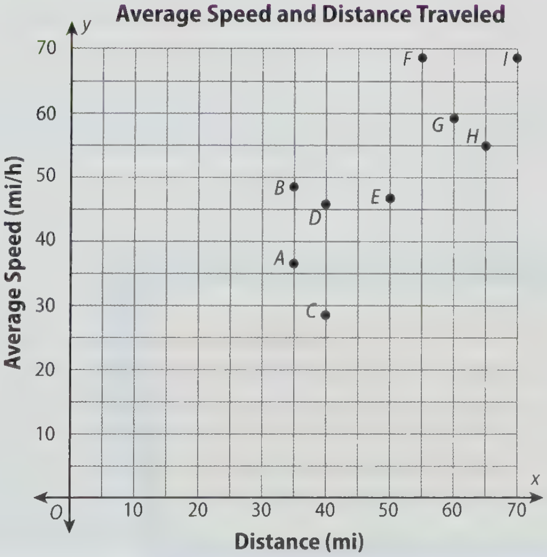
The graph shows the average speed and distance traveled by 9 different cars. Which point represents a car averaging 68 miles per hour for 55 miles?

Distance is the x-variable and average speed is the y-variable.

So look for the point (55, 68).

The point *F* represents a car averaging 68 miles per hour for 55 miles.

Average Speed and Distance Traveled



Different Ways to Represent a Function

You can represent a function as a set of ordered pairs, as a vertical or horizontal table of values, or as a graph.

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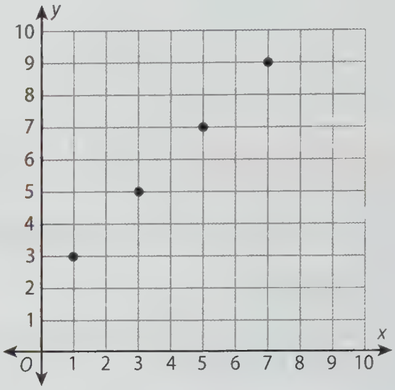
EXAMPLE 4

Graph each set of data.

a.

x	*y*
1	3
3	5
5	7
7	9

Plot the point (1, 3).
Plot the point (3, 5).
Plot the point (5, 7).
Plot the point (7, 9).



b.

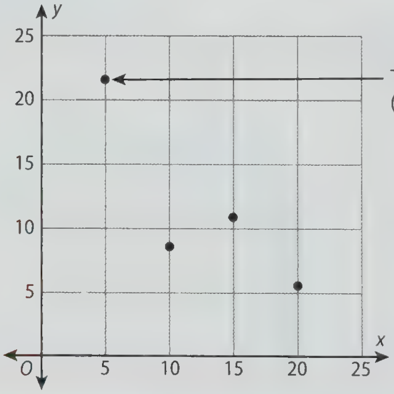
x	5	10	15	20
y	22	8	11	6

STEP 1 First choose a scale for each axis. The *x*-axis must include values up to 20 to fit the data. The *y*-axis must include values up to 22.

A good choice is for both axes to go to 25 with intervals of 5.

STEP 2 Plot each ordered pair. The ordered pairs are (5, 22), (10, 8), (15, 11), and (20, 6).

If a *y*-value does not fall on a grid line, estimate its position.



This is the graph of (5, 22).

Need More HELP

For help with scales of graphs, go to *Scatter Plots* in *Statistics and Probability* (p. 1264).

Functions can use variables other than *x* and *y*.

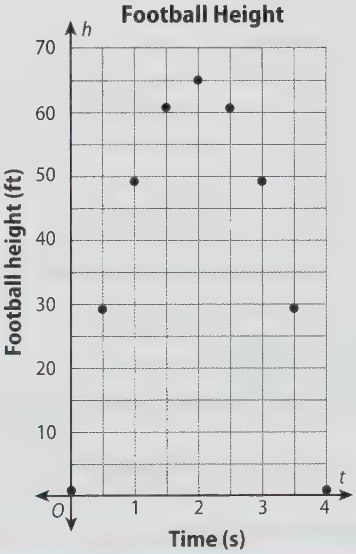
EXAMPLE 5

The height of a kicked football is a function of time. The height *h* in feet of a football after *t* seconds is given in the table below. Graph each ordered pair of values.

Time (s) ***t***	0	0.5	1	1.5	2	2.5	3	3.5	4
Height (ft) ***h***	1	29	49	61	65	61	49	29	1

The height of the football is a function of the time elapsed since it was kicked. Time is the independent variable, and height is the dependent variable. So show the time *t* on the horizontal axis and the height *h* on the vertical axis. Each point on the graph will represent an ordered pair (t, h).

Be sure the graph fits all the data.
The *t*-axis must show 0–4 seconds.
The *h*-axis must show 0–65 feet.



Try It This Way

If you have access to a graphing calculator, try its point graphing function. Use STAT and EDIT to enter values of *x* and *y* into lists 1 and 2. Then use the STAT PLOT function to display them.

Got to KNOW

Reading and Plotting Points

To read a point (x, y) on the coordinate plane, read the x-value on the x-axis and the y-value on the y-axis.

To plot a point (x, y), on the coordinate plane, move x units right. Then move y units up. Plot and label the point.

Linear Functions

Function Notation

Functions have their own notation. In **function notation**, the output value for the input x is written as $f(x)$ and read as "f of x." The notation $f(x) =$ means "the rule applied to x is."

Input	Rule	Output
x	$\longrightarrow$	$f(x)$

Equation: $y = x + 2$ — Meaning: The value of y equals the value of $x + 2$.

Function $f(x) = x + 2$ — Meaning: The input x in $f(x)$ gives the output $x + 2$.

Evaluating Functions

You can evaluate functions in the same way that you evaluate expressions.

EXAMPLE 1

Evaluate $f(x) = x^2 - 5$ for each value of x.

a. $x = 4$

$$\begin{aligned} f(x) &= x^2 - 5 \\ f(4) &= 4^2 - 5 \\ &= 16 - 5 \\ &= 11 \end{aligned}$$

b. $x = 7$

$$\begin{aligned} f(x) &= x^2 - 5 \\ f(7) &= 7^2 - 5 \\ &= 49 - 5 \\ &= 44 \end{aligned}$$

Input-Output Tables

One way to describe a function is to make an input-output table.

EXAMPLE 2

Make an input-output table for the function $f(x) = 2x - 5$.

Select values for x. Then evaluate $f(x)$.

Follow the rule: Multiply the input by 2 and then subtract 5.

$$\begin{aligned} \text{When } x = 1,\ f(1) &= 2(1) - 5 \\ &= 2 - 5 \\ &= -3 \end{aligned}$$

Input x	Rule $2x - 5$	Output $f(x)$
1	$2(1) - 5$	-3
2	$2(2) - 5$	-1
3	$2(3) - 5$	1
4	$2(4) - 5$	3
5	$2(5) - 5$	5

Got to KNOW!

Function Rules

Functions are often written as equations.

The statement $y = f(x)$ means that y is a function of x.

So the function

$$f(x) = x^2 - 5$$

can also be written as

$$y = x^2 - 5.$$

Graphing Functions

Often a function has a domain that is *continuous.* This type of function cannot be described by a set of ordered pairs because the domain is infinite.

A graph for this type of function is usually a line or curve. To graph a continuous function, plot the ordered pairs. Then connect the points by a line or smooth curve.

EXAMPLE 3

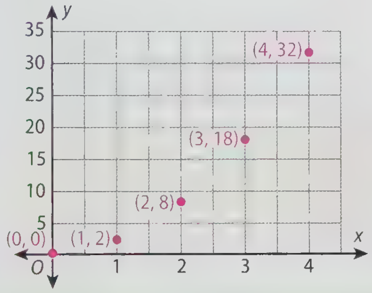
Graph the function $y = 2x^2$ for $x \geq 0$.

STEP 1 Make an input-output table.

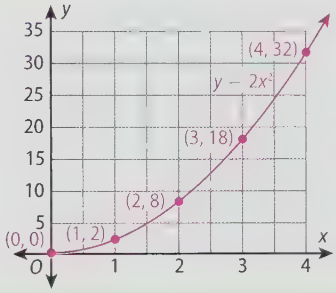
When $x = 0, y = 2(0)^2 = 0$
When $x = 1, y = 2(1)^2 = 2$
When $x = 2, y = 2(2)^2 = 8$

x	0	1	2	3	4
y	0	2	8	18	32

STEP 2 Plot the points as ordered pairs.



STEP 3 Connect the points with a smooth curve.



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Linear Functions

A **linear function** is any function with a graph that is a straight line.

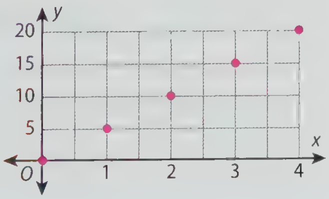
EXAMPLE 4

Graph the function $y = 5x$ for $x \geq 0$. Is the function linear?

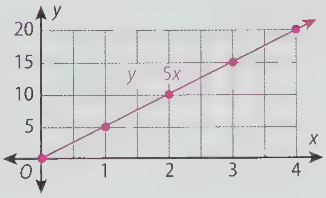
STEP 1 Make an input-output table.

x	0	1	2	3	4
y	0	5	10	15	20

STEP 2 Plot the points as ordered pairs.



STEP 3 Connect the points.



The graph is a straight line, so the function is linear.

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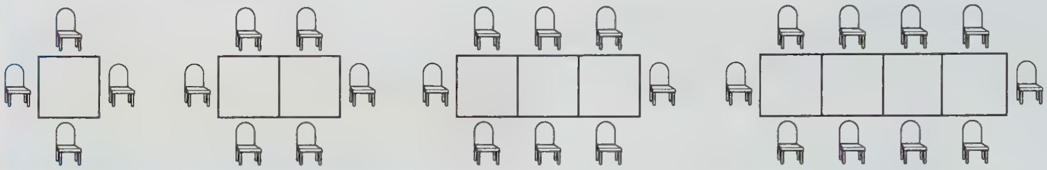
Using Patterns to Graph

Sometimes a problem relates two sets of numbers that follow a pattern. You can graph these relationships to determine whether they are linear functions.

EXAMPLE 5

The diagram shows the number of chairs that fit around a row of *x* tables. Graph the number of chairs *y* as a function of the number of tables *x*. Is the function linear?



METHOD 1

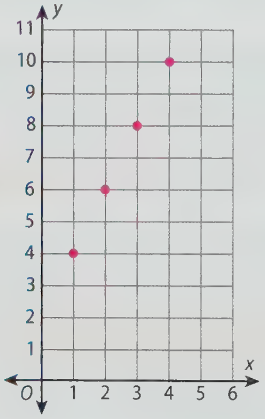
Make an Input-Output Table

STEP 1 Make an input-output table with the number of tables as the input x and the number of chairs as the output y.

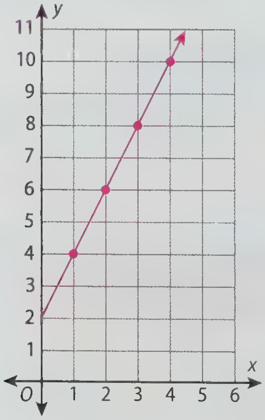
When $x = 1$, $y = 4$.
When $x = 2$, $y = 6$.
When $x = 3$, $y = 8$.
When $x = 4$, $y = 10$.

Tables x	1	2	3	4
Chairs y	4	6	8	10

STEP 2 Plot the points as ordered pairs.



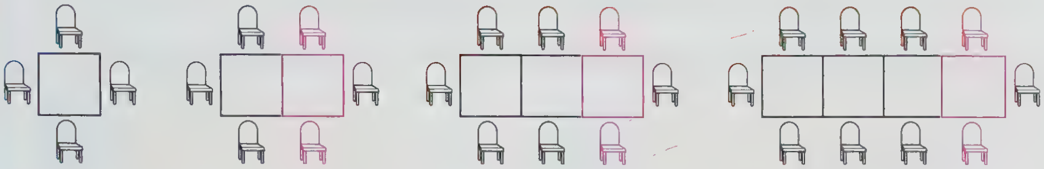
STEP 3 Connect the points.



The graph of the function is a straight line, so the pattern is linear. By extending the line, you can see how many chairs are needed for each additional table.

METHOD 2

Look for a Pattern

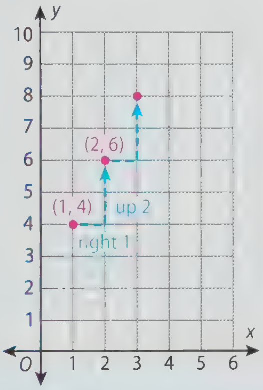


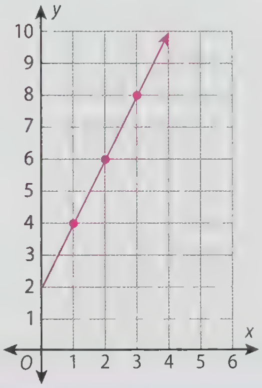
STEP 1 Notice that the first table is surrounded by 4 chairs. Plot the point (1, 4).

STEP 2 When each additional table is added, 2 more chairs are added. Move to the right on the graph 1 unit, and move up 2 units. Plot another point at (2, 6).

STEP 3 Continue moving right 1 and up 2 from each point, and plot another point.

STEP 4 Connect the points. The graph of the function is a straight line, so the pattern is linear.





Need More HELP

For help with graphing, go to *Graphing from an Equation* in *Algebra* (p. 1528).

Identifying Linear Functions

Linear functions are in the form $y = ax + b$, where a and b are real numbers.

These are linear functions.

$y = 3x$ $\quad a = 3$ and $b = 0$

$y = -2x + 7$ $\quad a = -2$ and $b = 7$

These are *not* linear functions.

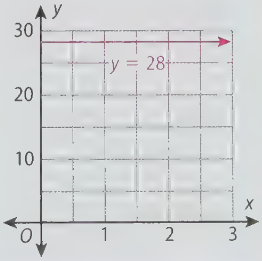
$y = x^2$ $\quad y = \sqrt{x}$

$y = 10^x + 5$

EXAMPLE 6

Is the function $y = 28$ linear?

Yes, the function is in the form $y = ax + b$, where $a = 0$ and $b = 28$. The graph of this function is a horizontal line at $y = 28$.



Got to KNOW!

Linear Functions

To graph a function, make an input-output table to find ordered pairs. Use at least two input values.

A linear function is any function with a graph that is a straight line.

Linear functions can be written in the form $y = ax + b$, where a and b are real numbers.

The Vertical Line Test

How Can You Use the Vertical Line Test?

Because a function has exactly one output for any input, you can use the vertical line test to decide whether a graph represents a function.

- If any vertical line can intersect the graph at more than one point, the graph does *not* represent a function.
- If no vertical line can intersect the graph at more than one point, the graph does represent a function.

Ways to REMEMBER

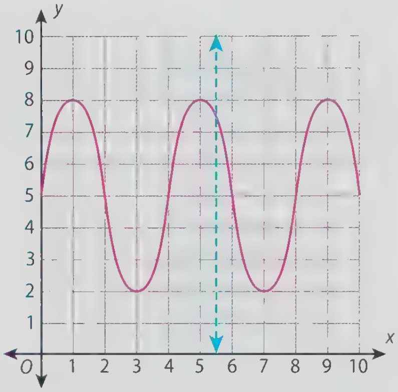
In basketball, it is illegal to jump with the ball and then not shoot it or pass it. This violation is called *up and down*. Remember, a graph does not represent a function if there are points "up and down."

EXAMPLE 1

Tell whether or not the graph represents a function.

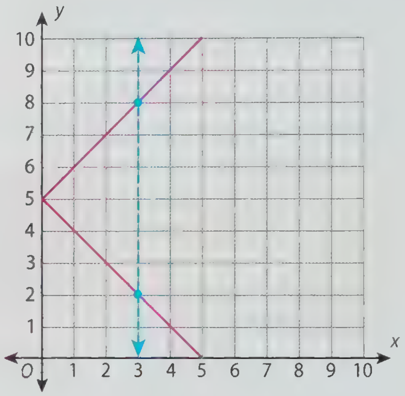
Use the vertical line test.

a.



No vertical line can cross the graph at more than one point. This graph does represent a function.

b.



The vertical line crosses the graph at more than one point. This graph does *not* represent a function.

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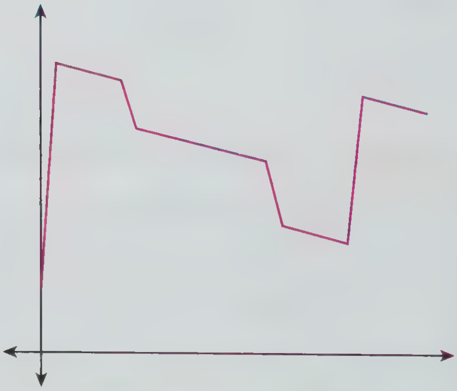
EXAMPLE 2

The graph shows the amount of water in a dog's bowl during 24 hours. Does the graph represent a function?

No vertical line can cross the graph at more than one point.

The graph does represent a function.



EXAMPLE 3

Show whether or not each relation is a function.

a. $y = 4$ **b.** $x = 7$

STEP 1 Make a table. Then use the table to make a graph.

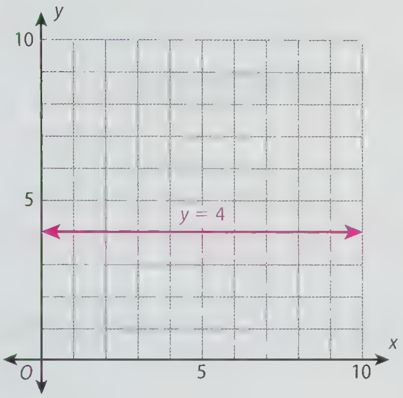
a. Rule: For any value of x, $y = 4$.

x	1	2	3	4
y	4	4	4	4

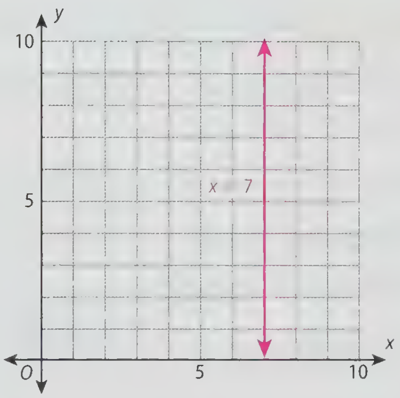
b. Rule: For any value of y, $x = 7$.

x	7	7	7	7
y	1	2	3	4

STEP 2 Use the vertical line test.

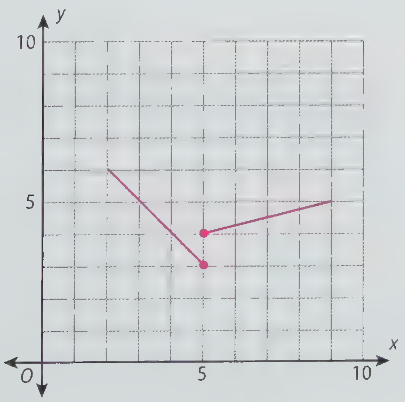


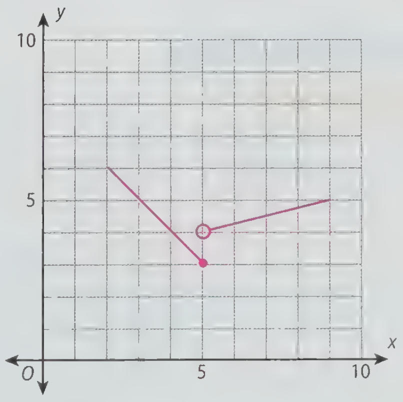
A vertical line cannot cross the graph at more than one point. The graph of $y = 4$ represents a function.



The graph is a vertical line. The graph of $x = 7$ does *not* represent a function.

An open circle at a point on a graph is *not* part of the graph. The graph on the left below does not represent a function because a vertical line can intersect both points shown. The graph on the right does represent a function because the upper segment does not contain the point at the open circle. So a vertical line cannot cross the graph at more than one point.





Got to KNOW!

The Vertical Line Test

If any vertical line intersects the graph at more than one point, the graph does not represent a function.

If no vertical line can intersect the graph at more than one point, the graph does represent a function.

Equation Concepts

True and False Equations

An equation is a mathematical statement that uses the symbol for equality, =, to state that two quantities are equal. An equation may be true or false.

True Equations		**False Equations**	
$2 + 2 = 4$	$5(4) + 1 = 21$	$1 + 1 = 3$	$25 \div 4 = 10$

If two quantities are *not* equal, use the symbol ≠, which means "is not equal to," to make a true statement.

$$1 + 1 \neq 3 \qquad 25 \div 4 \neq 10$$

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EXAMPLE 1

Place the correct symbol, = or ≠, in the box to make the statement true.

a. $3 + 3(7) \square 6(2 + 2)$

Simplify each expression. $3 + 3(7) \square 6(2 + 2)$

Use order of operations. $3 + 21 \square 6(4)$

The expressions are equal. $24 \boxed{=} 24$

$3 + 3(7) = 6(2 + 2)$

b. $59 - \left(-\frac{27}{3}\right) \square 2(3^2 + 4^2)$

Simplify each expression. $59 - \left(-\frac{27}{3}\right) \square 2(3^2 + 4^2)$

Use order of operations. $59 - (-9) \square 2(9 + 16)$

$68 \square 2(25)$

The expressions are *not* equal. $68 \boxed{\neq} 50$

$59 - \left(-\frac{27}{3}\right) \neq 2(3^2 + 4^2)$

Solving Equations

To solve an equation for an unknown quantity, find the missing value that makes the equation true. There are different ways to represent the missing value in an equation. You can use a question mark, an empty box, or a variable.

$$2 + \underline{?} = 5$$

$$2 + \square = 5$$

$$2 + x = 5$$

EXAMPLE 2

Solve each equation for the missing value.

a. $7 + ? = 10$

$7 + 3 = 10$

3 makes the equation true.

b. $10(4) = \square$

$10(4) = 40$

40 makes the equation true.

c. $x - 100 = 300$

$400 - 100 = 300$

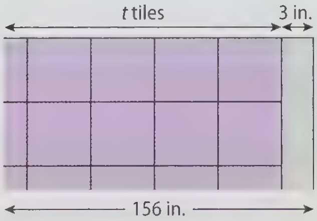
400 makes the equation true.

Using Guess-Check-Revise to Solve

Sometimes you can solve an equation by using the Guess-Check-Revise strategy.

EXAMPLE 3

The width of a room is 156 inches. After Tia covers the floor with as many 9-inch tiles as possible, a section of the floor that is 3 inches wide is left uncovered. How many tiles fit along the wall that measures 156 inches?



STEP 1 Write an equation to represent the problem.
Let *t* equal the unknown number of tiles.

9 times the number of tiles plus 3 in. equals 156 in.

$9(t) + 3 = 156$

STEP 2 Guess a value for *t*. Try 20. $9(20) + 3 \stackrel{?}{=} 156$

Too high $183 \neq 156$

STEP 3 Adjust the guess down from 20. ↓

Try 15. $9(15) + 3 \stackrel{?}{=} 156$

Too low $138 \neq 156$

STEP 4 Adjust the guess up from 15. ↑

Try 17. $9(17) + 3 \stackrel{?}{=} 156$

Got it! $156 = 156$ ✓

A total of 17 of the 9-inch tiles fit along the wall.

Watch Out !

To solve an equation, find the value that makes the equation true. The Guess-Check-Revise strategy may not work unless the answer is a simple value, such as a small whole number.

To learn more, see other solving methods in the following pages.

Need More HELP ?

The symbol $\stackrel{?}{=}$ means "does it equal?"

No Solution or Many Solutions

Variables on Both Sides

To solve an equation with variables on both sides, use inverse operations to write an equation with all the variable terms on the same side.

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EXAMPLE 1

Solve each equation.

a. $9x = 5x + 8$

Subtract $5x$ from both sides to get an equation with the variable term on the *left* side only.

$$\begin{array}{rcr} 9x &=& 5x + 8 \\ \underline{-5x} && \underline{-5x \quad} \\ 4x &=& 8 \end{array}$$

Divide both sides by 4.

$$\frac{4x}{4} = \frac{8}{4}$$
$$x = 2$$

CHECK Substitute 2 for x.

$$\begin{array}{r} 9(2) = 5(2) + 8 \\ 18 = 10 + 8 \checkmark \end{array}$$

The solution is $x = 2$.

b. $2x + 3 = 4x - 5$

METHOD 1

Subtract $4x$ from both sides to get an equation with the variable term on the *left* side only.

$$\begin{array}{rcr} 2x + 3 &=& 4x - 5 \\ \underline{-4x \qquad} && \underline{-4x \qquad} \\ -2x + 3 &=& -5 \end{array}$$

Subtract 3 from both sides to isolate the variable term.

$$\begin{array}{rcr} \underline{\qquad -3} && \underline{\qquad -3} \\ -2x &=& -8 \end{array}$$

Divide both sides by -2.

$$x = 4$$

METHOD 2

Subtract $2x$ from both sides to get an equation with the variable term on the *right* side only.

$$\begin{array}{rcr} 2x + 3 &=& 4x - 5 \\ \underline{-2x \qquad} && \underline{-2x \qquad} \\ 3 &=& 2x - 5 \end{array}$$

Add 5 to both sides to isolate the variable term.

$$\begin{array}{rcr} \underline{\qquad +5} && \underline{\qquad +5} \\ 8 &=& 2x \end{array}$$

Divide both sides by 2.

$$4 = x$$

CHECK Substitute 4 for x.

$$\begin{array}{r} 2(4) + 3 = 4(4) - 5 \\ 8 + 3 = 16 - 5 \\ 11 = 11 \checkmark \end{array}$$

The solution is $x = 4$.

How Many Solutions?

When an equation has a variable on both sides, it may have one solution, no solution, or infinitely many solutions.

EXAMPLE 2

Solve each equation.

a. $x + 10 = x$

Subtract x from both sides. $\quad x + 10 = x$

$\quad -x \quad -x$

This equation is always false! $\quad 10 = 0$ ✗

So, the original equation has no solution.

b. $3(x - 6) = 4x - 18 - x$

$3(x - 6) = 4x - 18 - x$

Use the Distributive Property. $\quad 3(x) - 3(6) = 4x - 18 - x$

Multiply. Use the Commutative Property. $\quad 3x - 18 = 4x - x - 18$

Combine like terms. $4x - x$ means $4x - 1x$. $\quad 3x - 18 = 3x - 18$

Subtract $3x$ from both sides. $\quad -3x \quad -3x$

This equation is always true! $\quad -18 = -18$ ✓

Any value of x can be a solution. Test with values of 0, −8, and 1.

Let $x = 0$.	Let $x = -8$.	Let $x = 1$.
$3(0 - 6) = 4(0) - 18 - 0$	$3(-8 - 6) = 4(-8) - 18 - (-8)$	$3(1 - 6) = 4(1) - 18 - 1$
$-18 = -18$ ✓	$-42 = -42$ ✓	$-15 = -15$ ✓

So, the original equation has infinitely many solutions.

Watch Out !

Whether you're dealing with numbers or variables, remember to use inverse operations to move them to the other side of the equation.

The equation $3x - 18 = 3x - 18$ cannot be rewritten as $6x - 18 = -18$.

Got to KNOW

Method to Solve Equations with Variables on Both Sides

- Use inverse operations to write an equation with all variable terms on one side of the equation.
- Combine like terms on each side of the equation.

The final equation you find will tell you the number of solutions the original equation has.

Resulting Equation	Sample	Number of Solutions
True for one value	$x = 2$	one solution
Never true	$3 = 2$	no solution
Always true	$3 = 3$	infinitely many solutions

Solving Word Problems with Equations

Translating Words to Equations

Need More HELP

For more review of translating words to algebraic expressions, go to *Translating English into Algebra* on page 1430.

To solve a word problem, follow these steps.

- Determine what number or numbers you need to find.
- Let a variable represent the number you are trying to find.
- Write an equation that describes the conditions of the problem.
- Solve the equation.
- Check the solution to see if it is reasonable in the problem.

EXAMPLE 1

The difference between three times a number and 15 is 60. Find the number.

STEP 1 Choose a variable.

Let k = the number.

STEP 2 Write an equation.

This is a problem about the difference between two values. The phrase "three times a number" occurs before the phrase "and 15," so write "three times a number" first.

$$\begin{array}{cccl} \underbrace{\text{Three times a number}} & \text{minus} & 15 & \text{is } 60. \\ 3k & - & 15 & = 60 \end{array}$$

STEP 3 Solve the equation.

Add 15 to both sides first to isolate the variable term on the left side.

$$\begin{array}{rcr} 3k - 15 & = & 60 \\ \underline{\quad + 15} & & \underline{+ 15} \\ 3k \quad\;\; & = & 75 \end{array}$$

Divide both sides by 3 to isolate k.

$$\frac{3k}{3} = \frac{75}{3}$$

The number is 25.

$$k = 25$$

CHECK Check the solution in the problem.

Read the word problem to see whether the answer is reasonable.

Replace words with numbers, where appropriate.

The difference between 3 times 25 and 15 is 60.

$$\begin{aligned} 3(25) - 15 &\overset{?}{=} 60 \\ 75 - 15 &\overset{?}{=} 60 \\ 60 &= 60 \checkmark \end{aligned}$$

The number 25 is correct.

EXAMPLE 2

The difference between 15 and three times a number is 60. Find the number.

STEP 1 Choose a variable, and translate the problem into an equation.

Let n = the number. Notice that 15 occurs before "three times a number."

Fifteen minus $\underbrace{\text{three times a number}}$ is 60.

$$15 \quad - \quad 3n \quad = 60$$

STEP 2 Solve the equation.

Subtract 15 from both sides first to isolate the variable term on the left side.

$$\begin{aligned} 15 - 3n &= 60 \\ -15 & -15 \\ \hline -3n &= 45 \end{aligned}$$

Divide both sides by -3 to isolate n.

$$\frac{-3n}{-3} = \frac{45}{-3}$$

The number is -15.

$$n = -15$$

CHECK Check the solution in the problem. Replace words with numbers, where appropriate.

The difference between 15 and 3 times -15 is 60.

$$\begin{aligned} 15 - 3(-15) &= 60 \\ 15 + 45 &\stackrel{?}{=} 60 \\ 60 &= 60 ✓ \end{aligned}$$

The number -15 is correct.

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EXAMPLE 3

When the sum of a number and 5 is divided by 3, the quotient is 14. Find the number.

STEP 1 Choose a variable, and translate the problem into an equation.

Let m = the number. Then break the statement into parts.

The quotient (when the sum of a number and 5 is divided by 3) is 14.

The quotient ... is 14. $\boxed{} = 14$

What is the quotient? the sum of a number and 5 divided by 3 $\dfrac{m+5}{3}$

Write the equation. $\dfrac{m+5}{3} = 14$

STEP 2 Solve the equation.

Multiply both sides of the equation by 3 to clear the fraction.

$$3\left(\frac{m+5}{3}\right) = 3(14)$$

Simplify.

$$m + 5 = 42$$

Subtract 5 from both sides to isolate the variable.

$$m + 5 - 5 = 42 - 5$$

The number is 37.

$$m = 37$$

CHECK Check the solution in the problem.

The quotient when the sum of 37 and 5 is divided by 3 is 14.

$$\begin{aligned} \frac{37+5}{3} &\stackrel{?}{=} 14 \\ \frac{42}{3} &\stackrel{?}{=} 14 \\ 14 &= 14 ✓ \end{aligned}$$

The number 37 is correct.

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Translating Problem Situations into Equations

EXAMPLE 4

Four-fifths of the students in a PE class chose to play dodgeball. After 2 more students joined the game, 26 were playing. How many students are in the PE class?

STEP 1 Choose a variable, and translate the problem into an equation.

Let s = the number of students in the PE class.

$$\underbrace{\text{Four-fifths of the students}}_{\frac{4}{5}s}\ \underset{+}{\text{plus}}\ \underbrace{\text{2 students}}_{2}\ \underset{=}{\text{is}}\ \underbrace{\text{26 students.}}_{26}$$

STEP 2 Solve the equation.

$$\frac{4}{5}s + 2 = 26$$

Subtract 2 from both sides of the equation.

$$\begin{aligned} \frac{4}{5}s + 2 &= 26 \\ -2 &\quad -2 \\ \hline \frac{4}{5}s &= 24 \end{aligned}$$

Multiply both sides of the equation by $\frac{5}{4}$.

$$\frac{5}{4}\left(\frac{4}{5}s\right) = \frac{5}{4}(24)$$

There are 30 students in the PE class.

$$s = 30$$

CHECK Check the solution in the problem.

Four-fifths of 30 students plus 2 students is 26 students.

$$\frac{4}{5}(30) + 2 \overset{?}{=} 26$$

$$24 + 2 \overset{?}{=} 26$$

The number of students, 30, is correct.

$$26 = 26 \checkmark$$

There are 30 students in PE class.

EXAMPLE 5

Carly paid for 3 tubes of lip gloss with a $10 bill and got $1.15 in change. How much does 1 tube of lip gloss cost?

Let c equal the cost in dollars of 1 tube of lip gloss. Translate to an equation and solve.

$$\underbrace{\text{3 times the cost of 1 tube}}_{3c}\ \underset{=}{\text{equals}}\ \underset{10}{\text{\$10}}\ \underset{-}{\text{minus}}\ \underbrace{\text{change of \$1.15}}_{1.15}$$

$$3c = 10 - 1.15$$

Simplify by subtracting. $3c = 8.85$

Divide both sides by 3. $\frac{3c}{3} = \frac{8.85}{3}$

$$c = 2.95$$

One tube of lip gloss costs $2.95.

EXAMPLE 6

A baseball team had played *g* games before January and had won 25% of them. Then they won the next 4 games. By then, the team had won 40% of their games.

a. How many games had the team played before January?

STEP 1 First convert the percents to decimals.

$$25\% = \frac{25}{100} = 0.25 \qquad 40\% = \frac{40}{100} = 0.4$$

STEP 2 Translate into an equation and solve.

$$\underbrace{\text{25\% of } g \text{ games}}_{0.25g}\ \text{plus}\ \underset{+\ 4}{4}\ \underset{=}{\text{equals}}\ \underset{0.4\ \cdot}{\text{40\% of}}\ \underbrace{\text{the number of games } g \text{ plus } 4}_{(g+4)}$$

$$\begin{array}{lr}
 & 0.25g + 4 = 0.4(g + 4)\\
\text{Multiply both sides by 100.} & 100(0.25g + 4) = 100(0.4)(g + 4)\\
\text{Use the Distributive Property and simplify.} & 25g + 400 = 40g + 160\\
\text{Subtract 160 from both sides.} & 25g + 400 - 160 = 40g + 160 - 160\\
 & 25g + 240 = 40g\\
\text{Subtract } 25g \text{ from both sides.} & 25g - 25g + 240 = 40g - 25g\\
 & 240 = 15g\\
\text{Divide both sides by 15.} & \dfrac{240}{15} = \dfrac{15g}{15}\\
 & 16 = g
\end{array}$$

The team had played 16 games before January.

b. How many games have they now played? How many have they won?

The team has now played $g + 4$ games. $16 + 4 = 20$

The team has now won 40% of its 20 games. $0.4(20) = 8$

The team has now played 20 games and has won 8 of them.

Try It This Way

To get the variable term on one side and the constant on the other, you can subtract 160 and $25g$ from both sides in a single step.

$$\begin{array}{rcl}
25g + 400 &=& 40g + 160\\
\underline{-25g - 160} && \underline{-25g - 160}\\
240 &=& 15g
\end{array}$$

Got to KNOW

Six Steps to Solving Word Problems

STEP 1 Read the problem carefully and determine what number or numbers you need to find.

STEP 2 Represent one unknown number with a variable.

STEP 3 Use a condition stated in the problem to write an expression for a second number.

STEP 4 Use a second condition to find two expressions that are equal. Then write these two expressions algebraically and connect them with an equals sign.

STEP 5 Solve the equation.

STEP 6 Check to make sure the result matches the conditions stated in the problem.

Graphing an Inequality as an Interval

Inequalities

An **inequality** is a mathematical statement that two quantities are not or may not be equal. An inequality may be true or false. There are five inequality symbols.

$\neq$	$<$	$>$	$\leq$	$\geq$
is not equal to	is less than	is greater than	is less than or equal to	is greater than or equal to

Ways to REMEMBER

The smaller end of the inequality symbol points to the smaller number.

The bigger end of an inequality symbol points to the larger number.

True Inequalities

$3 < 4$ $\quad 4 + 2 > 5$ $\quad 7(2) \geq 14$

$3 \leq 4$ $\quad 4 + 2 \geq 5$ $\quad 8 - 3 \neq 7$

False Inequalities

$7 < 5$ $\quad 7(2) > 14$ $\quad 10 - 3 \neq 7$

An **algebraic inequality** is an inequality with a variable. A value of the variable that makes the inequality true is a **solution of the inequality**.

The value 5 is a solution of $2x > 7$ because when $x = 5$, $2x > 7$ is true.

The value 0 is not a solution of $2x > 7$ because when $x = 0$, $2x > 7$ is false.

EXAMPLE 1

Tell whether 0.5 is a solution of each inequality.

a. $6x \leq 3$

$6(0.5) \leq 3$ Substitute 0.5 for x.

$3 \leq 3$ Simplify.

$3 \leq 3$ is true, so 0.5 is a solution.

b. $27 - 15x > 20$

$27 - 15(0.5) > 20$

$27 - 7.5 > 20$

$19.5 > 20$

$19.5 > 20$ is false, so 0.5 is *not* a solution.

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EXAMPLE 2

Evaluate $4x - 6$ and $2(x + 5)$ for $x = 7, 8$, and 9. Place the correct symbol, $<$, $=$, or $>$, in the box to make a true statement.

For $x = 7$:

$4x - 6 \boxed{\phantom{<}} 2(x + 5)$

$4(7) - 6 \qquad 2(7 + 5)$ Substitute.

$28 - 6 \qquad 2(12)$ Simplify.

$22 < 24$ Compare.

$4x - 6 \boxed{<} 2(x + 5)$

For $x = 8$:

$4x - 6 \boxed{} 2(x + 5)$

$4(8) - 6 \qquad 2(8 + 5)$ Substitute.

$32 - 6 \qquad 2(13)$ Simplify.

$26 = 26$ Compare.

$4x - 6 \boxed{=} 2(x + 5)$

For $x = 9$:

$4x - 6 \boxed{} 2(x + 5)$

$4(9) - 6 \qquad 2(9 + 5)$

$36 - 6 \qquad 2(14)$

$30 > 28$

$4x - 6 \boxed{>} 2(x + 5)$

Translating Words into Inequalities

EXAMPLE 3

Write an inequality for each situation.

a. The cable TV company offers more than two hundred channels.

The words "more than" indicate a *greater than* symbol.

Number of channels > 200

b. The express lane is for fifteen items or fewer.

The word "fewer" indicates a *less than* symbol or a *less than or equal to* symbol. The word "or" tells you that it should be *less than or equal to*.

Number of items ≤ 15

EXAMPLE 4

Translate each sentence into an inequality.

a. Seventy-two is no more than twice the number of children *c*.

$$\underbrace{\text{Seventy-two}}_{72}\ \underbrace{\text{is no more than}}_{\le}\ \underbrace{\text{twice}}_{2\,\cdot}\ \underbrace{\text{the number of children } c}_{c}.$$

The inequality is $72 \le 2c$.

b. A number *r* plus 2.75 has a maximum value of 6.25.

$$\underbrace{\text{A number } r}_{r}\ \underset{+}{\text{plus}}\ \underset{2.75}{2.75}\ \underbrace{\text{has a maximum value of}}_{\le}\ \underset{6.25}{6.25}.$$

The inequality is $r + 2.75 \le 6.25$.

Watch Out!

Maximum and *no more than* may make you think of "greater," but these are expressions that indicate something is *less* than or equal to another value.

No more than is the opposite of *more than.*

Got to KNOW!

Inequality Symbols

Look for key words to help you choose the inequality symbol.

Symbol	Key Words
$>$	is greater than, is more than, exceeds
$<$	is less than, are fewer than
$\ge$	is no less than, is at least, has a minimum of
$\le$	is no more than, is at most, has a maximum value of

Advanced Algebra

The seeds of a sunflower grow in spirals in opposite directions. The numbers of spirals in each direction are consecutive numbers in the Fibonacci sequence.

Coordinate System in Three Dimensions . . . 1752

Matrices 1770

Problem Solving and Variation 1818

Sequences and Series 1856

Complex Numbers 1892

Conic Sections 1908

Radicals, Exponents, Logarithms, and Rational Expressions 1950

Exponential and Logarithmic Functions . . . 1994

Rational and Radical Functions 2018

Writing Models for Functions 2040

Analyzing Functions 2060

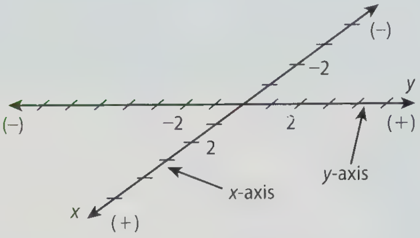
Plotting Points in Three Dimensions

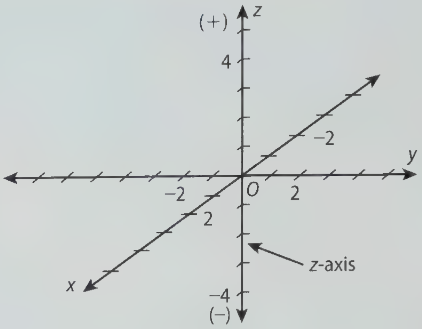
Using a Three-Dimensional Coordinate System

Need More **HELP**

To review two-dimensional coordinate systems, go to *Four Quadrants* in *Algebra* (p. 1524).

A three-dimensional coordinate system has three axes: the x-axis, the y-axis, and the z-axis.





In three dimensions, the x-axis and the y-axis are drawn differently than they are drawn in two dimensions. In three dimensions, the x-axis is shown as coming toward you. The y-axis is shown as a horizontal line.

The z-axis is shown as a vertical line. Each axis is perpendicular to the other two axes. The three axes meet at the point (0, 0, 0), which is the origin for a three-dimensional coordinate system.

The positive direction for the x-axis is toward you. For the y-axis, the positive direction is to the right. For the z-axis, the positive direction is up.

An **ordered triple** describes a point in a three-dimensional coordinate system. The numbers describe the x-, y-, and z-coordinates of the point in this order: (x, y, z).

EXAMPLE 1

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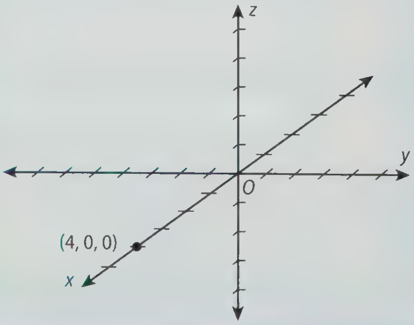
Each ordered triple represents a point on one of the axes. For each ordered triple, tell which axis contains the point and whether the point is on the positive or negative part of that axis. Then, plot the point.

a. (4, 0, 0)

Comparing (4, 0, 0) to (x, y, z) indicates that the point is on the x-axis. The value 4 is positive, so (4, 0, 0) is on the positive x-axis.

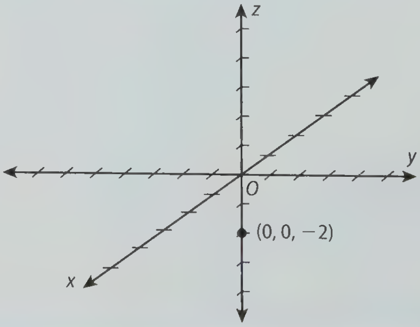
Plot the point by counting 4 units on the x-axis in the positive direction.



b. (0, 0, −2)

Comparing (0, 0, −2) to (x, y, z) indicates the point is on the negative z-axis.

Count 2 units on the z-axis in the negative direction.

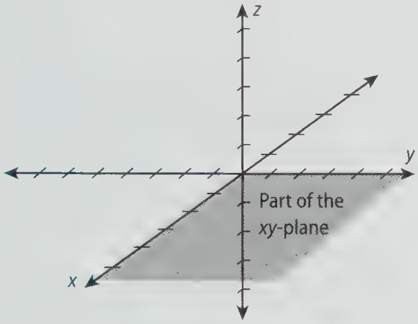


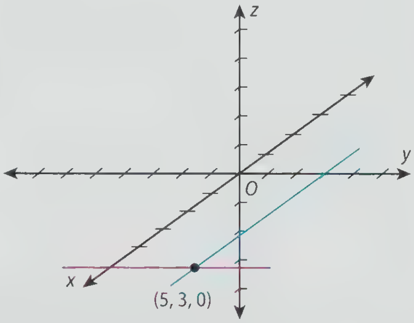
Example 1 showed how to plot an ordered triple if two of the numbers are zero. If just one number in an ordered triple is zero, plotting the point is similar to plotting points in a two-dimensional plane.

EXAMPLE 2

Plot the ordered triple (5, 3, 0).

STEP 1 Identify which of *x*, *y*, or *z* is zero. The *z*-value is zero, so the point lies on the plane determined by the *x*-axis and the *y*-axis.





STEP 2 Count 5 units in the positive direction on the *x*-axis. Draw a line parallel to the *y*-axis at that point.

STEP 3 Count 3 units in the positive direction on the *y*-axis. Draw a line parallel to the *x*-axis at that point.

STEP 4 The two lines and the *x*- and *y*-axes form a parallelogram. Locate the vertex of the parallelogram that is opposite the origin. That point represents the ordered triple (5, 3, 0).

GOT TO KNOW!

The Axes and the Origin

Each axis is perpendicular to the other two axes. Each axis has a positive direction and a negative direction. The three axes meet at the origin.

***x*-axis**	axis that comes toward you
***y*-axis**	horizontal axis
***z*-axis**	vertical axis
origin	(0, 0, 0)

EXAMPLE 3

In each of the following ordered triples, one of the numbers is zero. For each triple, tell whether the point is on the *xy*-plane, the *yz*-plane, or the *xz*-plane. Plot and label the point.

a. (−3, 0, 4)

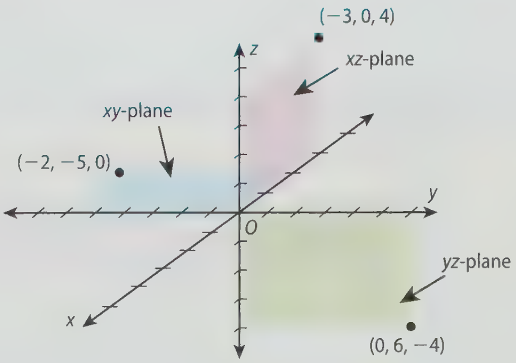
The *y*-value is zero, so the point is on the *xz*-plane.

b. (−2, −5, 0)

The *z*-value is zero, so the point is on the *xy*-plane.

c. (0, 6, −4)

The *x*-value is zero, so the point is on the *yz*-plane.



Plotting Ordered Triples

To plot an ordered triple when no value is zero, it is important to be accurate and to label your diagram.

EXAMPLE 4

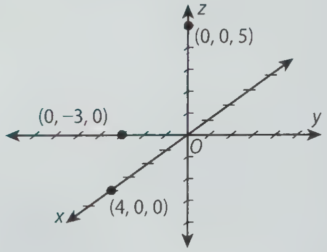
Plot the ordered triple (4, −3, 5).

METHOD 1

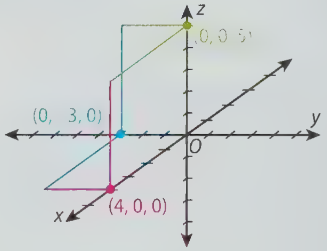
Start by finding related points on the axes.

STEP 1 Find the coordinates of the points on the axes that are related to (4, −3, 5). Plot those three points.

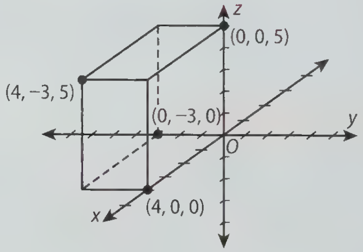
On the *x*-axis: (4, 0, 0)
On the *y*-axis: (0, −3, 0)
On the *z*-axis: (0, 0, 5)



STEP 2 Through each point, draw lines parallel to the "other two" axes. For this example, draw lines through (4, 0, 0) parallel to the *y*- and *z*-axes; draw lines through (0, −3, 0) parallel to the *x*- and *z*-axes; and draw lines through (0, 0, 5) parallel to the *x*- and *y*-axes.



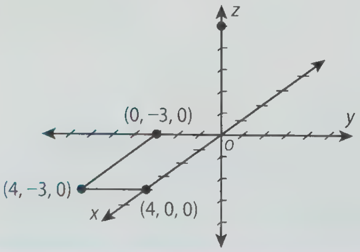
STEP 3 Those lines, along with the three axes, form part of a box. Complete the box to find the point (4, −3, 5).



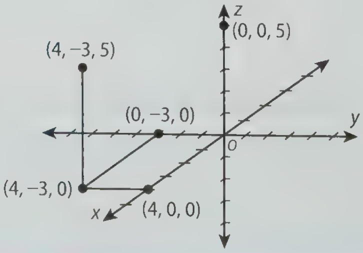
METHOD 2

Start by finding a related point on the *xy*-plane.

STEP 1 Use the fact that $z = 0$ on the *xy*-plane. If $z = 0$, the related ordered triple is (4, −3, 0). Plot the point (4, −3, 0).



STEP 2 The *z*-value is 5, so go 5 units in the positive *z* direction from (4, −3, 0). Label that point (4, −3, 5).



Try It This Way

You can make a physical model of a 3D coordinate grid to help see where a point is.

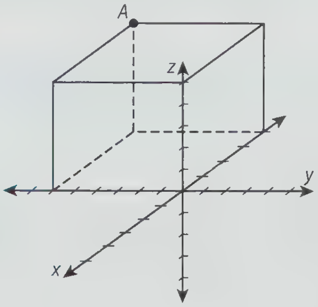
Use three straws, one for each axis. Set each straw perpendicular to the other two, and tie them together at the middle. Label each straw with axis numbers.

Then use your model and a marble (or other round object) to illustrate the position of the point (4, −3, 5).

Example 4 showed how to graph an ordered triple. Example 5 shows how to find the ordered triple for a point on a three-dimensional graph.

EXAMPLE 5

What are the coordinates of point *A*?



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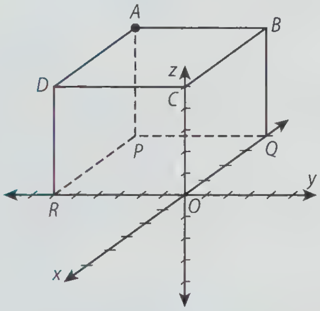
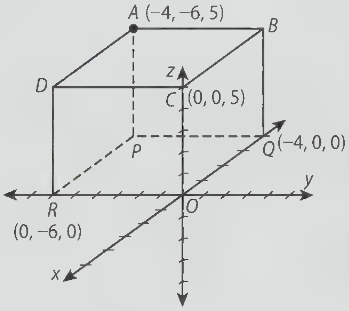
STEP 1 Label the other vertices of the box. Use the letter *O* to label the origin (0, 0, 0).

STEP 2 Find the first number in the ordered triple for point *A*. Notice that a vertex on the *x*-axis is *Q*, and the coordinates for *Q* are (−4, 0, 0). So the *x*-coordinate for point *A* is −4.





STEP 3 A vertex on the *y*-axis is *R*. Its coordinates are (0, −6, 0). So the *y*-coordinate for point *A* is −6.

STEP 4 A vertex on the *z*-axis is *C*. Its coordinates are (0, 0, 5). So the *z*-coordinate for point *A* is 5.

The coordinates for point *A* are (−4, −6, 5).

GOT TO KNOW!

Coordinates of Points on Axes, on Planes, and in Space

Axes	Plane Formed	Coordinates of Points
x-axis	—	(*a*, 0, 0)
y-axis	—	(0, *b*, 0)
z-axis	—	(0, 0, *c*)
x-axis and *y*-axis	*xy*-plane	(*a*, *b*, 0), with $z = 0$
y-axis and *z*-axis	*yz*-plane	(0, *b*, *c*), with $x = 0$
x-axis and *z*-axis	*xz*-plane	(*a*, 0, *c*), with $y = 0$
all three axes	none; points in 3D space	(*a*, *b*, *c*)

To plot the point (*a*, *b*, *c*), start by plotting and labeling the points (*a*, 0, 0), (0, *b*, 0), and (0, 0, *c*). Draw a box using those three points. The vertex of the box that is opposite the origin has the coordinates (*a*, *b*, *c*).

Graphing in Three Dimensions

Linear Equations and Planes

In a three-dimensional coordinate system, the graph of every linear equation in two or three variables is a plane. You will first see how to graph a linear equation in two variables and then how to graph a linear equation in three variables.

Graphing a Linear Equation in Two Variables

EXAMPLE 1

Graph the equation $2x + y = 3$ on a three-dimensional coordinate system. Explain how the graph is related to the fact that the equation does not have a *z*-variable.

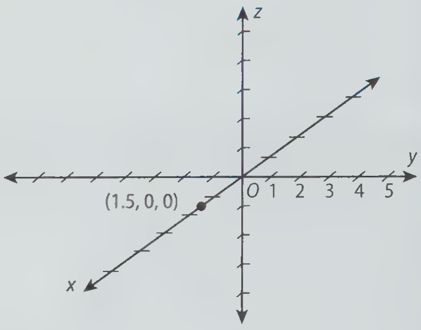
STEP 1 Find the *x*-intercept of the equation.

$2x + y = 3$

Set $y = 0$. $2x + (0) = 3$

Solve for *x*. $2x = 3$

$x = 1.5$



STEP 2 The *x*-intercept is 1.5, so plot the ordered triple (1.5, 0, 0). The graph is to the right.

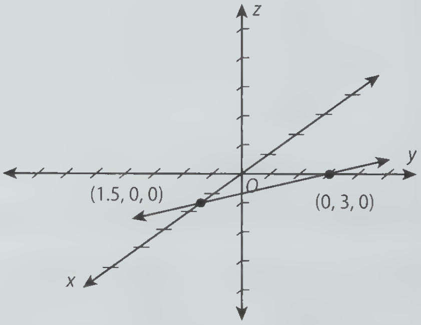
STEP 3 Find the *y*-intercept of the equation.

$2x + y = 3$

Set $x = 0$. $2(0) + y = 3$

Solve for *y*. $0 + y = 3$

$y = 3$



STEP 4 The *y*-intercept is 3, so plot the ordered triple (0, 3, 0). Then draw a line through the two intercepts. This line represents $2x + y = 3$ in the *xy*-plane in the graph to the right. Another way to describe the line is that it represents $2x + y = 3$ when $z = 0$.

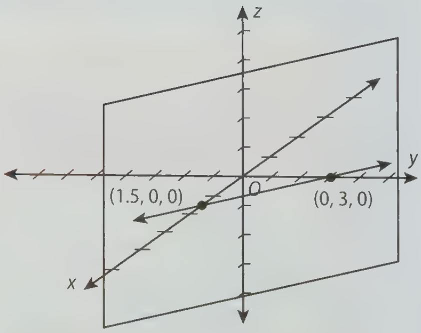
STEP 5 The plane for $2x + y = 3$ is related to three things:

- The line that connects the two intercepts

  The plane for $2x + y = 3$ intersects the *xy*-plane in the line $2x + y = 3$.
- The *xy*-plane

  The plane for $2x + y = 3$ is perpendicular to the *xy*-plane.
- The *z*-axis, whose variable is not included in the equation

  The plane for $2x + y = 3$ is parallel to the *z*-axis.



GOT TO KNOW!

Graphing an Equation in Two Variables in a Three-Dimensional Coordinate System

1. Find and plot the two intercepts given by the two variables.
2. Draw a line through the two intercepts.
3. Show the rest of the plane for the equation by drawing a plane parallel to the axis whose variable is not included in the equation.

Try It This Way

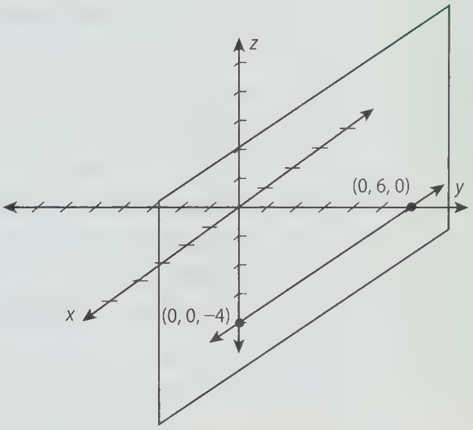
If you have difficulty remembering how to find an intercept, sketch an axis and draw a point that is on the axis. In other words, draw a point that intercepts the axis.

For example, in looking at a point that intercepts the *x*-axis, you can see that the point has an *x* value that depends on where the point is located. However, the *y*- and *z*-coordinates are always zero.

EXAMPLE 2

Graph each two-variable equation on a three-dimensional coordinate system. Identify the variable that is not in the equation, and describe the graph of the equation.

a. $2y - 3z = 12$



STEP 1 Find the y-intercept and the z-intercept.

For the y-intercept, $z = 0$: $2y - 3(0) = 12$

$y = 6$

For the z-intercept, $y = 0$: $2(0) - 3z = 12$

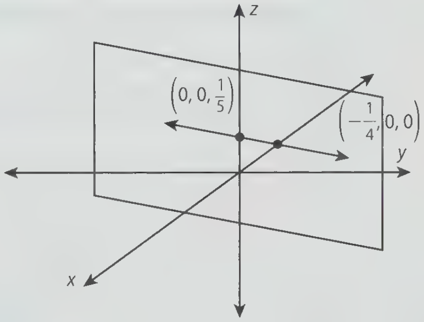
$z = -4$

The y-intercept is $(0, 6, 0)$. The z-intercept is $(0, 0, -4)$.

STEP 2 Graph the y- and z-intercepts and connect them with a line. Then show a plane through that line that is perpendicular to the yz-plane and parallel to the x-axis.

The graph of $2y - 3z = 12$ as shown above is a plane parallel to the x-axis and perpendicular to the yz-plane. It intersects the yz-plane in the line $2y - 3z = 12$.

b. $4x - 5z = -1$


z
y
x
$\left(0, 0, \frac{1}{5}\right)$
$\left(-\frac{1}{4}, 0, 0\right)$

STEP 1 Find the x-intercept and the z-intercept.

For the x-intercept, $z = 0$: $4x - 5(0) = -1$

$x = -\frac{1}{4}$

For the z-intercept, $x = 0$: $4(0) - 5z = -1$

$z = \frac{1}{5}$

The x-intercept is $\left(-\frac{1}{4}, 0, 0\right)$. The z-intercept is $\left(0, 0, \frac{1}{5}\right)$.

STEP 2 Graph the x- and z-intercepts and connect them with a line. Then show a plane through that line that is perpendicular to the xz-plane and parallel to the y-axis.

The graph of $4x - 5z = -1$ as shown above is a plane parallel to the y-axis and perpendicular to the xz-plane. It intersects the xz-plane in the line $4x - 5z = -1$.

GOT TO KNOW!

Graphing a Linear Equation in Two Variables

Relationship	Example Equation: $2x - z = 4$
The plane contains the line determined by the two intercepts for the equation.	The plane contains the line containing the intercepts $(2, 0, 0)$ and $(0, 0, -4)$.
The plane is perpendicular to the plane determined by the axes for the two variables in the equation.	The plane is perpendicular to the xz-plane.
The plane is parallel to the axis for the variable that is NOT in the equation.	The plane is parallel to the y-axis.

Solving Systems by Substitution

The Method of Substitution

Need More HELP

You can use many of the same strategies to solve systems of three equations with three variables that you used to solve systems of two equations with two variables. To review these strategies, go to *Solving a System by Substitution* in *Algebra* (p. 1568).

A system of three equations involves three variables. The value of each variable can be determined by using one strategy or a combination of several strategies.

The solution to a system of three equations in three variables is an ordered triple. The ordered triple has to satisfy all three equations.

The method of substitution is one strategy that can be used to solve a system of three equations.

- In one of the three equations a variable is solved for or isolated.
- The value of that variable is then substituted into the other two equations. That process converts the original system of three equations in three variables into a new system of equations. The new system has two equations in two variables.
- The process is then repeated for the system of two equations in two variables.

In the examples below, the original equations of a system are labeled (1a), (2a), and (3a). When an equation such as (1a) is changed, the new equation is labeled (1b).

EXAMPLE 1

Use the method of substitution to find the solution (x, y, z) for the system below. Note that one of the equations has just one variable.

(1a) $3x + 5y - 2z = 5$
(2a) $4x - y + 3z = 16$
(3a) $x = 2$

STEP 1 Equation (3a) is already solved for x. Substitute that value of x into equations (1a) and (2a).

$$\begin{aligned} &\text{(1a)} & 3x + 5y - 2z &= 5 \\ && 3(2) + 5y - 2z &= 5 \\ &\text{(1b)} & 5y - 2z &= -1 \end{aligned}$$

$$\begin{aligned} &\text{(2a)} & 4x - y + 3z &= 16 \\ && 4(2) - y + 3z &= 16 \\ &\text{(2b)} & -y + 3z &= 8 \end{aligned}$$

STEP 2 Equations (1b) and (2b) form a system of two equations in two variables. One way to solve this system is to use substitution. Solve one of the equations for one of the variables. Then, substitute that value into the other equation. For equations (1b) and (2b), it is easier to solve equation (2b) for y.

$$\begin{aligned} &\text{(2b)} & -y + 3z &= 8 \\ && y &= 3z - 8 \end{aligned}$$

$$\begin{aligned} &\text{(1b)} & 5y - 2z &= -1 \\ && 5(3z - 8) - 2z &= -1 \\ && 15z - 40 - 2z &= -1 \\ && 13z &= 39 \\ && z &= 3 \end{aligned}$$

STEP 3 It is now known that $x = 2$ and $z = 3$.
Substitute those values into either equation (1a) or equation (2a). The answer will be the same whichever equation you choose.

$$\begin{aligned} &\text{(1a)} & 3x + 5y - 2z &= 5 \\ && 3(2) + 5y - 2(3) &= 5 \\ && 6 + 5y - 6 &= 5 \\ && 5y &= 5 \\ && y &= 1 \end{aligned}$$

The solution to the system is (2, 1, 3).

STEP 4 Check the solution (2, 1, 3) in all three *original* equations.

$$\begin{aligned} &\text{(1a)} & 3x + 5y - 2z &= 5; \\ && 3(2) + 5(1) - 2(3) &\stackrel{?}{=} 5 \\ && 6 + 5 - 6 &= 5\ \checkmark \end{aligned}$$

$$\begin{aligned} &\text{(2a)} & 4x - y + 3z &= 16 \\ && 4(2) - (1) + 3(3) &\stackrel{?}{=} 16 \\ && 8 - 1 + 9 &= 16\ \checkmark \end{aligned}$$

$$\begin{aligned} &\text{(3a)} & x &= 2 \\ && 2 &= 2\ \checkmark \end{aligned}$$

EXAMPLE 2

Use the method of substitution to find the solution (x, y, z) for the system below. Note that one of the equations has just two variables.

(1a) $2x - y + 4z = 2$
(2a) $3x + 2y + 3z = 9$
(3a) $2y + z = -2$

STEP 1 Solve one of the equations for one of the variables. In this example, let's solve equation (3a) for z.

(3a) $2y + z = -2$
$z = -2y - 2$

STEP 2 Then, substitute that value into the other two equations to form a system with two new equations in two variables.

(1a)	$2x - y + 4z = 2$	(2a)	$3x + 2y + 3z = 9$
	$2x - y + 4(-2y - 2) = 2$		$3x + 2y + 3(-2y - 2) = 9$
	$2x - y - 8y - 8 = 2$		$3x + 2y - 6y - 6 = 9$
(1b)	$2x - 9y = 10$	(2b)	$3x - 4y = 15$

STEP 3 The new system of (1b) and (2b) has two equations and two variables. Solve for either variable in either equation. Let's solve for x in (1b).

(1b) $2x - 9y = 10$
$2x = 10 + 9y$
$x = 5 + 4.5y$

STEP 4 Substitute this value into the other two-variable equation, and solve for the variable.

(2b) $3x - 4y = 15$
$3(5 + 4.5y) - 4y = 15$
$15 + 13.5y - 4y = 15$
$9.5y = 0$
$y = 0$

STEP 5 Substitute $y = 0$ into either equation (1b) or (2b). Then, solve for x.

(1b) $2x - 9y = 10$
$2x - 9(0) = 10$
$2x = 10$
$x = 5$

STEP 6 Substitute $y = 0$ and $x = 5$ into any of the original equations. Solve for z.

(1a) $2x - y + 4z = 2$
$2(5) - (0) + 4z = 2$
$4z = -8$
$z = -2$

The solution is $(5, 0, -2)$.

STEP 7 Check the solution $(5, 0, -2)$ in all three *original* equations.

(1a)	$2x - y + 4z = 2$	(2a)	$3x + 2y + 3z = 9$	(3a)	$2y + z = -2$
	$2(5) - (0) + 4(-2) \overset{?}{=} 2$		$3(5) + 2(0) + 3(-2) \overset{?}{=} 9$		$2(0) + (-2) \overset{?}{=} -2$
	$10 - 0 - 8 = 2$ ✓		$15 + 0 - 6 = 9$ ✓		$0 - 2 = -2$ ✓

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Three Equations with Three Variables

EXAMPLE 3

Use the method of substitution to find the solution (*x, y, z*) for the system shown below. Note that all three equations have three variables.

(1a) $x + 2y - z = 3$
(2a) $3x - y + 4z = 2$
(3a) $5x + 3y + 4z = 12$

STEP 1 Solve one of the equations for one of the variables. In this example, let's solve equation (1a) for *z*.

$$\text{(1a)}\quad \begin{aligned} x + 2y - z &= 3 \\ z &= x + 2y - 3 \end{aligned}$$

STEP 2 Then, substitute the expression for *z* into the other two equations. The result is a system with two equations in two variables.

$$\text{(2a)}\quad \begin{aligned} 3x - y + 4z &= 2 \\ 3x - y + 4(x + 2y - 3) &= 2 \\ 3x - y + 4x + 8y - 12 &= 2 \\ 7x + 7y - 12 &= 2 \\ 7x + 7y &= 14 \\ \text{(2b)}\quad x + y &= 2 \end{aligned}$$

$$\text{(3a)}\quad \begin{aligned} 5x + 3y + 4z &= 12 \\ 5x + 3y + 4(x + 2y - 3) &= 12 \\ 5x + 3y + 4x + 8y - 12 &= 12 \\ 9x + 11y - 12 &= 12 \\ \text{(3b)}\quad 9x + 11y &= 24 \end{aligned}$$

STEP 3 The new system of (2b) and (3b) has two equations and two variables. Solve for either variable in either equation. You might choose to solve equation (2b) for *x*.

$$\text{(2b)}\quad \begin{aligned} x + y &= 2 \\ x &= -y + 2 \end{aligned}$$

STEP 4 Substitute this value into the other two-variable equation, and solve for the variable.

$$\text{(3b)}\quad \begin{aligned} 9x + 11y &= 24 \\ 9(-y + 2) + 11y &= 24 \\ -9y + 18 + 11y &= 24 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

STEP 5 Substitute $y = 3$ into either equation (2b) or (3b), and solve for *x*.

$$\text{(2b)}\quad \begin{aligned} x + y &= 2 \\ x + (3) &= 2 \\ x &= -1 \end{aligned}$$

STEP 6 Substitute $x = -1$ and $y = 3$ into one of the original equations. Solve for *z*.

$$\text{(1a)}\quad \begin{aligned} x + 2y - z &= 3 \\ (-1) + 2(3) - z &= 3 \\ -1 + 6 - z &= 3 \\ z &= 2 \end{aligned}$$

The solution is $(-1, 3, 2)$.

STEP 7 Check the solution $(-1, 3, 2)$ in all three original equations.

$$\text{(1a)}\quad \begin{aligned} x + 2y - z &= 3 \\ (-1) + 2(3) - (2) &\stackrel{?}{=} 3 \\ -1 + 6 - 2 &= 3 \checkmark \end{aligned}$$

$$\text{(2a)}\quad \begin{aligned} 3x - y + 4z &= 2 \\ 3(-1) - (3) + 4(2) &\stackrel{?}{=} 2 \\ -3 - 3 + 8 &= 2 \checkmark \end{aligned}$$

$$\text{(3a)}\quad \begin{aligned} 5x + 3y + 4z &= 12 \\ 5(-1) + 3(3) + 4(2) &\stackrel{?}{=} 12 \\ -5 + 9 + 8 &= 12 \checkmark \end{aligned}$$

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Watch Out !

It is important when checking your ordered triple that you substitute the solution into the original system of three equations instead of the new system of two equations. Using the original equations will enable you to find any mistakes you might have made in calculating the new system of two equations.

Exceptions to an Ordered Triple as a Solution

In the first three examples, the solution of the systems of equations is an ordered triple. However, sometimes there is no solution to a system of equations. And sometimes there is an infinite number of solutions.

EXAMPLE 4

Use the method of substitution to find the solution (x, y, z) for the system shown below.

(1a) $x + 3y - 2z = -1$
(2a) $2x - 4y + 6z = 5$
(3a) $x - 2y + 3z = 1$

STEP 1 Solve one of the equations for one of the variables. In this example, solve equation (3a) for x.

(3a) $x - 2y + 3z = 1$

$$x = 2y - 3z + 1$$

STEP 2 Then, substitute the expression for z into the other two equations to obtain a system with two equations in two variables.

(1a)
$$\begin{aligned} x + 3y - 2z &= -1 \\ 2y - 3z + 1 + 3y - 2z &= -1 \\ 5y - 5z &= -2 \end{aligned}$$

(2a)
$$\begin{aligned} 2x - 4y + 6z &= 5 \\ 2(2y - 3z + 1) - 4y + 6z &= 5 \\ 4y - 6z + 2 - 4y + 6z &= 5 \\ 2 &\neq 5 \end{aligned}$$

The substitution into one of the equations, 2(a), resulted in a false statement, so there are no solutions to this system of equations.

EXAMPLE 5

Use the method of substitution to find the solution (x, y, z) for the system shown below.

(1a) $x + 3y - 2z = -1$
(2a) $4x + 12y - 8z = -4$
(3a) $2x + 6y - 4z = -2$

Solve one of the equations for one of the variables. In this example, solve equation (1a) for x.

(1a) $x + 3y - 2z = -1$, so $x = -3y + 2z - 1$

When this expression is substituted for x in the other two equations to obtain a system with two equations in two variables, the result is $0 = 0$.

(2a)
$$\begin{aligned} 4x + 12y - 8z &= -4 \\ 4(-3y + 2z - 1) + 12y - 8z &= -4 \\ -12y + 8z - 4 + 12y - 8z &= -4 \\ -12y + 12y + 8z - 8z - 4 &= -4 \\ -4 &= -4 \\ 0 &= 0 \end{aligned}$$

(3a)
$$\begin{aligned} 2x + 6y - 4z &= -2 \\ 2(-3y + 2z - 1) + 6y - 4z &= -2 \\ -6y + 4z - 2 + 6y - 4z &= -2 \\ -6y + 6y + 4z - 4z - 2 &= -2 \\ -2 &= -2 \\ 0 &= 0 \end{aligned}$$

This result means that for any value of x, there are values of y and z that are solutions to the system. There are an infinite number of solutions to this system.

GOT TO KNOW!

Using Substitution to Solve a System of Three Equations in Three Variables

1. Solve one of the equations for one variable.
2. Substitute for that variable in the other two equations. The result is a system of two equations in two variables.
3. Solve the two-variable system for another variable.
4. Substitute the two variables you know into an original equation, and solve for the third variable.
5. Check the ordered triple in all three original equations.

Rows and Columns

The Dimensions of a Matrix

A **matrix** is a rectangular arrangement of numbers or values. Each number or value in a matrix is called an **element** of the matrix. The elements are arranged in horizontal rows and vertical columns. A matrix is usually indicated using square brackets. The plural of matrix is *matrices*.

The **dimensions** of a matrix refer to the number of rows and columns that it has. The dimensions of a matrix are usually written $m \times n$, read "m by n," where the first letter indicates the number of rows and the second number indicates the number of columns.

$$\begin{bmatrix} 2 & 5 \\ 8 & -7 \\ 0.5 & 16 \\ 20 & -2 \\ 0 & 0 \\ 0 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 7 & 3.5 & \frac{1}{2} \\ -1 & 0 & -2.3 & 6 \\ -\frac{3}{4} & -\frac{5}{2} & 8 & -10 \\ 1 & 2 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 25 & -10 & -7 & 8.5 & -\frac{3}{10} \end{bmatrix}$$

EXAMPLE 1

Tell the dimensions of the three matrices shown above.

STEP 1 Count the number of rows and columns. Left matrix: 6 rows and 2 columns
Middle matrix: 4 rows and 4 columns
Right matrix: 1 row and 5 columns

STEP 2 Use the $m \times n$ format. Left matrix: 6×2
Middle matrix: 4×4
Right matrix: 1×5

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EXAMPLE 2

Write a 5×3 matrix, where the elements in row m are $3m$, $5m$, and $10m$.

STEP 1 Set up a 5×3 matrix.

A 5×3 matrix has 5 rows and 3 columns. Prepare an outline for a 5×3 matrix, using $3m$, $5m$, and $10m$ to represent the elements in the matrix, as shown in the matrix to the right.

$$\begin{bmatrix} 3m & 5m & 10m \\ 3m & 5m & 10m \\ 3m & 5m & 10m \\ 3m & 5m & 10m \\ 3m & 5m & 10m \end{bmatrix}$$

STEP 2 Write the matrix.

The phrase "row m" means that $m = 1$ for row 1, $m = 2$ for row 2, and so on. Use the values of m to evaluate each element in the matrix.

$$\begin{bmatrix} 3(1) & 5(1) & 10(1) \\ 3(2) & 5(2) & 10(2) \\ 3(3) & 5(3) & 10(3) \\ 3(4) & 5(4) & 10(4) \\ 3(5) & 5(5) & 10(5) \end{bmatrix} \longrightarrow \begin{bmatrix} 3 & 5 & 10 \\ 6 & 10 & 20 \\ 9 & 15 & 30 \\ 12 & 20 & 40 \\ 15 & 25 & 50 \end{bmatrix}$$

Watch Out !

When a matrix is described as having dimensions $m \times n$, the first value *always* refers to the number of rows and the second value *always* refers to the number of columns.

Equal Matrices

Two matrices are **equal matrices** if they meet these conditions.

- The matrices have the same dimensions.
- Corresponding elements in the two matrices have the same value.

EXAMPLE 3

a. The two matrices below are equal. Use the definition of equal matrices to find the values of *x*, *y*, *z*, and *w*.

$$\begin{bmatrix} 3x-8 & -35 \\ 7 & \frac{w+8}{2} \end{bmatrix} = \begin{bmatrix} 1 & 3y-5 \\ 10-2z & 4 \end{bmatrix}$$

STEP 1 Corresponding elements are equal, so set up four equations.

$$3x-8=1 \qquad -35=3y-5 \qquad 7=10-2z \qquad \frac{w+8}{2}=4$$

STEP 2 Solve each of the four equations.

$$\begin{array}{rrrr} 3x-8=1 & -35=3y-5 & 7=10-2z & \frac{w+8}{2}=4 \\ 3x=9 & -30=3y & -3=-2z & w+8=8 \\ x=3 & y=-10 & z=1.5 & w=0 \end{array}$$

b. Use the results from part (a) to write all the elements in the following matrix as numbers, with no variables.

$$\begin{bmatrix} 2w-7 & x \div 0.5 & 100z & y-3 \\ x+2y & x \div 2 & 8w & x+y+z+w \end{bmatrix}$$

STEP 1 Rewrite the matrix using the values of *x*, *y*, *z*, and *w*.

$$\begin{bmatrix} 2(0)-7 & (3) \div 0.5 & 100(1.5) & (-10)-3 \\ (3)+2(-10) & (3) \div 2 & 8(0) & (3)+(-10)+(1.5)+(0) \end{bmatrix}$$

STEP 2 Evaluate each numerical expression in the matrix.

$$\begin{bmatrix} -7 & 6 & 150 & -13 \\ -17 & 1.5 & 0 & -5.5 \end{bmatrix}$$

Watch Out !

When you identify corresponding elements in two matrices, be sure the elements are in the same row and in the same column in each matrix.

GOT TO KNOW!

Writing a Matrix

$$\begin{bmatrix} 3 & 5 & 10 \\ 6 & 10 & 20 \\ 9 & 15 & 30 \\ 12 & 20 & 40 \\ 15 & 25 & 50 \end{bmatrix} \text{row} \qquad \begin{bmatrix} 3 & 5 & 10 \\ 6 & 10 & 20 \\ 9 & 15 & 30 \\ 12 & 20 & 40 \\ 15 & 25 & 50 \end{bmatrix} \text{column} \qquad \begin{bmatrix} 3 & 5 & 10 \\ 6 & 10 & 20 \\ 9 & 15 & 30 \\ 12 & 20 & 40 \\ 15 & 25 & 50 \end{bmatrix} \text{element}$$

dimensions: 5 (rows) × 3 (columns)

The elements in a matrix can: be positive or negative numbers,
be zero, or
contain variables.

Adding or Subtracting Matrices

Sums and Differences of Corresponding Elements

If two matrices have the same dimensions, you can **add** the two matrices by finding the sums of pairs of corresponding elements. You can **subtract** them by finding the differences of corresponding elements.

For example, if Matrix $A = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 2 & 6 \end{bmatrix}$ and Matrix $B = \begin{bmatrix} 4 & 10 & 0 \\ -5 & 0 & -2 \end{bmatrix}$,

then $A + B = \begin{bmatrix} 3+4 & -2+10 & 5+0 \\ 0+(-5) & 2+0 & 6+(-2) \end{bmatrix} = \begin{bmatrix} 7 & 8 & 5 \\ -5 & 2 & 4 \end{bmatrix}$

and $A - B = \begin{bmatrix} 3-4 & -2-10 & 5-0 \\ 0-(-5) & 2-0 & 6-(-2) \end{bmatrix} = \begin{bmatrix} -1 & -12 & 5 \\ 5 & 2 & 8 \end{bmatrix}$.

Watch Out !

When subtracting matrices, be sure to always subtract the element from the second matrix from the corresponding element in the first matrix, not the other way around.

For example, for $A - B$, every element in the resulting matrix is found by: element A − corresponding element B.

EXAMPLE 1

Using the matrices below, find $X + Z$, $Y - X$ and $X - (Z + Y)$.

$X = \begin{bmatrix} -3 & 2 \\ 4 & -3 \\ 8 & -1 \end{bmatrix}$, $Y = \begin{bmatrix} 7 & 10 \\ -3 & 0 \\ 5 & -2 \end{bmatrix}$, $Z = \begin{bmatrix} 8 & 0 \\ 0 & 0 \\ -1 & -1 \end{bmatrix}$

Write each expression, substitute the matrices, and evaluate.

$$X + Z = \begin{bmatrix} -3 & 2 \\ 4 & -3 \\ 8 & -1 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 0 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -3+8 & 2+0 \\ 4+0 & -3+0 \\ 8+(-1) & -1+(-1) \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 4 & -3 \\ 7 & -2 \end{bmatrix}$$

$$Y - X = \begin{bmatrix} 7 & 10 \\ -3 & 0 \\ 5 & -2 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 4 & -3 \\ 8 & -1 \end{bmatrix} = \begin{bmatrix} 7-(-3) & 10-2 \\ -3-4 & 0-(-3) \\ 5-8 & -2-(-1) \end{bmatrix} = \begin{bmatrix} 10 & 8 \\ -7 & 3 \\ -3 & -1 \end{bmatrix}$$

$$X - (Z + Y) = \begin{bmatrix} -3 & 2 \\ 4 & -3 \\ 8 & -1 \end{bmatrix} - \left(\begin{bmatrix} 8 & 0 \\ 0 & 0 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} 7 & 10 \\ -3 & 0 \\ 5 & -2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -3 & 2 \\ 4 & -3 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} 15 & 10 \\ -3 & 0 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} -18 & -8 \\ 7 & -3 \\ 4 & 2 \end{bmatrix}$$

GOT TO KNOW!

Identifying Corresponding Elements in Two Matrices

1. Make sure that the two matrices have the same dimensions.
2. Check that for each pair of corresponding elements, the row number and column number exactly match.

Adding and Subtracting Matrices A and B

Operation	Procedure
Addition: $A + B$	Add each element in A to its corresponding element in B.
Subtraction: $A - B$	Subtract each element in B from its corresponding element in A.

Always check that two matrices have the same dimensions before you try to add or subtract them.

EXAMPLE 2

Here are several matrices.

$$A=\begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix},\ B=\begin{bmatrix}6 & 8\\ 1 & 3\\ 2 & -2\end{bmatrix},\ C=\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix},\ D=\begin{bmatrix}2\\ 5\\ 2\\ 8\end{bmatrix},\ E=\begin{bmatrix}-3 & -2\\ -4 & 7\\ 0 & 1\end{bmatrix}$$

a. Determine whether it is possible to add or subtract matrices B and E. Explain. If it is possible, find the matrix for $B+E$ and find the matrix for $B-E$.

Yes, matrices B and E are both 3 × 2 matrices. They have the same dimensions so you can find the sum or difference for each pair of corresponding elements.

$$B+E=\begin{bmatrix}6 & 8\\ 1 & 3\\ 2 & -2\end{bmatrix}+\begin{bmatrix}-3 & -2\\ -4 & 7\\ 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 6\\ -3 & 10\\ 2 & -1\end{bmatrix},\ B-E=\begin{bmatrix}6 & 8\\ 1 & 3\\ 2 & -2\end{bmatrix}-\begin{bmatrix}-3 & -2\\ -4 & 7\\ 0 & 1\end{bmatrix}=\begin{bmatrix}9 & 10\\ 5 & -4\\ 2 & -3\end{bmatrix}$$

b. Is it possible to add or subtract matrices A and D? Explain. If it is possible, find the matrix for $A+D$ and find the matrix for $A-D$.

No, A is a 1 × 4 matrix and D is a 4 × 1 matrix. They do NOT have the same dimensions so you cannot identify pairs of corresponding elements to add or subtract.

c. Is it possible to add or subtract matrices A and C? Explain. If it is possible, find the matrix for $A+C$ and find the matrix for $A-C$.

Yes, matrices A and C are both 1 × 4 matrices. They have the same dimensions so you can find the sum or difference of corresponding elements.

$$A+C=\begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix}$$

$$A-C=\begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix}-\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix}$$

Watch Out !

A matrix with 1 row and 4 columns does NOT have the same dimensions as a matrix with 4 rows and 1 column.

Need More HELP

Remember: For the dimensions of a matrix, the first value always indicates the number of rows and the second number indicates the number of columns. So an $m \times n$ matrix has m rows and n columns.

EXAMPLE 3

Using the matrices in Example 2 above, find a matrix X so that $A+X=C$.

STEP 1 Write an equation. Use "?" symbols for the unknown elements in matrix X.

$$\begin{array}{ccccc} A & + & X & = & C\\ \begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix} & + & \begin{bmatrix}? & ? & ? & ?\end{bmatrix} & = & \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

STEP 2 Find the values for the unknown elements. For example, find out what number needs to be added to 2 for the sum to be 0.

$$\begin{bmatrix}2 & 0 & 5 & -3\end{bmatrix}+\begin{bmatrix}-2 & 0 & -5 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}$$

matrix $X=\begin{bmatrix}-2 & 0 & -5 & 3\end{bmatrix}$

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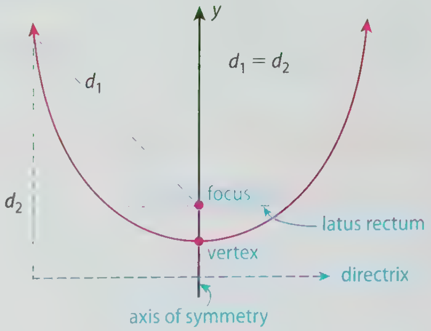
Graphing Parabolas

Parabolas



A parabola is the set of points equidistant from a fixed point, called the **focus**, and a fixed line, called the **directrix**. The **axis of symmetry** is the line that is perpendicular to the directrix and passes through the focus. The **vertex** of a parabola is the point on the parabola where it intersects the axis of symmetry. The **latus rectum (LR)** is the segment through the focus parallel to the directrix. The endpoints of the latus rectum are on the parabola.

The equation of a parabola whose vertex is at the origin is either $y^2 = 4px$ or $x^2 = 4py$.

- The constant p is the distance between the vertex and the focus and the vertex and the directrix. To solve for p in either equation, set $4p$ equal to the coefficient of the linear term x or y in the equation.
- To find the endpoints of the latus rectum (LR), substitute the value of p for x in the equation $y^2 = 4px$ and solve for y, or substitute the value of p for y in the equation $x^2 = 4py$ and solve for x.

Need More HELP

Remember that a quadratic equation can have two roots that are opposites of each other. For more review of quadratic equations, go to pages 1702–1713. On these pages, various methods of solving quadratic equations are explained.

EXAMPLE 1

Find the focus and the directrix of the parabola $y^2 = 8x$. Then graph the parabola.

STEP 1 The vertex of this parabola is at the origin (0, 0). The squared term is y^2, so the axis of symmetry is the x-axis.

STEP 2 Solve for p by setting $4p$ equal to 8, the coefficient of x. $4p = 8$, so $p = 2$

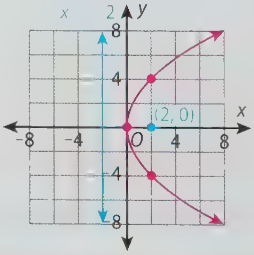
STEP 3 The constant $p > 0$, so the parabola opens to the right. The focus lies 2 units to the right of the vertex, and its coordinates are (2, 0).

STEP 4 The directrix is the vertical line p units from the vertex. Its equation is $x = -2$.

STEP 5 The LR is a vertical segment through the focus. To find the coordinates of its endpoints, substitute 2 for x into the equation and solve for y.

$$y^2 = 8(2) \longrightarrow y = \pm\sqrt{16} = \pm 4$$

The focus is at (2, 0). The directrix has the equation $x = -2$. The graph of the parabola $y^2 = 8x$ is shown to the right.



GOT TO KNOW!

Forms of the Equations of Parabolas with Vertex (0, 0)

Equation	Vertex	Axis of Symmetry	Directrix	Direction of Parabola
$y^2 = 4px$	(0, 0)	x-axis	Vertical	$p > 0$: opens to the right $p < 0$: opens to the left
$x^2 = 4py$	(0, 0)	y-axis	Horizontal	$p > 0$: opens up $p < 0$: opens down

EXAMPLE 2

Find the focus and directrix of the parabola $x^2 = -4y$. Then graph the parabola.

METHOD 1

STEP 1 The vertex of this parabola is at the origin (0, 0). The squared term is x^2, so the axis of symmetry is the y-axis.

STEP 2 Solve for p by setting $4p$ equal to -4.

$$4p = -4$$
$$p = -1$$

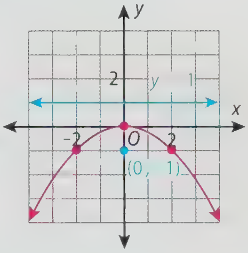
STEP 3 The constant $p < 0$, so the parabola opens down. The focus lies 1 unit below the vertex, and its coordinates are $(0, -1)$.

STEP 4 The directrix is the horizontal line p units above the vertex. Its equation is $y = 1$.

STEP 5 The LR is a vertical segment through the focus. To find the coordinates of its endpoints, substitute -1 for y into the equation and solve for x.

$$x^2 = -4(-1)$$
$$x = \pm\sqrt{4} = \pm 2$$

The focus is at $(0, -1)$. The equation of the directrix is $y = 1$. The graph of the parabola $x^2 = -4y$ showing its focus and directrix appears to the right.



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METHOD 2

You can use a table of values to graph parabolas whose equations are in this form.

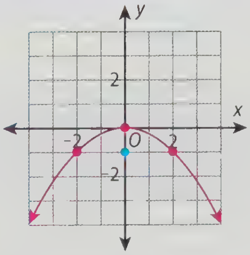
Rewrite the equation $x^2 = -4y$ solved for y.

$$x^2 = -4y$$
$$y = -\frac{1}{4}x^2$$

Set up a table of x and y values. Substitute at least three values of x into the equation and solve for y.

x	$-\frac{1}{4}x^2$	y
0	$-\frac{1}{4}(0)^2$	0
2	$-\frac{1}{4}(2)^2$	-1
-2	$-\frac{1}{4}(-2)^2$	-1

Plot the points and draw the parabola.



The coordinates of the focus are $(0, -1)$. The directrix is 1 unit above the vertex, and its equation is $y = 1$. The vertex of the graph is on (0, 0), and the parabola opens down.

Equations of Parabolas in Standard Form

Not all parabolas have a vertex located at (0, 0). The standard form of the equation of all parabolas can be written as $(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$. The point (h, k) is the vertex of the parabola, and p is the distance between the vertex and the focus and the vertex and the directrix.

- If the axis of symmetry is horizontal, the directrix is vertical and its equation is $x = h - p$. The graphs of these parabolas are not functions.
  - If $p > 0$, then the focus lies to the right of the vertex along the axis of symmetry.
  - If $p < 0$, the focus lies to the left of the vertex along the axis of symmetry.
- If the axis of symmetry is vertical, the directrix is horizontal and its equation is $y = k - p$. The graphs of these parabolas are functions.
  - If $p > 0$, then the focus lies above the vertex along the axis of symmetry.
  - If $p < 0$, the focus lies below the vertex along the axis of symmetry.
- If the vertex is located at the origin, then both h and k are 0, and the equations simplify to either $x^2 = 4py$ or $y^2 = 4px$.

Watch Out !

It's easy to make a mistake with the signs in these formulas. Remember that subtracting a negative number is the same thing as adding a positive number.

For example, if h is negative, then the expression is written as $x + h$.

If k is negative, then the expression is written as $y + k$.

EXAMPLE 3

Find the vertex, the focus, and the directrix of the parabola $(x + 3)^2 = -8(y - 1)$. Graph the parabola.

STEP 1 Find the vertex.

Rewrite the equation in standard form.

$$(x - h)^2 = 4p(y - k)$$

$$(x - (-3))^2 = -8(y - 1)$$

Use the equation to find the vertex.

$$(h, k) = (-3, 1)$$

STEP 2 Find the coordinates of the focus.

Solve for p.

$$4p = -8$$

$$p = -2$$

The x-term is squared, so the axis of symmetry is vertical and contains the focus.

Because $p = -2$, which is < 0, the focus lies 2 units below the vertex at $(-3, -1)$.

STEP 3 Find the directrix.

Because the axis of symmetry is vertical, the directrix is horizontal.

Substitute into the equation for a horizontal directrix; solve.

$$y = k - p$$

$$= 1 - (-2) = 3$$

The equation for the directrix is $y = 3$.

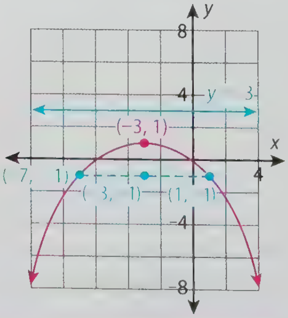
STEP 4 Use the LR and the vertex to graph the parabola. The y-coordinate of the LR is -1. To find the x-coordinates, substitute -1 for y into the equation of the parabola and solve the resulting quadratic equation for the two x-coordinates.

$$(x + 3)^2 = -8(-1 - 1) = 16$$

$$(x + 3) = \pm\sqrt{16} = \pm 4$$

$$x = +4 - 3 = 1$$

$$x = -4 - 3 = -7$$



EXAMPLE 4

Find the vertex, the focus, and the directrix of the parabola $(y + 1)^2 = -16(x - 1)$. Graph the parabola.

STEP 1 Find the vertex.

Rewrite the equation in standard form.
$$(y - k)^2 = 4p(x - h)$$
$$(y - (-1))^2 = -16(x - 1)$$

Use the equation to find the vertex.
$$(h, k) = (1, -1)$$

STEP 2 Find the coordinates of the focus.

Solve for p.
$$4p = -16$$
$$p = -4$$

The y-term is squared, so the axis of symmetry is horizontal and contains the focus. Because $p = -4$, which is < 0, the focus lies 4 units to the left of the vertex at $(-3, -1)$.

STEP 3 Find the directrix.

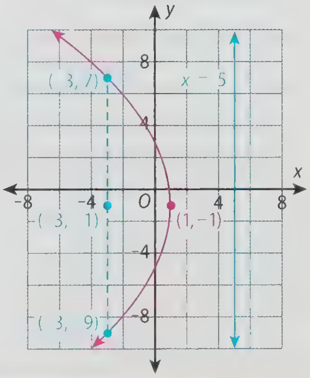
Because the axis of symmetry is horizontal, the directrix is vertical.

Substitute into the equation for a horizontal directrix; solve.
$$x = h - p$$
$$= 1 - (-4) = 5$$

The equation for the directrix is $x = 5$.

STEP 4 Use the LR and the vertex to graph the parabola. The x-coordinate of the LR is -3. To find the y-coordinates, substitute -3 for x into the equation of the parabola and solve the resulting quadratic equation for the two y-coordinates.

$$(y + 1)^2 = -16(-3 - 1) = 64$$
$$y + 1 = \pm\sqrt{64}$$
$$y = 8 - 1 \text{ and } y = -8 - 1$$
$$y = 7 \text{ and } y = -9$$



The graph of the parabola is shown to the right.

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GOT TO KNOW!

Standard Forms of the Equations of Parabolas with Vertex (h, k) and Their Properties

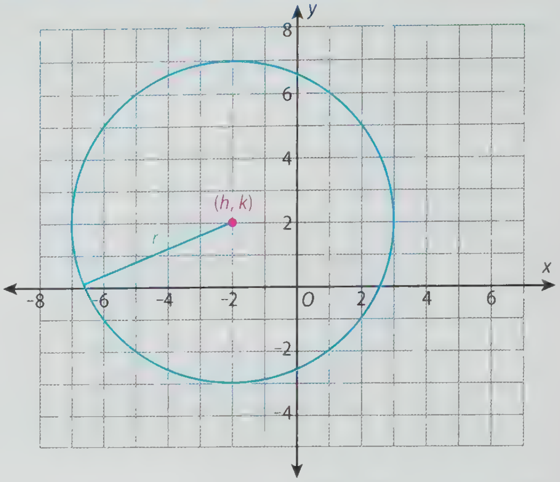
Equation	Vertex	Axis of Symmetry	Focus	Directrix	Length of the LR	Direction of Parabola
$(y - k)^2 = 4p(x - h)$	(h, k)	Horizontal	$(h + p, k)$	$x = h - p$	$\lvert 4p \rvert$	$p > 0$: opens to the right $p < 0$: opens to the left
$(x - h)^2 = 4p(y - k)$	(h, k)	Vertical	$(h, k + p)$	$y = k - p$	$\lvert 4p \rvert$	$p > 0$: opens up $p < 0$: opens down

Graphing Circles

Graphing Circles Using the Center and a Radius

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance from the center of the circle to any point on the circle is called the **radius**. The radius r of a circle is a positive number that is the length of a segment from the center to a point on the circle.

One way to graph a circle in the coordinate plane is to know the coordinates of its center (h, k) and its radius r.



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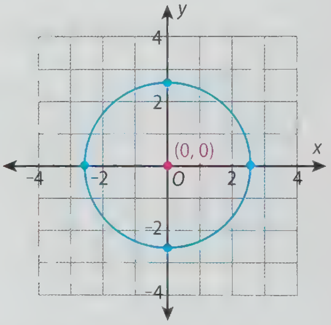
EXAMPLE 1

Graph the circle whose center is at (0, 0) and whose radius is 2.5.

STEP 1 Plot the point that is the center at the origin.

STEP 2 The radius is 2.5, so plot several points 2.5 units from the origin. Easy points to plot are (2.5, 0), (0, 2.5), (−2.5, 0), and (0, −2.5), which are on the x- and y-axes. Draw the circle through the four points.

The graph of the circle whose center is at (0, 0) and whose radius is 2.5 is shown at the right.



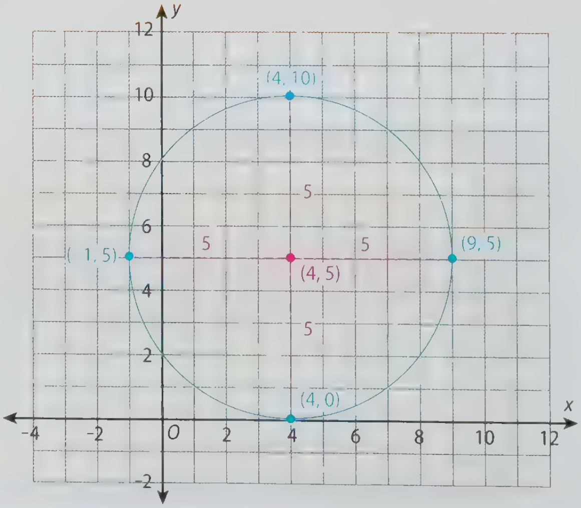
EXAMPLE 2

Graph the circle whose center is at (4, 5) and whose radius is 5.

STEP 1 Plot the center of the circle at (4, 5).

STEP 2 Starting at the center, plot several points that are 5 units from the center. You might move 5 units along the line $y = 5$ to the right and left of the center to the points (9, 5), and (−1, 5), and 5 units up and down along the line $x = 4$, to the points (4, 10) and (4, 0).

The graph of the circle whose center is at (4, 5) and whose radius is 5 is shown at the right.



Using the Center and a Point on the Circle

The Distance Formula can be used to find the distance d between the points (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. So, to graph a circle given the coordinates of its center and a point on the circle, find the radius by letting $d = r$ and solving for r, a positive number.

Need More **HELP**

For more help, go to *Distance Formula* in *Geometry* (Book 1, p. 818).

EXAMPLE 3

Graph the circle whose center is (−1, 0) and that contains the point (−4, −4).

STEP 1 Let $d = r$ and use the Distance Formula to calculate the distance between (−1, 0) and (−4, −4). Substitute the x- and y-coordinates of each point into the formula, and simplify the expression under the radical sign.

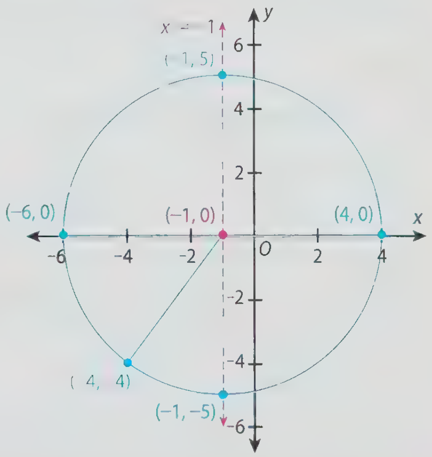
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$r = \sqrt{(-1 - (-4))^2 + (0 - (-4))^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

STEP 2 Plot the center. Draw the vertical line $x = -1$ through the center.

STEP 3 Plot several points that are 5 units from the center. Points easy to plot are those 5 units from (−1, 0) along the x-axis and those 5 units above and below (−1, 0) on the line $x = -1$.

STEP 4 Draw the circle through the points.

The graph of the circle is shown at the right.



EXAMPLE 4

Graph the circle whose center is (−3, −6) and that contains the point (9, −1).

STEP 1 Let $r = d$ and use the Distance Formula to calculate the distance between (−3, −6) and (9, −1). Substitute the x- and y-coordinates of the two points into the formula and simplify the radical expression.

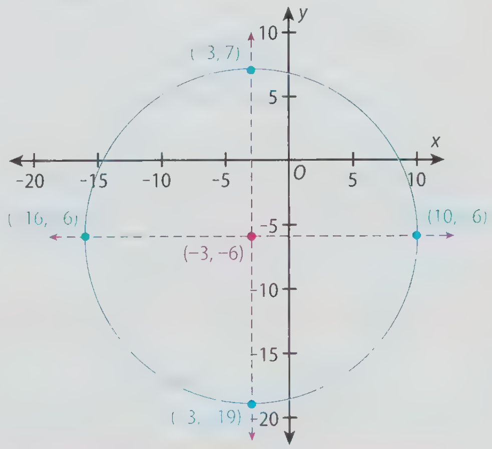
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$r = \sqrt{(9 - (-3))^2 + (-1 - (-6))^2}$$
$$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$
$$= \sqrt{169} = 13$$

STEP 2 Plot the center. Draw the vertical line $x = -3$ and the horizontal line $y = -6$.

STEP 3 Plot several points that are 13 units from the center. Points easy to plot are those along the lines $x = -3$ and $y = -6$: (10, −6), (−3, 7), (−16, −6) and (−3, −19).

STEP 4 Draw the circle through the points.

The graph of the circle is shown at the right.



Using the Endpoints of a Diameter

You can also graph a circle if you know the endpoints of a diameter of the circle.

- The radius of a circle equals half its diameter, so you can find the radius of a circle by finding half the distance of the diameter.
- Given the coordinates of the endpoints of a diameter, you can find the center of the circle by finding the midpoint of the diameter.
- Using this information, you can then draw the graph of the circle.

EXAMPLE 5

Graph the circle with a diameter whose endpoints are $(-9, 3)$ and $(-1, 3)$.

The horizontal line $y = 3$ contains the diameter. Draw this line, and plot the points $(-9, 3)$ and $(-1, 3)$.

METHOD 1

STEP 1 Find the length of the diameter d.

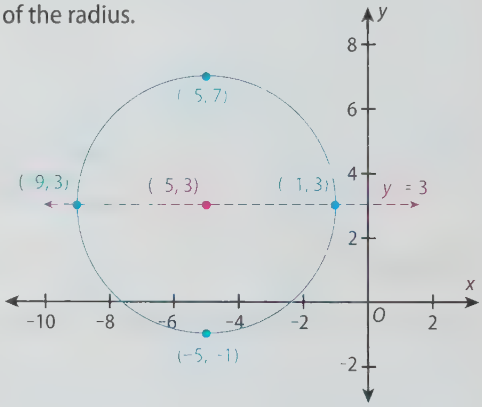
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-9 - (-1))^2 + (3 - 3)^2} \\ &= \sqrt{(-8)^2 + (0)^2} = \sqrt{64} = 8 \end{aligned}$$

STEP 2 From the length of the diameter, find the length of the radius.

$$\begin{aligned} r &= d \div 2 \\ &= 8 \div 2 = 4 \end{aligned}$$

STEP 3 The center of the circle is halfway between the endpoints of a diameter. So, plot the point 4 units between the endpoints $(-9, 3)$ and $(-1, 3)$. The center coordinates are $(-5, 3)$.

STEP 4 Use the center $(-5, 3)$ and $r = 4$ to draw the circle.



METHOD 2

STEP 1 The center of a circle is the midpoint of a diameter. The coordinates of the midpoint of a segment whose endpoints are (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Its midpoint coordinates are $\left(\frac{-9 + (-1)}{2}, 3\right) = (-5, 3)$.

STEP 2 The radius of the circle is the distance between the midpoint $(-5, 3)$ and either of its endpoints, $(-9, 3)$ or $(-1, 3)$. This distance is 4, so $r = 4$.

STEP 3 Use the center $(-5, 3)$ and $r = 4$ to draw the circle.

The graph of the circle is shown above.

Need More HELP

For more help, go to *Midpoint Formula* in *Geometry* (Book 1, p. 820).

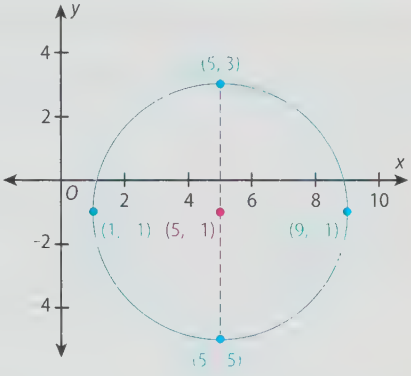
EXAMPLE 6

Graph the circle with a diameter whose endpoint coordinates are (5, 3) and (5, −5).

STEP 1 Draw the vertical line $x = 5$, which contains the diameter.

STEP 2 Find the length of the diameter.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5 - 5)^2 + (3 - (-5))^2} = \sqrt{0^2 + 8^2} = \sqrt{64} = 8$$



STEP 3 Find the length of the radius.

$$r = d \div 2$$
$$= 8 \div 2 = 4$$

STEP 3 Find the center of the circle.

The center of the circle is the midpoint of the diameter, so plot the point that is 4 units from either (5, 3) or (5, −5). The center coordinates are (5, −1).

STEP 4 Use the center (5, −1) and a radius of 4 to draw the circle.

The graph of the circle is shown above.

EXAMPLE 7

Graph the circle with a diameter whose endpoint coordinates are (2, 6) and (0, −4).

STEP 1 Plot the points that are the endpoints of the diameter. The diameter is neither horizontal nor vertical.

STEP 2 Use the Distance Formula to find the length of *d*, the diameter. Use estimation or a calculator for a better approximation.

$$d = \sqrt{(2 - 0)^2 + (6 - (-4))^2}$$
$$= \sqrt{2^2 + 10^2}$$
$$= \sqrt{104} \approx 10.2$$

STEP 3 Find the radius.

The radius is half the diameter, so $r \approx 5.1$.

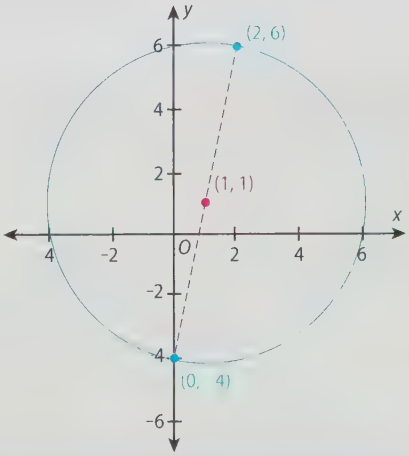
STEP 4 Find the center of the circle.

Use the midpoint formula to find the midpoint coordinates of the diameter.

$$\left(\frac{2 + 0}{2}, \frac{6 + (-4)}{2}\right) = (1, 1)$$

STEP 5 Use the center (1, 1) and a radius of about 5.1 and draw the circle.

The graph of the circle is shown to the right.



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Equations for Circles

The Standard Form of the Equation of a Circle

The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) are the coordinates of the center of the circle, and r is the radius of the circle. You can write the equation of a circle if you know the coordinates of its center and its radius.

EXAMPLE 1

Write the standard form of the equation of a circle whose center is at the origin and whose radius is 5.

STEP 1 Identify h and k.

The center of this circle is at the origin, so $h = 0$ and $k = 0$.

STEP 2 Write the equation of the circle.

Substitute values of h, k, and r into the equation for a circle.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 0)^2 + (y - 0)^2 = 5^2$$

Simplify.

$$x^2 + y^2 = 25$$

The equation of the circle is $x^2 + y^2 = 25$.

EXAMPLE 2

Write the standard form of the equation of a circle whose center is at (1, −5) and whose diameter is 2.

STEP 1 Identify h, k and r.

The center of this circle is at $(1, -5)$, so $h = 1$ and $k = -5$.
The diameter is $2r$, so $2 = 2r$, and $r = 1$.

STEP 2 Write the equation of the circle.

Substitute values of h, k, and r into the equation for a circle.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 1)^2 + (y - (-5))^2 = 1^2$$

Simplify.

$$(x - 1)^2 + (y + 5)^2 = 1$$

The standard form of the equation of the circle is $(x - 1)^2 + (y + 5)^2 = 1$.

Watch Out!

Remember that subtracting a negative number is the same as adding the positive number with the same absolute value as the negative number.

So, if $h < 0$, then $x - h = x + |h|$.

If $k < 0$, then $y - k = y + |k|$.

EXAMPLE 3

Write the equation of a circle whose center is at (−2, −6) and whose radius is $\sqrt{6}$.

STEP 1 Identify h and k.

The center of this circle is at $(-2, -6)$, so $h = -2$ and $k = -6$.

STEP 2 Write the equation of the circle.

Substitute values of h, k, and r into the equation for a circle.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-2))^2 + (y - (-6))^2 = (\sqrt{6})^2$$

Simplify.

$$(x + 2)^2 + (y + 6)^2 = 6$$

The equation of the circle is $(x + 2)^2 + (y + 6)^2 = 6$.

EXAMPLE 4

What is the standard form of the equation of a circle whose diameter has endpoint coordinates of (5, 8) and (−3, −6)?

STEP 1 Find the center of the circle.

The midpoint of the diameter is the center of the circle. Use the coordinates of the endpoints of the diameter and the Midpoint Formula $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$ to find the coordinates of the center: $\left(\frac{5 + (-3)}{2}, \frac{8 + (-6)}{2}\right) = (1, 1)$

METHOD 1 **(for step 2)**

STEP 2 Find r.

Use the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the length of the segment between one of the endpoints and the center of the circle. For this segment, $d = r$.

$$r = \sqrt{(5 - 1)^2 + (8 - 1)^2} = \sqrt{16 + 49} = \sqrt{65}$$

METHOD 2 **(for step 2)**

STEP 2 Find the length of the diameter and the radius.

Use the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the length of the diameter:

$$d = \sqrt{(5 - (-3))^2 + (8 - (-6))^2}$$
$$= \sqrt{64 + 196} = \sqrt{260} = \sqrt{4 \times 65} = 2\sqrt{65}$$

The radius is half the diameter, so $r = \frac{2\sqrt{65}}{2} = \sqrt{65}$

STEP 3 Write the equation of the circle.

Substitute values of h, k, and r into the equation for a circle. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 1)^2 = (\sqrt{65})^2$

Simplify. $(x - 1)^2 + (y - 1)^2 = 65$

The equation of the circle is $(x - 1)^2 + (y - 1)^2 = 65$.

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GOT TO KNOW!

Writing the Standard Form of the Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

h: x-coordinate of the center of the circle

k: y-coordinate of the center of the circle

r: radius of the circle

Writing the General Form from the Standard Form

The general form of the equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$, where D, E, and F are real numbers. The standard form of the equation of a circle $(x - h)^2 + (y - k)^2 = r^2$ can be expanded into its general form using algebra.

EXAMPLE 5

Rewrite the equation $(x + 4)^2 + y^2 = 3$ in general form.

STEP 1 Expand the binomial in x. $(x^2 + 8x + 16) + y^2 = 3$

STEP 2 Set the equation equal to 0.

Subtract 3 from both sides. $x^2 + 8x + 16 + y^2 - 3 = 0$

Simplify. $x^2 + 8x + y^2 + 13 = 0$

STEP 3 Rewrite the equation in general form, starting with the second degree terms, x^2 and y^2, followed by the linear terms, x and y, and then the constant.

Apply the Commutative Property of Addition. $x^2 + y^2 + 8x + 13 = 0$

The general form of the equation of this circle is $x^2 + y^2 + 8x + 13 = 0$.

Need More HELP

The Commutative Property of Addition states that the order of terms being added can be switched:

$a + b = b + a$

To review, go to *Properties in Algebra* (pp. 1406–1409).

EXAMPLE 6

Rewrite the equation $(x - 6)^2 + (y^2 + 2)^2 = 15$ in general form.

STEP 1 Expand the binomials in x and y. $(x^2 - 12x + 36) + (y^2 + 4y + 4) = 15$

STEP 2 Set the equation to 0.

Subtract 15 from both sides. $x^2 - 12x + 36 + y^2 + 4y + 4 - 15 = 0$

Simplify. $x^2 - 12x + y^2 + 4y + 25 = 0$

STEP 3 Rewrite the equation in general form, starting with the second degree terms, x^2 and y^2, followed by the linear terms, x and y, and then the constant.

Apply the Commutative Property of Addition $x^2 + y^2 - 12x + 4y + 25 = 0$

The general form of the equation of this circle is $x^2 + y^2 - 12x + 4y + 25 = 0$.

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GOT TO KNOW!

Changing from Standard Form to General Form for the Equation of a Circle

Standard Form:

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\downarrow$$

$$x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = r^2$$

$$\downarrow$$

$$x^2 - 2hx + h^2 + y^2 - 2ky + k^2 - r^2 = 0$$

$$\downarrow$$

$$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$$

$$\downarrow$$

General Form: $x^2 + y^2 + Dx + Ey + F = 0$, where $D = -2h$, $E = -2k$, and $F = (h^2 + k^2 - r^2)$

Writing the Standard Form from the General Form

The general form of the equation of a circle $x^2 + y^2 + Dx + Ey + F = 0$ can be converted into the standard form $(x - h)^2 + (y - k)^2 = r^2$ using a method known as completing the square.

Completing the square is based on factoring a quadratic trinomial expression $a^2 + 2ab + b^2$ into the square of a binomial $(a + b)^2$, such that $a^2 + 2ab + b^2 = (a + b)^2$. To convert a quadratic binomial into a perfect square trinomial, each of the following must be true.

- The coefficient of the quadratic term must equal 1.
- A perfect square equal to the square of half the coefficient of the linear term must be added to the expression.

If the general equation is written as $(x^2 + Dx) + (y^2 + Ey) + F = 0$, then the quadratic expressions are $(x^2 + Dx)$ and $(y^2 + Ey)$. To convert each of these binomials into perfect square trinomials, the squares to be added are equal to $\frac{1}{2}$ the coefficients of x and y; namely $\left(\frac{1}{2}D\right)^2$ and $\left(\frac{1}{2}E\right)^2$, respectively.

Need More HELP

To review the method of completing the square, go to *Completing the Square* in *Algebra* (p. 1706).

EXAMPLE 7

The general equation of a circle is $x^2 + y^2 + 6x + 2y + 6 = 0$.

a. What is the standard form of the equation of this circle?

STEP 1 Rewrite the equation arranging the like x and y terms together in descending order.

Use the Commutative Property of Addition. $x^2 + 6x + y^2 + 2y + 6 = 0$

Subtract 6 from both sides. $x^2 + 6x + y^2 + 2y = -6$

STEP 2 Use the Associative Property of Addition to separate the x and y expressions.

$$(x^2 + 6x) + (y^2 + 2y) = -6$$

STEP 3 Complete the square in both x and y expressions.

Calculate $\left(\frac{1}{2}D\right)^2$ and $\left(\frac{1}{2}E\right)^2$. $\left(\frac{1}{2} \cdot 6\right)^2 = 9$ and $\left(\frac{1}{2} \cdot 2\right)^2 = 1$

Add these values to both sides of the equation. $(x^2 + 6x + 9) + (y^2 + 2y + 1) = -6 + 9 + 1$

Simplify. $(x^2 + 6x + 9) + (y^2 + 2y + 1) = 4$

STEP 4 Factor each perfect square trinomial into two binomials. $(x + 3)^2 + (y + 1)^2 = 4$

The standard form of the equation is $(x + 3)^2 + (y + 1)^2 = 4$.

b. What are the coordinates of the center of the circle?

Find the coordinates of the center of the circle. $(x - h)^2 + (y - k)^2 = r^2$

$$(x - (-3))^2 + (y - (-1))^2 = 4$$

The coordinates of the center of the circle, (h, k), are $(-3, -1)$.

c. What is the radius of the circle?

Find the radius of the circle. $r^2 = 4$, so $r = 2$

The radius is 2.

Need More HELP

The Associate Property of Addition is the grouping property:

$(a + b) + c = a + (b + c)$.

To review, go to *Properties* in *Algebra* (pp. 1406–1409).

Graphing Ellipses

Graphing an Ellipse with Center at (0, 0)

An **ellipse** is the set of all points in a plane whose distances from two fixed points called the **foci** total a positive constant. The midpoint of the segment connecting the foci is the center of the ellipse.

An ellipse can be oriented horizontally or vertically. The perpendicular segments through the center of an ellipse are its **major axis** and its **minor axis**. The longer axis is the major axis.

There are two equations for an ellipse whose center is at the origin.

- If the major axis is horizontal, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the major axis is vertical, the equation is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. In each equation, $a^2 > b^2$.
- The length of the major axis is $2a$, and the length of the minor axis is $2b$, where a and b are positive numbers.
- The vertices of an ellipse are on its major axis, a units from the center.
- The foci of an ellipse are always located on the major axis, c units from the center, where $c^2 = a^2 - b^2$ and $\pm\sqrt{a^2 - b^2}$.

Watch Out !

When finding the square root of a number in a problem such as this example, the sign of the root depends on whether the answer is a location or a distance. A location can be either positive or negative, but a length is always positive. For example, if $c^2 = 16$, the foci are located at $+4$ and -4 units from the center. Both foci are at a distance of 4 units from the center.

EXAMPLE 1

Locate the foci and graph the ellipse whose equation is $\frac{x^2}{25} - \frac{y^2}{9} = 1$.

STEP 1 In this equation, the denominator of the x^2 term is greater than the denominator of the y^2 term, so the major axis is horizontal, and $a^2 = 25$.

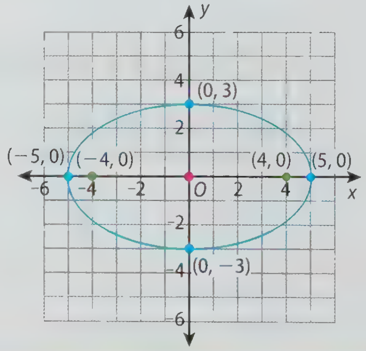
STEP 2 Because $a^2 = 25$, $a = \pm 5$. The coordinates of the vertices are 5 units to the left and right of the center at (5, 0) and (–5, 0).

Because $b^2 = 9$, $b = \pm 3$. The endpoints of the minor axis are 3 units above and below the center at (0, 3) and (0, −3).

STEP 3 To locate the foci of the ellipse, solve the equation $c^2 = a^2 - b^2$ for c.

$c^2 = 25 - 9 = 16; c = \pm 4$

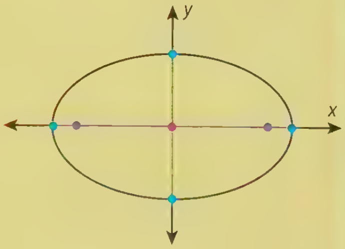
The foci are 4 units to the left and right of the center, at (4, 0) and (−4, 0).



GOT TO KNOW!

Parts of an Ellipse

Major axis length = $2a$
Minor axis length = $2b$
Focus: c units from the center
Vertex: a units from the center



Graphing an Ellipse with Center at (h, k)

The center of an ellipse can be any point (h, k) in the coordinate plane.

Equation:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	In both equations, $a^2 > b^2$.
Major axis:	horizontal	vertical	

EXAMPLE 2

Locate the foci and graph the ellipse whose equation is $\frac{(x+2)^2}{16}+\frac{(y-6)^2}{36}=1$.

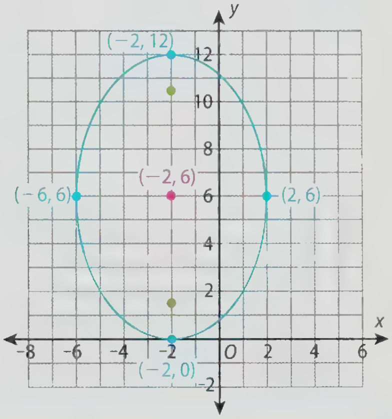
STEP 1 Find the center of the ellipse. $h = -2$ and $k = 6$, so $(h, k) = (-2, 6)$

STEP 2 Find the information needed to graph the ellipse: orientation, vertices, and minor axis.

Orientation: The denominator of the y^2-term is greater than the denominator of the x^2-term, so the major axis is vertical, and $a^2 = 36$.

Vertices: $a^2 = 36$, so the vertices are 6 units above and below the center at $(-2, 6 + 6)$ and $(-2, 6 - 6)$, or $(-2, 12)$ and $(-2, 0)$.

Minor axis: $b^2 = 16$, so the endpoints of the horizontal minor axis are 4 units to the left and right of the center at $(-2 - 4, 6)$ and $(-2 + 4, 6)$, or $(-6, 6)$ and $(2, 6)$.



STEP 3 Find the focus.

$c^2 = a^2 - b^2 = 36 - 16 = 20; c = \sqrt{20} \approx \pm 4.5$

The foci are about 4.5 units above and below the center.

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EXAMPLE 3

Locate the foci and graph the ellipse whose equation is $\frac{(x-1)^2}{64}+\frac{(y+3)^2}{49}=1$.

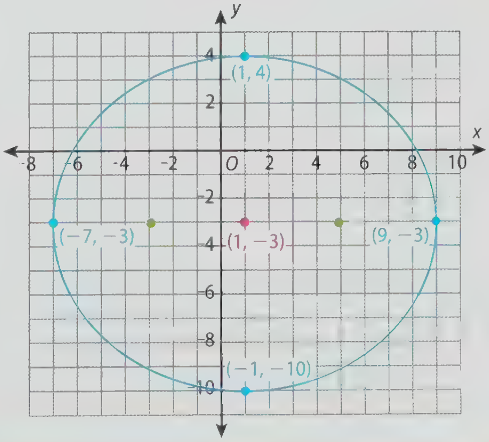
STEP 1 Find the center of the ellipse. $h = 1$ and $k = -3$, so $(h, k) = (1, -3)$.

STEP 2 Find the information needed to graph the ellipse: orientation, vertices, and minor axis.

Orientation: The denominator of the x^2-term is greater than the denominator of the y^2-term, so the major axis is horizontal and $a^2 = 64$.

Vertices: $a^2 = 64$, so the vertices are 8 units along the major axis, to the right and left of the center at $(1 - 8, -3)$ and $(1 + 8, -3)$, or $(-7, -3)$ and $(9, -3)$.

Minor axis: $b^2 = 49$, so the endpoints of the minor axis are 7 units above and below the center at $(1, -3 + 7)$ and $(1, -3 - 7)$, or $(1, 4)$ and $(1, -10)$.



STEP 3 Find the focus.

$c^2 = 64 - 49 = 15; c = \sqrt{15} \approx \pm 3.9$

The foci are about 3.9 units to the left of the center.

Equations for Ellipses

Equations for Ellipses with Center at (0, 0)

There are two standard forms for an equation for an ellipse whose center is at the origin.

- If the major axis is horizontal, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the major axis is vertical, $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. For both equations, $a^2 > b^2$, and a and b are positive numbers.
- The length of the major axis is $2a$, and the length of the minor axis is $2b$.
- The vertices of an ellipse are on its major axis, a units from the center.
- In both equations, the foci of an ellipse are located on the major axis, c units from the center, where $c^2 = a^2 - b^2$.

Watch Out !

Remember that distance is always a positive number. The distance between points on the x-axis equals $|x_1 - x_2|$. The distance between points on the y-axis equals $|y_1 - y_2|$.

EXAMPLE 1

The foci of an ellipse are $(-3, 0)$ and $(3, 0)$. Its vertices are $(-8, 0)$ and $(8, 0)$. What is its equation?

STEP 1 Find the orientation of the ellipse and the resulting equation.

The y-coordinates of the foci are both 0, so the foci lie on the x-axis where $y = 0$. This means that the major axis is horizontal and the standard form of the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

STEP 2 Find a^2.

The distance between the vertices is $|8 - (-8)| = 16$, which is the length of the major axis, $2a$. If $2a = 16$, then $a = 8$ and $a^2 = 64$.

STEP 3 Find b^2.

Both foci are 3 units from the origin, so $c = 3$. To find b^2, use the equation $c^2 = a^2 - b^2$.

$3^2 = 8^2 - b^2 \rightarrow 9 = 64 - b^2 \rightarrow 55 = b^2$

The equation of the ellipse is $\frac{x^2}{64} + \frac{y^2}{55} = 1$.

EXAMPLE 2

The foci of an ellipse are $(0, -2)$ and $(0, 2)$. Its x-intercepts are -2 and 2. What is the equation?

STEP 1 Find the orientation of the ellipse and the resulting equation.

The x-coordinates of the foci are 0, so the foci lie on the y-axis, whose equation is $x = 0$. Therefore, the major axis is vertical and the standard form of the equation is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

STEP 2 Find b^2.

The distance between the x-intercepts is $|-2 - 2| = 4$, which is the length of the minor axis, $2b$. So, if $2b = 4$, then $b = 2$ and $b^2 = 4$.

STEP 3 Find a^2.

Both foci are 2 units from the origin, so $c = 2$. To find a^2, use the equation $c^2 = a^2 - b^2$.

$2^2 = a^2 - 2^2 \rightarrow 4 = a^2 - 4 \rightarrow 8 = a^2$

The equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{8} = 1$.

Equations for Ellipses with Centers at (*h*, *k*)

The standard forms of an equation for an ellipse with center at (h, k) are:

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ if horizontal major axis $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ if vertical major axis

In both equations, $a^2 > b^2$, and a and b are positive numbers.

EXAMPLE 3

What are the vertices and the equation of the ellipse that has a vertical major axis of length 14, a minor axis of length 4, and center at (−4, −4)?

STEP 1 Determine the general equation of the ellipse.

The equation for an ellipse with a vertical major axis is $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$.

STEP 2 Find *h* and *k*.

The center of the ellipse is $(h, k) = (-4, -4)$, so $h = -4$ and $k = -4$.

STEP 3 Find a^2.

The length of the major axis is $2a$, which equals 14. So, $a = 7$ and $a^2 = 49$.

STEP 4 Find b^2.

The length of the minor axis is $2b$, which equals 4. So, $b = 2$ and $b^2 = 4$.

STEP 5 Determine the vertices.

The center of the ellipse is on the vertical line $x = -4$, and is 7 units from each vertex. So, the coordinates of the vertices are $(-4, -4 + 7)$ and $(-4, -4 - 7)$, which equal $(-4, 3)$ and $(-4, -11)$.

The vertices of the ellipse are $(-4, 3)$ and $(-4, -11)$, and its equation is $\frac{(x+4)^2}{4} + \frac{(y+4)^2}{49} = 1$.

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EXAMPLE 4

What is the equation of the ellipse if the coordinates of the endpoints of its major axis are (7, 9) and (7, 3), and the endpoints of its minor axis are (5, 6) and (9, 6)?

STEP 1 Find the orientation of the ellipse and the resulting general equation.

The vertices of the ellipse are on the line $x = 7$, so the ellipse is vertical. The equation for an ellipse with a vertical major axis is $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$.

STEP 2 Find a^2.

The length of the major axis is $2a$, which equals $|9 - 3| = 6$. So, $a = 3$ and $a^2 = 9$.

STEP 3 Find b^2.

The length of the minor axis is $2b$, which equals $|9 - 5| = 4$. So, $b = 2$ and $b^2 = 4$.

STEP 4 Find *h* and *k*.

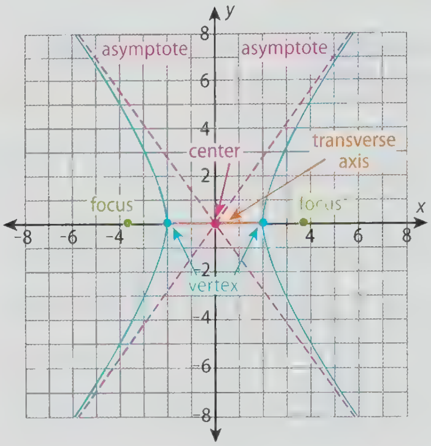
The center of the ellipse, (h, k), is where the major $(x = 7)$ and minor $(y = 6)$ axes intersect. They intersect at $(7, 6)$, so $h = 7$ and $k = 6$.

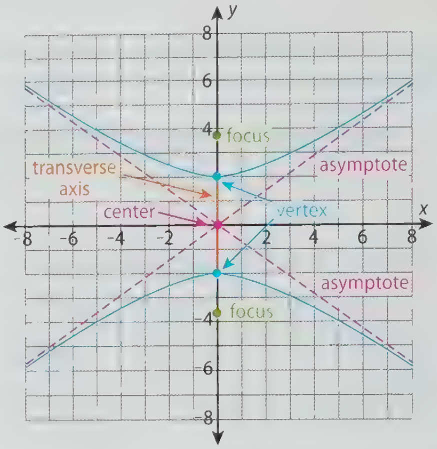
The equation of the ellipse is $\frac{(x-7)^2}{4} + \frac{(y-6)^2}{9} = 1$.

Graphing Hyperbolas

Hyperbolas with Center at (0, 0)

A hyperbola is the set of points in a plane the difference of whose distances from two fixed points, called the **foci**, is a constant. The line through the foci intersects the hyperbola at two points called **vertices**. The line that passes through both vertices is the **transverse axis**. The vertices are a units from the center of the hyperbola—the midpoint of the transverse axis. Each hyperbola also has **asymptotes**, which are lines that pass through the center of the hyperbola and approach intersecting the hyperbola near infinity.





If the center of the hyperbola is at the origin, and the transverse axis is on the x-axis, the standard form of the equation for the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the transverse axis is on the y-axis, the standard form of the equation is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

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EXAMPLE 1

Graph the hyperbola whose equation in standard form is $\frac{x^2}{36} - \frac{y^2}{25} = 1$.

STEP 1 Find the orientation of the hyperbola.

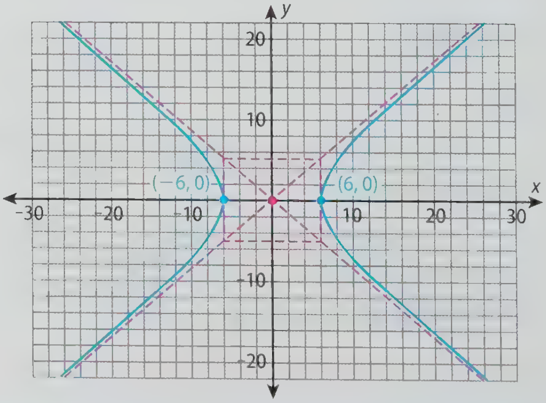
The x^2-term occurs first in the equation, so the transverse axis of the hyperbola is on the x-axis.

STEP 2 Locate and plot the vertices of the hyperbola.

The denominator of the x^2-term is a^2. Because $a^2 = 36$, $a = \pm 6$, and the vertices on the transverse axis are a units from the center at (6, 0) and (−6, 0). Plot these points on the graph.

STEP 3 Draw the asymptotes of the hyperbola.

Use dashed lines and draw the rectangle centered at the origin whose vertices are $(\pm a, 0)$ and $(0, \pm b)$.The coordinates of the vertices are $(\pm 6, 0)$ and $(0, \pm 5)$. Draw and extend the diagonals of the rectangle. These lines are the asymptotes of the hyperbola.



STEP 4 Start at the vertices of each hyperbola and draw the two branches of the hyperbola so that they approach the asymptotes but do not intersect them.

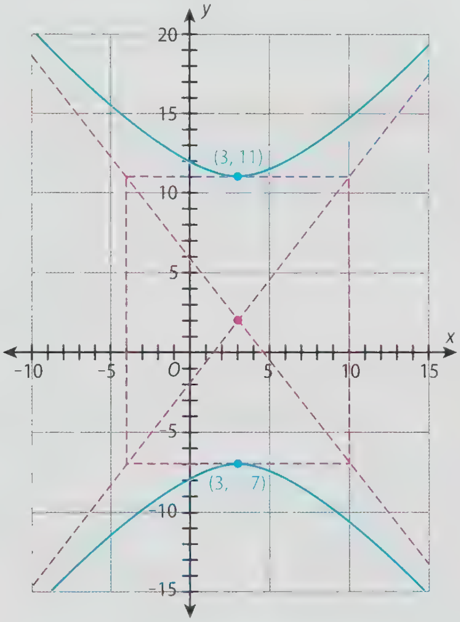
Hyperbolas with Centers at (*h*, *k*)

The standard form of the equations for a hyperbola with center at (h, k) is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, if the transverse axis is horizontal, and $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, if the transverse axis is vertical.

If the transverse axis is horizontal, the coordinates of the vertices are $(h - a, k)$ and $(h + a, k)$. If the transverse axis is vertical, the vertices are $(h, k - a)$ and $(h, k + a)$.

EXAMPLE 2

Graph the hyperbola whose equation in standard form is $\frac{(y-2)^2}{81} - \frac{(x-3)^2}{49} = 1$.



STEP 1 Find and plot the center of the hyperbola.

The center (h, k) of this hyperbola is $(3, 2)$. Plot this point on the graph.

STEP 2 Find the values of a and b.

The constant $a^2 = 81$, so $a = \pm 9$, and $b^2 = 49$, so $b = \pm 7$.

STEP 3 Determine the orientation of the hyperbola.

The y^2-term comes first in the equation, so the transverse axis of the hyperbola is vertical and passes through the center. Its equation is $x = 3$.

STEP 4 Find and plot the vertices.

Because the transverse axis is vertical, the vertices are located a units from the center at $(h, k + a)$ and $(h, k - a)$. Because $a = \pm 9$, the vertices are $(3, 2 + 9)$ and $(3, 2 - 9)$, or $(3, 11)$ and $(3, -7)$. Plot these points on the graph.

STEP 5 Draw the asymptotes of the hyperbola.

Use dashed lines and draw the rectangle that passes through points that are $a = 9$ units above and below the center and $b = 7$ units to the right and left of the center. The rectangle is defined by the points $(-4, -7)$, $(-4, 11)$, $(10, 11)$, and $(10, -7)$. Draw the diagonals of the rectangle using dashed lines and extend them to form the asymptotes of the hyperbola.

STEP 6 Start at the vertices of each hyperbola, and draw the two branches of the hyperbola so that they approach the asymptotes but do not intersect them.

Try It This Way

If you have difficulty drawing the rectangle that determines the asymptotes based on its vertices, draw it based on the distance of each side from the center. Draw the center first. Then locate the midpoints of each side of the rectangle. Draw the rectangle based on these midpoints.

GOT TO KNOW!

Graphing Hyperbolas

Transverse Axis	Center is (0, 0)		Center is (*h*, *k*)	
	Equation	Vertices	Equation	Vertices
Horizontal	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$(\pm a, 0)$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$(h \pm a, k)$
Vertical	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$(0, \pm a)$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$(h, k \pm a)$

Equations for Hyperbolas

Watch Out !

In the standard form of the equations for hyperbolas, there can be three possible relationships between a and b: $a > b$, $a < b$, or $a = b$.

Equations for Hyperbolas with Center at (0, 0)

There are two standard forms for an equation for a hyperbola whose center is at the origin.

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if horizontal transverse axis
- $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ if vertical transverse axis
- The length of the transverse axis is $2a$, where a is a positive number.
- The vertices of a hyperbola are on the transverse axis, a units from the center.
- The foci of a hyperbola are located on the transverse axis, c units from the center, where c is a positive number and $c^2 = a^2 + b^2$.

EXAMPLE 1

What is the standard form of the equation of a hyperbola whose foci are $(-5, 0)$ and $(5, 0)$ and whose vertices are $(-1, 0)$ and $(1, 0)$?

STEP 1 Find the general formula of the hyperbola from the orientation of the transverse axis.

The y-coordinates of the foci are both 0, so the foci lie on the x-axis. This means that the transverse axis is horizontal, and the standard form of the equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

STEP 2 Determine the values of a.

The coordinates of the vertices are $(\pm a, 0)$. The vertices are $(-1, 0)$ and $(1, 0)$, so $a = 1$.

STEP 3 Find the value of c.

The distance from the center to a focus is c. The foci are $(-5, 0)$ and $(5, 0)$, so $c = 5$.

STEP 4 Calculate b^2.

$c^2 = a^2 + b^2$, so $b^2 = c^2 - a^2 = 5^2 - 1^2 = 25 - 1 = 24$

The standard form of the equation of the hyperbola is $\frac{x^2}{1} - \frac{y^2}{24} = 1$, or $x^2 - \frac{y^2}{24} = 1$.

EXAMPLE 2

What is the standard form of the equation of a hyperbola whose foci are $(0, -4)$ and $(0, 4)$ and whose vertices are $(0, -2)$ and $(0, 2)$?

STEP 1 Find the general formula of the hyperbola from the orientation of the transverse axis.

The x-coordinates of the foci are both 0, so the foci lie on the y-axis. Therefore, the transverse axis is vertical and the standard form of the equation is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

STEP 2 Determine the value of a.

The coordinates of the vertices are $(0, \pm a)$. The vertices are $(0, -2)$ and $(0, 2)$, so $a = 2$.

STEP 3 Find the value of c.

The distance from the center to a focus is c. The foci are $(0, -4)$ and $(0, 4)$, so $c = 4$.

STEP 4 Calculate b^2.

$c^2 = a^2 + b^2$, so $b^2 = c^2 - a^2 = 4^2 - 2^2 = 16 - 4 = 12$

The standard form of the equation of the hyperbola is $\frac{y^2}{4} - \frac{x^2}{12} = 1$.

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Equations for Hyperbolas with Centers at (h, k)

If the transverse axis of a hyperbola is horizontal, the standard form of its equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. Its vertices are $(h - a, k)$ and $(h + a, k)$. If the transverse axis is vertical, its standard form is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Its vertices are $(h, k - a)$ and $(h, k + a)$.

EXAMPLE 3

What is the standard form of the equation of the hyperbola that has center at $(4, -2)$, one focus at $(7, -2)$, and one vertex at $(6, -2)$?

STEP 1 Find the values of h and k.

The center of the hyperbola (h, k) is $(4, -2)$, so $h = 4$, and $k = -2$.

STEP 2 Find the general formula of the hyperbola from the orientation of the transverse axis.

Both the center and the vertex lie on the line $y = -2$, so the transverse axis is horizontal. The equation has the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

STEP 3 Find a.

The distance from the center and a vertex is a, so $a = |6 - 4| = 2$.

STEP 4 Calculate c.

The distance between a focus and the center is c. So, $c = |7 - 4| = 3$.

STEP 5 Find b^2.

$c^2 = a^2 + b^2$, so $b^2 = c^2 - a^2 = 3^2 - 2^2 = 9 - 4 = 5$

The standard form of the equation of the hyperbola is $\frac{(x-4)^2}{4} - \frac{(y+2)^2}{5} = 1$.

Watch Out !

Distance is never negative. So, to find a distance, calculate the absolute value of a difference between coordinates.

EXAMPLE 4

What is the standard form of the equation of the hyperbola that has one focus at $(5, -11)$ and vertices at $(5, 3)$ and $(5, -9)$?

STEP 1 Find the general formula of the hyperbola from the orientation of the transverse axis.

The x-coordinates of the vertices are both 5, so the transverse axis lies on the vertical line $x = 5$, and the standard form of the equation of the hyperbola is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$.

STEP 2 Find a.

The distance between the vertices is $2a$. So, $2a = |-9 - 3| = 12$, and $a = 6$.

STEP 3 Find (h, k), the center of the hyperbola.

The center of the hyperbola lies 6 units from each vertex. So, 6 units below $(5, 3)$ is $(5, 3 - 6)$ and (h, k) is $(5, -3)$.

STEP 4 Calculate c.

Each focus is c units from the center. So, c is the absolute value of the difference between the y-coordinates of the focus and the center: $c = |-11 - (-3)| = 8$

STEP 5 Find b^2.

$c^2 = a^2 + b^2$, so $b^2 = c^2 - a^2 = 8^2 - 6^2 = 64 - 36 = 28$

The standard form of the equation of the hyperbola is $\frac{(y+3)^2}{36} - \frac{(x-5)^2}{28} = 1$.

Calculus

Architects use integrals to determine the amount of material needed to build curved structures.

Limits and Continuity 2108

Derivatives 2156

Applications of Derivatives 2188

Integration 2232

Applications of Integrals 2260

Differential Equations 2292

Logarithmic, Exponential,
and Transcendental Functions 2310

What Is a Limit?

GOT TO KNOW!

What is a Limit?

The answer to any limit problem is a y value.

Understanding Limits

In algebra, it is common to find an exact value of a function $f(x)$. Because $y = f(x)$, this value is sometimes called the "y value" of the function. When finding a specific y value of a function, all that you need is the corresponding x value.

But what happens if the function is more than just a simple curve? What if the function jumps around, has holes in it, or has fascinating behavior such as shooting up toward infinity or bouncing up and down forever? What if the function is undefined at a particular x value? What does "undefined" mean to different functions at different points? To describe the behavior of the y values of such functions, you can use a *limit.*

Intuitive Definition of a Limit

Let's start by understanding the meaning of limit. The **limit** of a function $f(x)$ is a number L on the y-axis such that if x is in the neighborhood of a number c, then $f(x)$ is in the neighborhood of L.

L is the limit as x approaches c. For a different number c, there may be a different L.

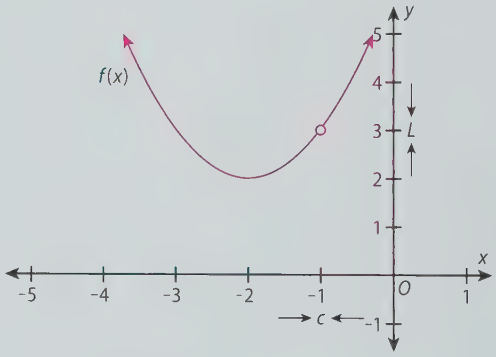
The notation for a limit looks like this:

$$\lim_{x \to c} f(x) = L$$

You should read the statement like this: "The limit of $f(x)$ as x approaches c is L."

EXAMPLE 1

Find $\displaystyle\lim_{x \to -1} \frac{x^3 + 5x^2 + 10x + 6}{x + 1}$.



To solve this problem, you need to find the y value of the function as x approaches -1.

The function is undefined at $x = -1$ because this value would cause division by zero. The exclusion of $x = -1$ creates a hole at the point $(-1, 3)$, as seen in the graph at the right, but the behavior of the graph can still be defined.

To find $\lim_{x \to -1} f(x)$, look at the y values of the curve as x approaches c from both the left and the right. (You will read about the importance of both directions in *Left- and Right-Hand Limits.*)

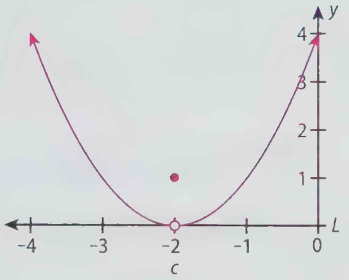
In this case, as the x values approach -1, the y value approaches 3.

Therefore, $\lim_{x \to -1} f(x) = 3$.

EXAMPLE 2

Given: $f(x) = \begin{cases} x^2 + 4x + 4, & x \neq -2 \\ 1, & x = -2 \end{cases}$

Find: $\lim\limits_{x \to -2} f(x)$



A limit and the function's value are not necessarily the same. In this example, the function is defined at *c*. The graph shows that $f(-2) = 1$, but the value that the function approaches as *x* gets closer and closer to -2 is 0.

Therefore, $\lim\limits_{x \to -2} f(x) = 0$.

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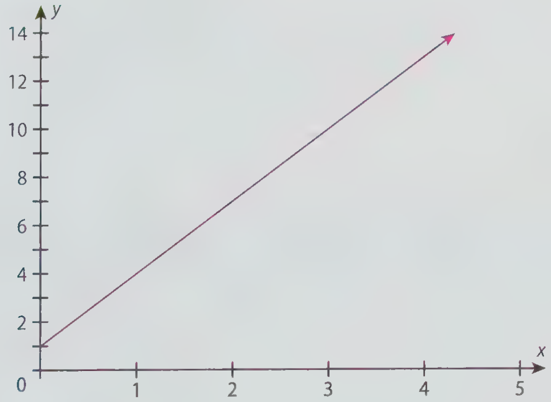
Simple Polynomials

There are still occasions when you use limits for smoothly changing functions. Simple polynomials, for example, have no undefined values or breaks in their graphs. Finding the limits of such functions is straightforward because the limit is the same as the function's value.

EXAMPLE 3

Find $\lim\limits_{x \to 4} 3x + 1$.



As seen in the graph above, $\lim\limits_{x \to 4} f(x) = 13$. Notice that $f(4)$ also equals 13. For all simple polynomials, $f(c) = L$, which means you can determine the limit by substituting *c* into the function.

GOT TO KNOW!

Greek Letters

The lower-case Greek letter epsilon:

ε

The lower-case Greek letter delta:

δ

Definition of a Limit

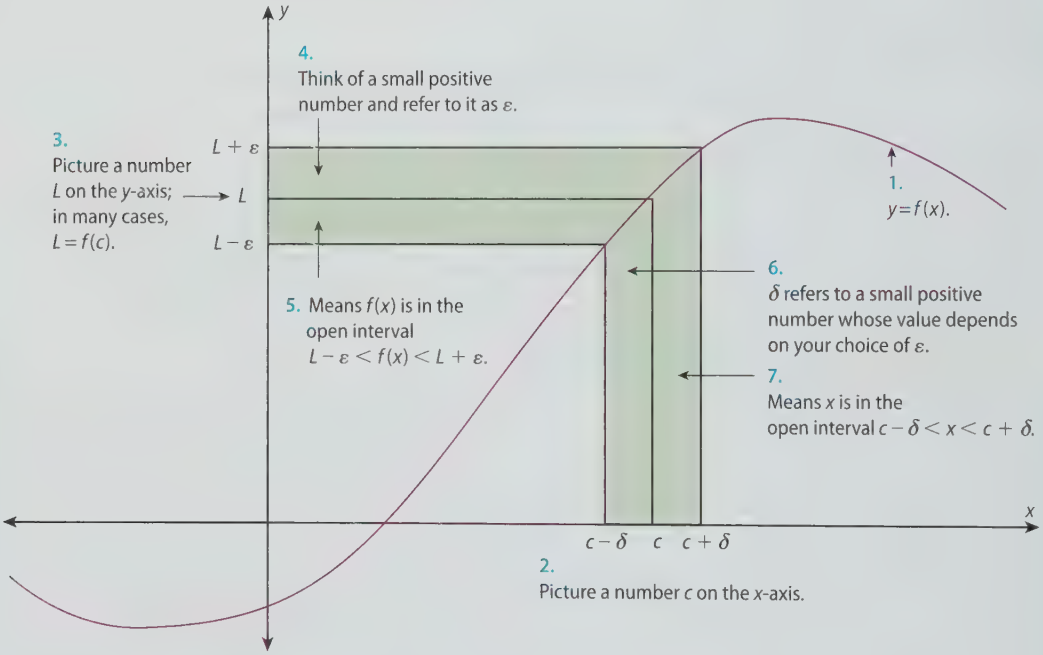
A formal definition of a limit is much more involved than our intuitive definition. As a result, people used some of the techniques of calculus to solve problems long before mathematicians could prove that the techniques were correct. It was only in the late nineteenth century that the mathematician A. L. Cauchy developed an early form of the Epsilon-Delta Definition of a limit, which is the backbone of calculus today. Briefly, this definition is as follows:

$$\underset{2}{\lim_{x \to c}}\ \underset{1}{f(x)} = \underset{3}{L} \text{ if and only if for each } \underset{4}{\varepsilon > 0},$$

$$\text{there exists } \underset{6}{\delta > 0} \text{ such that}$$

$$\text{if } \underset{7}{0 < |x - c| < \delta}, \text{ then } \underset{5}{|f(x) - L| < \varepsilon}.$$

What does the definition mean? Compare the blue numbers in the definition above to those in graph below.


y
4.
Think of a small positive number and refer to it as ε.
3.
Picture a number L on the y-axis; in many cases, $L = f(c)$.
$L + \varepsilon$
L
$L - \varepsilon$
1.
$y = f(x)$.
6.
δ refers to a small positive number whose value depends on your choice of ε.
5. Means $f(x)$ is in the open interval $L - \varepsilon < f(x) < L + \varepsilon$.
7.
Means x is in the open interval $c - \delta < x < c + \delta$.
x
$c - \delta$
c
$c + \delta$
2.
Picture a number c on the x-axis.

We use the Epsilon-Delta Definition of a limit to answer the question, "How close is close enough?" Regardless of how small the separation ε we choose along the y-axis, there is a corresponding separation δ along the x-axis such that if x is between $c - \delta$ and $c + \delta$, then $f(x)$ is between $L - \varepsilon$ and $L + \varepsilon$. To understand how this applies to problems, let's work a couple of examples.

Using the Formal Definition of a Limit

For formal proofs of limits using the Epsilon-Delta Definition, we must show that for any given positive number ε, we can find a positive number δ such that

$$|f(x) - L| < \varepsilon \quad \text{if} \quad 0 < |x - c| < \delta$$

As you work through the example below, notice that the goal is to manipulate the ε inequality so that it looks similar to the δ inequality.

You can then prove the limit by identifying the value of δ for which the Epsilon-Delta statement holds true.

EXAMPLE 4

Use the formal definition of a limit to prove the following:

$$\lim_{x \to 1} 2x + 1 = 3$$

STEP 1 Identify the terms $f(x)$, L, and c needed to apply the Epsilon-Delta Definition of a limit.

From the limit equation, we can identify:

$f(x) = 2x + 1$

$L = 3$

$c = 1$

We will show that given any positive number ε, a positive number δ can be found such that:

$$|\underbrace{(2x + 1)}_{f(x)} - \underbrace{3}_{L}| < \varepsilon \quad \text{if} \quad 0 < |x - \underbrace{1}_{c}| < \delta$$

STEP 2 Simplify and rearrange the ε inequality to a form similar to the δ inequality.

Write the ε inequality using $f(x)$ and L.	$\lvert(2x + 1) - 3\rvert < \varepsilon$
Simplify.	$\lvert 2x - 2\rvert < \varepsilon$
Factor out 2.	$\lvert 2(x - 1)\rvert < \varepsilon$
Divide both sides by 2.	$\lvert x - 1\rvert < \frac{\varepsilon}{2}$

STEP 3 Compare the ε inequality to the δ inequality, and choose a value of δ.

Write the revised the Epsilon-Delta Definition.	$(x - 1) < \frac{\varepsilon}{2}$ if $0 < \lvert x - 1\rvert < \delta$
Choose a value of δ that makes the δ inequality true.	$\delta = \frac{\varepsilon}{2}$

If we choose $\delta = \frac{\varepsilon}{2}$, then $|x - 1| < \delta$ becomes $|x - 1| < \frac{\varepsilon}{2}$, which is the same as the ε inequality.
According to the Epsilon-Delta Definition of limits, then, this proves the limit equation: $\lim_{x \to 1} 2x + 1 = 3$.

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Calculating Limits Algebraically

Direct Substitution

Sometimes the function value and the limit are the same. When this is the case, you can determine the limit by direct substitution of the x value into the given function. Functions of this type are *continuous* at c.

See the *Got To Know!* box at the bottom of the page for a list of limit rules.

Need More HELP

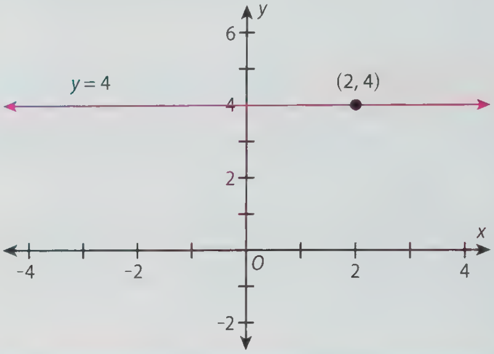
For more about continuous functions, go to *Continuity at a Point* on page 2144.

EXAMPLE 1

Find: $\lim_{x \to 2} 4$

Use the Constant Rule for limits. $\lim_{x \to 2} 4 = 4$

Look at the graph of the function at the right to see why the limit of the constant function as x approaches 2 is the same as the constant value. Notice that for the given constant function $y = 4$, the limit as x approaches *any* value is 4.



EXAMPLE 2

Find: $\lim_{x \to 1} 2(x + 1)$

Use the Constant Multiple Rule for limits. $\lim_{x \to 1} 2(x + 1) = 2 \lim_{x \to 1} (x + 1)$

Use direct substitution. $= 2(1 + 1)$

Simplify. $= 4$

GOT TO KNOW!

Limit Rules

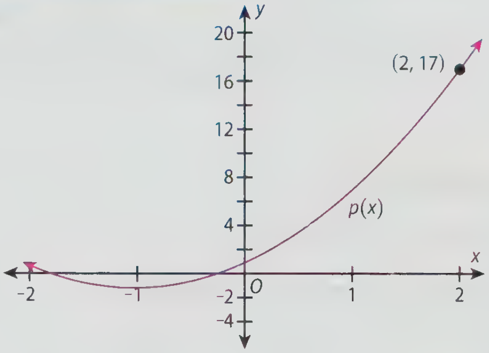
Let n be a positive integer, k be a constant, and f and g be functions which have limits at $x = c$. Then the following rules hold:

1. **Constant Rule**
   $\lim_{x \to c} k = k$
2. **Constant Multiple Rule**
   $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$
3. **Sum Rule**
   $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
4. **Difference Rule**
   $\lim_{x \to c} [f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$
5. **Product Rule**
   $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$
6. **Quotient Rule**
   $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, if $\lim_{x \to c} g(x) \neq 0$
7. **Power Rule**
   $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x)\right]^n$
8. ***n*th Root Rule**
   $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$, if $\lim_{x \to c} f(x) > 0$

Finding the Limit of Polynomial Functions

Polynomial functions are continuous functions whose graphs are smooth curves. The graph at the right shows the smooth curve of a polynomial function $p(x)$. Notice the point drawn on the graph at the point $x = 2$.

In Example 3, you can see how to find the limit of the polynomial function in the graph at $x = 2$ by applying several of the limit rules.



EXAMPLE 3

Find: $\lim_{x\to 2}(2x^2 + 4x + 1)$

Use the Sum Rule for limits.	$\lim_{x\to 2}(2x^2 + 4x + 1) = \lim_{x\to 2}2x^2 + \lim_{x\to 2}4x + \lim_{x\to 2}1$
Use the Constant Multiple Rule.	$= 2\lim_{x\to 2}x^2 + 4\lim_{x\to 2}x + \lim_{x\to 2}1$
Use the Power Rule.	$= 2\left(\lim_{x\to 2}x\right)^2 + 4\lim_{x\to 2}x + \lim_{x\to 2}1$
Use the Constant Rule.	$= 2\left(\lim_{x\to 2}x\right)^2 + 4\lim_{x\to 2}x + 1$
Use direct substitution.	$= 2(2)^2 + 4(2) + 1$
Simplify.	$= 17$

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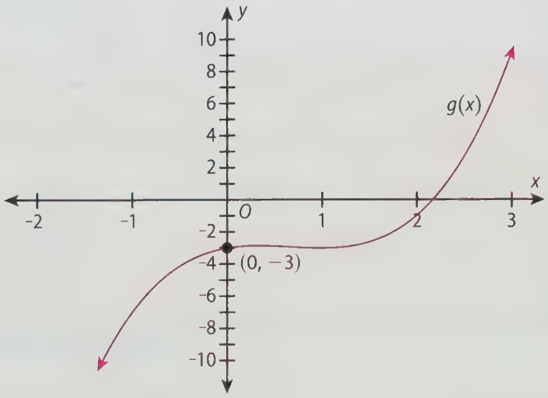
You can see that the limit as x approaches 2 in Example 3 is the same as the function's value at $x = 2$. This is always the case for limits of polynomial functions. You can use direct substitution to evaluate all polynomial limits.

EXAMPLE 4

Find: $\lim_{x\to 0}(x^3 - 2x^2 + x - 3)$

The graph of the function is shown at the right. Because you are finding the limit of a polynomial function, you can use direct substitution as follows.



Use direct substitution.	$\lim_{x\to 0}(x^3 - 2x^2 + x - 3) = (0)^3 - 2(0)^2 + (0) - 3$
Simplify.	$= -3$

Need More HELP

For help with using conjugates to rationalize a function, see *Dividing Radical Expressions* in *Advanced Algebra* (p. 1962).

Conjugate Technique

Another technique for finding the limit of indeterminate forms involves rationalizing the numerator. To do this, the rational function must be multiplied by a form of 1 that involves the conjugate of the numerator. Remember that the conjugate of a binomial such as $a + b$ is $a - b$.

EXAMPLE 8

Evaluate: $\lim\limits_{x \to -1} \dfrac{\sqrt{x+2} - 1}{x+1}$

METHOD 1 **Algebraic Rationalizing**

Direct substitution fails because it produces $\frac{0}{0}$.

Multiply by the conjugate form of 1.

$$\lim_{x \to -1} \frac{\sqrt{x+2} - 1}{x+1} = \lim_{x \to -1} \frac{\sqrt{x+2} - 1}{x+1} \cdot \frac{(\sqrt{x+2} + 1)}{(\sqrt{x+2} + 1)}$$

Use the Distributive Property of Multiplication.

$$= \lim_{x \to -1} \frac{(\sqrt{x+2})^2 + \sqrt{x+2} - \sqrt{x+2} - 1}{(x+1)(\sqrt{x+2} + 1)}$$

Simplify the numerator.

$$= \lim_{x \to -1} \frac{\cancel{x+1}}{(\cancel{x+1})(\sqrt{x+2} + 1)}$$

Cancel like factors.

$$= \lim_{x \to -1} \frac{1}{(\sqrt{x+2} + 1)}$$

Use direct substitution.

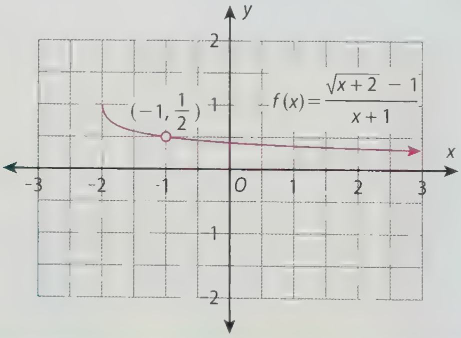
$$= \frac{1}{\left(\sqrt{(-1) + 2} + 1\right)}$$

Simplify.

$$= \frac{1}{2}$$

METHOD 2 **Graphing**

By studying the graph below, you can determine that the y value approaches 0.5 as the x values approach -1 from the left and from the right. Therefore, the limit as x approaches -1 is 0.5, as determined using the algebraic technique in Method 1.



EXAMPLE 9

Evaluate: $\lim\limits_{x\to 12} \dfrac{\sqrt{x-3}-3}{x-12}$

METHOD 1 **Algebraic Rationalizing**

Direct substitution fails because it produces $\frac{0}{0}$.

Multiply by the conjugate form of 1.

$$\lim_{x\to 12} \frac{\sqrt{x-3}-3}{x-12} = \lim_{x\to 12}\left[\frac{\sqrt{x-3}-3}{x-12} \cdot \frac{\sqrt{x-3}+3}{\sqrt{x-3}+3}\right]$$

Use the Distributive Property of Multiplication.

$$= \lim_{x\to 12} \frac{\left(\sqrt{x-3}\right)^2 + 3\sqrt{x-3} - 3\sqrt{x-3} - 9}{(x-12)\left(\sqrt{x-3}+3\right)}$$

Simplify the numerator.

$$= \lim_{x\to 12} \frac{x-12}{(x-12)\left(\sqrt{x-3}+3\right)}$$

Cancel like factors.

$$= \lim_{x\to 12} \frac{1}{\left(\sqrt{x-3}+3\right)}$$

Use direct substitution.

$$= \frac{1}{\left(\sqrt{(12)-3}+3\right)}$$

Simplify.

$$= \frac{1}{6}$$

METHOD 2 **Table of Values**

By looking at the table of values below, you can determine that the y values approach 0.1667, which is approximately $\frac{1}{6}$, as x approaches 12.

x	13	12.1	12.01	12.001	12	11.999	11.99	11.9	11
f(x)	0.1623	0.1662	0.1666	0.1667	?	0.1667	0.1667	0.1671	0.1716

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GOT TO KNOW!

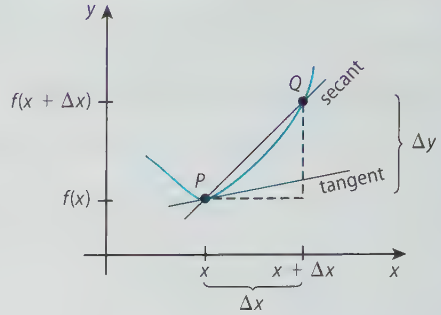
Strategies for Finding Indeterminate Limits

1. Direct substitution will fail, so try these methods.
2. Try algebraic factoring. Look for common factors that will cancel and allow for direct substitution.
3. If trigonometric functions are present, use identities and factoring to change the form of the function.
4. Rationalize the fraction by using conjugates.
5. If algebraic techniques fail, try graphing the function or creating a table of values to approximate the value of the limit by using values of x that are *very close to c*.

Definition of a Derivative

Slope of a Tangent

You can find the average rate of change between two points by drawing a secant between the points and calculating the slope, as shown on the graph at the right. Recall that the slope is Δy divided by Δx, which is sometimes called the rise divided by the run.



Calculus describes how to find the rate of change of a function at an instant. The instantaneous rate of change of $f(x)$ at point P is the slope of the tangent at P. Drawing the tangent with the correct slope would be difficult, but you can approximate the slope using a secant. On the graph, a secant is drawn from the point P to a nearby point Q. The slope of the secant m_{sec} is the change in the y values divided by the change in the x values.

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

From the graph, you can see that as Q moves closer to P (as Δx gets smaller), the secant gets closer to the tangent. The slope of the tangent, then, is the limit as Δx approaches zero.

$$m_{\tan} = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

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EXAMPLE 1

Find the slope of the tangent to $y = x^2$ at the points (1, 1) and (0, 0).

STEP 1 Find the equation for the slope.

Use the definition of the slope of the tangent. $$m_{\tan} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Substitute the function into the definition. $$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

Expand $(x + \Delta x)^2$. $$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

Subtract like terms. $$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

Use the Distributive Property. $$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

Cancel Δx. $$= \lim_{\Delta x \to 0} (2x + \Delta x)$$

Find the limit. $$= 2x$$

STEP 2 Find the value of the slope at (1, 1). $2x = 2(1) = 2$

STEP 3 Find the value of the slope at (0, 0). $2x = 2(0) = 0$

The slope of the tangent line to $y = x^2$ at (1, 1) is 2. The slope at (0, 0) is 0.

Try It This Way

To understand the slope of a tangent, draw a smooth curve on a piece of paper.

Mark a point of tangency on the curve. Place a ruler on the point of tangency and one other point on the curve. The edge of the ruler represents the secant.

Now move the second point closer to the point of tangency. Keep moving until the two points are essentially the same point. The edge of the ruler now represents the tangent.

Notice in Example 1 that the equation for the slope is the same at all points on the function, but the actual slope is different for different points along the function.

Definition of a Derivative

An important concept in calculus is instantaneous changes in functions. The slope of a tangent describes these changes. The slope, called the derivative of the function, is defined in the *Got To Know!* box. The process of finding a derivative is called **differentiation**. Often, the derivative is written as $f'(x)$, which is read "f prime of x."

GOT TO KNOW!

Common Derivative Notations

Notation	How It Is Read
$f'(x)$	f prime of x
y'	y prime
$\dot{y}$	y dot
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d}{dx}(f)$	the derivative of f with respect to x

EXAMPLE 2

Use the definition of a derivative to find $f'(x)$ if $f(x) = 5x + 6$.

Use the definition of a derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Substitute the function into the definition.

$$= \lim_{\Delta x \to 0} \frac{5(x + \Delta x) + 6 - (5x + 6)}{\Delta x}$$

Use the Distributive Property.

$$= \lim_{\Delta x \to 0} \frac{5x + 5\Delta x + 6 - 5x - 6}{\Delta x}$$

Subtract like terms.

$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x}$$

Cancel Δx and find the limit.

$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x} = 5$$

$f'(x) = 5$

Notice that the function described in Example 2, $f(x) = 5x + 6$, is a linear equation. The derivative is a constant, 5. Just as the slope of a linear equation is a constant, the derivative of a linear equation is a constant.

EXAMPLE 3

Use the definition of a derivative to find $f'(x)$ if $f(x) = x^2 - 4$.

Use the definition of a derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Substitute the function into the definition.

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 4 - (x^2 - 4)}{\Delta x}$$

Use the Distributive Property.

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 4 - x^2 + 4}{\Delta x}$$

Combine like terms.

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

Use the Distributive Property.

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

Cancel Δx. Then find the limit.

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

$f'(x) = 2x$

GOT TO KNOW!

Definition of a Derivative

The **derivative** of a function f at x is $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ provided the limit exists.

Derivative at a Point

The derivative of a function has the same *equation* for any point. For most functions, however, the *value* of the derivative is different for different points. The following examples show how to find the derivative at a certain point.

EXAMPLE 4

Use the definition of a derivative to find $f'(x)$ at (1, 10) and at $\left(\frac{1}{2}, 3\right)$ if $f(x) = 8x^3 + 2x$.

STEP 1 Find the derivative $f'(x)$.

Use the definition of a derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Substitute the function into the definition.

$$= \lim_{\Delta x \to 0} \frac{8(x + \Delta x)^3 + 2(x + \Delta x) - (8x^3 + 2x)}{\Delta x}$$

Use the Distributive Property. Subtract like terms of $8x^3$ and $-2x$.

$$= \lim_{\Delta x \to 0} \frac{24x^2\Delta x + 24x(\Delta x)^2 + 8(\Delta x)^3 + 2\Delta wx}{\Delta x}$$

Cancel Δx. Then find the limit.

$$= \lim_{\Delta x \to 0} 24x^2 + 24x\Delta x + 8(\Delta x)^2 + 2 = 24x^2 + 2$$

STEP 2 Find the value of the derivative at (1, 10).

$$f'(1) = 24(1)^2 + 2 = 26$$

STEP 3 Find the value of the derivative at $\left(\frac{1}{2}, 3\right)$.

$$f'\left(\frac{1}{2}\right) = 24\left(\frac{1}{2}\right)^2 + 2 = 8$$

The derivative of $f(x) = 8x^3 + 2x$ at (1, 10) is 26 and at $\left(\frac{1}{2}, 3\right)$ is 8.

EXAMPLE 5

Use the definition of a derivative to find $f'(x)$ at (3, 2) if $f(x) = \frac{6}{x}$.

STEP 1 Find the derivative $f'(x)$.

Use the definition of a derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Substitute the function into the definition.

$$= \lim_{\Delta x \to 0} \frac{\frac{6}{x + \Delta x} - \frac{6}{x}}{\Delta x}$$

Combine terms in the numerator.

$$= \lim_{\Delta x \to 0} \frac{\frac{6x - 6(x + \Delta x)}{x(x + \Delta x)}}{\Delta x}$$

Simplify.

$$= \lim_{\Delta x \to 0} \frac{-6\Delta x}{x(x + \Delta x)(\Delta x)}$$

Cancel Δx. Then find the limit.

$$= \lim_{\Delta x \to 0} \frac{-6}{x(x + \Delta x)} = -\frac{6}{x^2}$$

STEP 2 Find the derivative at (3, 2).

$$f'(3) = -\frac{6}{(3)^2} = -\frac{2}{3}$$

The derivative of $f(x) = \frac{6}{x}$ at (3, 2) is $-\frac{2}{3}$.

Instantaneous Rate of Change

Differentiation describes the rate of change of one variable with respect to another. This has many applications in everyday life, such as the rate of change of distance with respect to time, the rate of change in pressure with respect to volume, and the rate of change in cost with respect to availability.

EXAMPLE 6

The distance (in meters) along a straight path that an object moves from a starting point during time t (in seconds) is described by the function $f(t) = t^2 + 6t$. Use the definition of a derivative to find the instantaneous rate of change of the distance (in units m/s) at $t = 10$ s.

STEP 1 Find the derivative $f'(t)$.

Use the definition of a derivative.

$$f'(t) = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

Substitute the function into the definition.

$$= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^2 + 6(t + \Delta t) - (t^2 + 6t)}{\Delta t}$$

Simplify and cancel like terms. Then find the limit.

$$= \lim_{\Delta t \to 0} \frac{2t\Delta t + (\Delta t)^2 + 6\Delta t}{\Delta t} = 2t + 6$$

STEP 2 Find the value of the derivative at $t = 10$ s.

$$f'(10) = 2(10) + 6 = 26$$

The instantaneous rate of change of the distance at $t = 10$ seconds is 26 m/s.

EXAMPLE 7

The value V of a car decreases since the number of years t it was bought according to the equation $V(t) = 500t^2 - 4500t + 25{,}000$. What is the instantaneous rate of change (in dollars per year, \$/y) in the value of the car after 4 years?

STEP 1 Find the derivative $f'(t)$.

Use the definition of a derivative.

$$V'(t) = \lim_{\Delta t \to 0} \frac{V(t + \Delta t) - V(t)}{\Delta t}$$

Substitute the function into the definition.

$$= \lim_{\Delta t \to 0} \left[\frac{500(t + \Delta t)^2 - 4500(t + \Delta t) + 25{,}000 - (500t^2 - 4500t + 25{,}000)}{\Delta t} \right]$$

Simplify the numerator, and cancel like terms.

$$= \lim_{\Delta t \to 0} \frac{1000t\Delta t + 500(\Delta t)^2 - 4500\Delta t}{\Delta t}$$

Simplify. Then find the limit.

$$= \lim_{\Delta t \to 0} (1000t + 500\Delta t - 4500) = 1000t - 4500$$

STEP 2 Find the value of the derivative at $t = 4$ y.

$$V'(4) = 1000(4) - 4500 = -500$$

The instantaneous rate of change in the value of the car at $t = 4$ years is −\$500/y.

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Approximate Rate of Change

Using a Table to Find an Approximate Rate of Change

Differentiation is useful for finding the instantaneous rate of change of a function only if you know the exact form of the function. If you do not know the exact form of the function, you can still find its approximate rate of change using the difference quotient, $\frac{f(x + \Delta x) - f(x)}{\Delta x}$.

If you look back at the definition of a derivative, you see that the limit of the difference quotient is the derivative of the function. You can use the difference quotient without taking a limit to find an approximate value of a derivative.

Watch Out!

When you use the difference quotient with different methods, you should not expect to obtain the same answer. The answers will probably be different because they are approximations.

EXAMPLE 1

Find the approximate rate of change of the function $f(x)$ at $x = 2$ using the table at the right.

x	0	1	2	3	4
$f(x)$	2	5	14	29	50

METHOD 1

Use values of x to the left and right of $x = 2$. $\quad \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(3) - f(1)}{3 - 1}$

Substitute $f(3)$ and $f(1)$. Simplify the numerator. $\quad = \frac{29 - 5}{2}$

Simplify. $\quad = 12$

The approximate rate of change of $f(x)$ at $x = 2$ is 12.

METHOD 2

Use $x = 2$ and a value to the right of $x = 2$. $\quad \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(3) - f(2)}{3 - 2}$

Substitute $f(3)$ and $f(2)$. Simplify the numerator. $\quad = \frac{29 - 14}{1}$

Simplify. $\quad = 15$

The approximate rate of change of $f(x)$ at $x = 2$ is 15.

METHOD 3

Use $x = 2$ and a value to the left of $x = 2$. $\quad \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(2) - f(1)}{2 - 1}$

Substitute $f(2)$ and $f(1)$. Simplify the numerator. $\quad = \frac{14 - 5}{1}$

Simplify. $\quad = 9$

The approximate rate of change of $f(x)$ at $x = 2$ is 9.

GOT TO KNOW!

Difference Quotient

Use the difference quotient to find an approximate rate of change from a table of values.

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

EXAMPLE 2

Use the table at the right to find an approximate value of $f'(x)$ at $x = 9$.

x	0	2	4	8	10
$f(x)$	−1	15	55	207	319

Use $x = 8$ and $x = 10$ in the difference quotient.

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(10) - f(8)}{10 - 8}$$

Substitute $f(10)$ and $f(8)$. Simplify the numerator.

$$= \frac{319 - 207}{2}$$

Simplify.

$$= 56$$

The approximate value of $f'(x)$ at $x = 9$ is 56.

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EXAMPLE 3

The position x (in meters) of an object is a function of the time t (in seconds). The velocity of the object is the rate of change of its position with respect to time. Use the table of values to find the approximate velocity of the object at $t = 2.5$ s.

t(s)	1	2	3	4	5
x(m)	2	11	26	47	74

Use $t = 2$ and $t = 3$ in the difference quotient.

$$\frac{f(t + \Delta t) - f(t)}{\Delta t} = \frac{f(3) - f(2)}{3 - 2}$$

Substitute $f(3)$ and $f(2)$. Simplify the numerator.

$$= \frac{26 - 11}{1}$$

Simplify.

$$= 15$$

The approximate velocity at $t = 2.5$ s is 15 m/s.

EXAMPLE 4

The table shows the revenue r (in thousands of dollars) of a company as a function of time t (in years). Find the approximate rate of change of the revenue in the 30th year.

t(y)	1	5	10	15	30
r($1000)	100	300	245	250	439

Use $t = 15$ and $t = 30$ in the difference quotient.

$$\frac{f(t + \Delta t) - f(t)}{\Delta t} = \frac{f(30) - f(15)}{30 - 15}$$

Substitute $f(30)$ and $f(15)$. Simplify the numerator.

$$= \frac{439 - 250}{15}$$

Simplify.

$$= 13 \text{ (rounded to 2 significant figures)}$$

The approximate rate of change of the revenue in $t = 30$ is $13,000/y.

Need More HELP

For help with tangents, go to *Definition of a Derivative* on page 2158.

Using a Graph to Find an Approximate Rate of Change

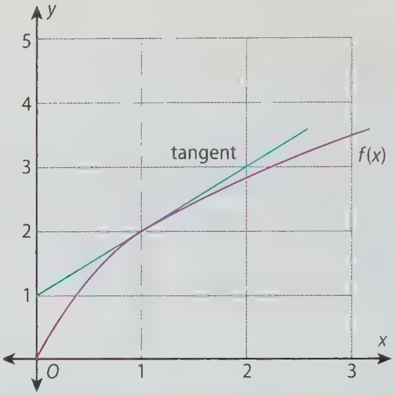
If you do not know the form of a function, but you have a graph of it, you can find the approximate rate of change at a point using the slope of the tangent.

In some cases, the tangent at a certain point is drawn on the graph, and the slope that you calculate will be the exact rate of change.

If the tangent is not drawn, you can place a ruler on the graph to estimate the tangent. The slope you determine by this method is the approximate rate of change. Since the derivative of a function is the slope of the tangent at a point, this method also provides an estimate of the derivative.

EXAMPLE 5

Use the graph at the right to find the rate of change of $f(x)$ at the point (1, 2).



STEP 1 Choose points on the tangent that have convenient values of Δy and Δx.

Use the points (2, 3) and (0, 1).

STEP 2 Calculate the slope of the line.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-1}{2-0} = 1$$

The approximate rate of change of $f(x)$ at (1, 2) is 1.

EXAMPLE 6

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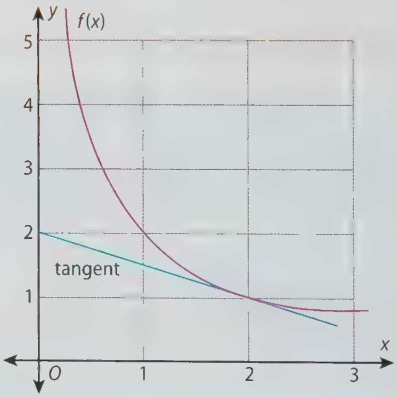
Use the graph at the right to find the derivative of $f(x)$ at the point (2, 1).



STEP 1 Choose points on the tangent that have convenient values of Δy and Δx.

Use the points (2, 1) and (0, 2).

STEP 2 Calculate the slope of the line.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{1-2}{2-0} = -\frac{1}{2}$$

The derivative of $f(x)$ at (1, 2) is $-\frac{1}{2}$.

EXAMPLE 7

Use the graph at the right to find the approximate rate of change of *f*(*x*) at the point (−1, −2).

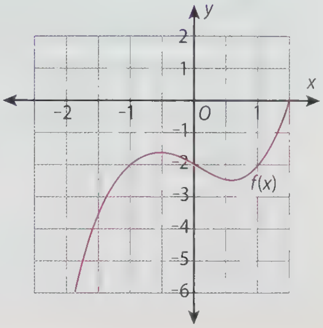
STEP 1 Use a ruler to connect two points that form a tangent at (−1, −2).

Use the points (−2, −4) and (0, 0).

STEP 2 Calculate the slope of the line.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{-4-0}{-2-0} = 2$$

The approximate rate of change of *f*(*x*) at (−1, −2) is 2.



Watch Out!

Be careful to place the ruler so that it touches the function curve at just one point.

EXAMPLE 8

Use the graph at the right to find the approximate derivative of *f*(*x*) at the point (−1, 0).

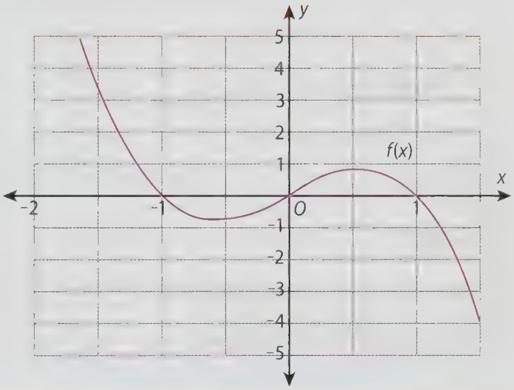
STEP 1 Use a ruler to connect two points that form a tangent at (−1, 0).

Use the points (−2, 4) and (0, −4).

STEP 2 Calculate the slope of the line.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{4-(-4)}{-2-0} = -4$$

The derivative of *f*(*x*) at (−1, 0) is −4.



EXAMPLE 9

Use the graph in Example 8 to find the approximate derivative of *f*(*x*) at the point (1, 0).

STEP 1 Use a ruler to connect two points that form a tangent at (1, 0).

Use the points $\left(\frac{1}{2}, 2\right)$ and $\left(\frac{3}{2}, -2\right)$.

STEP 2 Calculate the slope of the line.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2-(-2)}{\frac{1}{2}-\frac{3}{2}} = -4$$

The derivative of *f*(*x*) at (1, 2) is −4.

Differentiability and Continuity

Differentiability Implies Continuity

If a function is differentiable at a point *c*, then it is also continuous at that point. In order to prove that a function is continuous at a point, you can either show that it meets the three conditions for continuity at that point, or you can show that its derivative exists at that point.

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Conditions for continuity:

- $f(x)$ is defined
- $\lim\limits_{x \to c} f(x)$ exists
- $\lim\limits_{x \to c} f(x) = f(c)$

Definition of a Derivative:

$$f'(c) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

at $x = c$

EXAMPLE 1

Show that $f(x) = 3x^2 + 1$ is continuous at the point $x = 1$.

METHOD 1

Show that the function meets the conditions for continuity at $x = 1$.

Show that $f(1)$ is defined. $f(1) = 3(1)^2 + 1 = 4$

Show that $\lim\limits_{x \to 1} f(x)$ exists. $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1} (3x^2 + 1) = 3(1)^2 + 1 = 4$

Show that $\lim\limits_{x \to 1} f(x) = f(1)$ $4 = 4$

Because the function meets the three conditions for continuity, it is continuous at $x = 1$.

METHOD 2

Show that the derivative exists at $x = 1$.

STEP 1 Find the derivative.

Use the definition of a derivative. $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

Substitute $f(x) = 3x^2 + 1$. $= \lim\limits_{\Delta x \to 0} \dfrac{3(x + \Delta x)^2 + 1 - (3x^2 + 1)}{\Delta x}$

Simplify the numerator. Cancel Δx. $= \lim\limits_{\Delta x \to 0} \dfrac{6x\Delta x + 3(\Delta x)^2}{\Delta x}$

Simplify and find the limit. $= \lim\limits_{\Delta x \to 0} (6x + 3\Delta x) = 6x$

STEP 2 Find $f'(1)$.

$6(1) = 6$

Because the derivative exists at $x = 1$, the function is continuous at $x = 1$.

GOT TO KNOW!

Differentiability Implies Continuity

If a function is differentiable at a point, then the function is continuous at that point.

Continuity Does Not Imply Differentiability

Some functions are continuous but not differentiable at a point. Examples include functions that have a vertical tangent line at a point or functions that have a corner or cusp.

EXAMPLE 2

Show that $f(x) = |x|$ is continuous but not differentiable at $x = 0$.

STEP 1 Show that the function is continuous at $x = 0$.

Show that $f(0)$ is defined. $f(0) = |0| = 0$

Show that $\lim\limits_{x \to 0} f(x)$ exists by taking one-sided limits. $\lim\limits_{x \to 0^+} |0| = \lim\limits_{x \to 0^-} |0| = 0$

Show that $\lim\limits_{x \to 0} f(x) = f(0)$. $0 = 0$

Because the function meets the three conditions for continuity, it is continuous at $x = 0$.

STEP 2 Show that the function is not differentiable at $x = 0$.

Use the limit of the difference quotient. $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

Insert $f(x) = |x|$. $= \lim\limits_{\Delta x \to 0} \dfrac{|x + \Delta x| - |x|}{\Delta x}$

Evaluate for $f'(0)$ and simplify. $= \lim\limits_{\Delta x \to 0} \dfrac{|0 + \Delta x| - |0|}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{|\Delta x|}{\Delta x}$

Evaluate the left-hand limit. $\lim\limits_{\Delta x \to 0^-} \dfrac{|\Delta x|}{\Delta x} = -1$

Evaluate the right-hand limit. $\lim\limits_{\Delta x \to 0^+} \dfrac{|\Delta x|}{\Delta x} = 1$

Since the left- and right-hand limits do not agree, the derivative does not exist at $x = 0$.

Need More HELP

For more about one-sided limits, go to *Left- and Right-Hand Limits* on page 2132.

EXAMPLE 3

Show that $g(x) = \sqrt[3]{x}$ is continuous but not differentiable at $x = 0$.

STEP 1 Show that the function is continuous at $x = 0$.

Show that $g(0)$ is defined. $g(0) = \sqrt[3]{0} = 0$

Show that $\lim\limits_{x \to 0} g(x)$ exists. $\lim\limits_{x \to 0} g(0) = \lim\limits_{x \to 0} \sqrt[3]{0} = 0$

Show that $\lim\limits_{x \to 0} g(x) = g(0)$. $0 = 0$

Because the function meets the three conditions for continuity, it is continuous at $x = 0$.

STEP 2 Show that the function is not differentiable at $x = 0$.

Use the limit of the difference quotient. $g'(x) = \lim\limits_{\Delta x \to 0} \dfrac{g(x + \Delta x) - g(x)}{\Delta x}$

Insert $g(x) = \sqrt[3]{x}$. $= \lim\limits_{\Delta x \to 0} \dfrac{\sqrt[3]{x + \Delta x} - \sqrt[3]{x}}{\Delta x}$

Evaluate for $g'(0)$. $g'(0) = \lim\limits_{\Delta x \to 0} \dfrac{\sqrt[3]{0 + \Delta x} - \sqrt[3]{0}}{\Delta x}$

Simplify and find the limit. $= \lim\limits_{\Delta x \to 0} \dfrac{1}{(\Delta x)^{2/3}}$ Does Not Exist

The function is continuous but not differentiable at $x = 0$.

Need More HELP

For more about simplifying radical expressions, go to *Dividing Radical Expressions* in *Advanced Algebra*. (page 1962).

Constant, Sum, and Difference Rules

Simple Derivative Rules

Finding the derivative of a function using the limit of the difference quotient can be laborious. This method, however, reveals useful formulas for some common derivatives that make finding the derivative much simpler. For these special cases, you can use derivative rules such as the ones listed in the *Got To Know!* box below. It is useful when expressing these rules to use the derivative notation $\frac{d}{dx}f(x)$.

EXAMPLE 1

Find the derivative of $f(x) = 6$.

METHOD 1

Use the definition of a derivative.

Begin with the derivative of the function. $f'(6) = \lim\limits_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

The value of $f(x)$ is 6 everywhere. $= \lim\limits_{\Delta x \to 0} \frac{6 - 6}{\Delta x}$

Simplify. $= 0$

Therefore $f'(x) = 0$ for all values of x.

METHOD 2

Use the constant rule for derivatives. $\frac{d}{dx}(6) = 0$

This method also shows that $f'(x) = 0$ for all values of x.

Try It This Way

Graph the function $f(x) = 6$ on a graphing calculator. Notice that the slope of the constant line is always zero. This shows that the derivative of a constant function is zero.

GOT TO KNOW!

Constant, Sum, and Difference Differentiation Rules

Constant Rule: The derivative of any constant is zero.

If c is a real number, then

$\frac{d}{dx}(c) = 0.$

Constant Multiple Rule: The derivative of the product of a constant and a function is the constant times the derivative of the function.

If c is a real number and $f(x)$ is a differentiable function, then

$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x).$

Sum Rule: The derivative of sums is the sum of the derivatives.

If $f(x)$ and $g(x)$ are differentiable functions, then

$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$

Difference Rule: The derivative of a difference is the difference of the derivatives.

If $f(x)$ and $g(x)$ are differentiable functions, then

$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x).$

EXAMPLE 2

The derivative of $f(x) = 4x^2 - 9$ is $f'(x) = 8x$. Find the derivative of $7f(x)$.

Substitute $f(x)$. $\frac{d}{dx}[7f(x)] = \frac{d}{dx}[7(4x^2 - 9)]$

Use the constant multiple rule of derivatives. $= 7\frac{d}{dx}(4x^2 - 9)$

Insert the derivative of $4x^2 - 9$. Then simplify. $= 7(8x) = 56x$

The derivative of $7f(x)$ is $56x$.

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EXAMPLE 3

The derivative of $f(x) = x^6 + 4x$ is $f'(x) = 6x^5 + 4$, and the derivative of $g(x) = 2x^5 - 3x$ is $g'(x) = 10x^4 - 3$.

a. Find the derivative of $f(x) + g(x)$.

Substitute $f(x)$ and $g(x)$. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[(x^6 + 4x) + (2x^5 - 3x)]$

Use the sum rule of derivatives. $= \frac{d}{dx}(x^6 + 4x) + \frac{d}{dx}(2x^5 - 3x)$

Insert the derivatives. $= 6x^5 + 4 + 10x^4 - 3$

Simplify. $= 6x^5 + 10x^4 + 1$

The derivative of $f(x) + g(x)$ is $6x^5 + 10x^4 + 1$.

b. Find the derivative of $f(x) - g(x)$.

Substitute $f(x)$ and $g(x)$. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[(x^6 + 4x) - (2x^5 - 3x)]$

Use the difference rule of derivatives. $= \frac{d}{dx}(x^6 + 4x) - \frac{d}{dx}(2x^5 - 3x)$

Insert the derivatives. $= 6x^5 + 4 - (10x^4 - 3)$

Simplify. $= 6x^5 - 10x^4 + 7$

The derivative of $f(x) - g(x)$ is $6x^5 - 10x^4 + 7$.

EXAMPLE 4

The derivative of $f(x) = 4x^3 + 7x$ is $f'(x) = 12x^2 + 7$ and the derivative of $g(x) = 9x^2 - x$ is $g'(x) = 18x - 1$. Find the derivative of $8x^3 + 18x^2 + 12x - 18$.

Factor out 2 and rearrange to get $f(x)$ and $g(x)$. $\frac{d}{dx}[(8x^3 + 18x^2 + 12x - 18)] = \frac{d}{dx}2[(4x^3 + 7x) + (9x^2 - x) - 9]$

Use the constant multiple rule. $= 2\frac{d}{dx}[(4x^3 + 7x) + (9x^2 - x) - 9]$

Use the sum and difference rules. $= 2\left[\frac{d}{dx}(4x^3 + 7x) + \frac{d}{dx}(9x^2 - x) - \frac{d}{dx}9\right]$

Insert the derivatives. Use the constant rule. $= 2[12x^2 + 7 + 18x - 1 - 0]$

Simplify. $= 2[12x^2 + 18x + 6]$

Use the distributive property. $= 24x^2 + 36x + 12$

The derivative of $8x^3 + 18x^2 + 12x - 9$ is $24x^2 + 36x + 12$.

Power, Product, and Quotient Rules

Power Rule for Derivatives

One of the most powerful formulas for simplifying the differentiation process is the power rule for derivatives. This formula allows you to quickly take the derivative of any power without using the definition of a derivative equation.

When you take the derivative of a power function, multiply the function by the exponent, and then reduce the exponent by one. This means that the instantaneous rate of change of a quartic function is a cubic function, the instantaneous rate of change of a cubic function is a quadratic function, and so on.

EXAMPLE 1

Find: $\frac{d}{dx}(x^7)$

Use the power rule for derivatives. $\frac{d}{dx}(x^7) = 7\frac{d}{dx}(x^{7-1})$

Simplify. $= 7x^6$

EXAMPLE 2

Find: $\frac{d}{dx}\left(x^{\frac{3}{2}}\right)$

Use the power rule for derivatives. $\frac{d}{dx}\left(x^{\frac{3}{2}}\right) = \frac{3}{2}x^{\frac{3}{2}-1}$

Simplify. $= \frac{3}{2}x^{\frac{1}{2}}$

Ways to REMEMBER

Product Rule: Try the phrase "1d2 plus 2d1" to remember the formula.

Quotient Rule: Try the phrase "low-dee-high minus high-dee-low all over low low" to remember the formula.

GOT TO KNOW!

Power, Product, and Quotient Differentiation Rules

1) **Power Rule**

   If n is a real number then

   $\frac{d}{dx}(x^n) = nx^{n-1}$.

2) **Product Rule**

   If $f(x)$ and $g(x)$ are differentiable functions, then

   $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$.

3) **Quotient Rule**

   If $f(x)$ and $g(x)$ are differentiable functions where $g(x) \neq 0$ then

   $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$.

EXAMPLE 3

Find: $\frac{d}{dx}\left(\frac{1}{x}\right)$

Rewrite the fraction as a power of *x*. $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1})$

Use the power rule for derivatives. $= -1x^{-1-1}$

Simplify and rewrite. $= -x^{-2} = -\frac{1}{x^2}$

EXAMPLE 4

Find: $\frac{d}{dx}\sqrt{x}$

Rewrite the fraction as a power of *x*. $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$

Use the power rule for derivatives. $= \frac{1}{2}x^{\frac{1}{2}-1}$

Simplify. $= \frac{1}{2}x^{-\frac{1}{2}}$

EXAMPLE 5

Find: $\frac{d}{dx}(3x^3)$

Use the constant multiple rule for derivatives. $\frac{d}{dx}(3x^3) = 3\frac{d}{dx}(x^3)$

Use the power rule for derivatives. $= 3 \cdot 3x^{3-1}$

Simplify. $= 9x^2$

EXAMPLE 6

Find the derivative of $y = 3x^2 + 2x + 1$.

Use the sum rule for derivatives. $\frac{dy}{dx} = \frac{d}{dx}(3x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(1)$

Use the constant rule for derivatives. $= \frac{d}{dx}(3x^2) + \frac{d}{dx}(2x) + 0$

Use the constant multiple rule for derivatives. $= 3\frac{d}{dx}(x^2) + 2\frac{d}{dx}(x)$

Use the power rule for derivatives. $= 3(2x) + 2(1)$

Simplify. $= 6x + 2$

Notice that the derivative of the quadratic function in Example 6 is a linear function.

Need More HELP

For more about fractional exponents, see *Properties of Exponents* in *Advanced Algebra* (p. 1970).

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Product Rule for Derivatives

You can use the product rule to find the derivative of a function that involves the multiplication of two functions. For some functions, it is easier to apply the product rule before you simplify. For other functions, simplifying is easier than using the product rule.

Watch Out !

The derivative of a product is not a product of the derivatives.

EXAMPLE 7

Find the derivative of $y = 3x^{-3}(2x + 1)$.

Insert the function into the equation. $\frac{dy}{dx} = \frac{d}{dx}[3x^{-3}(2x + 1)]$

Use the product rule for derivatives. $= 3x^{-3}\frac{d}{dx}[(2x + 1)] + (2x + 1)\frac{d}{dx}(3x^{-3})$

Use the sum rule for derivatives. $= 3x^{-3}\frac{d}{dx}(2x) + 3x^{-3}\frac{d}{dx}(1) + (2x + 1)\frac{d}{dx}(3x^{-3})$

Use the constant multiple rule and the constant rule. $= 6x^{-3}\frac{d}{dx}x + 0 + 3(2x + 1)\frac{d}{dx}(x^{-3})$

Use the power rule for derivatives. $= 6x^{-3}(1) + 3(2x + 1)(-3)x^{-4}$

Simplify. $= -12x^{-3} - 9x^{-4}$

$= \frac{-12}{x^3} - \frac{9}{x^4}$

EXAMPLE 8

Find the derivative of $y = (3x^{-3})(x^{-6} - 3x)$.

METHOD 1

Simplify the function before taking the derivative.

Insert the function into the equation. $\frac{dy}{dx} = \frac{d}{dx}[3x^{-3}(x^{-6} - 3x)]$

Use the distributive property. $= \frac{d}{dx}(3x^{-9} - 9x^{-2})$

Use the difference and the constant rules. $= 3\frac{d}{dx}(x^{-9}) - 9\frac{d}{dx}(x^{-2})$

Use the power rule for derivatives. $= -27x^{-10} + 18x^{-3}$

METHOD 2

Use the product rule before simplifying.

Insert the function into the equation. $\frac{dy}{dx} = \frac{d}{dx}[3x^{-3}(x^{-6} - 3x)]$

Use the product rule for derivatives. $= (3x^{-3})\frac{d}{dx}(x^{-6} - 3x) + (x^{-6} - 3x)\frac{d}{dx}(3x^{-3})$

Use the constant multiple and difference rules. $= (3x^{-3})\frac{d}{dx}(x^{-6}) - 3(3x^{-3})\frac{d}{dx}(x) + 3(x^{-6} - 3x)\frac{d}{dx}(x^{-3})$

Use the power rule for derivatives. $= (-6)(3x^{-3})x^{-7} - 3(3x^{-3})(1) + (-3)(3)(x^{-6} - 3x)x^{-4}$

Simplify. $= -27x^{-10} + 18x^{-3}$

$= \frac{-27}{x^{10}} + \frac{18}{x^3}$

Quotient Rule for Derivatives

If you apply the quotient rule, you might find that the procedure gets complicated. Like the product rule, however, the quotient rule can greatly simplify the differentiation process for some functions.

EXAMPLE 9

Use the quotient rule to find the derivative of $\frac{x}{\sqrt{x}}$.

Insert the function. Rewrite the radical as an exponent.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{x}{x^{\frac{1}{2}}}\right)$$

Use the quotient rule for derivatives.

$$= \frac{x^{\frac{1}{2}}\frac{d}{dx}(x) - x\frac{d}{dx}\left(x^{\frac{1}{2}}\right)}{\left(x^{\frac{1}{2}}\right)^2}$$

Use the power rule for derivatives.

$$= \frac{x^{\frac{1}{2}}(1) - \frac{1}{2}x\left(x^{-\frac{1}{2}}\right)}{\left(x^{\frac{1}{2}}\right)^2}$$

Simplify.

$$= \frac{1}{2}x^{\frac{1}{2}}$$

$$= \frac{1}{2x^{\frac{1}{2}}}$$

EXAMPLE 10

Find the derivative of $y = \frac{12x^3}{(x^4 + 2)}$.

Insert the function in the equation.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{12x^3}{(x^4 + 2)}\right]$$

Use the quotient rule for derivatives.

$$= \frac{(x^4 + 2)\frac{d}{dx}(12x^3) - 12x^3\frac{d}{dx}(x^4 + 2)}{(x^4 + 2)^2}$$

Use the sum rule for derivatives.

$$= \frac{(x^4 + 2)\frac{d}{dx}(12x^3) - 12x^3\frac{d}{dx}(x^4) - 12x^3\frac{d}{dx}(2)}{(x^4 + 2)^2}$$

Use the distributive property.

$$= \frac{x^4\frac{d}{dx}(12x^3) + 2\frac{d}{dx}(12x^3) - 12x^3\frac{d}{dx}(x^4) - 12x^3\frac{d}{dx}(2)}{(x^4 + 2)^2}$$

Use the constant multiple rule.

$$= \frac{12x^4\frac{d}{dx}(x^3) + 24\frac{d}{dx}(x^3) - 12x^3\frac{d}{dx}(x^4) - 12x^3\frac{d}{dx}(2)}{(x^4 + 2)^2}$$

Use the power rule and constant rule.

$$= \frac{36x^4x^2 + 72x^2 - 48x^3x^3 - 0}{(x^4 + 2)^2}$$

Simplify the numerator.

$$= \frac{-12x^6 + 72x^2}{(x^4 + 2)^2}$$

Expand the denominator.

$$= \frac{-12x^6 + 72x^2}{x^8 + 4x^4 + 4}$$

SEARCH

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Chain Rule

Derivatives of Composite Functions

A composite is a function that is composed of another function: $f(g(x))$. The rules for derivatives presented so far will not work for many composite functions. To differentiate composite functions, first identify the outer and inner functions.

outer function
$f(g(x))$
inner function

The Chain Rule describes how to find the derivative of a composite function. Let $u = g(x)$ be the inner function. Then the derivative of the composite function is the derivative of the outer function with respect to u multiplied by the derivative of the inner function with respect to x.

EXAMPLE 1

Find the derivative of $y = (x + 4)^2$.

METHOD 1

Expand the function and then find the derivative.

Use the FOIL method to expand the function. $y = (x + 4)^2 = x^2 + 8x + 16$

Use the Power Rule to find the derivative. $\frac{dy}{dx} = 2x + 8$

METHOD 2

Use the Chain Rule to find the derivative.

STEP 1 Determine the layers of functions. $y = u^2$ $u = x + 4$

STEP 2 Differentiate the outer and inner functions.

Differentiate the outer function. $\frac{dy}{du} = \frac{d}{du}(u^2) = 2u$

Differentiate the inner function. $\frac{du}{dx} = \frac{d}{dx}(x + 4) = 1$

STEP 3 Apply the Chain Rule. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Substitute the derivatives. $= (2u)(1) = 2u$

Substitute for u, and simplify. $= 2(x + 4) = 2x + 8$

Using both methods, the derivative of $y = (x + 4)^2$ is $2x + 8$.

Ways to REMEMBER

When taking the derivative of a composite function, think about eating a chocolate covered peanut. The outer function is the chocolate and the inner function is the peanut. The derivative of this delicious treat is the derivative of the outer chocolate times the derivative of the inner peanut.

GOT TO KNOW!

Chain Rule

For a composite function $y = f(g(x))$, let $y = f(u)$, and $u = g(x)$. If $f(u)$ is differentiable at u, and $g(x)$ is differentiable at x, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \text{ or equivalently, } \frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).$$

EXAMPLE 2

Use the Chain Rule to find the instantaneous rate of change of $y = \sqrt{x^2 + 2x + 3}$.

STEP 1 Determine the layers of functions.

$$y = u^{\frac{1}{2}} \qquad u = x^2 + 2x + 3$$

STEP 2 Differentiate the outer and inner functions.

Differentiate the outer function. $\frac{dy}{du} = \frac{d}{du}(u^{\frac{1}{2}}) = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$

Differentiate the inner function. $\frac{du}{dx} = \frac{d}{dx}(x^2 + 2x + 3) = 2x + 2$

STEP 3 Apply the Chain Rule.

Insert the derivatives. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2\sqrt{u}}(2x + 2)$

Substitute for *u*, and simplify. $= \left(\frac{1}{2\sqrt{x^2 + 2x + 3}}\right)(2x + 2) = \frac{x + 1}{\sqrt{x^2 + 2x + 3}}$

Watch Out !

Remember that a derivative is always taken relative to some variable. When applying the Chain Rule, the inner function is treated like a variable. The derivative of the outer function is then taken relative to the inner function, not relative to *x*.

The following example uses the Chain Rule to find a tangent line. Recall that the slope at a point is the derivative. You can therefore use the point-slope equation to find the tangent line equation. Notice the use of the notation $\left.\frac{dy}{dx}\right|_c$, which means the derivative at a point *c*.

EXAMPLE 3

Write the equation of the tangent line to $f(x) = \frac{1}{(3x^4 - 5)^2}$ at $x = 0$.

STEP 1 Determine the layers of functions.

$$y = u^{-2} \qquad u = 3x^4 - 5$$

STEP 2 Differentiate the outer and inner functions.

Differentiate the outer function. $\frac{dy}{du} = \frac{d}{du}(u^{-2}) = -2u^{-3} = \frac{-2}{u^3}$

Differentiate the inner function. $\frac{du}{dx} = \frac{d}{dx}(3x^4 - 5) = 12x^3$

STEP 3 Apply the Chain Rule.

Insert the derivatives. Then substitute *u*. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{-2}{u^3}(12x^3) = \frac{-24x^3}{(3x^4 - 5)^3}$

STEP 4 Evaluate the derivative at $x = 0$. $\left.\frac{dy}{dx}\right|_0 = \frac{-24(0)^3}{(3(0)^4 - 5)^3} = 0$

STEP 5 Find the equation of the tangent line.

Use $x_1 = 0$, $y_1 = f(0) = \frac{1}{(-5)^2}$, and $m = \left.\frac{dy}{dx}\right|_0$ in the point-slope equation. $y - \frac{1}{(-5)^2} = 0(x - 0)$

Simplify. $y = \frac{1}{(-5)^2} = \frac{1}{25}$

The equation of the tangent line at $x = 0$ is $y = \frac{1}{25}$.

SEARCH

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Derivatives of Multiple-Function Composites

You can extend the Chain Rule for a composite of multiple functions. The derivative of the composite function is the product of the derivative of each function with respect to the next inner function.

EXAMPLE 4

Find the derivative of $y = \left(3 - \frac{1}{x}\right)^{-1}$.

STEP 1 Determine the layers of functions.

$$y = u^{-1} \qquad u = 3 - v \qquad v = \frac{1}{x}$$

STEP 2 Differentiate each function.

Differentiate y with respect to u. $\quad \frac{dy}{du} = \frac{d}{du}(u^{-1}) = -u^{-2}$

Differentiate u with respect to v. $\quad \frac{du}{dv} = \frac{d}{dv}(3 - v) = -1$

Differentiate v with respect to x. $\quad \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -x^{-2}$

STEP 3 Apply the Chain Rule.

Insert the derivatives.

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dv}\,\frac{dv}{dx}$$

$$= (-u^{-2})(-1)(-x^{-2})$$

Substitute u and v.

$$= \left(\frac{-1}{\left(3 - \frac{1}{x}\right)^2}\right)(-1)\left(\frac{-1}{x^2}\right)$$

Simplify.

$$= \frac{-1}{x^2\left(3 - \frac{1}{x}\right)^2}$$

Need More HELP

For more about composite functions, go to *Composition of Functions* in *Advanced Algebra* (p. 1502).

GOT TO KNOW!

Chain Rule for a Three-Function Composite

For a composite function $y = f(g(h(x)))$, let $y = f(u)$, and $u = g(v)$, and $v = h(x)$. If $f(u)$ is differentiable at u, $g(v)$ is differentiable at v, and $h(x)$ is differentiable at x, then

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dv}\,\frac{dv}{dx},$$

or equivalently, $\frac{d}{dx}[f(g(h(x)))] = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$.

Using Multiple Derivative Rules

The following examples show how to take the derivative of functions using more than one derivative technique.

EXAMPLE 5

Find the derivative of $f(x) = (x^2 + 12)\left(\sqrt{2x^2 - 1}\right)$.

Use the Product Rule.

$$f'(x) = (x^2 + 12)\frac{d}{dx}\left(\sqrt{2x^2 - 1}\right) + \left(\sqrt{2x^2 - 1}\right)\frac{d}{dx}(x^2 + 12)$$

Use the Chain Rule and the Power Rule.

$$= (x^2 + 12)\left[\frac{1}{2}(2x^2 - 1)^{-\frac{1}{2}}(4x)\right] + \left(\sqrt{2x^2 - 1}\right)(2x)$$

Simplify in radical form.

$$= \frac{2x(x^2 + 12)}{\sqrt{2x^2 - 1}} + 2x\sqrt{2x^2 - 1}$$

Write with a common denominator.

$$= \frac{2x(x^2 + 12)}{\sqrt{2x^2 - 1}} + \frac{2x(2x^2 - 1)}{\sqrt{2x^2 - 1}}$$

Combine fractions.

$$= \frac{2x(x^2 + 12) + 2x(2x^2 - 1)}{\sqrt{2x^2 - 1}}$$

Simplify.

$$= \frac{6x^3 + 22x}{\sqrt{2x^2 - 1}}$$

EXAMPLE 6

Find the derivative of $f(x) = \frac{x}{(x^4 - 1)^3}$.

Use the Quotient Rule.

$$f'(x) = \frac{(x^4 - 1)^3\frac{d}{dx}(x) - (x)\frac{d}{dx}(x^4 - 1)^3}{[(x^4 - 1)^3]^2}$$

Use the Power Rule and the Chain Rule.

$$= \frac{(x^4 - 1)^3(1) - (x)[3(x^4 - 1)^2(4x^3)]}{[(x^4 - 1)^3]^2}$$

Simplify.

$$= \frac{(x^4 - 1)^3 - (12x^4)(x^4 - 1)^2}{(x^4 - 1)^6}$$

Factor out the common factor.

$$= \frac{(x^4 - 1)^2[(x^4 - 1) - (12x^4)]}{(x^4 - 1)^6}$$

Cancel the common factor.

$$= \frac{(x^4 - 1) - (12x^4)}{(x^4 - 1)^4}$$

Simplify.

$$= \frac{-11x^4 - 1}{(x^4 - 1)^4}$$

Need More HELP

The previous examples demonstrate how to apply the Chain Rule by explicitly showing each step. Once you are familiar with the Chain Rule, you can skip most of the steps and simply write down the derivative, as shown on the examples on this page.

SEARCH

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A

absolute maximum (p. 2202) The highest point on a closed interval.

absolute minimum (p. 2202) The lowest point on a closed interval.

absolute value (p. 1387) The distance of a number from zero on a number line.

absolute value of a complex number (p. 1114) The absolute value of a complex number $a + bi$ is $\sqrt{a^2 + b^2}$.

acre (p. 482) A unit of area equal to 43,560 square feet.

acute angle (pp. 140, 624, 894) An angle with measure greater than 0° and less than 90°.

acute triangle (p. 156) A triangle with three acute angles.

add or subtract from a known fact (p. 70) A way to find the product of two numbers.

addends (p. 82) The numbers that are added to form a sum.

addition (pp. 56, 82) Mathematical operation in which the values of two or more numbers are combined.

addition method (p. 1572) A method of solving a system of equations that involves adding the equations to eliminate a variable.

additive identity (p. 1414) The number zero; the sum of zero and any number is that number.

additive inverse (p. 1415) The opposite of a given number.

adjacent angles (pp. 146, 625) Coplanar angles that share only an endpoint and one side.

adjacent arcs (p. 728) Arcs of the same circle that share exactly one endpoint.

algebraic expression (p. 1422) A mathematical phrase that contains one or more numbers, one or more variables, and one or more arithmetic operations.

algebraic inequality (p. 1492) A mathematical phrase that contains an inequality with a variable.

alternate exterior angles (p. 150) Two non-adjacent exterior angles on opposite sides of a transversal.

alternate interior angles (pp. 150, 630) Two non-adjacent interior angles on opposite sides of a transversal.

altitude of a triangle (pp. 537, 872) The perpendicular distance from a vertex to the line containing the side opposite that vertex.

amplitude (p. 952) The absolute value of half the difference between the maximum and minimum values of a trigonometric function.

angle (pp. 136, 894) A figure formed by two rays with a common endpoint.

angle bisector (p. 868) A ray that divides an angle into two adjacent angles that are congruent.

angle of depression (p. 924) The angle between a horizontal line and the line of sight from an observer to an object at a lower level.

angle of elevation (p. 924) The angle between a horizontal line and the line of sight from an observer to an object at a higher level.

angle of rotation See *rotation*.

angular velocity (p. 942) The number of degrees (radians, or revolutions) per unit time through which a point travels along a circular path.

apothem (p. 690) The distance from the center to a side of a regular polygon.

arc (p. 728) Part of a circle between two points on a circle.

arc of a chord (p. 736) The minor arc cut off by a chord of a circle.

area (pp. 482, 504, 528) The size of the surface of a flat figure or shape.

arithmetic pattern (p. 438) A pattern that grows when the same number is added to each subsequent term.

arithmetic sequence (p. 1859) A sequence in which consecutive terms differ by a constant amount *d*.

arithmetic series (p. 1862) The sum of the terms of an arithmetic sequence.

array (pp. 69, 75) An arrangement of objects in rows and columns where each row has the same number of objects.

asymptote (pp. 968, 1932, 1996, 2020) A line that a graph gets closer and closer to, but never touches or crosses.

average value of a function (p. 2270) If f is a continuous function on $[a, b]$, then the average value $f(c)$ is $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$.

axis of a cone (p. 713) The line segment whose endpoints are the vertex of the cone and the center of the base.

axis of revolution (p. 2278) The line about which a two-dimensional figure revolves that results in a three-dimensional figure, called a *solid of revolution.*

axis of symmetry (pp. 1688, 1910) The line that can be drawn through a parabola so that one side of the parabola is a reflection of the other. It is perpendicular to the directrix and passes through the focus.

B

bar graph (pp. 188, 1270) A graph that uses horizontal or vertical bars to display data in categories.

base angles of a trapezoid (p. 656) Each pair of angles that lie on the same base of a trapezoid.

base angles of an isosceles triangle (p. 784) The two angles adjacent to the base of an isosceles triangle.

base design (p. 706) A drawing that shows the base of a figure made of cubes and the number of cubes in each position.

base of a power (pp. 294, 1396) A number that is used as a factor a given number of times.

base of an isosceles triangle (p. 784) The side that is not a leg, or any side if the triangle is equilateral.

base(s) of a geometric figure See *cone*, *cylinder*, *prism*, *pyramid*, *trapezoid.*

benchmark (p. 316) A reference number such as $0, \frac{1}{2}$, and 1 to which other numbers are compared.

biased sample (p. 1222) A sample that overrepresents or underrepresents some part of the population.

biconditional statement (p. 616) A statement of the form *p if and only if q. p* is true if and only if *q* is true.

binomial (pp. 1348, 1600) A polynomial that has two terms.

binomial coefficient (p. 1349) The formula for n items taken k at a time, ${}_nC_k = \frac{n!}{k!(n-k)!}$.

binomial experiment (p. 1344) An experiment that has exactly two possible outcomes, one of which is called a success.

Symbols

Foundations of Mathematics

$>$	is greater than	46
$<$	is less than	46
$=$	is equal to	46
$^\circ$	degrees of an angle	50
$+$	addition	56
$-$	subtraction	62
$\times$	multiplication	68
$\div$	division	74
$\overline{)}$	division	76
10^2	exponent; power of ten	94
$3 \cdot 4; 3(4)$	multiplication	96
$\parallel$	is parallel to	129
$\perp$	is perpendicular to	130
$\overline{AB}$	line segment *AB*	132
AB	length of $\overline{AB}$	132
$\overleftrightarrow{AB}$	line *AB*	132
$\overrightarrow{AB}$	ray *AB*	133
$\angle A$	angle *A*	136
$^\circ$	degrees	137
⌐	right angle symbol	156
$\vert$	tally mark	182
%	percent	196

Numbers and Operations

$.$	decimal point	216
$>$	greater than	222
$<$	less than	222
$=$	equal to	222
R	real numbers	268
Q	rational numbers	268
Z	integers numbers	268
W	whole numbers	268
N	natural numbers	268
$\sqrt{4}$	square root	268
0.3	repeating decimal; 0.33 . . .	270
3^4	exponent	294
$\frac{1}{a}$	reciprocal	346
$a : b, \frac{a}{b}$	ratio	364
%	percent	380
. . .	and so on	434

Measurement

$^\circ F$	degrees Fahrenheit	496
$^\circ C$	degrees Celsius	518
π	pi; $\frac{22}{7}$, about 3.14	544
$\approx$	is approximately equal to	544

Geometry

$\rightarrow$	maps into	612, 834
$\sim p$	not *p*	614
$\leftrightarrow$	if and only if	616
$\cong$	is congruent to	621
$\angle$	angle	621
$^\circ$	degrees	624
$m\angle C$	measure of angle *C*	629
$\parallel$	is parallel to	630
$\overleftrightarrow{AB}$	line *AB*	630
$\perp$	is perpendicular to	633
$\overline{AB}; AB$	line segment *AB*; length of $\overline{AB}$	639
$\overset{\frown}{AB}$	arc *AB*	728
$\odot A$	circle *A*	729
$\triangle ABC$	triangle *ABC*	734
$\sim$	is similar to	790
A'	image of *A*; *A* prime	834

Trigonometry

$'$ $''$	minutes seconds	896
DMS	degrees/minutes/seconds	896
$\sin A$	sine of $\angle A$	902
$\cos A$	cosine of $\angle A$	902
$\tan A$	tangent of $\angle A$	902
θ	theta (angle measure)	903
$\csc A$	cosecant of $\angle A$	904
$\sec A$	secant of $\angle A$	904
$\cot A$	cotangent of $\angle A$	904
$\sin^{-1} x$	inverse sine	915
$\cos^{-1} x$	inverse cosine	915
$\tan^{-1} x$	inverse tangent	915
$\overrightarrow{AB}$ or $\mathbf{v}$	vector	1066
$\Vert \mathbf{a} \Vert$	magnitude of a vector	1066
$\langle x, y \rangle$	ordered pair vector	1070
$\mathbf{u} \cdot \mathbf{v}$	dot product of two vectors	1082
(r, θ)	polar coordinates	1093

Statistics and Probability

P(event)	probability of the event	1232
$n!$	factorial	1245
${}_nP_r$	permutations	1246
${}_nC_r$	combinations	1249
|	tally mark	1260
2 | 6	stem-and-leaf plot key	1276
$\in$	element of a set	1290
$\subset$	subset	1290
$\cap$	intersection of a set	1291
$\cup$	union of a set	1291
$\sim$	complement of a set	1291
$\bar{x}$	mean of a set	1324
Σ	sigma: summation	1324
σ	lowercase sigma	1324
μ	mu: mean	1340
$\ddot{p}$	sample proportion	1350
z_c	*z*-score	1350
$P(B \mid A)$	probability of event *B* given event *A*	1360

Algebra

$\lvert a \rvert$	absolute value of *a*	1387
$-x$	opposite of *x*	1387
a^b	$a =$ base; $b =$ exponent	1394
$\sqrt{a}$	principal square root	1400
$\pm$	plus or minus	1400
$\{1, 2, \ldots\}$	set notation	1402
( ), [ ]	for grouping	1419
x	variable or input	1422
x_1, x_2, y_1, y_2	specific values of a variable	1429
(x, y)	ordered pair	1436
y	output variable	1437
$f(x)$	"*f* of *x*", function value at *x*	1450
$=$	is equal to	1458
$\neq$	is not equal to	1458
$\overset{?}{=}$	does it equal?	1459
$<$	is less than	1492
$>$	is greater than	1492
$\leq$	is less than or equal to	1492
$\geq$	is greater than or equal to	1492
m	slope	1542
b	*y*-intercept	1542
i	imaginary number, $\sqrt{-1}$	1641

Advanced Algebra

(x, y, z)	ordered triple	1754
$m \times n$	matrix, *m* rows, *n* columns	1772
$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix	1772
$\det(M)$	determinant	1794
$A^{\ 1}$	inverse of matrix *a*	1798
a_n	general term of sequence	1862
S_n	partial sum of sequence	1863
$\sum\limits_{n=1}^{3}$	summation	1864
i	imaginary unit, $\sqrt{-1}$	1894
$a + bi$	complex number	1896
e	eccentricity	1940
$\sqrt[n]{a}$	principal *n*th root	1954
a^n	*n*th power of *a*	1966
a^{-n}	$\frac{1}{a^n}$	1966
$a^{\frac{1}{n}}$	$\sqrt[n]{a}$	1967
$\log b$	common logarithm, base 10	1974
$\ln x$	natural logarithm, $\log_e x$	1975, 2004
e	base for natural log; ≈ 2.718	1975, 2004
$\log_b x$	logarithm of *x*, base *b*	2006
$(a, b], [a, b)$	interval notation	2014
$(f \circ g)(x)$	composition, $f(g(x))$	2063
$f^{-1}(x)$	inverse of a function	2090

Calculus

$\lim\limits_{x \to c} f(x)$	limit	2110
δ, ε	Greek delta, Greek epsilon	2112
$[x]$	greatest integer function	2132
DNE	does not exist	2132
$\infty, -\infty$	infinity, negative infinity	2136
$\therefore$	therefore	2146
$[0, \pi)$	half open interval	2151
Δx	change in *x*	2158
$f'(x)$	derivative of *f*	2159
$\left.\frac{dy}{dx}\right\rvert_c$	derivative at point *c*	2177
$f''(x)$	second derivative of *f*	2184
$f^{(n)}(x)$	*n*th derivative of *f*	2184
$\int f(x)dx$	integral	2234
$\int_a^b f(x)dx$	definite interval	2246
$F(x)\rvert_a^b$	endpoints of an interval	2246

Basic Addition Facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Basic Subtraction Facts

−	0	1	2	3	4	5	6	7	8	9	10
0	0										
1	1	0									
2	2	1	0								
3	3	2	1	0							
4	4	3	2	1	0						
5	5	4	3	2	1	0					
6	6	5	4	3	2	1	0				
7	7	6	5	4	3	2	1	0			
8	8	7	6	5	4	3	2	1	0		
9	9	8	7	6	5	4	3	2	1	0	
10	10	9	8	7	6	5	4	3	2	1	0

Basic Addition Facts

0 + 0 = 0	1 + 0 = 1	2 + 0 = 2	3 + 0 = 3	4 + 0 = 4	5 + 0 = 5	6 + 0 = 6	7 + 0 = 7	8 + 0 = 8	9 + 0 = 0	10 + 0 = 10
0 + 1 = 1	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10	10 + 1 = 11
0 + 2 = 2	1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7	6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11	10 + 2 = 12
0 + 3 = 3	1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8	6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12	10 + 3 = 13
0 + 4 = 4	1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9	6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13	10 + 4 = 14
0 + 5 = 5	1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10	6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14	10 + 5 = 15
0 + 6 = 6	1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11	6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15	10 + 6 = 16
0 + 7 = 7	1 + 7 = 8	2 + 7 = 9	3 + 7 = 10	4 + 7 = 11	5 + 7 = 12	6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16	10 + 7 = 17
0 + 8 = 8	1 + 8 = 9	2 + 8 = 10	3 + 8 = 11	4 + 8 = 12	5 + 8 = 13	6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17	10 + 8 = 18
0 + 9 = 9	1 + 9 = 10	2 + 9 = 11	3 + 9 = 12	4 + 9 = 13	5 + 9 = 14	6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18	10 + 9 = 19
0 + 10 = 10	1 + 10 = 11	2 + 10 = 12	3 + 10 = 13	4 + 10 = 14	5 + 10 = 15	6 + 10 = 16	7 + 10 = 17	8 + 10 = 18	9 + 10 = 19	10 + 10 = 20

Basic Subtraction Facts

0 − 0 = 0										
1 − 0 = 1	1 − 1 = 0									
2 − 0 = 2	2 − 1 = 1	2 − 2 = 0								
3 − 0 = 3	3 − 1 = 2	3 − 2 = 1	3 − 3 = 0							
4 − 0 = 4	4 − 1 = 3	4 − 2 = 2	4 − 3 = 1	4 − 4 = 0						
5 − 0 = 5	5 − 1 = 4	5 − 2 = 3	5 − 3 = 2	5 − 4 = 1	5 − 5 = 0					
6 − 0 = 6	6 − 1 = 5	6 − 2 = 4	6 − 3 = 3	6 − 4 = 2	6 − 5 = 1	6 − 6 = 0				
7 − 0 = 7	7 − 1 = 6	7 − 2 = 5	7 − 3 = 4	7 − 4 = 3	7 − 5 = 2	7 − 6 = 1	7 − 7 = 0			
8 − 0 = 8	8 − 1 = 7	8 − 2 = 6	8 − 3 = 5	8 − 4 = 4	8 − 5 = 3	8 − 6 = 2	8 − 7 = 1	8 − 8 = 0		
9 − 0 = 9	9 − 1 = 8	9 − 2 = 7	9 − 3 = 6	9 − 4 = 5	9 − 5 = 4	9 − 6 = 3	9 − 7 = 2	9 − 8 = 1	9 − 9 = 0	
10 − 0 = 10	10 − 1 = 9	10 − 2 = 8	10 − 3 = 7	10 − 4 = 6	10 − 5 = 5	10 − 6 = 4	10 − 7 = 3	10 − 8 = 2	10 − 9 = 1	10 − 10 = 0

Basic Multiplication Facts

×	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Basic Division Facts

÷	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

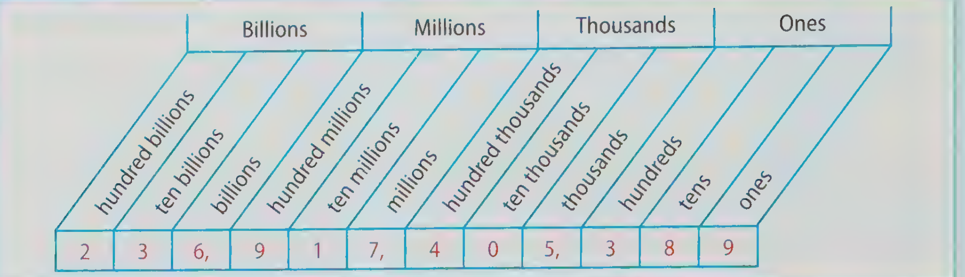
Basic Multiplication Facts

zeros	0 × 0 = 0	1 × 0 = 0	2 × 0 = 0	3 × 0 = 0	4 × 0 = 0	5 × 0 = 0	6 × 0 = 0	7 × 0 = 0	8 × 0 = 0	9 × 0 = 0	10 × 0 = 0
ones	0 × 1 = 0	1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9	10 × 1 = 10
twos	0 × 2 = 0	1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18	10 × 2 = 20
threes	0 × 3 = 0	1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27	10 × 3 = 30
fours	0 × 4 = 0	1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36	10 × 4 = 40
fives	0 × 5 = 0	1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45	10 × 5 = 50
sixes	0 × 6 = 0	1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54	10 × 6 = 60
sevens	0 × 7 = 0	1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63	10 × 7 = 70
eights	0 × 8 = 0	1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72	10 × 8 = 80
nines	0 × 9 = 0	1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81	10 × 9 = 90
tens	0 × 10 = 0	1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80	9 × 10 = 90	10 × 10 = 100

Basic Division Facts

ones	0 ÷ 1 = 0	1 ÷ 1 = 1	2 ÷ 1 = 2	3 ÷ 1 = 3	4 ÷ 1 = 4	5 ÷ 1 = 5	6 ÷ 1 = 6	7 ÷ 1 = 7	8 ÷ 1 = 8	9 ÷ 1 = 9	10 ÷ 1 = 10
twos	0 ÷ 2 = 0	2 ÷ 2 = 1	4 ÷ 2 = 2	6 ÷ 2 = 3	8 ÷ 2 = 4	10 ÷ 2 = 5	12 ÷ 2 = 6	14 ÷ 2 = 7	16 ÷ 2 = 8	18 ÷ 2 = 9	20 ÷ 2 = 10
threes	0 ÷ 3 = 0	3 ÷ 3 = 1	6 ÷ 3 = 2	9 ÷ 3 = 3	12 ÷ 3 = 4	15 ÷ 3 = 5	18 ÷ 3 = 6	21 ÷ 3 = 7	24 ÷ 3 = 8	27 ÷ 3 = 9	30 ÷ 3 = 10
fours	0 ÷ 4 = 0	4 ÷ 4 = 1	8 ÷ 4 = 2	12 ÷ 4 = 3	16 ÷ 4 = 4	20 ÷ 4 = 5	24 ÷ 4 = 6	28 ÷ 4 = 7	32 ÷ 4 = 8	36 ÷ 4 = 9	40 ÷ 4 = 10
fives	0 ÷ 5 = 0	5 ÷ 5 = 1	10 ÷ 5 = 2	15 ÷ 5 = 3	20 ÷ 5 = 4	25 ÷ 5 = 5	30 ÷ 5 = 6	35 ÷ 5 = 7	40 ÷ 5 = 8	45 ÷ 5 = 9	50 ÷ 5 = 10
sixes	0 ÷ 6 = 0	6 ÷ 6 = 1	12 ÷ 6 = 2	18 ÷ 6 = 3	24 ÷ 6 = 4	30 ÷ 6 = 5	36 ÷ 6 = 6	42 ÷ 6 = 7	48 ÷ 6 = 8	54 ÷ 6 = 9	60 ÷ 6 = 10
sevens	0 ÷ 7 = 0	7 ÷ 7 = 1	14 ÷ 7 = 2	21 ÷ 7 = 3	28 ÷ 7 = 4	35 ÷ 7 = 5	42 ÷ 7 = 6	49 ÷ 7 = 7	56 ÷ 7 = 8	63 ÷ 7 = 9	70 ÷ 7 = 10
eights	0 ÷ 8 = 0	8 ÷ 8 = 1	16 ÷ 8 = 2	24 ÷ 8 = 3	32 ÷ 8 = 4	40 ÷ 8 = 5	48 ÷ 8 = 6	56 ÷ 8 = 7	64 ÷ 8 = 8	72 ÷ 8 = 9	80 ÷ 8 = 10
nines	0 ÷ 9 = 0	9 ÷ 9 = 1	18 ÷ 9 = 2	27 ÷ 9 = 3	36 ÷ 9 = 4	45 ÷ 9 = 5	54 ÷ 9 = 6	63 ÷ 9 = 7	72 ÷ 9 = 8	81 ÷ 9 = 9	90 ÷ 9 = 10
tens	0 ÷ 10 = 0	10 ÷ 10 = 1	20 ÷ 10 = 2	30 ÷ 10 = 3	40 ÷ 10 = 4	50 ÷ 10 = 5	60 ÷ 10 = 6	70 ÷ 10 = 7	80 ÷ 10 = 8	90 ÷ 10 = 9	100 ÷ 10 = 10

Place Value


Billions
Millions
Thousands
Ones
hundred billions
ten billions
billions
hundred millions
ten millions
millions
hundred thousands
ten thousands
thousands
hundreds
tens
ones
2
3
6,
9
1
7,
4
0
5,
3
8
9

Do Your BEST!

Forms of a Number

Standard form	236,917,405,389
Expanded form	200,000,000,000 + 30,000,000,000 + 6,000,000,000 + 900,000,000 + 10,000,000 + 7,000,000 + 400,000 + 5,000 + 300 + 80 + 9
Word form	two hundred thirty-six billion, nine hundred seventeen million, four hundred five thousand, three hundred eighty-nine
Short word form	236 billion, 917 million, 405 thousand, 389

Hints on Memorizing Facts

1. Focus on the ways to memorize facts that are listed on the next page.
2. Memorize the facts by saying the facts aloud.
   - For 3 + 8, say "Three plus eight equals eleven."
   - For 8 − 3, say "Eight minus three equals five."
   - For 8 × 7, say "Eight times seven equals fifty-six."
   - For 56 ÷ 8, say "Fifty-six divided by eight equals seven."
3. Memorize a few facts at a time. Work on 2 or 3 facts per day. Start with the easier facts.
4. Practice saying the facts in reverse order, so you don't have to memorize so many facts.

   $8 + 5 = 13, 5 + 8 = 13 \qquad 9 - 4 = 5, 9 - 5 = 4$
5. Memorize one set of times tables at a time. Work on the set until you have mastered it. Then move to another set.
6. Use the relationship of multiplication to division. To remember $10 \div 2 = 5$, think $5 \cdot 2 = 10$.
7. Play fact games.
   - Toss two dice and add the dots that come up. Then have a friend toss two dice and add the dots. Then subtract the smaller sum from the larger sum.
   - Play multiplication facts bingo with some friends.
   - Play "Beat the Clock."
8. Make flash cards and use them on your own or with friends and family. Time how long it takes you to say a set of facts. Try this again the next day and see if you can beat your previous time.
9. Every day, review the facts you know or usually know. Then work on two new facts and play games to practice them.
10. Use a calculator. Enter the fact. Before you press ENTER, say the answer. Then press ENTER to see if you are correct.

Go For It!

Addition Facts: Memorize Fewer Than 121 Facts

There are 121 basic facts, but here are some ways to cut down the task.

- 0 plus any number is that number.
- 1 plus any number is the next number when you count.
- 2 plus any even number is the next even number.
- 2 plus any odd number is the next odd number.
- 10 plus any one-digit number is a teen number with the 1 in the tens place and the one-digit number in the ones place.
- If you know $2 + 3$, you know $3 + 2$. This takes care of half of the 121 facts.
- Once you memorize the doubles, the near doubles are easy to remember.
- Any number plus 9 is one less than that number plus ten.

Subtraction Facts: Memorize Fewer Than 121 Facts

There are 121 basic facts, but here are some ways to cut down the task.

- Any number minus 0 is itself.
- Any number minus itself is 0.
- Any number minus 1 is the previous counting number.
- Any even number minus 2 is the previous even number.
- Any odd number minus 2 is the previous odd number.
- Any teen number minus 10 is the digit in the ones place of the teen number.
- If you know $8 - 3 = 5$, you also know $8 - 5 = 3$. This takes care of half of the subtraction facts.
- Nine minus any number is one less than 10 minus the same number.

Multiplication Facts: Memorize Fewer Than 121 Facts

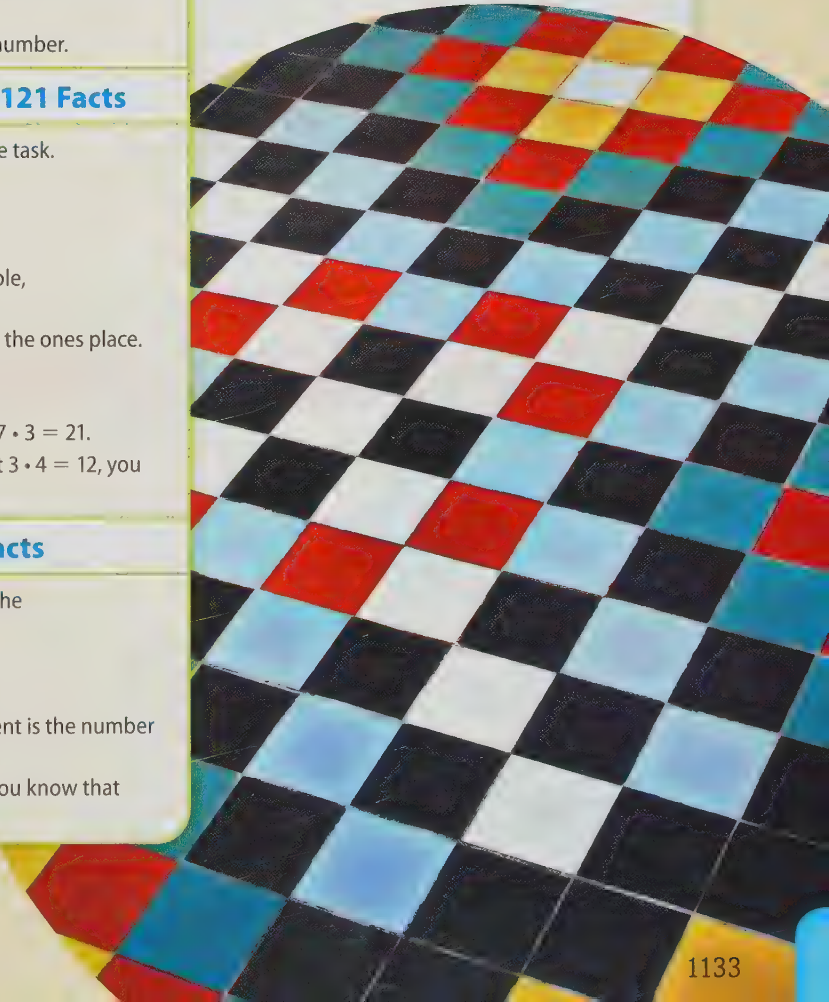
There are 121 basic facts, but here are some ways to cut down the task.

- Any number times 0 is 0.
- Any number times 1 is itself.
- Any number times 10 is that number with a zero to its right.
- Any number times 2 is the double of that number. For example, $2 \cdot 3 = 3 + 3$, or 6.
- The product of any number times 5 will always have 0 or 5 in the ones place.
- Four times a number is its double times 2. For example, $4 \cdot 3 = 2(2 \cdot 3) = 2(6) = 12$.
- Flip the numbers. If you know $3 \cdot 7 = 21$, you also know that $7 \cdot 3 = 21$.
- Think "one group more" or "one group less." If you know that $3 \cdot 4 = 12$, you know that $4 \cdot 4 = 12 + 4 = 16$.

Division Facts: Memorize Fewer Than 100 Facts

There are 100 facts to remember. Here are some ways to reduce the number of facts you need to memorize.

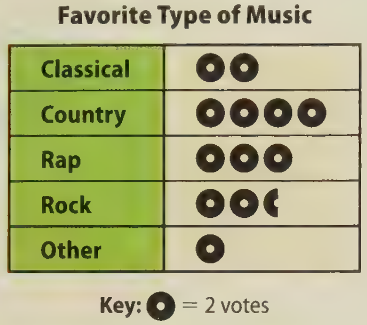
- Any number divided by 1 is the number itself.
- Any number divided by itself is 1.
- When a number that ends in zero is divided by 10, the quotient is the number without the zero. For example, $50 \div 10 = 5$.
- Pairs of facts are related. If you know that $15 \div 3 = 5$, then you know that $15 \div 5 = 3$.



Data Graphs

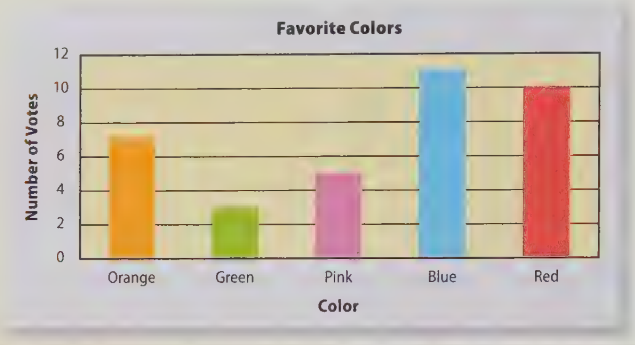
Pictographs

A *pictograph* uses pictures or symbols to represent data. You can use a pictograph to compare amounts.



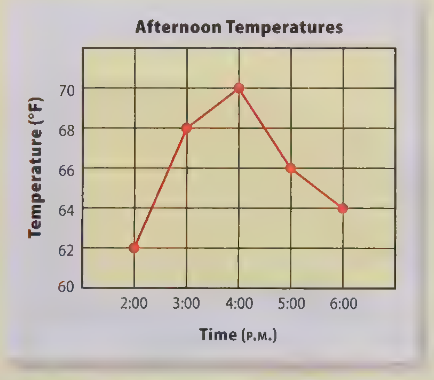
Bar Graphs

A *bar graph* is a type of data display that uses bars to organize information. You can compare the information on a bar graph by comparing the lengths of the bars.



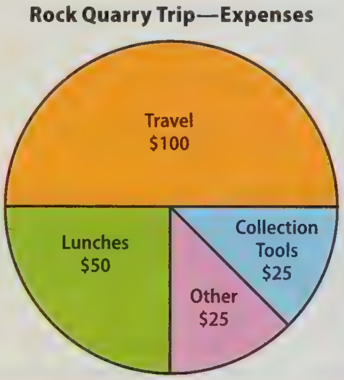
Line Graphs

A *line graph* is a data display that shows information as data points connected by line segments. A line graph usually shows change over time.



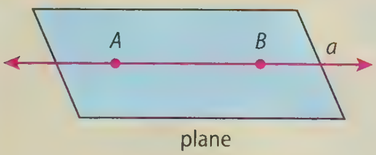
Circle Graphs

A *circle graph* uses sections of a circle to display a set of data as parts of a whole. The whole circle represents 100% of the data.

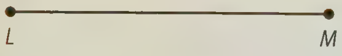


Lines

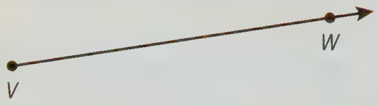
Points, Lines, Planes



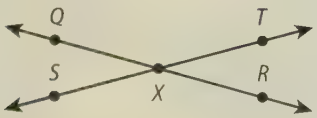
Line Segment



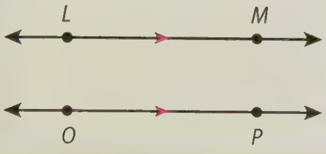
Ray



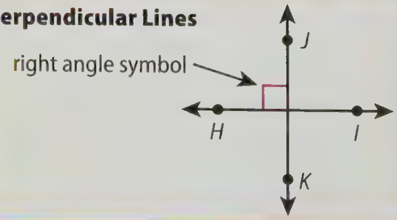
Intersecting Lines



Parallel Lines

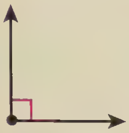


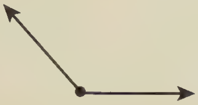
Perpendicular Lines

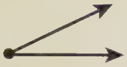


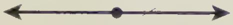
Angles

Angles Classified by Measure

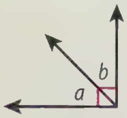

right angle


obtuse angle

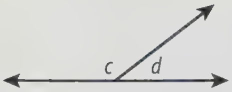

acute angle


straight angle

Complementary and Supplementary Angles



Angles *a* and *b* are complementary.

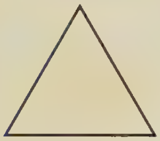


Angles *c* and *d* are supplementary.

Polygons

Regular Polygons

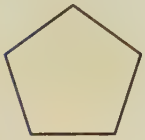
Triangle


3 sides

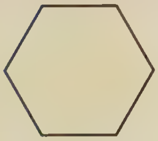
Quadrilateral

4 sides

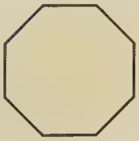
Pentagon


5 sides

Hexagon

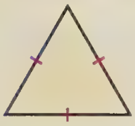

6 sides

Octagon

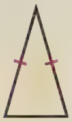

8 sides

Triangles

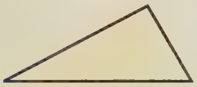
Equilateral


3 equal sides

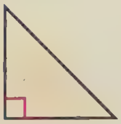
Isosceles


2 equal sides

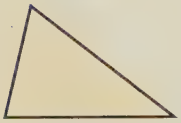
Scalene


no equal sides

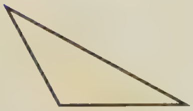
Right


1 right angle

Acute

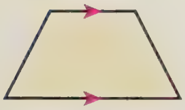

3 acute (<90°) angles

Obtuse

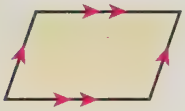

1 obtuse (>90°) angle

Quadrilaterals

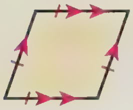
Trapezoid


One pair of parallel sides

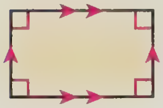
Parallelogram


Two pairs of parallel sides

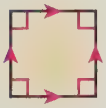
Rhombus


Parallelogram with 4 congruent sides

Rectangle

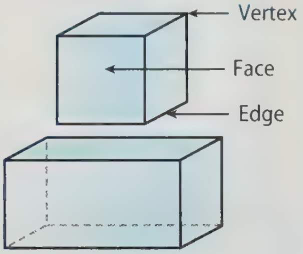

Parallelogram with 4 right angles

Square

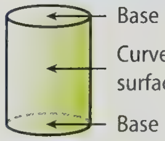

Rectangle with 4 congruent sides

Three-Dimensional Figures

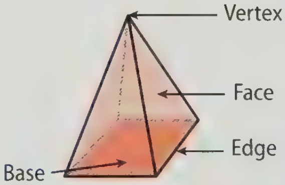
Cube and Prism


Vertex
Face
Edge

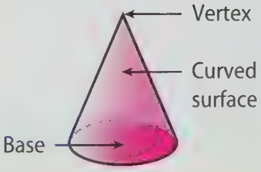
Cylinder


Base
Curved surface
Base

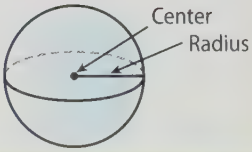
Pyramid


Vertex
Face
Edge
Base

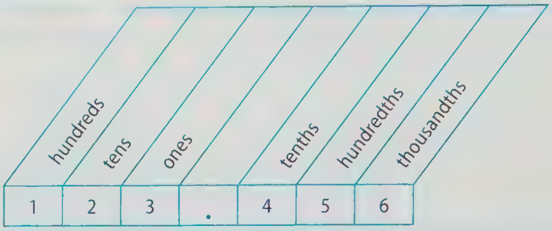
Cone


Vertex
Curved surface
Base

Sphere


Center
Radius

Decimal Place Value



Forms of a Number

Standard form	123.456
Expanded form	100 + 20 + 3 + 0.4 + 0.05 + 0.006
Word form	one hundred twenty-three and four hundred fifty-six thousandths
Short word form	123 and 456 thousandths
Point form	one twenty-three point four five six

Tests for Divisibility

Divisible by	Rule
2	The digit in the ones place is an even digit: 0, 2, 4, 6, or 8.
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The digit in the ones place is 0 or 5.
6	The number is divisible by both 2 and 3.
8	The number formed by the last three digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The digit in the ones place is 0.

You are a
WINNER!

Powers of 10

Power of 10	Value	Zeros
10^0	1	0
10^1	10	1
10^2	100	2
10^3	1,000	3
10^4	10,000	4
10^5	100,000	5
10^6	1,000,000	6
10^7	10,000,000	7

Four-Step Problem Solving Plan

Step 1	**Read**	• Read the problem carefully. You can restate the problem in your own words. • Determine what the problem is asking you to find. • Identify what you know. This is the information given in the problem. • Identify what you need to find out to solve the problem. This is the question asked. You can restate it in your own words.
Step 2	**Plan**	• Plan how to find the answer to the question. • Think about other similar problems you have solved. Think about how the information you know can help you find the answer. • Choose a problem solving strategy.
Step 3	**Solve**	• Solve the problem using your plan. As you work, you can revise your plan if you need to. • Make sure you write a sentence that states your answer.
Step 4	**Check**	• Check your work. • Make sure you have answered the question that was asked. • Make sure your answer makes sense by using another problem solving strategy to check your work.

Formulas, Properties, and Theorems

The Fundamental Theorem of Arithmetic	Every positive integer greater than 1 has exactly one set of prime factors (one prime factorization).
Cross Products Property	The cross products of two fractions are the products of the numerator of one fraction and the denominator of the other fraction. If the cross products of two ratios (fractions) are equal, then the ratios form a proportion.
Percent: Find What Number Is a Given Percent of Another Number	$n = p \times b$ number (n) is a given percent (p) of a given base number (b)
Percent: Find What Percent One Number Is of Another Number	$p = \frac{n}{b}$ percent (p) one number (n) is of another number (b)
Percent: Find a Number When Given a Certain Percent of the Number	$b = \frac{n}{p}$ base number (b) when given a number (n) and the percent (p) the number is of the base number
Percent of Increase	$p\% = \frac{\text{amount of increase}}{\text{original amount}}$ amount of increase = new amount − original amount
Percent of Decrease	$p\% = \frac{\text{amount of decrease}}{\text{original amount}}$ amount of decrease = larger amount − the smaller amount
Simple Interest	$I = prt$ where I is the interest earned or paid, p is the principal amount, r is the interest rate per year, t is the time in years

Ways to Check Answers

Addition

- Reverse the addition order. Add from the bottom of a column to the top of the column, if you had added from top to bottom to solve the problem.
- Use a calculator to enter the computation. Check the calculator result with your answer.
- Use an estimation strategy (*see list below*).

Subtraction

- Reverse the operation. Add the difference (answer) to the subtrahend (number being subtracted).
- Use a calculator to enter the computation. Check the calculator result with your answer.
- Use an estimation strategy (*see list below*).

Multiplication

- Reverse the order of the factors.
- Factor the multiplier, and use the factors to multiply.
- Use a calculator to enter the computation. Check the calculator result with your answer.
- Use an estimation strategy (*see list below*).

Division

- Reverse the operation. Multiply the quotient (answer) by the divisor.
- Factor the divisor and divide each factor separately.
- Use a calculator to enter the computation. Check the calculator result with your answer.
- Use an estimation strategy (*see list below*).

Methods for Estimating

Rounding	• Round numbers so that you can use mental math to perform the computation. • For whole numbers and decimals, add 1 if the digit in the place to the right of the one you're rounding to is greater than or equal to 5. Add 0 if it's less than 5.
Benchmarks	• Rounding to a benchmark instead of a place value sometimes gives you an estimate that is closer to the actual answer. • There are benchmarks for both fractions and decimals.
Compatible Numbers	• Compatible numbers are values that are close to the numbers given in a computation. Choose compatible numbers that make it easy to use mental math. • For division, choose the number for the divisor first. Then find a compatible number for the dividend.
Front-end Estimation	• Use only the whole number parts of decimals or mixed numbers to perform a computation. • You can use the decimal or fraction parts to get a closer estimate.

Benchmarks for Fractions

If the numerator is . . .	Round to . . .
Much less than one-fourth of the denominator	0
About one-fourth of the denominator	$\frac{1}{4}$
About one-half of the denominator	$\frac{1}{2}$
About three-fourths of the denominator	$\frac{3}{4}$
Much greater than three-fourths of the denominator	1

Benchmarks for Decimals

To make addition or subtraction easier, round to . . .
0
0.25
0.5
0.75
1

Fraction-Decimal-Percent Equivalents

$\frac{1}{100}$	$= 0.01$	$= 1\%$	$\frac{8}{16}, \frac{6}{12}, \frac{5}{10}, \frac{4}{8}, \frac{3}{6}, \frac{2}{4}, \frac{1}{2}$	$= 0.5$	$= 50\%$
$\frac{1}{16}$	$= 0.0625$	$= 6\frac{1}{4}\%$	$\frac{5}{9}$	$= 0.5555...$	$= 55\frac{5}{9}\%$
$\frac{1}{12}$	$= 0.0833...$	$= 8\frac{1}{3}\%$	$\frac{9}{16}$	$= 0.5625$	$= 56\frac{1}{4}\%$
$\frac{1}{10}$	$= 0.1$	$= 10\%$	$\frac{4}{7}$	$= 0.571428...$	$= 57\frac{1}{7}\%$
$\frac{1}{9}$	$= 0.1111...$	$= 11\frac{1}{9}\%$	$\frac{7}{12}$	$= 0.5833...$	$= 58\frac{1}{3}\%$
$\frac{2}{16}, \frac{1}{8}$	$= 0.125$	$= 12\frac{1}{2}\%$	$\frac{10}{16}, \frac{5}{8}$	$= 0.625$	$= 62\frac{1}{2}\%$
$\frac{1}{7}$	$= 0.142857...$	$= 14\frac{2}{7}\%$	$\frac{8}{12}, \frac{6}{9}, \frac{4}{6}, \frac{2}{3}$	$= 0.6666...$	$= 66\frac{2}{3}\%$
$\frac{2}{12}, \frac{1}{6}$	$= 0.1666...$	$= 16\frac{2}{3}\%$	$\frac{11}{16}$	$= 0.6875$	$= 68\frac{3}{4}\%$
$\frac{13}{16}$	$= 0.1875$	$= 18\frac{3}{4}\%$	$\frac{7}{10}$	$= 0.7$	$= 70\%$
$\frac{2}{10}, \frac{1}{5}$	$= 0.2$	$= 20\%$	$\frac{5}{7}$	$= 0.714285...$	$= 71\frac{3}{7}\%$
$\frac{2}{9}$	$= 0.2222...$	$= 22\frac{2}{9}\%$	$\frac{12}{16}, \frac{9}{12}, \frac{6}{8}, \frac{3}{4}$	$= 0.75$	$= 75\%$
$\frac{4}{16}, \frac{3}{12}, \frac{2}{8}, \frac{1}{4}$	$= 0.25$	$= 25\%$	$\frac{7}{9}$	$= 0.7777...$	$= 77\frac{7}{9}\%$
$\frac{2}{7}$	$= 0.285714...$	$= 28\frac{4}{7}\%$	$\frac{8}{10}, \frac{4}{5}$	$= 0.8$	$= 80\%$
$\frac{3}{10}$	$= 0.3$	$= 30\%$	$\frac{13}{16}$	$= 0.8125$	$= 81\frac{1}{4}\%$
$\frac{5}{16}$	$= 0.3125$	$= 31\frac{1}{4}\%$	$\frac{10}{12}, \frac{5}{6}$	$= 0.8333...$	$= 83\frac{1}{3}\%$
$\frac{4}{12}, \frac{3}{9}, \frac{2}{6}, \frac{1}{3}$	$= 0.3333...$	$= 33\frac{1}{3}\%$	$\frac{6}{7}$	$= 0.857142...$	$= 85\frac{5}{7}\%$
$\frac{6}{16}, \frac{3}{8}$	$= 0.375$	$= 37\frac{1}{2}\%$	$\frac{14}{16}, \frac{7}{8}$	$= 0.875$	$= 87\frac{1}{2}\%$
$\frac{4}{10}, \frac{2}{5}$	$= 0.4$	$= 40\%$	$\frac{8}{9}$	$= 0.8888...$	$= 88\frac{8}{9}\%$
$\frac{5}{12}$	$= 0.4166...$	$= 41\frac{2}{3}\%$	$\frac{9}{10}$	$= 0.9$	$= 90\%$
$\frac{3}{7}$	$= 0.428571...$	$= 42\frac{6}{7}\%$	$\frac{11}{12}$	$= 0.9166...$	$= 91\frac{2}{3}\%$
$\frac{7}{16}$	$= 0.4375$	$= 43\frac{3}{4}\%$	$\frac{15}{16}$	$= 0.9375$	$= 93\frac{3}{4}\%$
$\frac{4}{9}$	$= 0.4444...$	$= 44\frac{4}{9}\%$	$\frac{16}{16}, \frac{12}{12}, \frac{10}{10}, \frac{8}{8}, \frac{6}{6}, \frac{4}{4}, \frac{2}{2}$	$= 1.000$	$= 100\%$

STAY Focused

Measurement

U.S. Customary System	Metric System
Length	
1 mile (mi) = 1,760 yards = 5,280 feet	**1 kilometer (km)** = 1,000 m
1 yard (yd) = 3 feet = 36 inches	0.001 km = **1 meter (m)** = 100 cm = 1,000 mm
1 foot (ft) = 12 inches	0.01 m = **1 centimeter (cm)** = 10 mm
$\frac{1}{36}$ yard = $\frac{1}{12}$ foot = **1 inch (in.)**	0.001 m = 0.1 cm = **1 millimeter (mm)**
Area	
1 square mile (mi²) = 640 acres	**1 square kilometer (km²)** = 1,000,000 m^2
1 acre (a) = 4,840 yd^2	**1 square meter (m²)** = 10,000 cm^2
1 square yard (yd²) = 9 ft^2	**1 square centimeter (cm²)** = 100 mm^2
1 square foot (ft²) = 144 $in.^2$	0.01 cm^2 = **1 square millimeter (mm²)**
$\frac{1}{144}$ ft^2 = **1 square inch (in.²)**	
Volume	
1 cubic yard (yd³) = 27 ft^3	**1 cubic meter (m³)** = 1,000,000 cm^3
1 cubic foot (ft³) = 1,728 $in.^3$	**1 cubic centimeter (cm³)** = 1,000 mm^3
$\frac{1}{1,728}$ ft^3 = **1 cubic inch (in.³)**	0.001 cm^3 = **1 cubic millimeter (mm³)**
Capacity (Liquid)	
1 gallon (gal) = 4 qt = 8 pt = 16 c = 128 fl oz	**1 kiloliter (kL)** = 1,000 L
1 quart (qt) = 2 pt = 4 c = 32 fl oz	0.001 kL = **1 liter (L)** = 100 cL = 1,000 mL
1 pint (pt) = 2 c = 16 fl oz	0.01 L = **1 centiliter (cL)** = 10 mL
1 cup (c) = 8 fl oz	0.001 L = 0.1 cL = **1 milliliter (mL)**
$\frac{1}{8}$ cup = **1 fluid ounce (fl oz)**	
Weight	**Mass**
1 ton (t) = 2,000 pounds	**1 metric ton (t)** = 1,000 kg
1 pound (lb) = 16 ounces	0.001 t = **1 kilogram (kg)** = 1,000 g
$\frac{1}{16}$ pound = **1 ounce (oz)**	0.001 kg = **1 gram (g)** = 100 cg = 1,000 mg
	0.01 g = **1 centigram (cg)** = 10 mg
	0.001 g = 0.1 cg = **1 milligram (mg)**
Temperature	
32°F = freezing point of water	0°C = freezing point of water
98.6°F = normal body temperature	37°C = normal body temperature
212°F = boiling point of water	100°C = boiling point of water
$F = \frac{9}{5}C + 32$ or $F = 1.8C + 32$	$C = \frac{5}{9}(F - 32)$

Do Your BEST!

Time

60 seconds (s) = **1 minute (min)**	365 days = **1 year (yr)**
60 minutes = **1 hour (h)**	52 weeks (approx.) = 1 year
24 hours = **1 day (d)**	12 months = 1 year
7 days = **1 week (wk)**	10 years = 1 decade
4 weeks (approx.) = **1 month (mo)**	100 years = 1 century

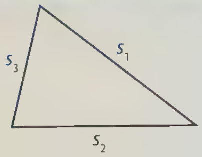
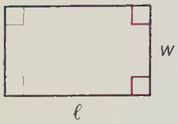
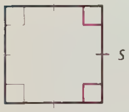
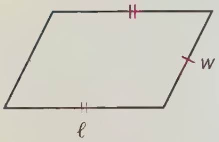
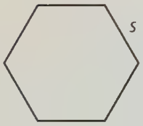
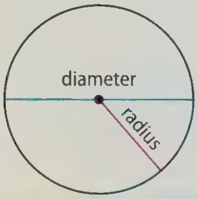
Equivalents

1 acre = 43,560 square feet = 4,840 square yards
1 bushel (U.S.) = 2,150.42 cubic inches
= 32 quarts
1 cord = 128 cubic feet
1 cubic centimeter = 0.061 cubic inch
1 cubic foot = 7.481 gallons = 1,728 cubic inches
1 cubic inch = 0.554 fluid ounce
= 16.387 cubic centimeters
1 cubic meter = 1.308 cubic yards
1 cubic yard = 0.765 cubic meter = 27 cubic feet
1 cup = 8 fluid ounces = 0.5 liquid pint
1 gallon (U.S.) = 231 cubic inches
= 128 U.S. fluid ounces
= 4 liquid quarts
1 liter = 1.057 liquid quarts
1 meter = 39.37 inches = 1.094 yards
1 micron = 0.001 millimeter = 0.00003937 inch
1 mile, nautical = 1.852 kilometers
= 1.151 statute miles
= 6,076.1155 feet
1 milliliter = 0.061 cubic inch
1 pint, dry = 33.600 cubic inches = 0.551 liter
1 pint, liquid = 28.875 inches = 0.473 liter
= 2 cups = 16 fluid ounces
1 pound, avoirdupois = 7,000 grains = 16 ounces
= 453.59237 grams
1 quart, dry (U.S.) = 67.201 cubic inches
= 1.101 liters
1 quart, liquid (U.S.) = 57.75 cubic inches
= 0.946 liter
= 2 pints = 32 fluid ounces
1 square foot = 929 square centimeters
= 144 square inches
1 square inch = 6.45 square centimeters
1 square kilometer = 0.386 square miles
= 247.105 acres
1 square meter = 1.196 square yards
= 10.764 square feet
1 square mile = 640 acres
1 square yard = 0.836 square meter
= 9 square feet
= 1,296 square inches
1 tablespoon = 3 teaspoons = 0.5 fluid ounce
1 ton, metric = 2,204.623 pounds
= 1.102 net tons
1 ton, net or short = 2,000 pounds
= 0.907 metric ton
1 yard = 0.9144 meter = 3 feet = 36 inches

Conversions

To Convert	Into	Multiply By
angstroms	microns	0.0001
centimeters	feet	0.03281
centimeters	inches	0.3937
cubic cm	cubic inches	0.06102
cubic feet	cubic meters	0.02832
days	seconds	86,400.0
degrees (angle)	radians	0.01745
fathoms	feet	6.0
feet	centimeters	30.48
feet	meters	0.3048
feet/min.	cm/sec.	0.5080
feet/sec.	knots	0.5921
feet/sec.	statute mi./hr.	0.6818
furlongs/hr.	statute mi./hr.	0.125
furlongs	feet	660.0
gallons (liq.)	liters	3.785
gal. of water	pounds of water	8.3453
grams	oz. (avoirdupois)	0.03527
grams	pounds	0.002205
hours	days	0.04167
hours	weeks	0.005952
inches	centimeters	2.540
kilograms	pounds	2.205
kilometers	feet	3,280.8
kilometers	mi. (statute)	0.6214
knots	feet/hr.	6080.0
knots	nautical mi./hr.	1.0
knots	statute mi./hr.	1.151
liters	gallons (liq.)	0.2642
liters	pints (liq.)	2.113
meters	feet	3.281
meters	mi. (nautical)	0.0005396
meters	mi. (statute)	0.0006214
microns	meters	0.000001
mi. (nautical)	feet	6,076.115
mi. (statute)	feet	5,280.0
mi. (nautical)	kilometers	1.852
mi. (statute)	kilometers	1.609
mi. (nautical)	mi. (statute)	1.1508
mi. (statute)	mi. (nautical)	0.8684
mi. (statute)/hr.	feet/min.	88.0
millimeters	inches	0.03937
oz. (avoirdupois)	grams	28.3495
oz. (avoirdupois)	lb. (avoirdupois)	0.0625
pints (liq.)	gallons (liq.)	0.125
pints (liq.)	quarts (liq.)	0.5
lb. (avoirdupois)	kilograms	0.4536

Measurement

Length Formulas		Perimeter P, Circumference C
$P = s_1 + s_2 + s_3$	**Triangle** s_1, s_2, s_3 are lengths of sides	
$P = 2\ell + 2w$ or $P = 2(\ell + w)$	**Rectangle** ℓ is length w is width	
$P = 4s$	**Square** s is length of each side	
$P = 2\ell + 2w$ or $P = 2(\ell + w)$	**Parallelogram** ℓ is length w is width	
$P = ns$	**Regular Polygon** n is number of sides s is length of each side	
$C = \pi d$ or $C = 2\pi r$	**Circle** π is pi, a number about 3.14 or $\frac{22}{7}$ d is length of diameter r is length of radius	

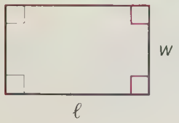
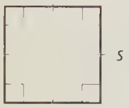
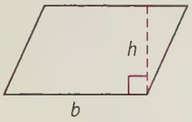
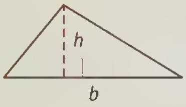
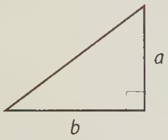
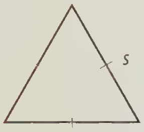
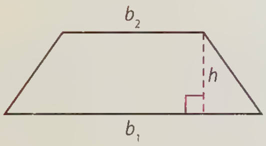
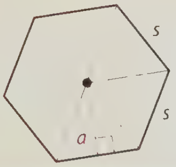
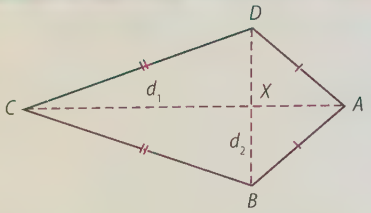
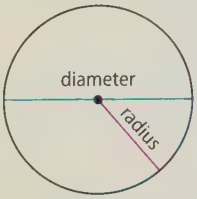
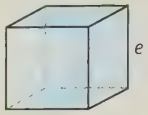
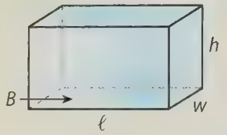
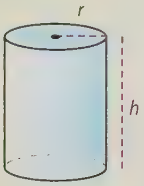
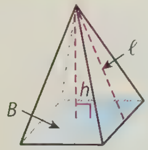
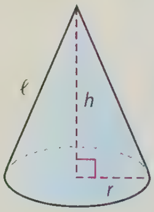
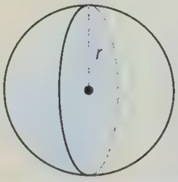
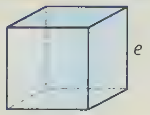
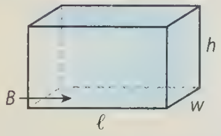
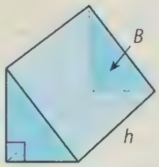
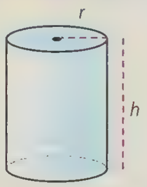
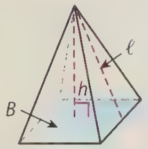
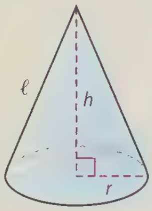
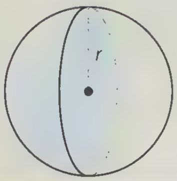
Area Formulas		**Area A, Surface Area SA**
$A = \ell w$ or $A = bh$	**Rectangle** ℓ is length w is width b is length of the base h is height	
$A = s^2$	**Square** s is length of each side	
$A = bh$	**Parallelogram** b is length of the base h is height	
$A = \frac{1}{2}bh$	**Triangle** b is length of the base h is height	
$A = \frac{1}{2}ab$	**Right Triangle** a is length of one leg b is length of the other leg	
$A = \frac{s^2}{4}\sqrt{3}$	**Equilateral Triangle** s is length of each side	
$A = \frac{1}{2}h(b_1 + b_2)$	**Trapezoid** h is height b_1 is length of one parallel side b_2 is length of other parallel side	
$A = \frac{1}{2}ap$	**Regular Polygon** a is length of an apothem p is perimeter of polygon	

Table continued on next page

Area Formulas		Area A, Surface Area SA
$A = \frac{1}{2}d_1d_2$	**Kite** d_1 is length of one diagonal d_2 is length of other diagonal	
$A = \pi r^2$	**Circle** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius	
$SA = 6e^2$	**Cube** e is length of each edge	
$SA = Ph + 2B$	**Rectangular Prism** P is perimeter of the base h is height B is area of the base	
$SA = 2\pi rh + 2\pi r^2$ or $SA = 2\pi r(h + r)$	**Right Cylinder** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius of the base h is height	
$SA = \frac{1}{2}P\ell + B$	**Pyramid** P is perimeter of the base ℓ is slant height B is area of the base	
$SA = \pi r\ell + \pi r^2$ or $SA = \pi r(\ell + r)$	**Right Cone** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius of the base ℓ is slant height	
$SA = 4\pi r^2$	**Sphere** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius	

Volume Formulas		Volume V
$V = e^3$	**Cube** e is length of each edge	
$V = \ell wh$	**Rectangular Prism** ℓ is length w is width h is height	
$V = Bh$	**Prism** B is area of the base h is height	
$V = \pi r^2 h$	**Right Cylinder** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius of the base h is height	
$V = \frac{1}{3}Bh$	**Pyramid** B is area of the base h is height	
$V = \frac{1}{3}\pi r^2 h$	**Right Cone** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius of the base h is height	
$V = \frac{4}{3}\pi r^3$	**Sphere** π is pi, a number about 3.14 or $\frac{22}{7}$ r is length of the radius	

Statistics and Probability

Think Success

Formulas

Probability (theoretical)	$P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$
Probability (experimental)	$P(\text{event}) = \dfrac{\text{number of times an event occurs}}{\text{number of trials}}$
Complement C of an Event E	$P(C) = P(\text{not } E)$ $P(C) = 1 - P(E)$ $P(\text{not } E) = 1 - P(E)$ $P(E) + P(\text{not } E) = 1$
Independent Events	$P(A \text{ and } B) = P(A) \cdot P(B)$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Dependent Events	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
Odds for an Event	$\text{Odds in favor} = \dfrac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$
Odds Against an Event	$\text{Odds against} = \dfrac{\text{number of unfavorable outcomes}}{\text{number of favorable outcomes}}$
Permutations	${}_nP_r = \dfrac{n!}{(n-r)!}$
Combinations	${}_nC_r = \dfrac{n!}{r!(n-r)!}$
Binomial Probability	$P(k \text{ successes in } n \text{ trials}) = {}_nC_r \cdot p^k(1-p)^{n-k}$
Outlier	$n < Q1 - 1.5(IQR)$ or $n > Q3 + 1.5(IQR)$
Linear Interpolation and Extrapolation	$y = y_1 + \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$
Variance	$\sigma^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n}$
Standard Deviation	$\sigma = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n}}$
Margin of Error	$ME = z_c\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ $\quad ME = z_c \cdot \dfrac{\sigma}{\sqrt{n}}$
Confidence Interval	$\hat{p} \pm ME$ $\quad \bar{x} \pm ME$

Theorems

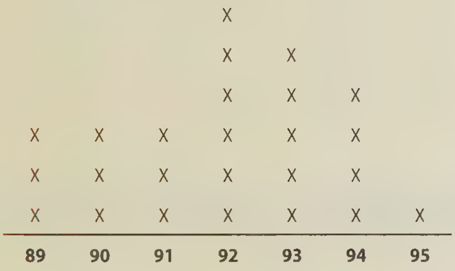
Fundamental Counting Principle	If there are m ways to choose a first item and n ways to choose a second item after the first item has been chosen, then there are $m \times n$ ways to choose both items.
Binomial Theorem	$(a+b)^n = ({}_nC_0)a^n + ({}_nC_1)a^{n-1}b + ({}_nC_2)a^{n-2}b^2 + \ldots + ({}_nC_{n-1})ab^{n-1} + ({}_nC_n)b^n$ $(a+b)^n = \sum_{k=0}^{n} {}_nC_k a^{n-k}b^k$

Data Graphs

Line Plots

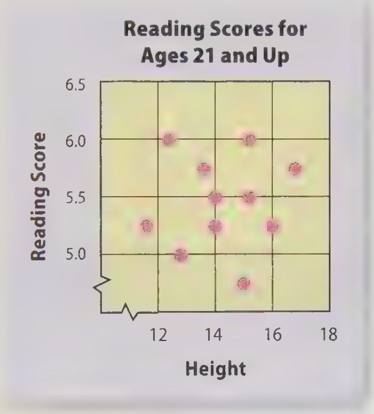
A *line plot* displays data along a number line. Each number in the data set is represented by an X or another symbol.

Midterm Scores in Science



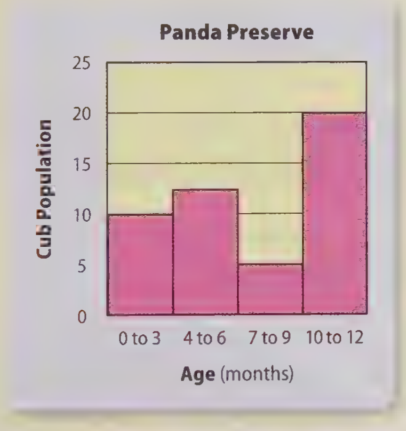
Scatter Plots

A *scatter plot* is a graph that shows data points on a coordinate grid. Each data point represents a pair of values.



Histograms

A *histogram* displays the frequency of a set of data in equal-size intervals, or ranges, of data.



Stem-and-Leaf Plots

A *stem-and-leaf plot* is a data display that can be used to show how data are distributed. The data are in numerical order in two columns.

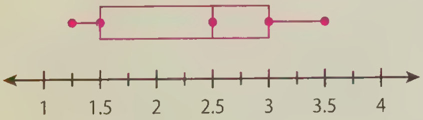
Cars Parked (Past 10 Days)

Stem	Leaf
0	1 2 3 6
1	0 0 3 6
2	0 9

Key: 2 | 9 = 29

Box-and-Whisker Plots

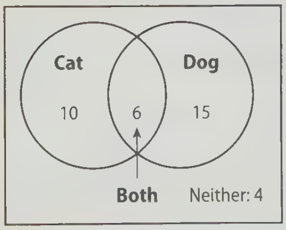
A *box-and-whisker plot* is a data display that divides a set of data into four equal parts. It shows how the data are distributed among these parts.



Venn Diagrams

A *Venn diagram* is a graphic display that uses overlapping circles (or squares) to show the relationship between sets and subsets of data.

Pet Owners



Algebra

Properties of Equality

Addition	If $a = b$, then $a + c = b + c$.
Subtraction	If $a = b$, then $a - c = b - c$.
Multiplication	If $a = b$, then $a \cdot c = b \cdot c$.
Division	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive	$a = a$
Symmetric	If $a = b$, then $b = a$.
Transitive	If $a = b$ and $b = c$, then $a = c$.

Properties of Inequality

Addition	If $a < b$, then $a + c < b + c$.
Subtraction	If $a < b$, then $a - c < b - c$.
Multiplication	If $a < b$ and $c > 0$, then $a \cdot c < b \cdot c$. If $a < b$ and $c < 0$, then $a \cdot c > b \cdot c$.
Division	If $a < b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.
Transitive	If $a < b$ and $b < c$, then $a < c$.

These properties are also true for $>$, $\leq$, and $\geq$.

Properties of Real Numbers

Commutative Property	$a + b = b + a$ $a \cdot b = b \cdot a$
Associative Property	$(a + b) + c = a + (b + c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive Property	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$
Closure for Addition	$a + b$ is a real number.
Closure for Multiplication	$a \cdot b$ is a real number.
Identity Property	$a + 0 = a$ $a \cdot 1 = a$
Inverse Property	$a + (-a) = 0$ $a \cdot \frac{1}{a} = 1$
Multiplication by 0	$a \cdot 0 = 0$
Multiplication by -1	$a \cdot (-1) = -a$
Zero Product	If $ab = 0$, then $a = 0$, $b = 0$ or both a and $b = 0$.

Properties of Exponents

Zero exponent	$a^0 = 1$
Negative exponent	$a^{-1} = \frac{1}{a}$
Product of Powers	$a^m \times a^n = a^{m+n}$
Product to a Power	$(ab)^n = a^n \times b^n$
Power of a Power	$(a^m)^n = a^{m \cdot n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Quotient to a Power	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Translating Word Problems into Algebra

1. Read the problem carefully, and determine what number or numbers need to be found.
2. Represent this unknown number with a variable, for example, x.
3. Identify key words that show what operations are used with the numbers and variables in the problem.
4. Words like *equals* or *is* tell you to write an equation rather than just an expression.
5. Use numbers, variables, operation symbols, and a relationship stated in the problem to write any expression needed.
6. To write an equation, write two expressions and connect them with an equals sign.

Solving Word Problems

Step 1 Read the problem carefully and determine what number or numbers you need to find.

Step 2 Represent one unknown number with a variable.

Step 3 Use a condition stated in the problem to write an expression for a second number.

Step 4 Use a second condition to find two expressions that are equal. Then write these two expressions algebraically and connect them with an equals sign.

Step 5 Solve the equation.

Step 6 Check to make sure the result matches the conditions stated in the problem.

Operations with Signed Numbers

Adding

If the numbers have the same signs, add the absolute values and use the sign of the numbers.	$5 + 11 = 16$	$-5 + (-11) = -16$
If the numbers have different signs, subtract their absolute values and use the sign of the number with the greater absolute value.	$-7 + 11 = 4$ $7 + (-11) = -4$	$-15 + 6 = -9$ $15 + (-6) = 9$

Subtracting

To subtract a number, add its opposite.	$8 - 6 = 8 + (-6)$	$-1 - 7 = -1 + (-7)$

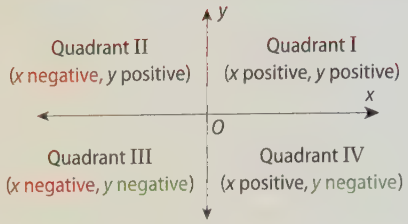
Multiplying or Dividing

If two numbers have the same sign, their product or quotient is positive.	$(+)(+) = (+)$ $(-)(-) = (+)$	$(+) \div (+) = (+)$ $(-) \div (-) = (+)$
If two numbers have different signs, their product or quotient is negative.	$(+)(-) = (-)$ $(-)(+) = (-)$	$(+) \div (-) = (-)$ $(-) \div (+) = (-)$

Algebra

Graphing (x, y) on the Coordinate Plane

Value of x	Value of y	Location of Point
$+$	$+$	Quadrant I
$-$	$+$	Quadrant II
$-$	$-$	Quadrant III
$+$	$-$	Quadrant IV
0	$+$ or $-$	on the y-axis
$+$ or $-$	0	on the x-axis


y
Quadrant II
(x negative, y positive)
Quadrant I
(x positive, y positive)
x
O
Quadrant III
(x negative, y negative)
Quadrant IV
(x positive, y negative)

Formulas

Polynomials

Binomial squares	$(a+b)^2 = a^2 + 2ab + b^2$ $(a-b)^2 = a^2 - 2ab + b^2$
Difference of squares	$a^2 - b^2 = (a+b)(a-b)$
Difference of cubes	$a^3 - b^3 = (a-b)(a^2+ab+b^2)$
Sum of cubes	$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Linear Equations

Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form	$y = mx + b$
Point-Slope Form	$y - y_1 = m(x - x_1)$
Standard Form	$Ax + By = C$

Quadratic Equations and Functions

Standard Form	$ax^2 + bx + c = 0$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Discriminant	$b^2 - 4ac$
Vertex Form of a Function	$f(x) = a(x-h)^2 + k$
Line of Symmetry	$x = -\frac{b}{2a}$

Distance

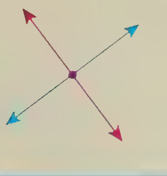
horizontal distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane	$\lvert x_2 - x_1 \rvert$
vertical distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane	$\lvert y_2 - y_1 \rvert$

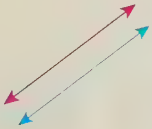
Theorems

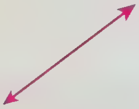
Rational Zeros Theorem	If the polynomial function has integer coefficients, then every rational zero has this form. p ← a factor of the constant term q ← a factor of the leading coefficient
Rational Roots Theorem	If a polynomial equation $P(x) = 0$ has integer coefficients, then every rational root has this form. p ← a factor of the constant term q ← a factor of the leading coefficient
The Fundamental Theorem of Algebra and Its Corollary	*Theorem:* If $P(x)$ is a polynomial of degree n, then the equation $P(x) = 0$ has at least one root, which is complex. *Corollary:* If $P(x)$ is a polynomial of degree n, then the equation $P(x) = 0$ has exactly n roots when multiplicities are taken into account.
The Remainder Theorem	If a polynomial function $P(x)$ is divided by $x - a$, then the remainder r is equal to $P(a)$.
Descartes' Rule of Signs for Negative Real Zeros	The number of negative real zeros of the polynomial function $P(x)$ is either: • equal to the number of sign changes in $P(-x)$ or • less than the number of sign changes in $P(-x)$ by an even number.
Irrational Zeros Theorem	If $P(x)$ has a zero of the form $a + b\sqrt{c}$, it also has a zero of the form $a - b\sqrt{c}$.

Systems of Two Linear Equations

One Solution	**No Solution**	**Infinitely Many Solutions**
consistent system	inconsistent system	consistent system
independent system		dependent system
intersecting lines	parallel lines	same line
different slopes	same slope	same slope
	different y-intercepts	same y-intercept







Geometry

Formulas

Distance Formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Lateral Area of a Regular Pyramid	$LA = \frac{1}{2}Ps$
Midpoint Formula	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Lateral Area of a Regular Cone	$LA = \pi rs$
Slope m	$m = \frac{y_2 - y_1}{x_2 - x_1}$	Euler's Formula	$F + V = E + 2$
Slope-Intercept Form of an Equation for a Line	$y = mx + b$	Length of an Arc	$\ell = 2\pi r\left(\frac{x°}{360°}\right)$
Point-Slope Form of an Equation for a Line	$y - y_1 = m(x - x_1)$	Area of a Sector	$A = \pi r^2\left(\frac{x°}{360°}\right)$
Equation of a Circle	$(x - h)^2 + (y - k)^2 = r^2$ with center (h, k) and radius r		

Perimeter, Area, and Volume

Figure	**Perimeter Formula**	**Area Formula**
Regular Polygon	$P = ns$	$A = \frac{1}{2}ap$
Triangle	$P = s_1 + s_2 + s_3$.	$A = \frac{1}{2}bh$
Square	$P = 4s$	$A = s^2$
Rectangle	$P = 2\ell + 2w$ or $2(\ell + w)$	$A = \ell \cdot w$
Parallelogram	$P = 2(s_1 + s_2)$	$A = b \cdot h$
Trapezoid	$P = s_1 + s_2 + s_3 + s_4$	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	circumference: $C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$
Figure	**Surface Area Formula**	**Volume Formula**
Cube	$SA = 6s^2$	$V = s^3$
Rectangular Prism	$SA = Ph + 2B$	$V = Bh$
Pyramid	$SA = \frac{1}{2}P\ell + B$	$V = \frac{1}{3}Bh$
Cylinder	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Cone	$SA = \pi r\ell + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
Sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Congruence Properties

Property	Segments	Angles
Reflexive	$\overline{AB} \cong \overline{AB}$	$\angle X \cong \angle X$
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.	If $\angle X \cong \angle Y$, then $\angle Y \cong \angle X$.
Transitive	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.	If $\angle X \cong \angle Y$ and $\angle Y \cong \angle Z$, then $\angle X \cong \angle Z$.

Logic Statements

Statement	In Words	In Symbols
Conditional	If p, then q.	$p \rightarrow q$
Inverse	If not p, then not q.	$\sim p \rightarrow \sim q$
Converse	If q, then p.	$q \rightarrow p$
Contrapositive	If not q, then not p.	$\sim q \rightarrow \sim p$
Biconditional	p if and only if q.	$p \leftrightarrow q$

Math SMART!

Truth Table for $p \rightarrow q$

Hypothesis p	Conclusion q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Biconditional $p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Laws of Logic

The Law of Detachment

If $p \rightarrow q$ is true and p is true, then q is true.

Law of Syllogism

If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ must be true.

Types of Reasoning

Deductive Reasoning	• Uses properties, rules, definitions, theorems, given facts, and the laws of logic to arrive at a conclusion. • The conclusion must be true if the hypotheses are true.
Inductive Reasoning	• Uses observations and patterns. • The conclusion is not necessarily true.

Angles

Angle Addition Postulate: If point M is in the interior of $\angle JKL$, then $m\angle JKM + m\angle MKL = m\angle JKL$.

Linear Pair Theorem: If two angles form a linear pair, then they are supplementary.

Congruent Supplements Theorem: If two angles are supplementary to the same angle (or to congruent angles), then the angles are congruent.

Congruent Complements Theorem: If two angles are complementary to the same angle (or to congruent angles), then the angles are congruent.

Vertical Angles Theorem: Vertical angles are congruent.

Right Angle Congruence Theorem: All right angles are congruent.

Theorem (p. 628): If two angles are congruent and supplementary, then each angle is a right angle.

Corresponding Angles Postulate: If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Converse of the Corresponding Angles Postulate: If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

Alternate Interior Angles Theorem: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Converse of the Alternate Interior Angles Theorem: If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Alternate Exterior Angles Theorem: If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

Converse of the Alternate Exterior Angles Theorem: If two lines are cut by a transversal and alternate exterior angles are congruent, then the lines are parallel.

Same-Side Interior Angles Theorem: If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

Converse of the Same-Side Interior Angles Theorem: If two lines are cut by a transversal and same-side interior angles are supplementary, then the lines are parallel.

Polygons

Polygon Interior Angle-Sum Theorem: The sum of the measures of the interior angles of a polygon with n sides is $(n - 2)180°$.

Polygon Exterior Angle-Sum Theorem: The sum of the measures of the exterior angles of a polygon is 360°.

Parallel and Perpendicular Lines

Theorem (p. 633): In a plane, if two lines are perpendicular to the same line, then they are parallel.

Theorem (p. 814): If two non-vertical lines are parallel, then their slopes are equal.

Theorem (p. 814): If the slopes of two non-vertical lines are equal, then the lines are parallel.

Theorem (p. 816): If two non-vertical lines are perpendicular, then their slopes are negative reciprocals.

Theorem (p. 816): If the slopes of two non-vertical lines are negative reciprocals, then the lines are perpendicular.

Parallel Postulate: Given a line and a point not on the line, there is exactly one line through the point parallel to the line.

Circles

Arc Addition Postulate: The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$

Theorem (p. 728): In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

Theorem (p. 734): In the same circle (or in congruent circles), congruent chords have congruent central angles.

Converse of Theorem (p. 734): In the same circle (or in congruent circles), congruent central angles have congruent chords.

Theorem (p. 736): In the same circle (or in congruent circles), congruent arcs have congruent chords.

Converse of Theorem (p. 736): In the same circle (or congruent circles), congruent chords have congruent arcs.

Theorem (p. 738): If a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

Theorem (p. 738): If a radius bisects a chord that is not a diameter, then it is perpendicular to the chord.

Theorem (p. 738): The perpendicular bisector of a chord passes through the center of the circle.

Theorem (p. 739): If two chords of a circle are congruent, then they are equidistant from the center.

Theorem (p. 739): If two chords of a circle are equidistant from the center, then they are congruent.

Theorem (p. 740): If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Theorem (p. 740): If a line in the plane of a circle is perpendicular to the radius at a point on the circle, then the line is tangent to the circle.

Two-Tangent Theorem: Two tangent segments drawn from a point not on a circle are congruent.

Inscribed Angle Theorem: The measure of an inscribed angle is half the measure of its intercepted arc.

Corollary: If two inscribed angles intercept the same arc, then they are congruent.

Corollary: An angle inscribed in a semicircle is a right angle.

Corollary: If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Tangent-Chord Angle Theorem: The measure of an angle whose sides are a tangent and a chord equals half the measure of the intercepted arc.

Tangent-Tangent Angle Theorem: The measure of the angle formed by two tangents drawn from a point outside a circle equals half the difference of the measures of the intercepted arcs.

Corollary: The measure of the angle formed by two tangents drawn from a point outside a circle is 180° minus the measure of the intercepted minor arc.

Chord-Chord Angle Theorem: If two chords intersect in a circle, the measure of an angle formed is half the sum of the measures of arcs intercepted by the angle and its vertical angle.

Secant-Secant Angle Theorem: The measure of the angle formed by two secants drawn from a point outside a circle equals half the difference of the measures of the intercepted arcs.

Secant-Tangent Theorem: The measure of the angle formed by a secant and a tangent drawn from a point in the exterior of a circle equals half the difference of the measures of the intercepted arcs.

Theorem (p. 758): If two chords intersect inside a circle, then the product of the segment lengths of one chord is equal to the product of the segment lengths of the other.

Theorem (p. 760): If two secants intersect outside a circle, then the product of the lengths of one secant segment and its external segment is equal to the product of the other secant segment and its external segment.

Theorem (p. 762): If a secant and a tangent intersect outside a circle, then the product of the lengths of the secant segment and its external segment is equal to the length of the tangent segment squared.

Geometry

Quadrilaterals

Theorem (p. 644): If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem (p. 645): If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Theorem (p. 650): The diagonals of a rectangle are congruent.

Theorem (p. 651): If a parallelogram has one right angle, then it is a rectangle.

Theorem (p. 652): The diagonals of a rhombus are perpendicular.

Theorem (p. 652): The diagonals of a rhombus bisect the angles of the rhombus.

Trapezoid Midsegment Theorem: The midsegment of a trapezoid is parallel to each of the bases, and its length is half the sum of the lengths of the bases.

Theorem (p. 658): The base angles of an isosceles trapezoid are congruent.

Theorem (p. 659): The diagonals of an isosceles trapezoid are congruent.

Theorem (p. 660): If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Theorem (p. 661): The diagonals of a kite are perpendicular.

Triangles

Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.

Hinge Theorem: If two sides of one triangle are congruent to two sides of another triangle, but their included angles are not congruent, then the side opposite the larger included angle is longer than the side opposite the smaller included angle.

Converse of Hinge Theorem: If two sides of one triangle are congruent to two sides of another triangle, but the third sides are not congruent, then the angle opposite the longer side is larger than the angle opposite the shorter side.

Triangle Angle-Sum Theorem: The sum of the angle measures of a triangle is 180°.

Corollary: The acute angles of a right triangle are complementary.

Corollary: The measure of each angle of an equiangular triangle is 60°.

Exterior Angle Theorem: The measure of an exterior angle of a triangle equals the sum of the measures of its remote interior angles.

Pythagorean Inequality Theorem: Given: a, b, and c are the side lengths of a triangle with c the length of the longest side:

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.
If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

Side-Side-Side Congruence Postulate (SSS): If three sides of one triangle are congruent to the corresponding three sides of another triangle, then the triangles are congruent.

Side-Angle-Side Congruence Postulate (SAS): If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.

Angle-Side-Angle Congruence Postulate (ASA): If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent.

Angle-Angle-Side Congruence Theorem (AAS): If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

Angle-Angle Similarity Postulate (AA): If two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

Triangles (continued)

CPCTC: Corresponding Parts of Congruent Triangles are Congruent.

Theorem (p. 784): If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Theorem (p. 784): If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Theorem (p. 786): If a triangle is equilateral, it is also equiangular.

Theorem (p. 786): If a triangle is equiangular, it is also equilateral.

Side-Side-Side Similarity Theorem (SSS): If three sides of one triangle are proportional to the corresponding sides of another triangle, then the triangles are similar.

Side-Angle-Side Similarity Theorem (SAS): If two sides of one triangle are proportional to the corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Theorem (p. 804): If a segment that is parallel to a side of a triangle intersects the other two sides, then it divides those two sides into proportional segments.

Theorem (p. 804): If a segment divides two sides of a triangle into proportional segments, then it is parallel to the third side.

Theorem (p. 805): An angle bisector of a triangle divides the opposite sides into two segments that are proportional to the other two sides of the triangle.

Theorem (p. 806): A midsegment of a triangle is parallel to one side of the triangle and is half as long as that side.

Theorem (p. 831): The lines that contain the altitudes of a triangle are concurrent.

Challenge YOURSELF!

Right Triangles

Pythagorean Theorem: The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$

Converse of the Pythagorean Theorem: If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

45°–45°–90° Triangle Theorem: In a 45°–45°–90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

30°–60°–90° Triangle Theorem: In a 30°–60°–90° triangle, the hypotenuse is twice the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Leg-Leg Congruence (LL): If both legs of a right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.

Leg-Angle Congruence (LA): If one leg and an acute angle of a right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.

Hypotenuse-Angle Congruence (HA): If the hypotenuse and an acute angle of a right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the triangles are congruent.

Hypotenuse-Leg Congruence (HL): If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

Theorem (p. 796): If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Corollary: The length of the altitude drawn to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments into which it divides the hypotenuse.

Corollary: The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the adjacent segment of the hypotenuse.

Advanced Algebra

Formulas

Conic Sections

Circles	$(x-h)^2+(y-k)^2=r^2$	$\frac{(x-h)^2}{r^2}+\frac{(y-k)^2}{r^2}=1$
	Horizontal Axis	Vertical Axis
Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Hyperbola	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}-\frac{(y-k)^2}{a^2}=1$
Parabola	$x-h=\frac{1}{4p}(y-k)^2$	$y-k=\frac{1}{4p}(x-h)^2$

Variation

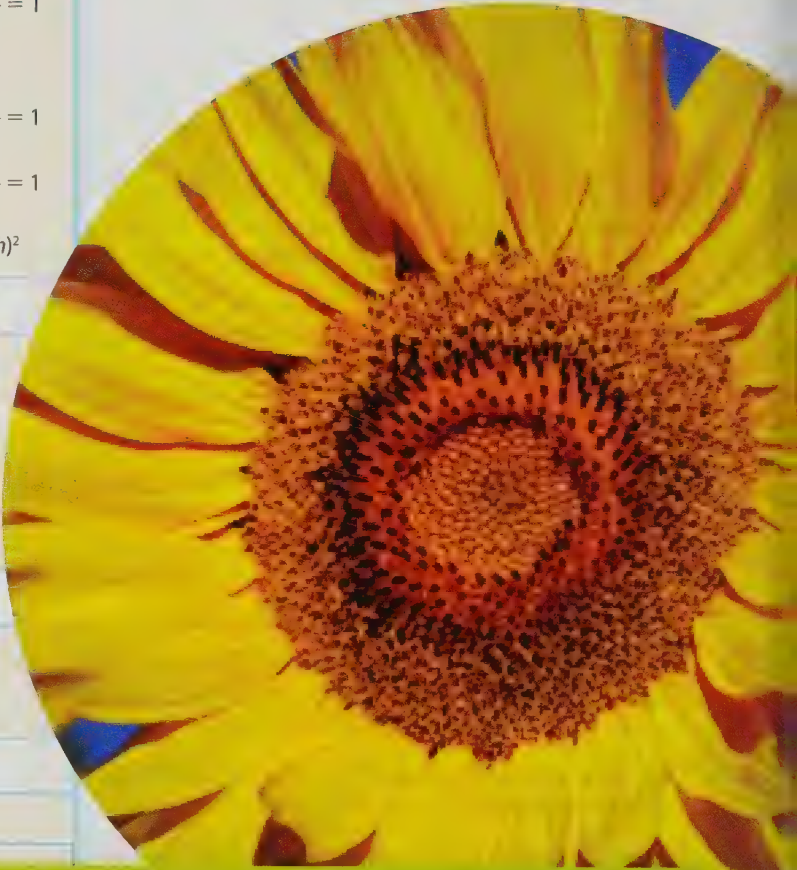
Inverse Variation	$y=\frac{k}{x}$ or $k=xy$
Direct Variation	$y=kx$, or $k=\frac{y}{x}$
Joint Variation	$y=kxz$ or $k=\frac{y}{xz}$
Power Function	$y=ax^b$

Interest

Simple Interest	$I=Prt$
Compound Interest	$A=P\left(1+\frac{r}{n}\right)^{nt}$

Exponential Models

Growth	$A(t)=a(1+r)^t$
Decay	$A(t)=a(1-r)^t$



Sequences and Series

	Arithmetic	Geometric
First term	a or a_1	a or a_1
Number of terms	n	n
Constant	difference, d	ratio, r
Sequence, explicit	$a_n=a_1+(n-1)d$	$a_n=a_1r^{n-1}$, if $r\neq 0$
Sequence, recursive	$a_n=a_{n-1}+d$	$a_n=r\cdot a_{n-1}$
Partial Sum of Sequence (Sum of finite series)	$S_n=\frac{n(a_1+a_n)}{2}$; $S_n=\frac{n[2a_1+(n-1)d]}{2}$	$S_n=\frac{a_1(1-r^n)}{1-r}$, if $r\neq 1$
Sum of Infinite Series	None	$S=\frac{a_1}{1-r}$, if $\lvert r\rvert<1$
Fibonacci sequence	$a_n=\frac{(1+\sqrt{5})^n-(1-\sqrt{5})^n}{2^n\sqrt{5}}$	

Cramer's Rule

2 equations in 2 variables	$x = \frac{\det(M_x)}{\det(M_c)} = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$ and $y = \frac{\det(M_y)}{\det(M_c)} = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$	3 equations in 3 variables	$x = \frac{\det(M_x)}{\det(M_c)}, y = \frac{\det(M_y)}{\det(M_c)}, z = \frac{\det(M_z)}{\det(M_c)}$

Properties of Logarithms

	Exponential	**For all *b***	**Common logs**	**Natural logs**
Zero	$b^0 = 1, b \neq 0$	$\log_b 1 = 0$	$\log 1 = 0$	$\ln 1 = 0$
One	$b^1 = b$	$\log_b b = 1$	$\log 10 = 1$	$\ln e = 1$
Product	$b^x \cdot b^y = b^{x+y}$	$\log_b(xy) = \log_b x + \log_b y$	$\log(xy) = \log x + \log y$	$\ln(xy) = \ln x + \ln y$
Quotient	$b^{\frac{x}{y}} = b^{x-y}, y \neq 0$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log\left(\frac{x}{y}\right) = \log x - \log y$	$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
Reciprocal	$b^{\frac{1}{y}} = b^{-y}, y \neq 0$	$\log_b\left(\frac{1}{y}\right) = -\log_b y$	$\log\left(\frac{1}{y}\right) = -\log y$	$\ln\left(\frac{1}{y}\right) = -\ln y$
Power	$(b^x)^y = b^{xy}$	$\log_b x^y = y \log_b x$	$\log x^y = y \log x$	$\ln x^y = y \ln x$
Change of Base		$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$		

Properties of Radicals

Exponents	$a^{\frac{1}{n}} = \sqrt[n]{a}, n \neq 0$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}, n \neq 0$
Product Rule	$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
Quotient Rule	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, b \neq 0$	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, b \neq 0$

Go For It!

Transformations of Functions

Reflections

$g(x) = -f(x)$	over *x*-axis
$g(x) = f(-x)$	over *y*-axis

Dilations

$g(x) = f(cx)$	horizontal $0 < c < 1$: stretch $c > 1$: shrink
$g(x) = c \cdot f(x)$	vertical $0 < c < 1$: shrink $c > 1$: stretch

Translations

$g(x) = f(x - h) + k$	$h > 0$: right *h* units
$g(x) = ab^{x-h} + k$	$h < 0$: left *h* units
$g(x) = \frac{a}{x - h} + k$	$k > 0$: up *k* units
$g(x) = \sqrt{x - h} + k$	$k < 0$: down *k* units

Advanced Algebra

Solving Word Problems

Step 1	Read the problem carefully. Determine what number or numbers need to be found.
Step 2	Represent one unknown number with a suitable variable.
Step 3	Use conditions stated in the problem to write expressions for other numbers.
Step 4	Use additional conditions to find the relationship between expressions. Write the expressions algebraically. Connect them with the appropriate sign ($=$, $<$, $>$, $\leq$, or $\geq$.)
Step 5	Solve the equation or inequality.
Step 6	Check to see if the resulting solution matches the conditions stated in the problem.

Word Problem Formulas

Distance	$d = rt$	d = distance r = rate t = time
Consecutive Number	$A = n + (n + 1)$ $B = n(n + 2)$	n = the first of two consecutive numbers $n + 1$ = the next consecutive number $n + 2$ = the next consecutive odd or even number A = sum of two consecutive numbers B = product of two consecutive odd (or even) numbers
Digit	$A(u + t + h) = u + 10t + 100h$	A = number multiplied by the sum of the digits u = the ones digit of the given number t = the tens digit of the given number h = the hundreds digit of the given number
Coin	$0.25q + 0.1d + 0.05n + 0.01p = A$	q = number of quarters d = number of dimes n = number of nickels p = number of pennies A = total worth
Lever	$\ell_1 w_1 = \ell_2 w_2$	ℓ_1 = length from fulcrum to first weight w_1 = first weight ℓ_2 = length from fulcrum to second weight w_2 = second weight
Work	$\frac{1}{a} + \frac{1}{b} = \frac{1}{n}$	a = length of time it takes person A to complete a task b = length of time it takes person B to complete a task n = length of time it would take A and B to complete the task together

Word Problem Formulas (continued)

Mixture

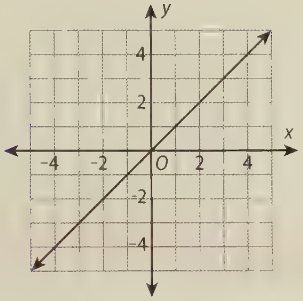
Use this table format.

Concentration	Volume	Percent Substance	Total Volume Substance
1	V	$a\%$	$0.01(a)(V)$
2	Total – V	$b\%$	$0.01(b)(\text{Total} - V)$
Final	Total	$c\%$	$0.01(c)(\text{Total})$

Parent Functions

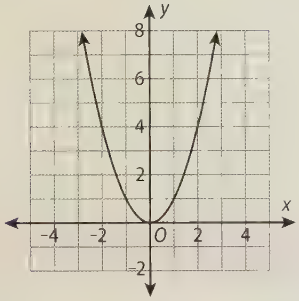
Linear (Identity)

$y = x$



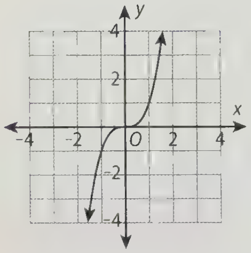
Quadratic (Squared)

$y = x^2$



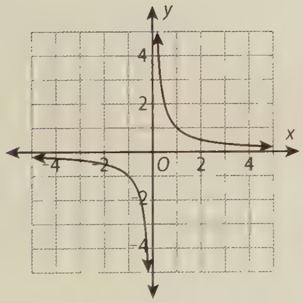
Cubic (Cubed)

$y = x^3$



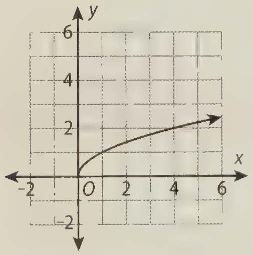
Rational (Reciprocal)

$y = \frac{1}{x}$



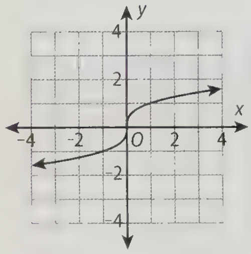
Radical (Square Root)

$y = \sqrt{x}$



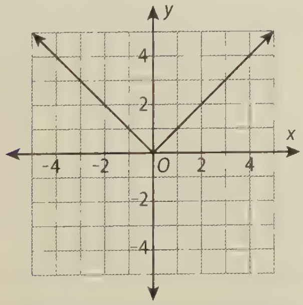
Cube Root

$y = \sqrt[3]{x}$



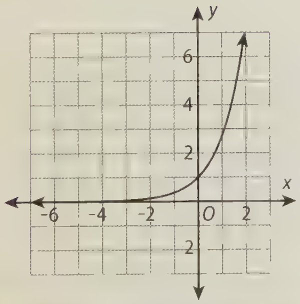
Absolute Value

$y = |x|$



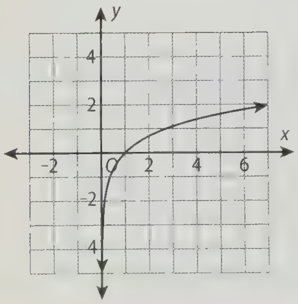
Exponential

$y = e^x$



Natural Logarithm

$y = \ln x$



Trigonometry

Trigonometric Ratios

$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$	$\cos A = \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}}$	$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A}$
$\csc A = \frac{\text{hypotenuse}}{\text{leg opposite } \angle A}$	$\sec A = \frac{\text{hypotenuse}}{\text{leg adjacent } \angle A}$	$\cot A = \frac{\text{leg adjacent } \angle A}{\text{leg opposite } \angle A}$

Trigonometric Values of the Special Angles

Degrees	Radians	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0	0	0	1	0	undefined	1	undefined
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90	$\frac{\pi}{2}$	1	0	undefined	1	undefined	0

Conversion Formulas

Degrees to Radians:	y radians $= x° \cdot \frac{\pi \text{ radians}}{180°}$	Radians to Degrees:	$y° = x$ radians $\cdot \frac{180°}{\pi \text{ radians}}$
Rectangular coordinates to Polar coordinates:		Polar coordinates to Rectangular coordinates:	
$P(x, y)$	$r = \pm\sqrt{x^2 + y^2}$ and $\cos q = \frac{x}{r}$ and $\sin q = \frac{y}{r}$	$P(r, q)$	$x = r \cos q$ and $y = r \sin q$

Rules and Properties

Inverse sine	$y = \sin^{-1} x$ if and only if $x = \sin y$
Inverse cosine	$y = \cos^{-1} x$ if and only if $x = \cos y$
Inverse tangent	$y = \tan^{-1} x$ if and only if $x = \tan y$
Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$ $\quad b^2 = a^2 + c^2 - 2ac \cos B$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$
DeMoivre's Theorem	Let $z = r \cos \theta + (r \sin \theta)i$ and n be a positive integer. $\quad z^n = r^n \cos(n\theta) + (r^n \sin(n\theta))i$
Magnitude of a Vector in the Plane	$\lVert \overline{PQ} \rVert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Angle Between Two Vectors	$\mathbf{u} \cdot \mathbf{v} = \lVert \mathbf{u} \rVert \, \lVert \mathbf{v} \rVert \cos \theta$
Dot Product of Two Vectors	$\mathbf{u} \cdot \mathbf{v} = x_1 x_2 + y_1 y_2$

Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Negative Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta} \qquad \sec\theta = \pm\sqrt{1 + \tan^2\theta} \qquad \csc\theta = \pm\sqrt{1 + \cot^2\theta}$$

Cofunction Identities

$$\sin\theta = \cos(90^\circ - \theta) \qquad \tan\theta = \cot(90^\circ - \theta) \qquad \sec\theta = \csc(90^\circ - \theta)$$

$$\cos\theta = \sin(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

Coterminal Identities

$$\sin\theta = \sin(360^\circ + \theta) \qquad \cos\theta = \cos(360^\circ + \theta) \qquad \tan\theta = \tan(360^\circ + \theta)$$

$$\csc\theta = \csc(360^\circ + \theta) \qquad \sec\theta = \sec(360^\circ + \theta) \qquad \cot\theta = \cot(360^\circ + \theta)$$

Sum and Difference Identities

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Identities

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

Identities (continued)

Half-Angle Identities

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}},\ \cos\alpha \neq -1$$

$$\tan\frac{\alpha}{2} = \frac{1 - \cos\alpha}{\sin\alpha},\ \sin\alpha \neq 0$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1 + \cos\alpha},\ \cos\alpha \neq -1$$

Product-to-Sum Identities

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum-to-Product Identities

$$\sin x + \sin y = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

Trigonometry

Trigonometric Functions

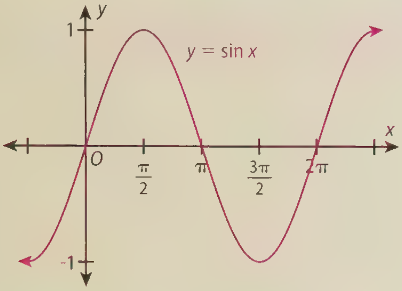
Angle	**Radians**	**Sine**	**Cosine**	**Tangent**	**Cotangent**	**Secant**	**Cosecant**		
0°	0.0000	0.0000	1.0000	0.0000	undefined	1.0000	undefined	1.5708	**90°**
1	0.0175	0.0175	0.9998	0.0175	57.2900	1.0002	57.2987	1.5533	**89**
2	0.0349	0.0349	0.9994	0.0349	28.6363	1.0006	28.6537	1.5359	**88**
3	0.0524	0.0523	0.9986	0.0524	19.0811	1.0014	19.1073	1.5184	**87**
4	0.0698	0.0698	0.9976	0.0699	14.3007	1.0024	14.3356	1.5010	**86**
5	0.0873	0.0872	0.9962	0.0875	11.4301	1.0038	11.4737	1.4835	**85**
6	0.1047	0.1045	0.9945	0.1051	9.5144	1.0055	9.5668	1.4661	**84**
7	0.1222	0.1219	0.9925	0.1228	8.1443	1.0075	8.2055	1.4486	**83**
8	0.1396	0.1392	0.9903	0.1405	7.1154	1.0098	7.1853	1.4312	**82**
9	0.1571	0.1564	0.9877	0.1584	6.3138	1.0125	6.3925	1.4137	**81**
10	0.1745	0.1736	0.9848	0.1763	5.6713	1.0154	5.7588	1.3963	**80**
11	0.1920	0.1908	0.9816	0.1944	5.1446	1.0187	5.2408	1.3788	**79**
12	0.2094	0.2079	0.9781	0.2126	4.7046	1.0223	4.8097	1.3614	**78**
13	0.2269	0.2250	0.9744	0.2309	4.3315	1.0263	4.4454	1.3439	**77**
14	0.2443	0.2419	0.9703	0.2493	4.0108	1.0306	4.1336	1.3265	**76**
15	0.2618	0.2588	0.9659	0.2679	3.7321	1.0353	3.8637	1.3090	**75**
16	0.2793	0.2756	0.9613	0.2867	3.4874	1.0403	3.6280	1.2915	**74**
17	0.2967	0.2924	0.9563	0.3057	3.2709	1.0457	3.4203	1.2714	**73**
18	0.3142	0.3090	0.9511	0.3249	3.0777	1.0515	3.2361	1.2566	**72**
19	0.3316	0.3256	0.9455	0.3443	2.9042	1.0576	3.0716	1.2392	**71**
20	0.3491	0.3420	0.9397	0.3640	2.7475	1.0642	2.9238	1.2217	**70**
21	0.3665	0.3584	0.9336	0.3839	2.6051	1.0711	2.7904	1.2043	**69**
22	0.3840	0.3746	0.9272	0.4040	2.4751	1.0785	2.6695	1.1868	**68**
23	0.4014	0.3907	0.9205	0.4245	2.3559	1.0864	2.5593	1.1694	**67**
24	0.4189	0.4067	0.9135	0.4452	2.2460	1.0946	2.4586	1.1519	**66**
25	0.4363	0.4226	0.9063	0.4663	2.1445	1.1034	2.3662	1.1345	**65**
26	0.4538	0.4384	0.8988	0.4877	2.0503	1.1126	2.2812	1.1170	**64**
27	0.4712	0.4540	0.8910	0.5095	1.9626	1.1223	2.2027	1.0996	**63**
28	0.4887	0.4695	0.8829	0.5317	1.8807	1.1326	2.1301	1.0821	**62**
29	0.5061	0.4848	0.8746	0.5543	1.8040	1.1434	2.0627	1.0647	**61**
30	0.5236	0.5000	0.8660	0.5774	1.7321	1.1547	2.0000	1.0472	**60**
31	0.5411	0.5150	0.8572	0.6009	1.6643	1.1666	1.9416	1.0297	**59**
32	0.5585	0.5299	0.8480	0.6249	1.6003	1.1792	1.8871	1.0123	**58**
33	0.5760	0.5446	0.8387	0.6494	1.5399	1.1924	1.8361	0.9948	**57**
34	0.5934	0.5592	0.8290	0.6745	1.4826	1.2062	1.7883	0.9774	**56**
35	0.6109	0.5736	0.8192	0.7002	1.4281	1.2208	1.7434	0.9599	**55**
36	0.6283	0.5878	0.8090	0.7265	1.3764	1.2361	1.7013	0.9425	**54**
37	0.6458	0.6018	0.7986	0.7536	1.3270	1.2521	1.6616	0.9250	**53**
38	0.6632	0.6157	0.7880	0.7813	1.2799	1.2690	1.6243	0.9076	**52**
39	0.6807	0.6293	0.7771	0.8098	1.2349	1.2868	1.5890	0.8901	**51**
40	0.6981	0.6428	0.7660	0.8391	1.1918	1.3054	1.5557	0.8727	**50**
41	0.7156	0.6561	0.7547	0.8693	1.1504	1.3250	1.5243	0.8552	**49**
42	0.7330	0.6691	0.7431	0.9004	1.1106	1.3456	1.4945	0.8378	**48**
43	0.7505	0.6820	0.7314	0.9325	1.0724	1.3673	1.4663	0.8203	**47**
44	0.7679	0.6947	0.7093	0.9657	1.0355	1.3902	1.4396	0.8029	**46**
45°	0.7854	0.7071	0.7071	1.0000	1.0000	1.4142	1.4142	0.7854	**45°**
		Cosine	**Sine**	**Cotangent**	**Tangent**	**Cosecant**	**Secant**	**Radians**	**Angle**

Parent Functions

Sine

$y = \sin x$

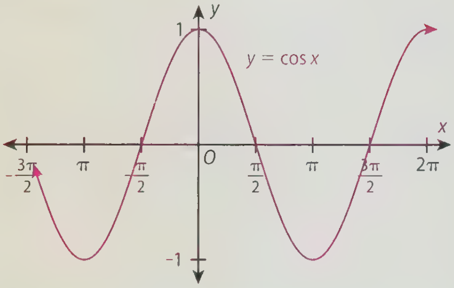
period $= 2\pi$; amplitude $= 1$



Cosine

$y = \cos x$

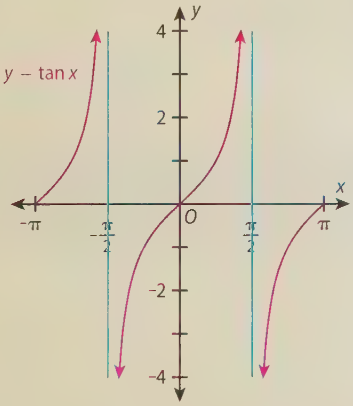
period $= 2\pi$; amplitude $= 1$



Tangent

$y = \tan x$

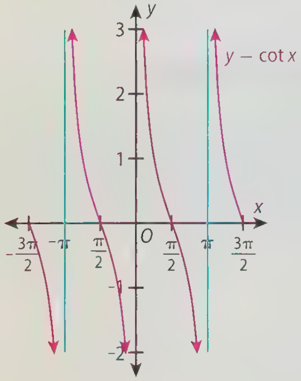
period $= \pi$; asymptotes: $x = \frac{\pi}{2} + k\pi$



Cotangent

$y = \cot x$

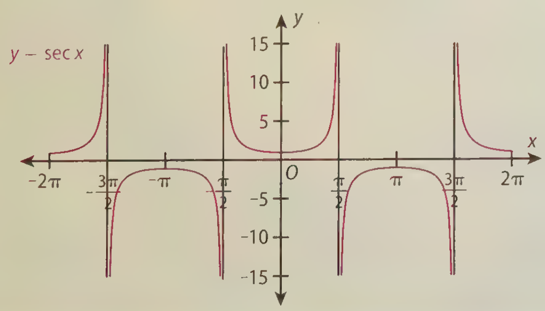
period $= \pi$; asymptotes: $x = k\pi$



Secant

$y = \sec x$

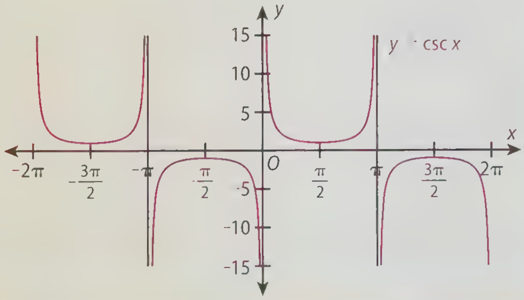
period $= \pi$; asymptotes: $x = \frac{\pi}{2} + k\pi$



Cosecant

$y = \csc x$

period $= \pi$; asymptotes: $x = k\pi$



Calculus

Limit Rules

Constant Rule	$\lim_{x\to c} k = k$
Constant Multiplier Rule	$\lim_{x\to c} kf(x) = k\lim_{x\to c} f(x)$
Sum Rule	$\lim_{x\to c} [f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
Difference Rule	$\lim_{x\to c} [f(x) - g(x)] = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$
Product Rule	$\lim_{x\to c} [f(x) \cdot g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$
Quotient Rule	$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, if $\lim_{x\to c} g(x) \neq 0$
Power Rule	$\lim_{x\to c} [f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$
*n*th Root Rule	$\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$, if $\lim_{x\to c} f(x) > 0$
Limits of Radicals	$\lim_{x\to c} \sqrt[n]{x} = \sqrt[n]{c}$, for all c when n is odd, and for $c > 0$ when n is even.
Limits of Composite Functions	If $\lim_{x\to c} g(x) = L$ and $\lim_{x\to c} f(x) = f(L)$, then $f(g(x)) = f\left(\lim_{x\to c} g(x)\right) = f(L)$.

For Derivative Rules, see the table of Derivatives on the next page, or go to pages 2170 and 2172 for constant, sum, difference, power, product, and quotient rules.

For Integral Rules, see the table of Integrals on the next page, or go to page 2234 for constant, constant multiplier, sum, difference, and power rules.

Volume of a Solid of Revolution

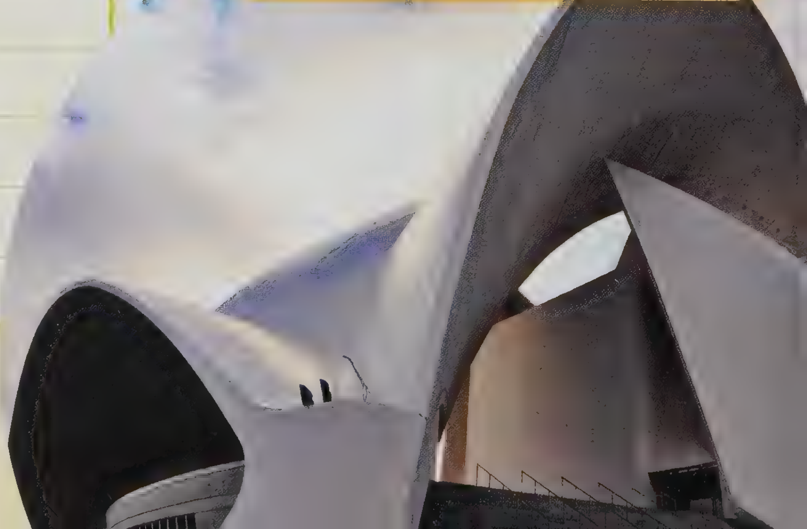
Disk Method (*x*-axis)	$V = \pi\int_a^b [R(x)]^2\,dx$
Disk Method (*y*-axis)	$V = \pi\int_a^b [R(y)]^2\,dy$
Washer Method (*x*-axis)	$V = \pi\int_a^b (f(x))^2 - (g(x))^2\,dx$
Washer Method (*y*-axis)	$V = \pi\int_a^b (f(y))^2 - (g(y))^2\,dy$
Shell Method (*x*-axis)	$V = 2\pi\int_a^b yf(y)dx$
Shell Method (*y*-axis)	$V = 2\pi\int_a^b xf(x)dx$

Formulas, Theorems, and Definitions

Special Trigonometric Limits	$\lim_{a\to 0} \frac{\sin a}{a} = 1;\ \lim_{a\to 0} \frac{1 - \cos a}{a} = 0$
Definition of a Derivative	$f'(x) = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
The Mean Value Theorem	$f'(c) = \frac{f(b) - f(a)}{b - a}$
Area Between Two Curves	$A = \int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]dx$
Fundamental Theorem of Calculus	$\int_a^b f(x)dx = F(b) - F(a)$
Second Fundamental Theorem of Calculus	$\frac{d}{dx}\int_a^x f(t)dt = f(x)$
The Mean Value Theorem for Integrals	$\int_a^b f(x)dx = f(c)(b - a)$
Average Value of a Function	$f(c) = \frac{1}{b - a}\int_a^b f(x)dx$
Integration by Parts	$\int u\,dv = uv - \int v\,du$

Volumes of Solids with Known Cross Sections

Square	$V = \int (\text{base})^2\,dx$
Semicircle	$V = \frac{\pi}{8}\int (\text{base})^2\,dx$
Right Isosceles Triangle	$V = \frac{1}{2}\int (\text{base})^2\,dx$
Equilateral Triangle	$V = \frac{\sqrt{3}}{4}\int (\text{base})^2\,dx$



Derivatives

Function, f	Derivative, $\frac{df}{dx}$	Function, f	Derivative, $\frac{df}{dx}$	Function, f	Derivative, $\frac{df}{dx}$
c	0	$\frac{u}{v}$	$\frac{(vu' - uv')}{v^2}$	$\arctan x$	$\frac{1}{(1 + x^2)}$
x	1	$f[u(x)]$	$\frac{df}{du} \cdot \frac{du}{dx}$	$\operatorname{arc\,cot} x$	$-\frac{1}{(1 + x^2)}$
kx	k	$\sin x$	$\cos x$	$\operatorname{arc\,sec} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2 - 1}}$
x^n	nx^{n-1}	$\cos x$	$-\sin x$	$\operatorname{arc\,csc} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2 - 1}}$
$k[g(x)]$	$kg'(x)$	$\tan x$	$\sec^2 x$	e^x	e^x
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\cot x$	$-\csc^2 x$	a^x	$a^x \ln a$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$	$\sec x$	$\sec x \tan x$	$\ln x$	$\frac{1}{x}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\csc x$	$\csc x \cot x$	$\log_a x$	$\frac{1}{x \ln a}$
$u + v$	$u' + v'$	$\arcsin x$	$\frac{1}{\sqrt{1 - x^2}}$	$f^{-1}(x)$	$\frac{1}{\frac{d}{dx}(f(f^{-1}(x)))}$
uv	$uv' + vu'$	$\arccos x$	$-\frac{1}{\sqrt{1 - x^2}}$		

Integrals

Function, f	Integral, $\int f\,dx$	Function, f	Integral, $\int f\,dx$	Function, f	Integral, $\int f\,dx$
k	kx	e^x	e^x	$\sec^2 x$	$\tan x$
x	$\frac{1}{2}x^2$	$\ln x$	$x \ln x - x$	$\csc^2 x$	$-\cot x$
x^n	$\frac{x^{n+1}}{(n+1)}$ $n \neq -1$	$\sin x$	$-\cos x$	$\sec x \tan x$	$\sec x$
$\frac{1}{x}$	$\ln x$	$\cos x$	$\sin x$	$\csc x \cot x$	$-\csc x$
$\frac{1}{x^n}$	$-\frac{1}{(n-1)x^{n-1}}$ $n \neq 1$	$\tan x$	$\ln \sec x$	$\frac{1}{(ax + b)}$	$\left(\frac{1}{a}\right)\ln(ax + b)$
$\sqrt{x}$	$\frac{2}{3}x^{\frac{3}{2}}$	$\cot x$	$\ln \sin x$	$\frac{1}{(x^2 + a^2)}$	$\left(\frac{1}{a}\right)\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x}}$	$2\sqrt{x}$	$\sec x$	$\ln(\tan x + \sec x)$	$\frac{1}{(x^2 - a^2)}$	$\left(\frac{1}{2a}\right)\ln \frac{(y - a)}{(x + a)}$
$x^{\frac{n}{r}}$	$\frac{rx^{\left(\frac{n}{r}\right)+1}}{r + n}$	$\csc x$	$\ln(\csc x - \cot x)$	$\frac{1}{\sqrt{(a^2 - x^2)}}$	$\arcsin\left(\frac{x}{a}\right)$
$x^{-\frac{n}{r}}$	$\frac{r}{(r - n)x^{\left(\frac{n}{r}\right)-1}}$	$\sin^2 x$	$\frac{1}{2}x - \frac{1}{2}\sin x \cos x$	$\frac{1}{\sqrt{(x^2 \pm a^2)}}$	$\ln\left(x + \sqrt{(x^2 \pm a^2)}\right)$

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SCIENCE

SOUTHWESTERN
ADVANTAGE

Southwestern Advantage

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Preface

Welcome to *Southwestern Advantage Science.* We are pleased to bring you this unique, user-friendly reference book. It has been designed in such a way that students can spend "more time learning, less time looking." The pages are open and inviting and organized into information boxes, bulleted lists, and other easily usable and understandable pieces.

How Does That Work boxes contain capsule summaries of particular processes or other information. **The Basics** boxes outline information essential to the topic at hand. **How Do We Know That** boxes give brief recaps of how scientists discovered certain things or formulated or proved various theories. **FYI** boxes give additional small nuggets or bits of information.

Students (or their parents) can also go to SWAdvantage.com, where they will find additional, more in-depth information on a wide range of subject matter.

We hope you will find this book both useful and enjoyable. Every effort has been made to ensure that the information in this book is as accurate as possible. If errors should be found, however, we would appreciate hearing from you. Please share your comments or suggestions with us at 1-888-551-5901 or send them to Editor c/o Southwestern Advantage, P.O. Box 305142, Nashville, TN 37230.

How to Use Southwestern Advantage

How to Use This Book

This book has been designed so that information can be accessed easily. Science has been divided into six "strands": Biology; Chemistry; Earth Science; Ecology; Physics; and Space Science. Each strand is then divided into smaller units.

The first navigational tool is the detailed, color-coded Table of Contents. The contents pages also indicate separately where the special features of the book can be found, such as the Table of Physics Formulas and Constants and the Space Science time line.

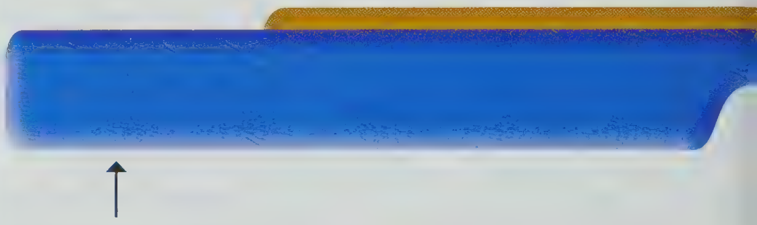
BIOLOGY

CHEMISTRY

EARTH SCIENCE

Next, above the heading on the right-hand text pages, you will see color bars that tell you exactly where you are in the book. The bar that extends all the way to the edge of the page is the color of the unit you are in; the other bar denotes the strand you are in. The strand color is repeated in a tab at the bottom of the page. When the book is closed, you can tell at a glance where each strand and unit begins and ends.

When the book is open, headings on the pages also help to tell you exactly where you are in the book, for example, the Life section of Biology.



Strand color bar

fyi! *The human body has more than 10 trillion cells!*

How to Use Southwestern Advantage Online (SWAdvantage.com)

An integral part of Southwestern Advantage is the accompanying Web site. Organized by subject areas, it is a comprehensive suite of online study helps, additional in-depth subject matter, tips for parents, and coaching for students on how to get better at life.

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Cells LIFE BIOLOGY

Unit within Biology

Name of strand Biology strand

Unit color bar

Spotlight on... GREGOR JOHANN MENDEL

1822–1884

Gregor Johann Mendel was an Austrian botanist and monk who formulated the basic laws of heredity; he

- used the gardens of the abbey to pursue his passion for studying nature

THE basics

CELLS

All cells have certain characteristics in common.

✔ cells are alive—as alive as you are

HOW DOES THAT WORK ?

RNA IS FORMED

Messenger RNA is a copy of the DNA blueprint for the polypeptide chain of the protein to be produced. mRNA forms in three steps:

1. First, the DNA splits lengthwise between its bases. Half the ladder serves as a mold to form mRNA.

FRANKENFOODS

Today, scientists genetically modify a variety of crops and animals, called *GMOs* (genetically modified organisms).

Features

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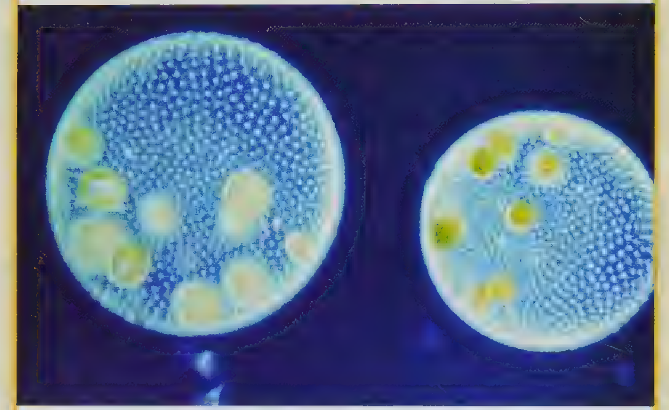

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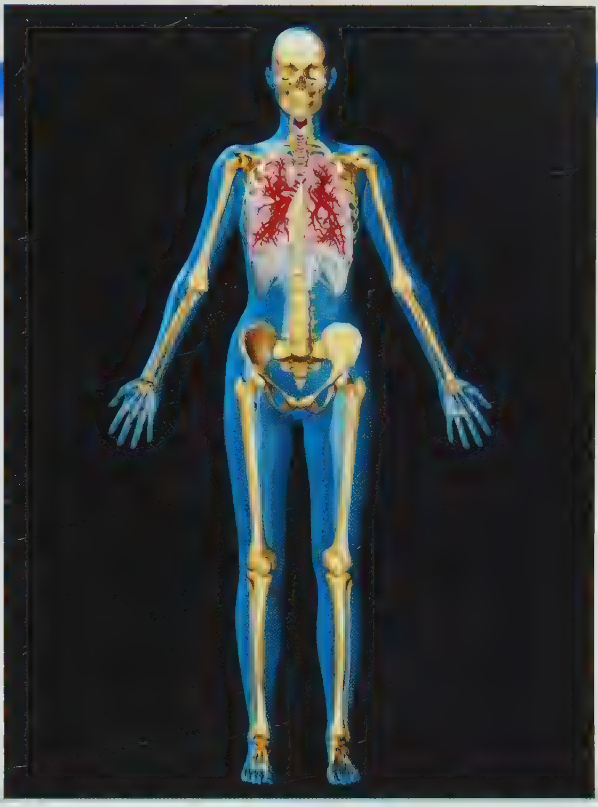

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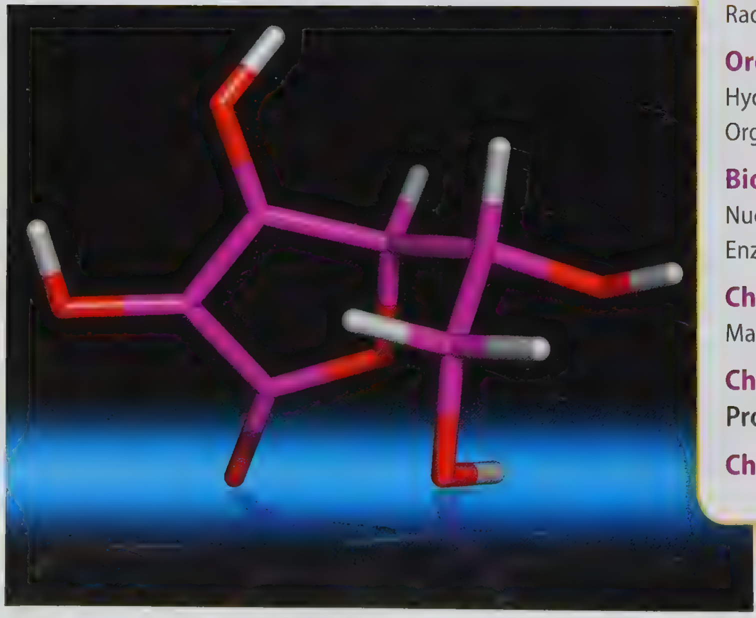

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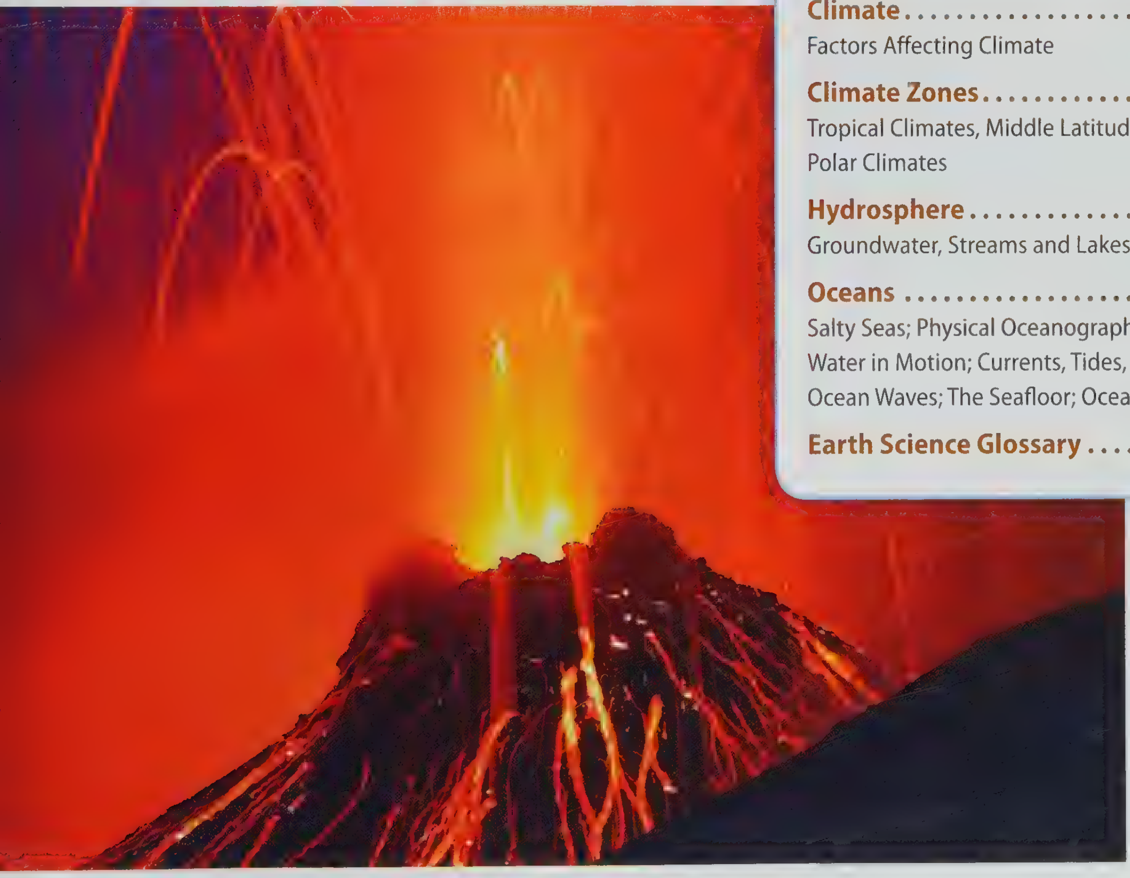

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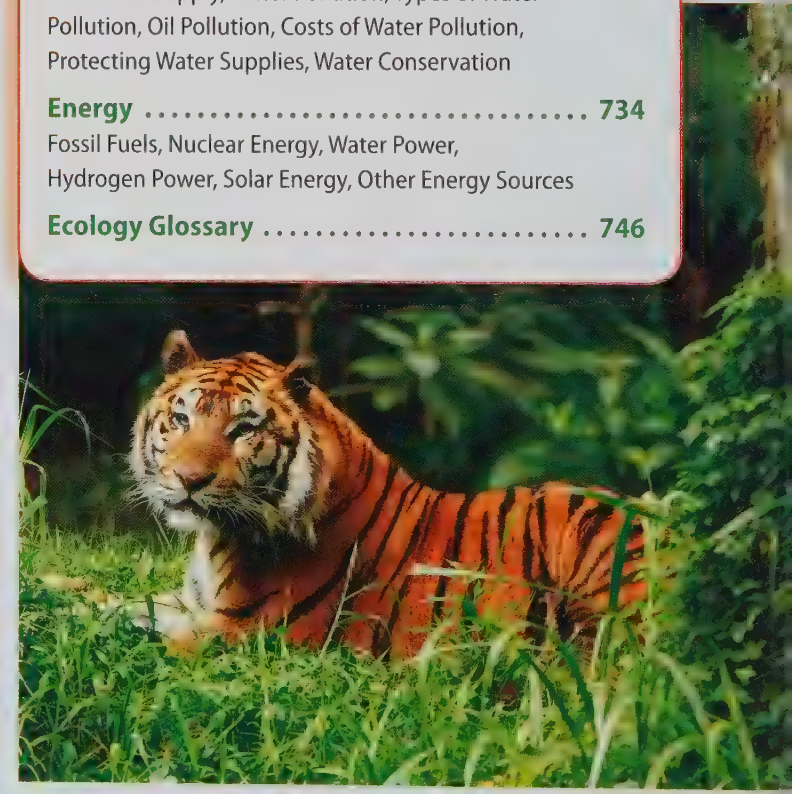

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Science Advantage

Biology

Chemistry

Earth Science

Ecology

Physics

Space Science

Scientific Methods

Scientists use a number of methods in making discoveries and in developing theories. Most scientific research involves some or all of these methods. They include the following:

1. **observing nature**
2. **classifying data**
3. **using logic**
4. **conducting experiments**
5. **forming a hypothesis (proposed explanation)**
6. **expressing findings mathematically**
7. **modeling with computers**

HAPPY ACCIDENT

It's not an official method, but even chance can play a role in the scientific process. Sir Alexander Fleming, a British bacteriologist, discovered penicillin accidentally in 1928, when he noticed that a bit of mold of the genus *Penicillium* had contaminated a laboratory dish containing bacteria. Examining the dish, Fleming saw that the bacteria around the mold had been killed.

Observing Nature

Observing nature is one of the oldest scientific methods. For example, the ancient Egyptians and Babylonians studied the motions of heavenly bodies and so learned to predict the changes of seasons and the best times to plant and harvest crops.

In the 1830s, Charles Darwin carefully observed plants and animals in many parts of the world while serving as a naturalist with a British scientific expedition aboard the H.M.S. *Beagle.* Study of the specimens collected on the voyage helped Darwin develop his theory that modern species had evolved from a few earlier ones. (See pages 36–39.)

Classifying Data

The relationships among observed facts can be revealed through the classification of data. In the mid-1800s, Dmitri Mendeleev, a Russian chemist, classified the elements into families or groups in a chart called the *periodic table.* On the table, elements with similar properties appeared at regular intervals. Gaps in the table indicated elements that were not yet known. Scientists later proved the importance of Mendeleev's systematic classification when they discovered the existence and chemical properties of new elements that filled the gaps. (See pages 320–323.)

The ancient Egyptians carefully observed the motions of the objects in the sky.

The Periodic Table

12.0111
$1s^2 2s^2 2p^2$
15
280
44.1
1.82
P
Phosphorus
30.9738
$(Ne) 3s^2 3p$
$1s^2 2s^2 2p^1$
14
3267
1414
2.33
Si
Silicon
28.086
$(Ne) 3s^2 3p^2$
2520
660
2.70
Al
Aluminum
26.9815
$(Ne) 3s^2 3p^1$
33
As
Arsenic
74.922
31
2205
29.8
5.91
Ga
Gallium
69.72
$(Ar) 3d^{10} 4s^2 4p^1$
32
2834
938.4
5.32
Ge
Germanium
72.59
$(Ar) 3d^{10} 4s^2 4p^2$
51
49
2073
157
7.31
In
Indium
767
321.11
50
2603
232
7.30
Sn
Tin
118.69
$(Kr) 4d^{10} 5s^2 5p^2$

Using Logic

Scientists use logic to draw conclusions from existing information. In the late 1800s, a German physicist named Wilhelm Wien studied the relationship between temperature and the energy radiated by heated solids and liquids. After studying many specific examples, he noted that multiplying the temperature of a heated solid or liquid by the wavelength of greatest intensity radiated at that temperature always produced the same number. Although Wien could not test all solids and liquids, he used inductive reasoning (reasoning from specific cases to a general rule) to conclude that this number was a universal constant which was the same for all heated solids and liquids, regardless of their physical or chemical makeup.

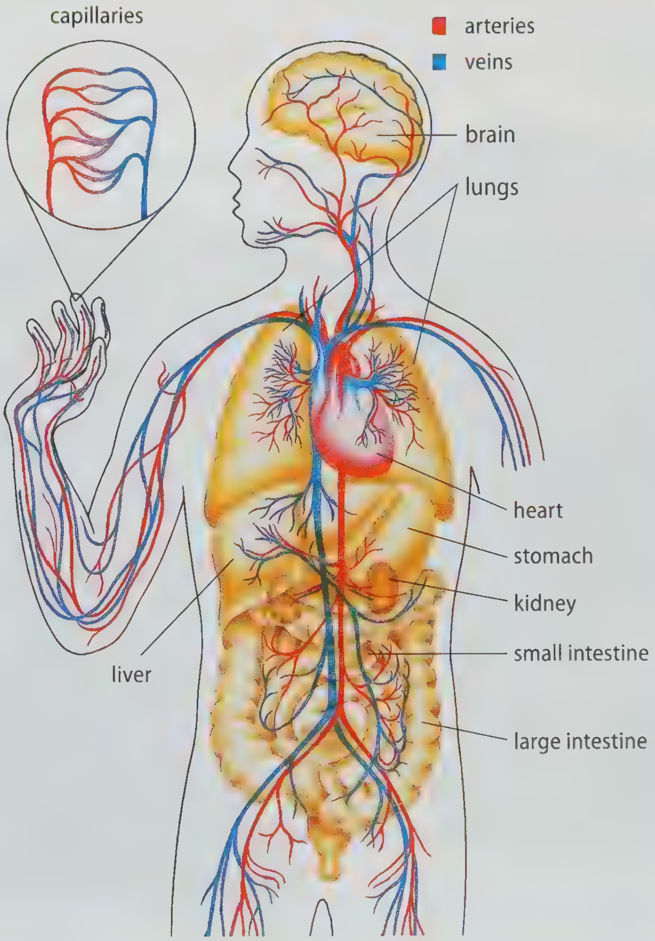
Conducting Experiments

Conducting experiments is one of the most important tools in developing and testing scientific theories. The Italian astronomer and physicist Galileo was one of the first scientists to recognize that systematic experimentation could help reveal the laws of nature. During the late 1500s, Galileo began performing carefully designed experiments to study the basic properties of matter in motion. By rolling balls of different weights down inclined planes, Galileo discovered that all objects fall to the ground with the same acceleration (rate of increase in speed), unless air resistance or some other force slows them down.

AN EXPERIMENT WITH HEART

In the early 1600s, William Harvey, an English physician, used the experimental method to learn how blood circulates through the body.

Harvey made careful studies of the human pulsebeat and heartbeat and dissected (cut up) human and animal corpses for examination. Harvey concluded that the heart pumps blood through the arteries to all parts of the body and that the blood returns to the heart through the veins.



The Circulatory System ▶

Scientific Methods

Forming a Hypothesis

A *hypothesis* is a proposed explanation for a scientific question. Forming a hypothesis requires talent, skill, and creativity. Scientists base their proposed explanations on existing information. They strive to form hypotheses that help explain, order, or unify related facts. They then use experimentation and other means to test their hypotheses.

The discovery of the planet Neptune in the mid-1800s resulted from the formation of a hypothesis. Astronomers noticed that Uranus, which they thought was the most distant planet, was not always in the position predicted for it by the laws of gravitation and motion.

Some astronomers concluded that the laws did not hold at such great distances from the sun. But other astronomers hypothesized that the force of gravity from an unknown planet might cause the variations in the orbit of Uranus. By calculating where such a planet would have to be located to affect the orbit, astronomers eventually discovered Neptune. (See page 932.)

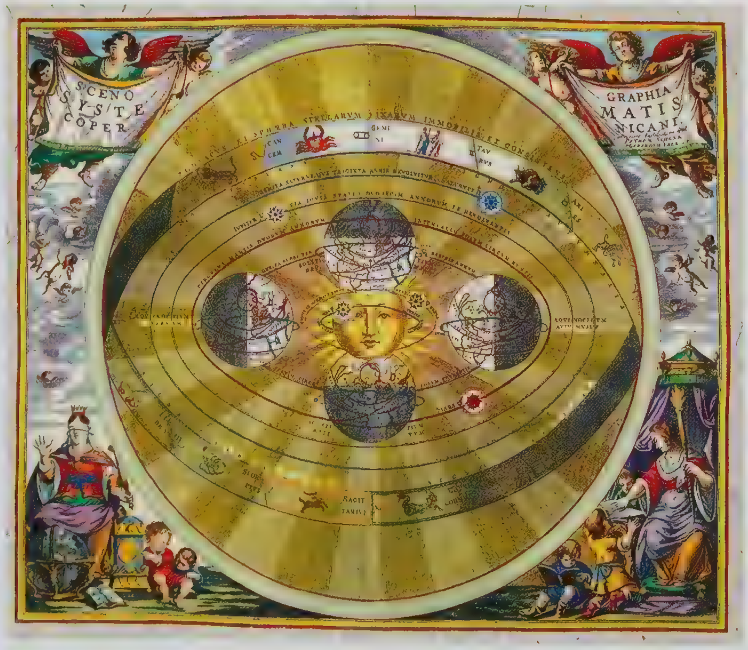
Expressing Findings Mathematically

Expressing findings mathematically can yield valuable insights about how the world works. Galileo used mathematics to express the results of his experiments with falling bodies and to enable him to determine the distance an object would fall in a certain amount of time.

The English scientist Sir Isaac Newton developed a mathematical theory of gravitation in the 1600s that explained many types of motion, both on Earth and throughout the universe. (See pages 762–763.)

fyi!

In addition to the development of the law of universal gravitation, Isaac Newton also discovered that sunlight is a mixture of light of all colors and invented the branch of mathematics called the calculus.



▲ **The heliocentric theory** was published by Copernicus in 1543.

WHAT IS A THEORY?

In our everyday use of the term, *theory* means something unproven, perhaps an educated guess. When scientists use the term *theory,* they mean something quite different.

In science, a theory is a set of statements or concepts based on a vast number of observations about nature that explains many related facts. When a set of concepts has withstood many scientific challenges and has solid evidence supporting it, scientists begin to refer to those concepts as a theory.

Like all scientific knowledge, theories may be refined by future findings or discoveries. A scientific theory represents the most reasonable explanation science has to offer about how nature works.

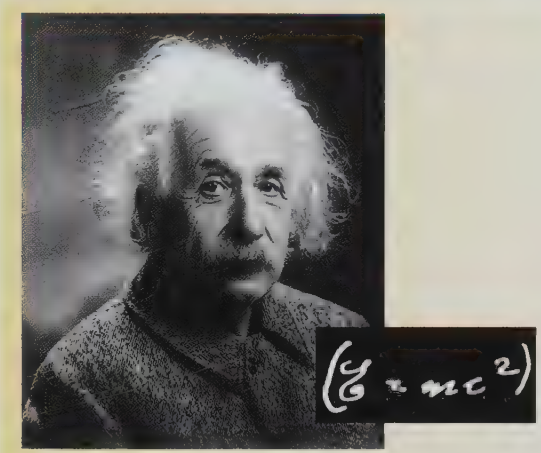
One important criterion for a theory is that it must be testable—that is, we can specify what observations or experiments would show that the theory is false. Theories also help scientists find fruitful new questions to investigate.

Modeling with Computers

Modeling with computers helps scientists quickly analyze large amounts of data. A *model* is a set of mathematical equations that describes relationships between data. In the past, scientists computed these equations on paper or with a calculator. Many models were too difficult or time-consuming to attempt. But the development of highly powerful computers in the late 1970s enabled scientists to formulate complex models at great speeds.

Using computer models, scientists can easily vary data to test scientific hypotheses. This use of a model is known as *simulation*. Scientists commonly simulate experiments that would be impossible to carry out in a laboratory.

For example, meteorologists simulate the development of thunderstorms to test how changes in atmospheric pressure affect cloud movement. An engineer may simulate an airplane's flight to find ways of improving its design. Simulations are also used to predict voting results, population growth, and stock market prices.



EUREKA!

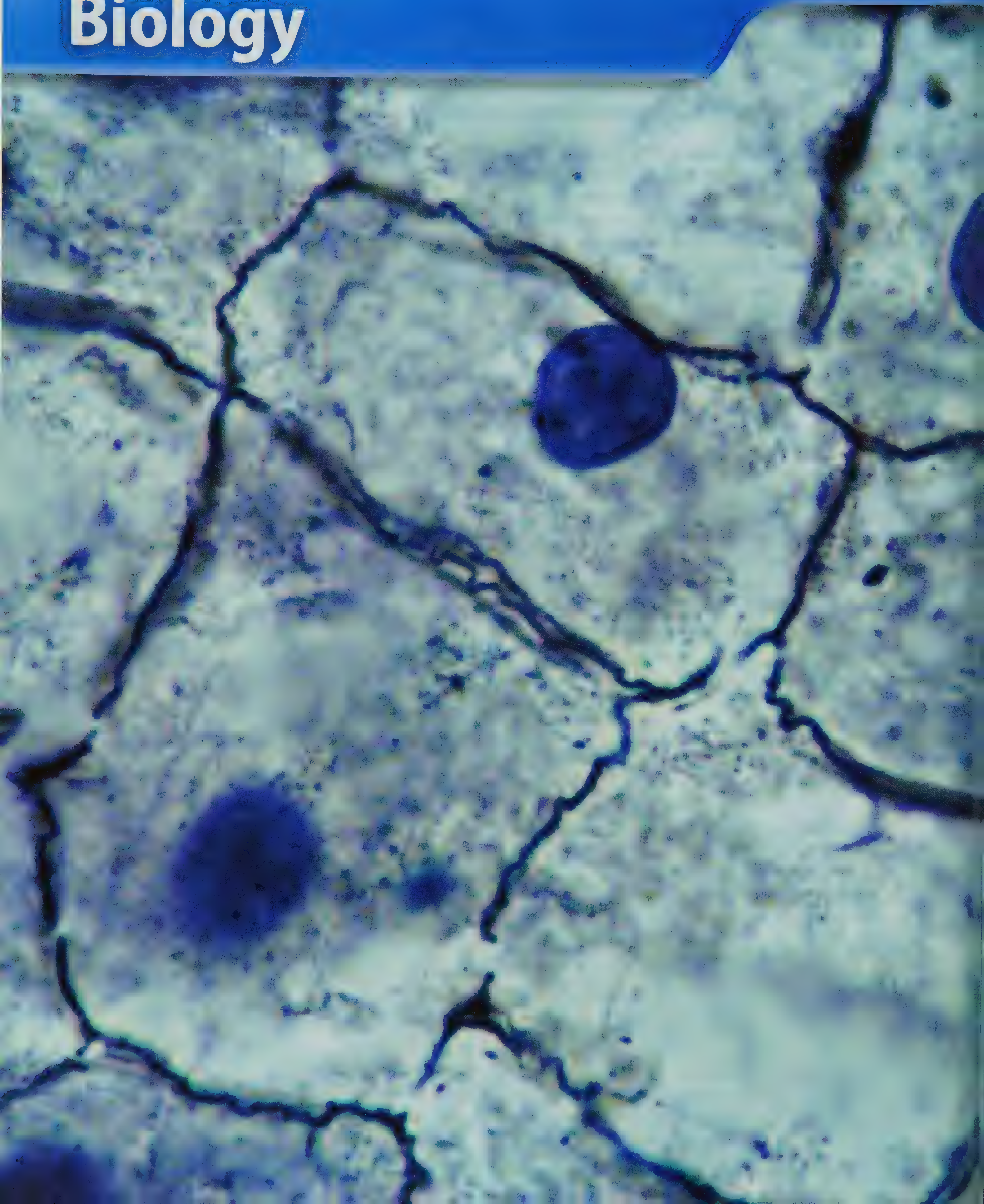
In the early 1900s, the German-born physicist Albert Einstein found that mass is related to energy by the mathematical equation $E=mc^2$. The equation states that energy (E) is equivalent to mass (m) multiplied by the speed of light squared (c^2). This equation later provided the basis for the development of nuclear energy. (See page 853.)

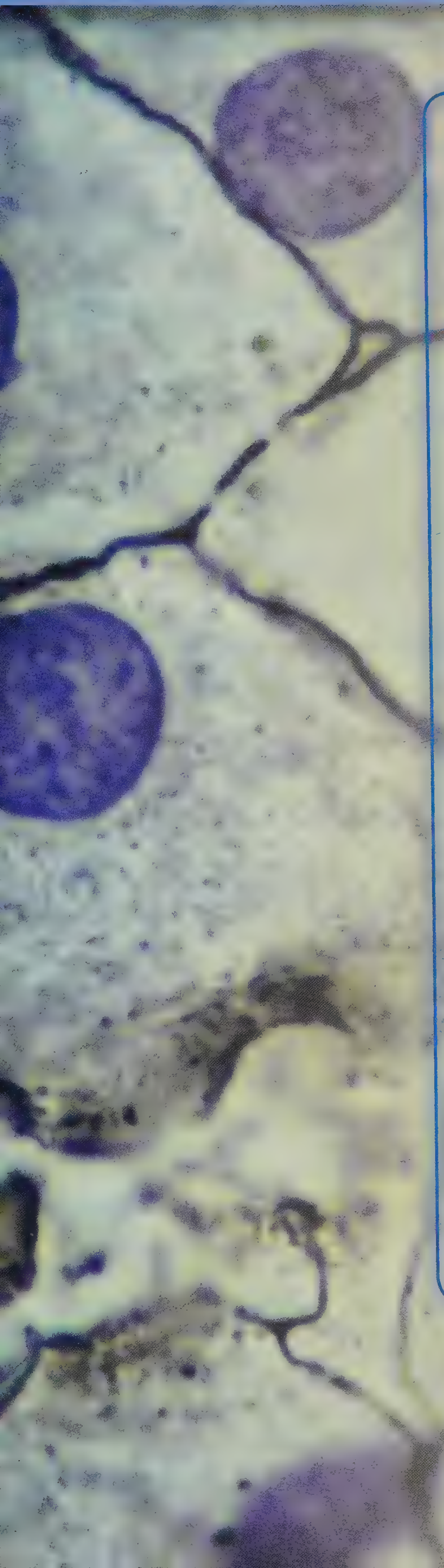
Examples of successful scientific theories include the *heliocentric theory* (the theory that the sun is the center of the solar system), the *germ theory of disease* (the theory that diseases can be caused by microorganisms), and *cell theory* (the theory that the cell is the basic structural and functional unit of all living things).

The Germ Theory. Advancements in the mid-1800s led to the germ theory of disease. ▶

▲ **Cell theory** was developed in the 1830s.

Biology





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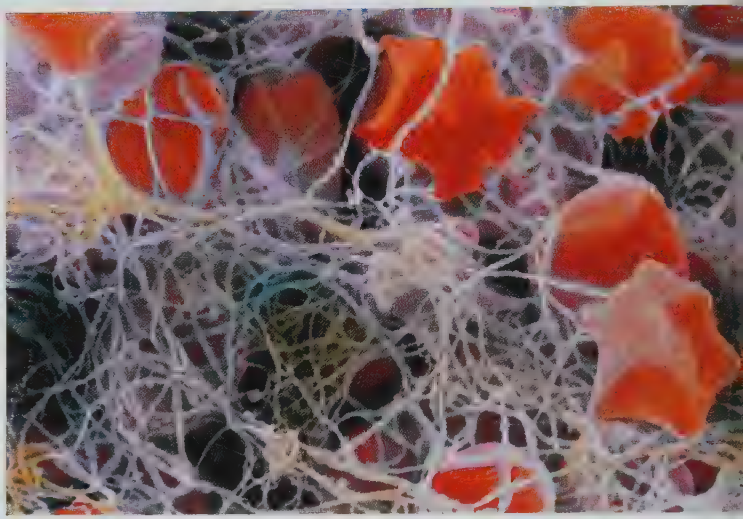
LIFE ON EARTH

Origins and Development

Throughout history people have attempted to explain how and why the universe, Earth, and life exist. Explanations include both natural and supernatural phenomena. Several of these explanations are explored in the following pages.

Below are summaries of these explanations: scientific theories on how life originated on Earth, the theory of evolution, creationism, and intelligent design.



Theories on the Origin of Life

Most scientists favor the idea that life originated on Earth through a process of chemical evolution: simple organic compounds formed naturally early in Earth's history.

These simple compounds combined with one another to form the complex structures that make up living things. There are several theories for how chemical evolution may have taken place.

Oparin-Haldane Hypothesis. States that the first living organism arose out of large quantities of organic chemicals that were present in the oceans of the primitive Earth.

Miller-Urey Experiments. Re-created the Oparin-Haldane model of Earth's early atmosphere in a laboratory. The experiments resulted in the creation of amino acids that are necessary for life.

Panspermia. States that life began elsewhere in the solar system or universe and was carried to Earth from outer space, on a comet or meteorite.

Clay-Life Hypothesis. States that life developed in deposits of clay.

The Theory of Evolution

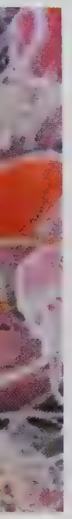
Evolution is a process of change over time. The basic idea behind the theory of evolution states that species undergo changes in their inherited characteristics over time.

Causes. Mutations produce random variation in species, causing permanent changes in the hereditary material of organisms. Through a process called natural selection, the organisms best suited to their environment are most likely to leave offspring.

New Species. New species evolve based on three factors.

1. Reproductive isolating factors, which are biological factors that prevent different species from mating.
2. Geographically isolated groups, which evolve differently because their environments differ and different mutations occur.
3. Rapid speciation, which often occurs when a population settles in a new habitat.

Evidence. Evidence that supports the theory of evolution is found in fossils, in the way species are distributed, in organisms' anatomical similarities and differences, and in direct observation of rapid evolutionary change.



◀ **Irreducible Complexity.** According to intelligent design, the system by which blood clots is so complex that it could not have evolved without an intelligent designer.

Fossils. According to evolution, the fossil record shows a progression from one-celled life to multicelled organisms, and to the many simple and complex organisms living now. This is the fossilized arm of an extinct arthropod. Its spikes (the points on the arm's underside) made it a fearsome predator. ▶

Creationism

Creationism is a set of beliefs based on the idea that a Supreme Being brought into existence Earth and all its life through a direct act of creation. Most proponents of creationism base their beliefs on the Bible's account of the Creation.

Creationism states that all life forms remain relatively unchanged over time. Each species remains essentially as it was created.

Strict Creationism. States that a Supreme Being created the universe as described in the Bible's book of Genesis. According to Genesis, God created the universe less than ten thousand years ago, and He created all life forms in a 6-day period.

Scientific Creationism. States that a Supreme Being has always existed, and that this being created the universe, Earth, and all life. States that:

1. The universe was created in a state of perfect organization and complexity. Changes can only maintain or decrease that degree of organization.
2. The universe operates within a set of fixed laws at a relatively uniform rate.
3. The perfect order of the universe was disrupted sometime after its creation, causing negative changes that lead to imperfections in the world.

Intelligent Design

Intelligent design is the idea that, because of the complexity of certain features of nature, an "intelligent designer" must have played a role in the development of life. Supporters of intelligent design argue that living beings are too complex to have developed through purely natural processes.

Irreducible Complexity. States that any organism that possesses irreducible complexity could not have evolved through natural selection. Irreducible complexity describes a system within a living thing that is made up of parts that work together and have no necessary function on their own.

Specified Complexity. States that if a natural occurence is both complex and specified, it must have come from an intelligent designer. Defines *complex* as a pattern that is unlikely to occur by chance, and *specified* as something requiring a complex set of instructions that is not part of a pattern.

A Designed Universe. Intelligent design states that the laws the universe operates under— factors like electromagnetism, nuclear forces, and the gravity that holds fundamental particles together—are so complex that they cannot be explained by chance.

EVOLUTION is a process of change over time. The word evolution may refer to various types of change. For example, many astronomers think that the stars and planets evolved from a huge cloud of hot gases. Anthropologists study the evolution of human culture from hunting and gathering societies to complex, industrialized societies.

Most commonly, however, evolution refers to the formation and development of life on Earth. The idea that all living things evolved from simple organisms and changed through the ages to produce millions of species is known as the *theory of evolution*.

FACT OR FICTION?

The theory of evolution is supported by evidence from many scientific fields. Some people, however, reject the theory of evolution because of their religious beliefs. They believe the theory conflicts with religious accounts, which they interpret to mean that all forms of life were created essentially as they exist today. These views lie outside the scope of scientific inquiry. The material presented here deals with the scientific theory of evolution.

Theory of Evolution

The basic idea behind evolutionary theory states that species undergo changes in their inherited characteristics over time.

Competition and Fitness

The most important mechanism of evolution is *natural selection,* a process by which the individuals better suited to their environment tend to leave more descendants. All living organisms must compete for a limited supply of food, water, space, mates, and other things they need to successfully reproduce. Scientists use the term *fitness* to refer to an individual's overall ability to reproduce.

Main Types of Change

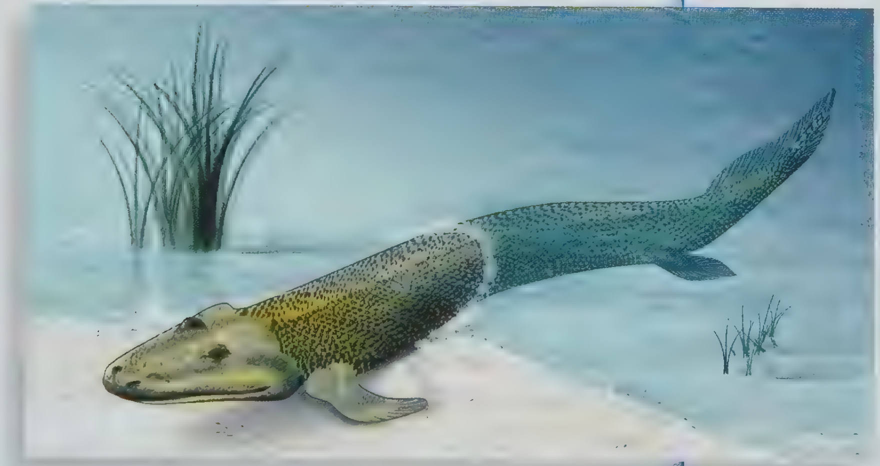
There are two main types of change in organic evolution.

- *anagenesis:* change within species
- *cladogenesis:* the splitting of one species into two

Anagenesis refers to changes that occur within a single species over time. Because of anagenetic change, the forms and traits of many species today differ from the forms and traits of their ancestors.

Cladogenesis refers to the splitting of one species into two or more descendant species. This branching process, also called *speciation,* can be repeated to create many species.

Current evolutionary theory holds that all species evolved from a single form of life that lived more than 3.5 billion years ago. Over time, repeated speciation events and anagenetic changes have produced the more than 10 million species inhabiting Earth today.



◀ **Out of the Sea.** In 2006, scientists found 375-million-year-old fossils of a species they named *Tiktaalik*. Scientists believe that *Tiktaalik* spent most of its life in the water but used its leglike fins to crawl onto land to find food or escape predators.

Common Ancestors

Related to speciation is the idea of *common ancestry*. Because all organisms evolved from one basic life form, any two species once had a common ancestor. Closely related species share a more recent common ancestor, but distantly related species must trace their ancestry far into the past to find a common ancestor.

For example, mammals evolved from a common ancestor—a shrewlike animal—that lived about 150 million years ago, while the common ancestor of mammals and reptiles lived about 300 million years ago.

The Tempo of Change

The rate at which evolutionary changes occur depends on many factors. Scientists have identified two patterns related to the tempo of evolutionary change.

Gradualism is the idea that some evolutionary changes occur continuously over long stretches of time.

Punctuated equilibrium is the idea that some evolutionary changes take place in relatively short periods, from tens to hundreds of generations, followed by longer periods of little change called *stasis*.

Both gradual and punctuated patterns can occur for different traits. The theory of evolution holds that evolution continues today at rates comparable to those of the past.

◀ **Natural selection** favors hummingbirds with bills that are the correct size and shape for the flowers in its environment.

THE basics

THEORY OF EVOLUTION

Theory of Evolution: species undergo changes in their inherited characteristics over time. All species developed from a single form of life that lived more than 3.5 billion years ago.

Causes

- Mutations: permanent changes in hereditary material of organisms produce random variation in the biology of a species
- Natural Selection: organisms best suited to their environment are most likely to leave offspring

Evolution of New Species

- Reproductive Isolating Factors: species remain distinct within an area because of factors that prevent different species from mating
- Geographically Isolated Groups: populations isolated geographically evolve differently because their environments differ and different mutations occur
- Rapid Speciation: the relatively rapid development of a new species; often occurs when a population settles in a new habitat

Evidence

- Fossils: show progression from earliest types of one-celled life to first simple, multicelled organisms, and then to the many simple and complex organisms living today
- Distribution of Species: oceanic islands—never connected to continents—provide important evolutionary evidence
- Anatomy: structural similarities and differences in organisms help scientists determine evolutionary history
- Direct Observation: evolutionary change that occurs rapidly allows scientists to directly observe the process

Causes of Change

Much evolutionary change results from the interaction of two processes.

1. **Mutation** produces random variation in the biological makeup of a species or a population—that is, individuals of the same species living in the same area.
2. **Natural selection** sorts out these random changes according to their value in enhancing the individual's reproduction and survival.

Mutations

Mutation is a permanent change in the hereditary material of an organism. Mutations may produce changes in the inherited characteristics of an organism. To understand how mutations produce these changes, one must understand how characteristics are inherited. (See also pages 68–69.)

How Characteristics Are Inherited. Hereditary characteristics of organisms are carried by threadlike structures called *chromosomes* in cells. Chromosomes carry large numbers of *genes,* the basic units of heredity.

Among most animals and plants, each body cell has a full set of paired chromosomes. Human body cells, for example, have 46 chromosomes arranged in 23 pairs. Offspring inherit half a set of chromosomes from each parent.

Fertilization. Parents pass on their chromosomes to their offspring during sexual reproduction. Egg and sperm cells have half the number of chromosomes found in all other cells in the body. During reproduction, a sperm and an egg unite in the process called *fertilization,* and the fertilized egg then has the full number of chromosomes.

Recombination. Sometimes, the genes from one of a pair of chromosomes change places with genes on the other pair as a sperm or egg cell forms. This change in the arrangement of genes, called *recombination,* can result in new combinations of inherited traits.

GOOD MUTATIONS

Mutations that help an organism survive will be passed on. A plant in a dry area might have a mutant gene that causes it to grow longer roots. Since this plant's roots could reach deeper for water, it would have a better chance of survival than others of its species. And because it has a better chance of survival, it is more likely to reproduce and pass its genes on to the next generation.

How Mutations Change a Species. Mutations introduce new hereditary characteristics. For this reason, they are the building blocks of evolutionary change and of the development of new species.

Mutations may be caused by environmental factors, such as chemicals and radiation, which alter the DNA in genes, or by errors in the copying of DNA during cell division.

After a gene has changed, it duplicates itself in its changed form. If these mutant genes are present in the egg or sperm cells of an organism, they may alter some inherited characteristics.

Unfavorable Traits. Mutations occur regularly but are usually infrequent, and most of them produce unfavorable traits. In most cases, such mutant genes are eliminated by natural selection because most individuals that inherit them die before producing any offspring.

Favorable Traits. Some mutations, however, help organisms adapt better to their environment. These beneficial mutations provide the raw material for evolutionary change.

Natural Selection

The theory of *natural selection* was first explained in detail in the 1850s by the British naturalist Charles Darwin. Natural selection is a process in nature by which the organisms best suited to their environment are the ones most likely to leave offspring. This process has been called *survival of the fittest,* though a more accurate term would be *reproduction of the fittest.*

Plants and animals produce many offspring, but some of the young die before they can become parents. According to Darwin's theory, natural selection determines which members of a species die prematurely and which ones survive and reproduce.

Variation. The theory of natural selection is based on the great variation among even closely related individuals. In most cases, no two members of a species are exactly alike. Each has a unique combination of such traits as size, color, and ability to withstand cold or other harsh conditions.

Competition. Only a limited supply of food, water, and other necessities of life exists for all the organisms that are produced. Therefore, the organisms must constantly compete for these necessities. They also struggle against such dangers as being destroyed by animals that prey on them or by unfavorable weather.

In any environment, some members of a species have combinations of traits that help them. Other members have traits that are less suitable for that environment. The organisms with the favorable traits are most likely to survive, reproduce, and pass on those traits to their young. Organisms that are less able to compete are likely to die prematurely or to produce few or inferior offspring. As a result, the favorable traits replace the unfavorable ones in the species. (See pages 216–217.)

Different Environments. If the environment changes, different traits or combinations of traits may become favorable to survival, and the overall character of a species might change. In this way, a species adapts to its environment and avoids extinction. If two populations of a species live in different environments, they will probably develop differently. Eventually, they may differ so much that they become two separate species.

Competition. In any environment, the members of a species with traits that allow them to catch food—or escape predators—will be more likely to survive and reproduce. ▼

EXPLAINING EXISTENCE

Creationism and *intelligent design* are two related explanations for how the universe, Earth, and the life on it came to be. Both explanations state that a Supreme Being, or intelligent designer, created the universe.

Creationism states that the universe was created in a state of perfect organization and complexity. Since it began in a perfect state, this organization and complexity can only be maintained or decreased.

Intelligent design states that the universe, Earth, and life are too complex to have developed through natural processes. An intelligent being must have played a role.

Theory of Creationism

Creationism is a set of beliefs based on the idea that a Supreme Being brought into existence Earth and all its life through a direct act of creation. Most proponents of creationism base their beliefs on the Bible's account of the Creation.

There is considerable variation in creationist beliefs, but there are two main types of creationism.

- strict creationism
- scientific creationism

Strict Creationism

Strict creationism states that a Supreme Being created the universe as described in the Bible's Book of Genesis. According to Genesis, God created the universe less than 10 thousand years ago, and He created all life forms in a 6-day period.

Strict creationism—like all creationism—states that all life forms remain relatively unchanged over time. Each species remains essentially as it was created. Creationism does not agree that life evolved from simple beings into more complex beings.

Some proponents of creationism interpret the Bible more loosely. For example, some think the universe is millions or billions of years old, but that human beings were created thousands of years ago.

Earth's Surface. According to creationism, the formations on Earth's surface were formed by a devastating worldwide flood that occurred thousands of years ago. ▼

SOME TENETS OF SCIENTIFIC CREATIONISM

- The universe, including space, time, matter, and energy, has not always existed. A Supreme Being is the only thing that has existed for eternity. The Supreme Being created the universe supernaturally.
- Life did not arise by natural processes from nonliving matter. The Supreme Being created life supernaturally.
- Each species was created functionally complete. No species evolved from some other species. Changes that have occurred in certain species from the time of their creation have been either horizontal changes, which add variation within a species, or downward changes, such as harmful mutations or extinctions.
- Human beings were created fully formed. They did not evolve from an animal lineage. The spiritual side of humans—self-awareness, morality, abstract reasoning, language, will, and religious nature—is distinct from the biological side. The spiritual side of human beings was created supernaturally.
- Earth's surface has been mainly formed by short-term, intense catastrophes. Earth's history, as recorded in rock layers and fossils, cannot be explained as the result of gradual, uniform natural processes. There is scientific evidence that Earth and the universe were created relatively recently. There is scientific evidence that rock layers and the fossil record were formed and deposited by a worldwide flood.

Scientific Creationism

Scientific creationism attempts to use scientific evidence to explain the existence of Earth and all its life in a way that remains consistent with creationist beliefs. Scientific creationism states that a Supreme Being has always existed, and that this being created the universe, Earth, and all life.

Complexity. According to scientific creationism, the basic systems of nature were brought into existence fully formed in a state of organization and complexity. Since the universe was created by a Supreme Being, it began in a state of perfection. All changes that have occurred since that time have either maintained that degree of organization and complexity, or decreased it.

Fixed Laws. Today the natural world operates within a set of fixed natural laws at a relatively uniform rate, so the level of organization and complexity generally does not decrease at a dramatic rate.

Harmful genetic mutations can cause changes of decreased complexity within a species. Global catastrophes can cause larger changes, such as extinction.

Disrupted Order. According to creationism, the perfect order was disrupted sometime after the creation. Aging, disease, extinction, and other imperfections in the natural world are caused by the negative changes that occur as a result of this disruption.

▲ **Creationism** states that all life forms remain relatively unchanged over time, existing just as the Supreme Being created them.

COMPLEXITY, MORE OR LESS

The theory of evolution states that all living things developed from simple organisms. Natural processes such as mutations cause both increases and decreases in complexity.

According to creationism, the universe was created in perfection by a Supreme Being, so the level of complexity can only remain the same or decrease. The rate of decrease might be very small during peaceful times, and very high during catatstrophes.

INTELLIGENT DESIGN is the idea that, because of the complexity of certain features of nature, an "intelligent designer" must have played a role in the development of life. Supporters of intelligent design argue that living beings are too complex to have developed through purely natural processes.

Intelligent design differs from creationism in that it does not base its tenets on the Biblical story of creation. It does not assert that Earth is less than 10 thousand years old and does not attempt to explain the nature of the intelligent designer.

Irreducible Complexity

Irreducible complexity describes a system within an organism that is made up of multiple parts that do not work if all parts are not present. According to intelligent design, any organism that possesses irreducible complexity could not have evolved through natural selection.

- An irreducibly complex system is made up of individual parts that work together and have no necessary function on their own.
- The individual parts would not have been useful to an organism that had not fully evolved the entire system.
- The useless parts would not have survived through the generations necessary to evolve the entire complex system.

Complex Examples. Supporters of intelligent design point to several examples in nature that they understand to be irreducibly complex (see table below).

System	Description	Irreducible Complexity Argument
Bacterial flagellum	*Flagella* are tail-like structures that project from the body of certain cells. Bacterial flagella are made up of 30 to 35 different proteins, organized into a rotor that spins like a propeller, rotating the bacteria's tail and allowing it to move.	According to intelligent design, there are many parts that work together intricately, and if any one of these parts is removed, the entire system becomes useless.
Blood clotting	*Blood clotting* is the coagulation of blood that causes a wound to stop bleeding. It is a complex process requiring several different proteins. When a blood vessel is damaged, each of the proteins responds, causing platelets to stick to the damaged surface and to one another, forming a plug.	Clotting is intricate and complex. If blood clots did not form, a tiny cut could cause a person to bleed to death. If clots formed at the wrong time, they would block circulation and cause heart attacks and strokes. According to intelligent design, the clotting system is so complex that it could not have evolved without an intelligent designer.
The immune system in mammals	The *immune system* is a collection of cells, molecules, and tissues that help defend the body against diseases and other harmful invaders, such as bacteria, fungi, parasites, and viruses. The immune system consists of a complex collection of interconnected parts that must work together for the system to function properly.	According to intelligent design, each of the individual parts of the immune system would not have been necessary to an organism except as part of the system. Therefore, the immune system is an example of irreducible complexity.

Specified Complexity

Specified complexity is the argument that if something that occurs in nature is both *complex* and *specified,* it must have originated from an intelligent designer. Intelligent design defines these terms as follows:

- complex: describes a pattern that is unlikely to occur by chance
- specified: something that requires a complicated set of instructions but is not part of a pattern

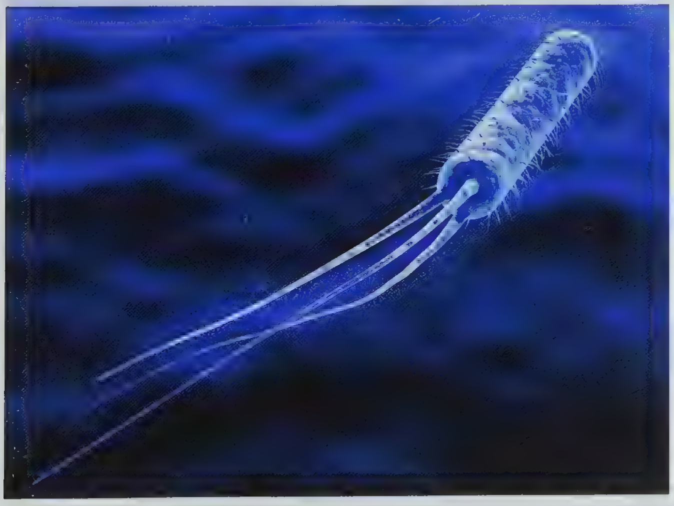
The example often used to illustrate specified complexity is the written word. An individual letter is specified but not complex. A long string of random letters is complex but not specified. A poem is both specified and complex.

According to intelligent design, details within living things, such as the sequence of genetic information contained within DNA, exhibit specified complexity. Therefore, living things—and DNA—must have been created by an intelligent designer.

A LONG HISTORY

The intelligent design argument in its current form gained popularity in the late 1980s and 1990s. But the central ideas of intelligent design date back thousands of years.

Ancient Greek philosophers Plato and Aristotle both argued that the first cause of the cosmos must be an intelligent creator. Marcus Tullius Cicero of ancient Rome wrote that evidence of a divine power could be seen throughout the natural world.



The Fine-Tuned Universe

Intelligent design goes beyond the origins of life to argue that the universe and the laws it operates under are so complex that they cannot be explained by chance.

This argument states that factors like electromagnetism, nuclear forces, and the gravity that holds fundamental particles together must have been set in motion by an intelligent designer.

If any of these features were slightly different, it would have been impossible for galaxies, solar systems, and planets to form in the first place.

COSMIC WATCHMAKER

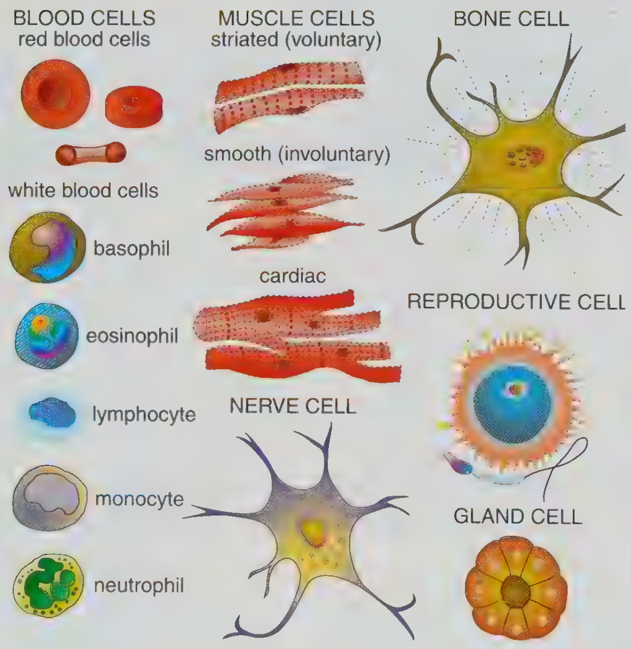
In the 1800s, theologian William Paley stated that if a person who had never seen a watch found one on the ground, then based on the complicated inner workings of the watch, the person would conclude that it could not exist by accident—that an intelligent being must have designed it.

Paley used this example to argue that a similar conclusion can be drawn from looking at the complexity of the world.

◀ **The bacterial flagellum,** according to intelligent design, is an example of irreducible complexity. The tail-like structure acts as a rotary motor, allowing bacteria to move about.

CELLS are the basic units of all life. Every living organism consists of either a single cell or a collection of many interdependent cells. Bacteria are examples of single-celled, or *unicellular,* organisms. *Multicellular* organisms, composed of many cells, include most plants, fungi, and animals.

While some cells—frog eggs and certain nerve cells, for example—can be seen with the naked eye, most cells can only be seen using a microscope. It would take about 40,000 red blood cells to fill this letter *O*. It takes millions of cells to make up the skin on the palm of your hand.


BLOOD CELLS
red blood cells
white blood cells
basophil
eosinophil
lymphocyte
monocyte
neutrophil
MUSCLE CELLS
striated (voluntary)
smooth (involuntary)
cardiac
NERVE CELL
BONE CELL
REPRODUCTIVE CELL
GLAND CELL

Cell Theory

Human beings, and every multicellular organism, develop from a single cell. That cell grows to a certain size and then divides. Each new cell grows and divides, over and over again, until it forms a complete organism.

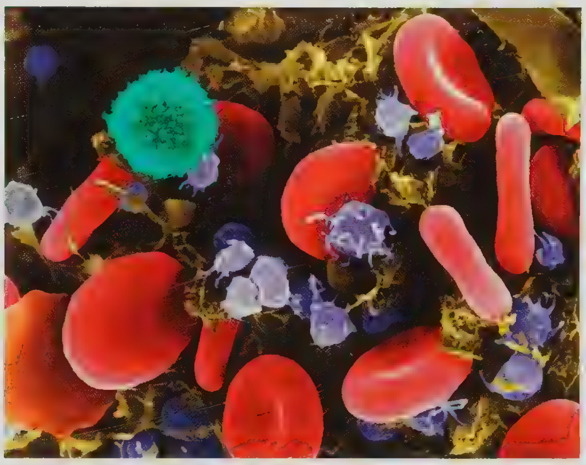
The theory that every form of life is made up of cells was developed in the 1800s. Cell theory consists of three basic principles:

1. The cell is the fundamental unit of all life.
2. All living organisms are made up of cells.
3. Every cell is the product of the division of a previously existing cell.

Today, many different scientific specialists study the cell. Cell biologists deal with cells on every level. Cytologists study cell structure. Biochemists and biophysicists investigate cell *physiology,* or function. Histologists study tissues, which are collections of similar cells, such as muscles and blood.

Cell Size	
human nerve cell	up to 2 m in length
chicken egg cell	approx. 30 mm in diameter
frog egg cell	approx. 1 mm in diameter
human egg cell	approx. 100 µm* in diameter
human red blood cell	approx. 8 µm in diameter
average eukaryotic cell	approx 10–100 µm in diameter
average plant body cell	approx. 30–50 µm in diameter
average animal body cell	approx 10–20 µm in diameter
average true bacteria	approx. 1 µm in diameter
average prokaryotic cell	approx. 1–10 µm in diameter
mycoplasma	approx. 0.16 µm in diameter
*µm = micron (one micron = 0.000001 meters)	

Microscopes allow scientists to study the details of a cell. The scanning electron microscope (*left*) can magnify an image up to 200,000 times. At this magnification, the minuscule features of human blood (*below*) are clear, bright, and vivid.



THE basics

CELLS

All cells have certain characteristics in common.

- ✓ cells are alive—as alive as you are
- ✓ cells "breathe," take in food, get rid of wastes, grow, reproduce, and, in time, die
- ✓ cells have a *membrane,* a thin covering that encloses every cell
- ✓ complete contents of a cell are called the *protoplasm*
- ✓ most cells have a structure called the *nucleus,* which contains the cell's genetic program, the master plan that controls almost everything the cell does
- ✓ the part of the protoplasm outside the nucleus is called the *cytoplasm*

fyi! *The human body has more than 10 trillion cells!*

HOW DO WE KNOW THAT ?

CELLS

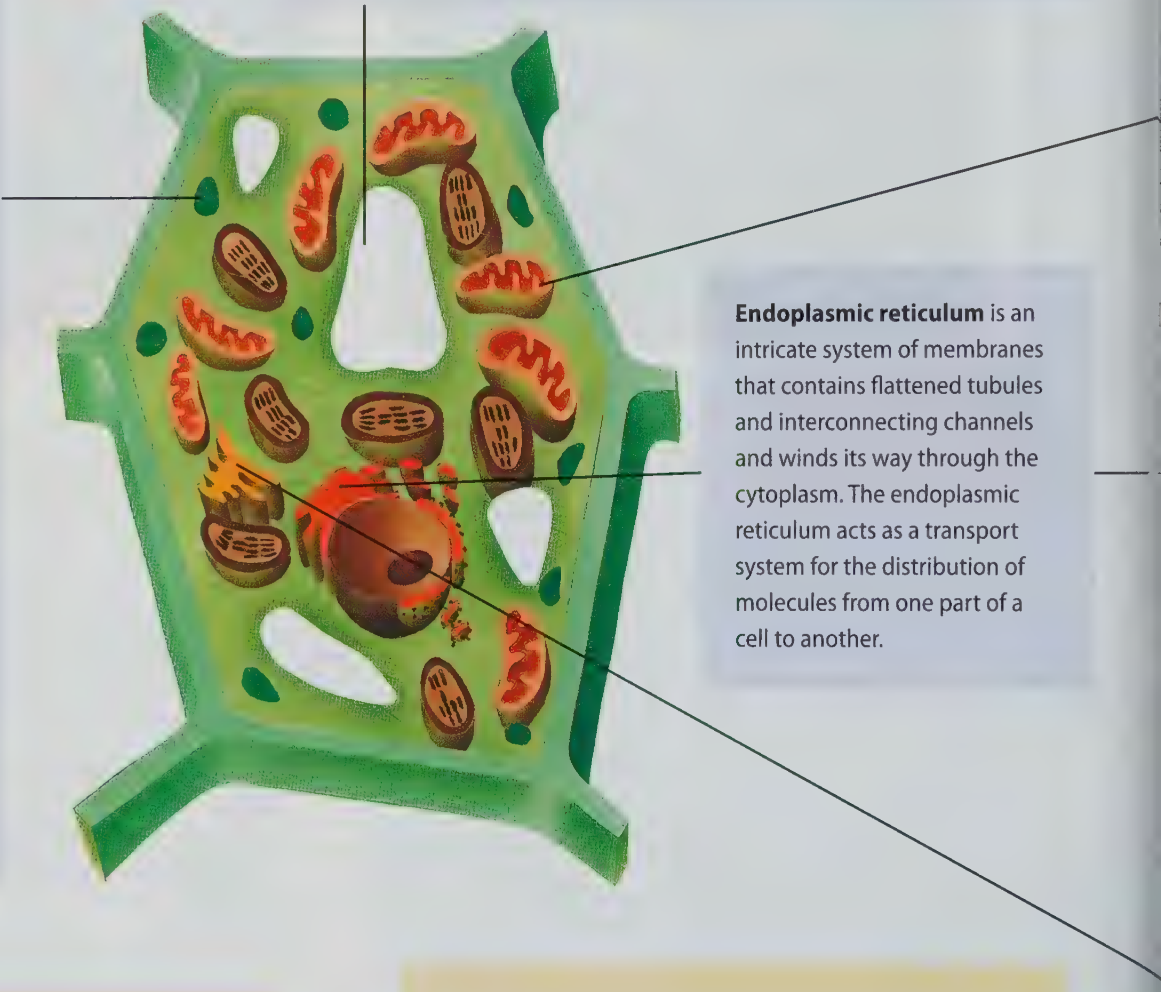
- **1665** English physicist Robert Hooke coins the term "cell" after observing that a thin section of cork he was examining through a primitive microscope was made up of tiny chambers, like cells in a monastery.
- **1674** Dutch naturalist Anton van Leeuwenhoek observes unicellular organisms, recognizing that the miniscule cells were not plants but functioning animals. He calls the bacteria and protozoa "animalcules."
- **1805** German naturalist Lorenz Oken claims that every form of life is made up of cells.
- **1838** German botanist Matthias Jakob Schleiden studies plants under a microscope, shows that all plants are made up of cells.
- **1838** German zoologist Theodor Schwann confirms that animals are also made of cells and that living things grow when cells multiply.

In the Cytoplasm

The organelles contained within the cytoplasm of a cell each have a certain function to perform.

Vacuoles are membrane-bound cavities in the cytoplasm that vary greatly in size. Many plant cells contain one or more large vacuoles; in some, a single large vacuole takes up almost the entire interior space. Plant vacuoles may contain water, stored food, pigments, salts, gases, sugars, and other materials. Substances toxic to the cell may also be safely stored there. Vacuoles in animal cells are generally much smaller than those found in plants. Materials may enter and leave the cell by way of vacuoles.

TYPICAL PLANT CELL



Plastids. Found in most plant cells and in photosynthetic unicellular organisms, plastids are intricate, self-replicating organelles containing their own DNA, RNA, and ribosomes. The most important plastids are *chloroplasts,* where photosynthesis occurs in plants. *Chromoplasts* manufacture and store the pigments that give fruits, vegetables, leaves, and flowers their color. *Leucoplasts* are the sites where fats and proteins are stored and glucose is converted to starch.

Endoplasmic reticulum is an intricate system of membranes that contains flattened tubules and interconnecting channels and winds its way through the cytoplasm. The endoplasmic reticulum acts as a transport system for the distribution of molecules from one part of a cell to another.

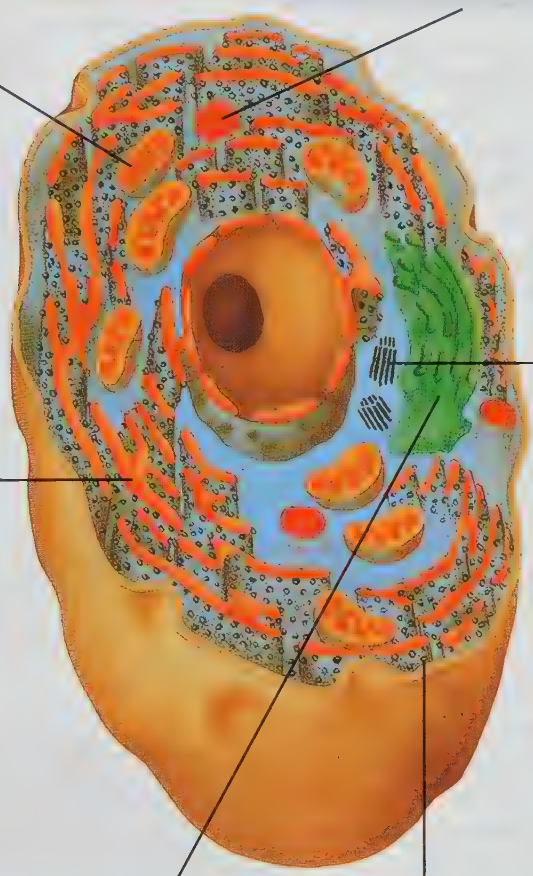
Microfilaments, *(not shown)* or *fibrils,* are much thinner than microtubules. They are thought to be composed of fibers of protein and seem to have the ability to contract like muscle. Microfilaments are found in some plant and animal cells and may account for the movement of the cytoplasm and the movement of the cell itself.

Microtubules *(not shown)* are a series of long, thin cylinders that support, stiffen, and give shape to cells. They consist primarily of the protein *tubulin* and are found throughout the cytoplasm. In addition to providing internal structural support for the cell, microtubules are thought to play a role in the sorting out of chromosomes during cell division. They are the basic components of centrioles, kinetosomes, cilia, and flagella.

Mitochondria are the power plants of the cell. They are relatively large, sausage-shaped, fluid-filled sacs scattered throughout the cytoplasm. They provide the cell with its supply of chemical energy in the form of adenosine triphosphate (ATP). A double-layered membrane separates each mitochondrion from the surrounding cytoplasm. Each membrane encapsulates an inner membrane that is highly convoluted to form partitions called *cristae*. The cristae provide a large surface area on which chemical reactions take place.

Lysosomes are small membranous sacs of varying size and shape containing powerful digestive enzymes. These enzymes break down carbohydrate, protein, and fat molecules into simpler substances. Lysosomes can fuse with and digest foreign particles or organisms that invade the cell. For example, it is the lysosomes in white blood cells that fight pathogenic bacteria by digesting it.

TYPICAL ANIMAL CELL



Centrioles are located near the nucleus and found in the cells of animals, most unicellular eukaryotes, fungi, and lower plants. They are a pair of small, rodlike bodies that lie at right angles to each other. The centrioles assist in the distribution of the chromosomes during the process of cell reproduction.

Golgi Apparatus. Located near the nucleus and continuous with the endoplasmic reticulum, the Golgi apparatus is a stacked arrangement of small, flattened membrane-bounded sacs. They synthesize carbohydrates and package proteins in *vesicles,* or membranous sacs, for transport to the surface of the cell, where they are secreted.

Ribosomes. Attached to membranes of the endoplasmic reticulum, ribosomes are the extremely small sites within the cytoplasm where proteins are synthesized. The number of ribosomes in a cell varies with the amount of protein the cell is required to produce. In some cells, there are millions of ribosomes. They attach themselves to messenger RNA from the nucleus and construct proteins from amino acids according to the coded instructions of the messenger RNA.

Mitosis vs. Meiosis

Mitosis

All eukaryotic cells except the sex cells divide through the duplication of the nucleus with its DNA and chromosomes. This process is called mitosis. The time between cell divisions is called *interphase.* Mitosis then takes place in four phases: *prophase, metaphase, anaphase,* and *telophase.*

Interphase. The cell grows and carries out its regular functions. Each cell begins to grow to mature size. Each chromosome duplicates itself to form identical sister chromatids. The DNA is duplicated and proteins are synthesized. The cell becomes ready to divide and interphase ends.

Prophase. The chromosomes condense or coil into short, thick structures. The nuclear membrane breaks down and disappears. The centrioles migrate to opposite regions of the cell, forming two poles. A football-shaped structure, the spindle, made of tubelike fibers, then forms between the poles.

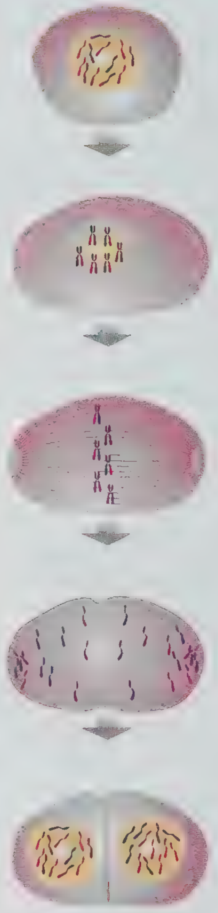
Metaphase. The double-stranded chromosomes migrate to the center of the cell, apparently pushed or pulled along by the spindle fibers.

Anaphase. The sister chromatids separate from each other. Each daughter chromosome, as they are now called, migrates toward opposite poles.

Telophase. The daughter chromosomes are at opposite poles of the spindle, and the spindle starts to break down. The individual chromosomes become thinner and longer. The nuclear membrane re-forms around each set of chromosomes.

Cytokinesis, the division of the cytoplasm, usually begins during telophase. A furrow or groove appears in the membrane of animal cells and a cell plate forms in plant cells. In animal cells, the furrow curves inward until a complete membrane separates the two daughter cells. In plant cells, the plate expands until a wall is formed that separates the two cells. A new interphase then begins in each daughter cell.

Mitosis



Parent Cell

Prophase
Chromatin condenses into chromosomes. Nuclear envelope disappears.

Metaphase
Chromosomes align at the equatorial plate.

Anaphase
Sister chromatids separate. Centromeres divide.

Telophase
Chromatin expands. Cytoplasm divides.

Two identical daughter cells

THE basics

MITOSIS

- All eukaryotic cells except sex cells divide through mitosis.
- The nucleus divides and forms two identical nuclei.
- The cytoplasm usually divides soon after mitosis.
- End result is two daughter cells with identical nuclei.
- Takes place in four stages:
  prophase
  metaphase
  anaphase
  telophase

Meiosis

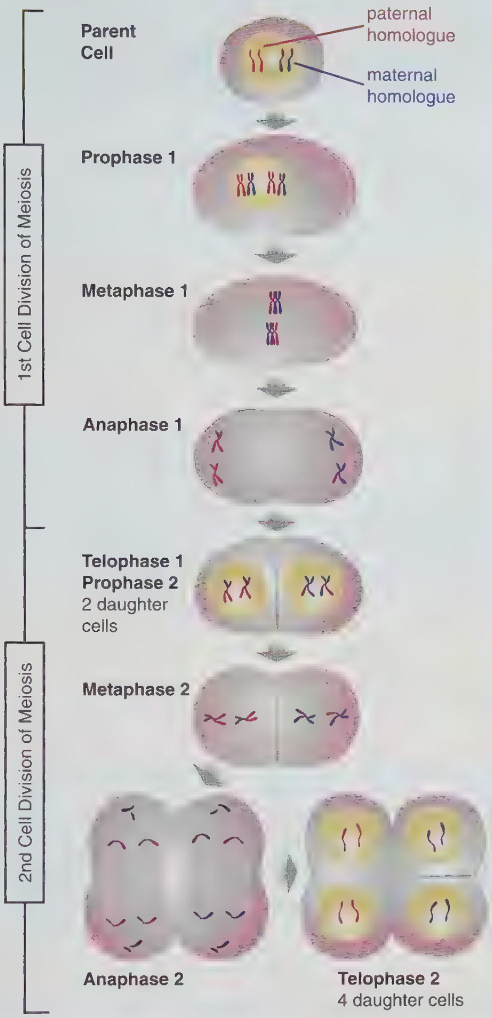
Organisms that form through sexual reproduction receive DNA from two separate parents. These organisms produce specialized cells called *gametes,* or sex cells. A male gamete, called a *sperm,* unites with a female gamete, called an *egg,* to create a new individual. Gametes are produced through a special kind of cell division called meiosis.

Meiosis differs from mitosis in a number of ways. In meiosis, there are two cell divisions resulting in four daughter cells. The chromosomes can exchange parts. The chromosomes duplicate only once, even though there are two cell divisions. The chromosomes line up and randomly move to either pole. In this way, each gamete is unique and has one-half the chromosome number of its species. Meiosis occurs in two stages.

First Meiotic Division. Each chromosome has duplicated itself. Each replicated chromosome seeks out its sister. The duplicated chromosomes pair up, forming a four-stranded group called a tetrad. Exchange of genetic material may happen at this time. As tetrads are forming, the nuclear membrane is breaking down and spindle fibers are forming. The tetrads migrate toward the center and line up randomly. Complete chromosomes gather at each pole and the cytoplasm divides either by furrowing or forming a cell plate. Each new cell now has half the chromosome number of its species. Humans, for example, have 46 chromosomes, so each gamete has 23 chromosomes after the first division.

Second Meiotic Division. This phase is very similar to mitosis. Spindle fibers form, the paired chromosomes line up at the center of the cell, and the spindle fiber begins to draw the chromosomes apart. The chromosomes group and migrate together at each pole. The cytoplasm again divides. Four daughter cells have formed, each with half the chromosome number. Usually in male organisms the cytoplasm divides evenly and all four cells function as sperm cells. In females, the cytoplasm divides unequally with one cell receiving the majority of the cytoplasm. This cell becomes the functioning gamete.

Meiosis

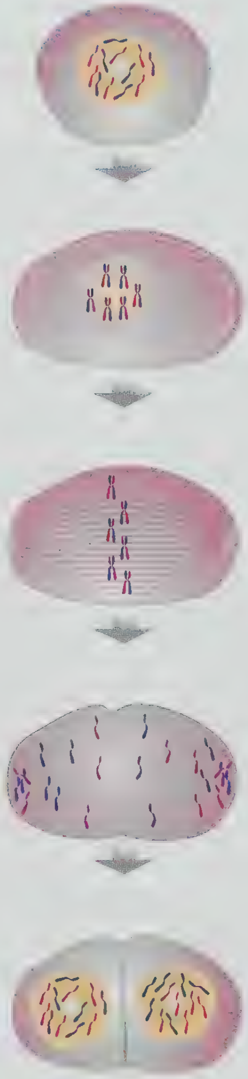


MEIOSIS

- Only sex cells divide through meiosis.
- Sex cells are called gametes.
- The female gamete is the egg and the male gamete is the sperm.
- Consists of two cell divisions that result in four daughter cells.
- Each daughter cell has half the chromosomes of the species.
- Those chromosomes will combine with the chromosomes of another gamete during reproduction.
- Meiosis produces organisms that are genetically different.

Mitosis vs. Meiosis

Mitosis



Parent Cell

Prophase
Chromatin condenses into chromosomes. Nuclear envelope disappears.

Metaphase
Chromosomes align at the equatorial plate.

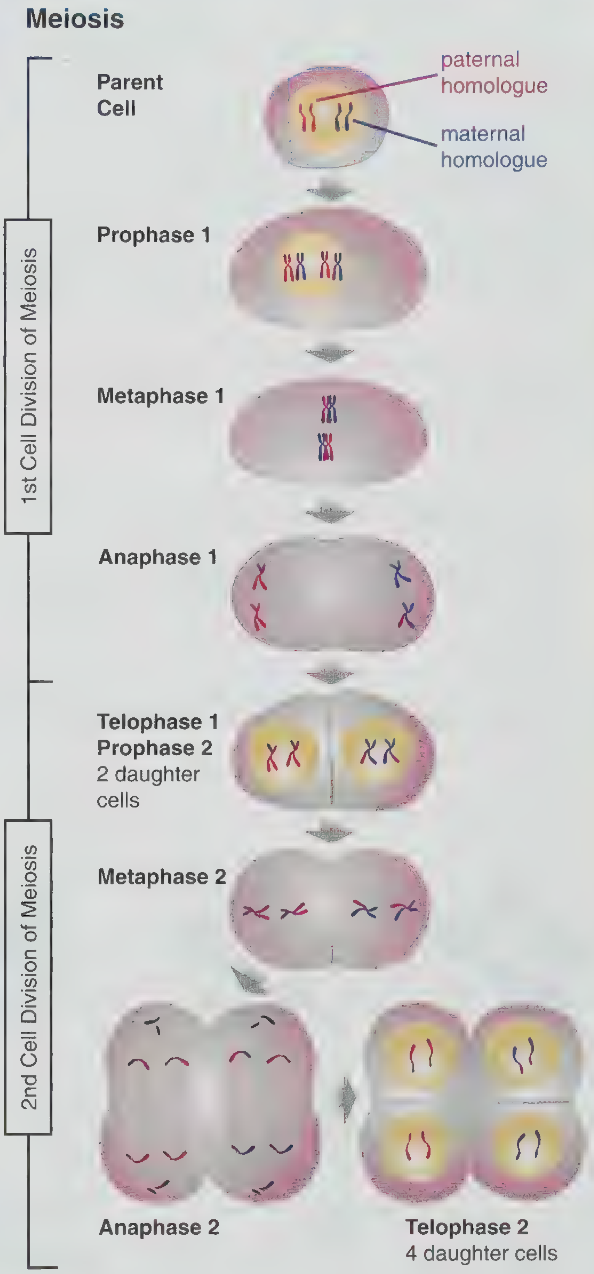
Anaphase
Sister chromatids separate. Centromeres divide.

Telophase
Chromatin expands. Cytoplasm divides.

Two identical daughter cells

Mitosis
Produces body (somatic) cells with two sets of chromosomes (diploid cells)
Homologous (paired) chromosomes line up independently at the center (equatorial plate) of the cell.
Occurs in all cells
Uses spindle fibers
Genetic code of daughter cells is identical.
Two daughter cells produced
One nuclear division per cycle
Chromosome pairs are replicated before mitosis.
Products (daughter cells) of mitosis are capable of further division.

Meiosis
Produces sex (sperm and egg) cells with one set of chromosomes (haploid cells)
Homologous chromosomes or tetrads line up together at the equatorial plate.
Occurs only in sex organs or gonads
Uses spindle fibers
Genetic code varies due to crossing over and random assortment of chromosomes.
Four daughter cells produced in males; one daughter cell produced in females
Two nuclear divisions per cycle
Chromosome pairs are replicated before meiosis
Meiotic products are not capable of further meiotic division but may divide again through mitosis.


Meiosis
Parent Cell
paternal homologue
maternal homologue
Prophase 1
Metaphase 1
Anaphase 1
Telophase 1
Prophase 2
2 daughter cells
Metaphase 2
Anaphase 2
Telophase 2
4 daughter cells
1st Cell Division of Meiosis
2nd Cell Division of Meiosis

Mendel's Laws

Mendel's studies of pea plants in the garden of his monastery led him to formulate the first correct theory of heredity. His theory had two principles called *Mendel's laws of heredity.*

- Law of Segregation
- Law of Independent Assortment

Law of Segregation

Mendel's first law, the *Law of Segregation,* has three parts:

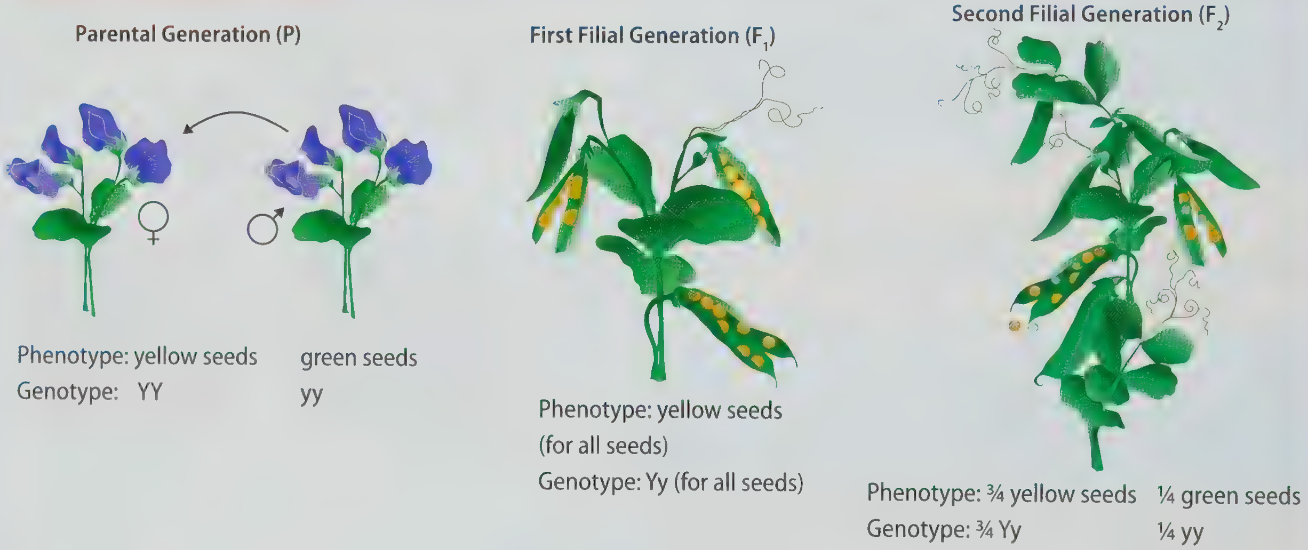
1. Hereditary characteristics are determined by separate units (now called genes).
2. These units occur in pairs.
3. The genes in a pair *segregate,* or separate, during the division of *gametes,* or sex cells. Each sperm or egg is *haploid,* meaning it receives only one member of the pair. Once the gametes unite to form the *zygote,* or fertilized egg, the zygote is called *diploid,* or double.

Law of Independent Assortment

Mendel's second law is called the *Law of Independent Assortment.* It states that each pair of genes behaves independently of all other pairs in the production of sex cells. Therefore, each gene pair is inherited independently of all other genes.

Geneticists now know that independent assortment applies only to genes that are on different chromosomes or far apart on the same chromosome. Genes that are linked, or near each other on the same chromosome, tend to be inherited together.

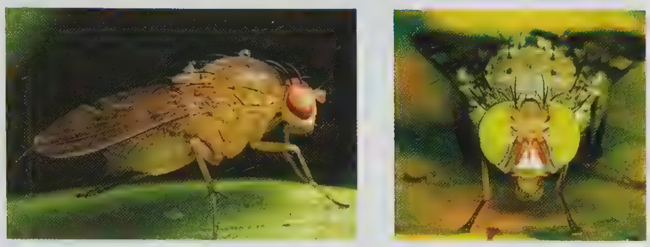
LAW OF SEGREGATION


Parental Generation (P)
♀
♂
Phenotype: yellow seeds
Genotype: YY
green seeds
yy
First Filial Generation (F_1)
Phenotype: yellow seeds
(for all seeds)
Genotype: Yy (for all seeds)
Second Filial Generation (F_2)
Phenotype: ¾ yellow seeds
Genotype: ¾ Yy
¼ green seeds
¼ yy

Each true-breeding parent plant carries a pair of identical alleles. The yellow seed allele is dominant (YY). The green seed allele is recessive (yy). The allele pairs separate when sex cells form. Each sex cell contains a single Y or a single y allele.

The offspring receive one allele from each parent. All seeds in this generation have a genotype of Yy. They are all yellow, since green is recessive.

Plants from F_1 are now crossed. The possible genotype combinations for the second filial generation are YY, Yy, yY, and yy. There is a one in four chance that two recessive y alleles will combine, so only one offspring out of four will have green seeds.



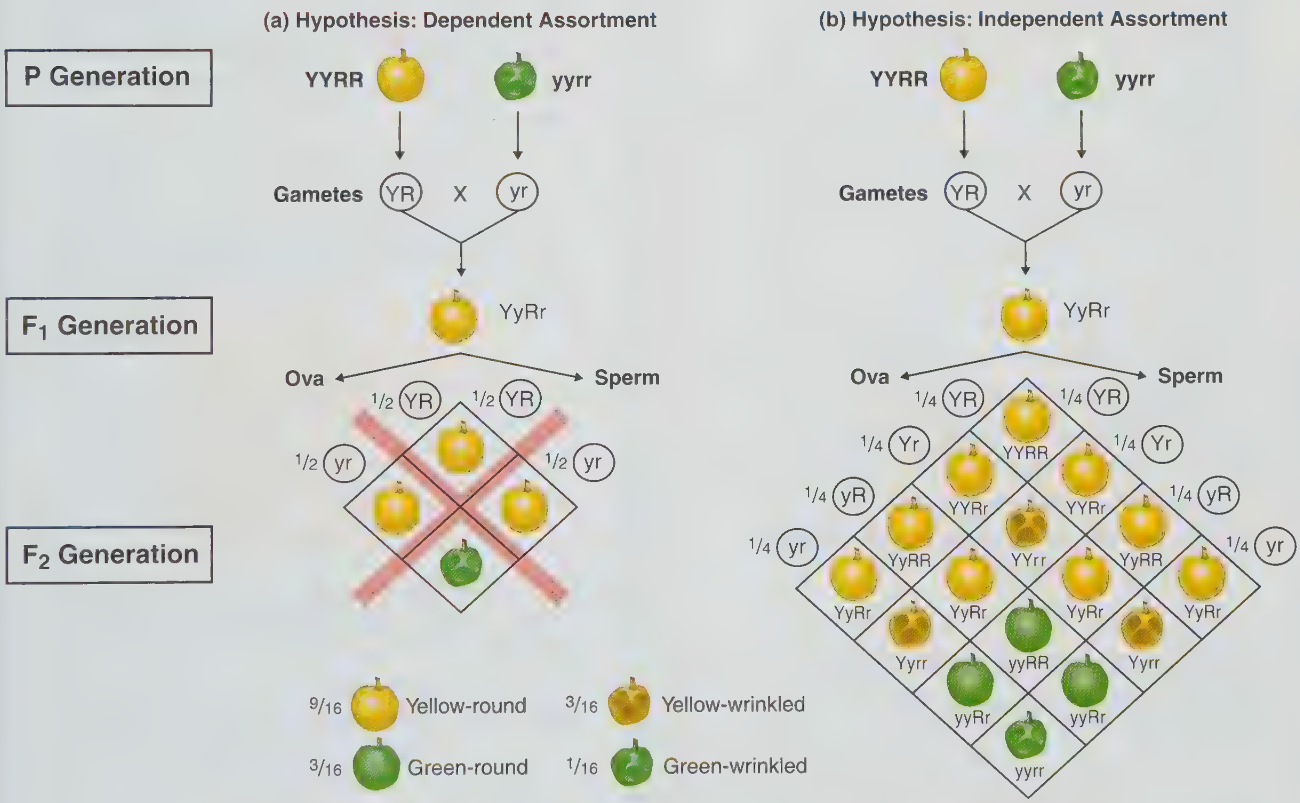
Fruit Flies. In 1908, geneticist Thomas Hunt Morgan and his team began breeding fruit flies. They studied the inheritance of eye color and wing shape, making important genetic discoveries, such as crossing over and sex-linked traits.

MENDEL LOST AND FOUND

Mendel published his work in 1866, but it went unnoticed for years. Then, in 1900, three European botanists each rediscovered Mendel's work. They conducted their own experiments that each supported Mendel's conclusions.

Many important genetic discoveries followed in the early 1900s. Scientists studying fruit flies showed that genes were located on chromosomes, made the first genetic map, and demonstrated the inheritance of genes on sex chromosomes.

LAW OF INDEPENDENT ASSORTMENT


(a) Hypothesis: Dependent Assortment
(b) Hypothesis: Independent Assortment
P Generation
YYRR
yyrr
Gametes
YR
X
yr
F1 Generation
YyRr
Ova
Sperm
1/2 YR
1/2 YR
1/2 yr
1/2 yr
F2 Generation
1/4 YR
1/4 Yr
1/4 yR
1/4 yr
YYRR
YYRr
YYRr
YyRR
YYrr
YyRR
YyRr
YyRr
YyRr
YyRr
Yyrr
yyRR
Yyrr
yyRr
yyRr
yyrr
9/16 Yellow-round
3/16 Yellow-wrinkled
3/16 Green-round
1/16 Green-wrinkled

Two sets of true-breeding parents carry two pair of identical alleles. One is for color: YY (yellow) or yy (green). One is for shape: RR (round) or rr (wrinkled).

(a) If the two traits were inherited together, the F_1 hybrids can only produce two different sex cells. The phenotype ratio of the F_2 generation would be 3:1.

(b) If the two traits are inherited separately, the F_1 hybrids will produce four different gametes. The phenotype ratio of the F_2 generation will be 9:3:3:1.

Mendel's experiments supported the Independent Assortment hypothesis.

Genetics Practice Problems

1. Sunflowers come in 2 varieties. The tall variety is determined by a dominant allele (T), while a dwarf variety is determined by a recessive allele (t). Remember that most organisms carry 2 alleles for each trait. For the following genotypes, describe whether the phenotype will be tall or dwarf.

a) Tt ____________ **b)** tt ____________

c) tT ____________ **d)** TT ____________

2. Is there any difference between the heterozygous genotypes Tt and tT?

____

3. Consider a cross between two sunflowers. One is a tall heterozygous (Tt) sunflower while the other is a homozygous recessive dwarf variety (tt). When only 1 trait is examined, it is called a monohybrid cross. For this cross, answer the following questions.

a) How many different types of sex cells can the tall sunflower produce? What alleles do they contain?

____

b) Fill in the Punnett square below to illustrate this cross. Indicate the genotype of 1 parent on the horizontal axis. Indicate the genotype of the other parent on the vertical axis on the left. How many different offspring phenotypes are possible from such a cross? How many different offspring genotypes are possible from such a cross?

____

c) Among the offspring, what is the expected ratio of tall plants to dwarf plants?

____

Tt × tt

4. Howler monkeys come in 2 varieties. There is a long-tailed variety and a short-tailed variety. These characteristics are caused by the action of a single gene. The long-tailed variety is dominant (L), while the short-tailed variety is recessive (l).

In the Punnett square below, illustrate a cross between a homozygous long-tailed monkey (LL) and a heterozygous long-tailed monkey (Ll). Then answer the following questions:

LL × Ll

a) Out of 16 offspring from this cross, how many would be expected to have long tails?

____

b) Out of 16 offspring from this cross, how many would be expected to have short tails?

____

c). Out of 16 offspring from this cross, how many would be expected to be heterozygous?

____

Howler monkeys use their long, powerful tails for balance and sometimes for swinging.

5. In howler monkeys, black fur is a dominant trait (B). Brown fur is a recessive trait (b). Consider a cross between two heterozygous (Bb) howler monkeys and answer the following questions.

a) What is the phenotype of each parent in terms of fur color?

b) Each sex cell produced by each parent contains only 1 allele. How many different sex cell varieties can each parent produce? What are they?

c) Illustrate this cross in the Punnett square below. Out of 16 offspring, how many can be expected to have black fur? Do you expect any of the offspring to have brown fur?

d) Out of 16 offspring, how many can be expected to have a heterozygous genotype? How many offspring can be expected to have a homozygous genotype?

Bb × Bb

6. In a cross between two homozygous dominant howler monkeys (BB), what percentage of the offspring can be expected to have black fur?

Illustrate your answer in the Punnett square below. Will any offspring have brown fur?

BB × BB

7. In a cross between two heterozygous howler monkeys (Bb × Bb), how many genotypes can result from this cross? Illustrate your answer in the Punnett square below.

Bb × Bb

a) Express the number of genotypes as a ratio:

____ BB ____ Bb ____ bb

b) Express the number of phenotypes as a ratio:

____ Black ____ Brown

c) Remember, the Punnet square illustrates all possible outcomes of a cross. Is it possible that all of the offspring of this cross have brown fur?

Genetics Problems — Answers

1. a) Tt *tall* **b)** tt *dwarf*

c) tT *tall* **d)** TT *tall*

Explanation: A dominant allele will always mask the effects of a recessive allele. To determine the phenotype from the genotype, simply check to see if one of the two alleles is a dominant (T) allele. The presence of even a single dominant allele will result in a tall genotype. If no dominant allele is present, the sunflower will have 2 copies of the recessive dwarf allele. The phenotype for this plant will be dwarf, since no dominant allele is present.

2. *No. They are identical.*

Explanation: The position of each allele in a pair has no effect. The only factor that will affect phenotype is the combination of dominant and recessive alleles. However, the uppercase (dominant allele) is always stated first, so Tt is expressed correctly, while tT is not.

3. a) *2; T and t*

Explanation: The tall sunflower is heterozygous, so it has one of each of the two alleles (T and t). When producing sex cells for reproduction, each parent contributes only one allele to each sex cell. Thus, the heterozygous sunflower can contribute either a tall allele (T) or a dwarf allele (t).

b) *2; 2*

Explanation:

Tt × tt

	T	*t*
t	*Tt*	*tt*
t	*Tt*	*tt*

c) *1 : 1*

Explanation: As seen in the Punnett square, there are only 2 possible genotypes for the offspring in such a cross. One genotype is heterozygous (Tt) and will produce only tall flowers. This is due to the presence of the dominant tall allele (T). The other possible genotype is homozygous recessive (tt). Since the dominant tall allele is not present, this will produce only dwarf flowers.

4.

LL × Ll

	L	*L*
L	*LL*	*LL*
l	*Ll*	*Ll*

a) *16*

Explanation: The Punnett square illustrates all possible outcomes from the cross in each of the four boxes. Since individual outcomes occur randomly, each outcome has a 1 in 4 chance of occurring. For 16 offspring, you would expect 4 offspring to possess the genotype of each box within the Punnett square. However, in this particular cross, each box in the Punnett square contains at least one dominant allele (L). Thus, the phenotype for all four genotypes possible will be the same. The phenotype for all offspring will be the same—the dominant long tail.

b) *0*

Explanation: See above. Since a dominant allele (L) is present in all four possible outcomes of this cross, none of the offspring can show the recessive phenotype (ll) of a short tail. Only homozygous recessive individuals will have a short tail.

4. (cont'd.)

c). 8

Explanation: Here we must distinguish between the genotype and the phenotype among the outcomes in the Punnett square. While all of the offspring will have the same phenotype (long tails), they will not have the same genotype. As seen in the boxes, one half of the possible outcomes will have one of each allele (Ll). These are *heterozygous*. One half will have two identical alleles (LL) and are called *homozygous*. Since half of the expected outcomes are heterozygous, we should expect half of the actual outcomes to have a heterozygous genotype. The actual number of outcomes, however, may be different, since outcomes are random.

5.

Bb × Bb

	B	*b*
B	*BB*	*Bb*
b	*Bb*	*bb*

a) *black*

Explanation: If each parent is heterozygous, they each possess one dominant allele (B). The dominant allele will mask the expression of any recessive allele present. So, both parents will have the black fur phenotype.

b) *2; B, b*

Explanation: Each parent is heterozygous, meaning they possess one of each of the two alleles (Bb). Each parent contributes only one allele to each offspring, so each parent can contribute either the dominant (B) allele or the recessive (b) allele. The alleles are randomly selected from each parent.

5. (cont'd.)

c) *12; yes, 4*

Explanation: The Punnett square shows all possible outcomes from such a cross. Remember, the outcomes of Punnett squares are probabilities. Each outcome is random and has a 1 in 4 chance of actually occurring. Thus, out of 16 offspring, we would expect $^3/_4$ of them to possess at least one dominant (B) allele. These outcomes will have black fur. Three-fourths of 16 is 12. Of the possible outcomes, $^1/_4$ are homozygous recessive. These individuals will have brown fur, as the recessive trait is not masked by a dominant allele. One-fourth of 16 is 4. So, out of 16 offspring, we would expect 4 to have brown fur.

d) *8; 8*

Explanation: Each outcome is random and has a 1 in 4 chance of actually occurring. Of the four outcome boxes, two are homozygous (BB and bb) and two are heterozygous (Bb and bB). Thus, out of 16 offspring, we would expect half (8) to be homozygous for either allele and half (8) to be heterozygous, possessing one of both alleles.

6. *100%; no*

BB × BB

	B	*B*
B	*BB*	*BB*
B	*BB*	*BB*

Explanation: There is only one possible outcome from such a cross. Each parent can only contribute one dominant allele (B) to the offspring. Thus, the offspring must be homozygous for the dominant allele. Since no parent can contribute a recessive (b) allele for brown fur, there is no possibility that any offspring will show this trait.

Plants and Animals

Kingdom Plantae

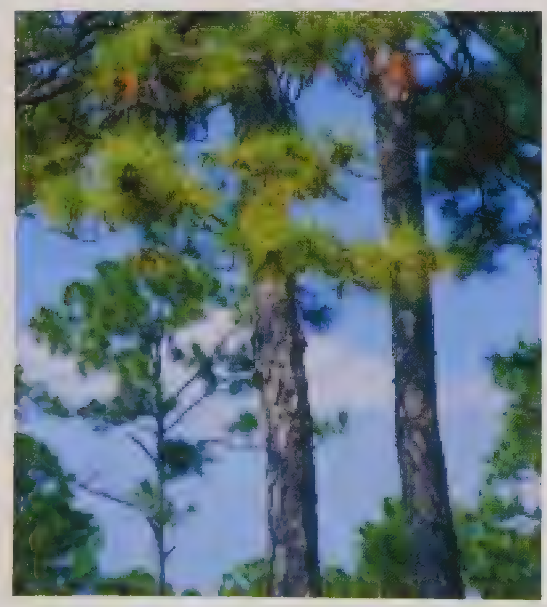
This kingdom consists of multicellular, photosynthetic organisms that range in size from tiny mosses to giant sequoia trees. Reproduction among the plants is both sexual and asexual.

Phylum Bryophyta. The *bryophytes* are small, nonflowering plants.

- made up of Class Musci (mosses), Class Hepaticae (liverworts), and Class Antherocerotae (hornworts)
- bryophytes are *nonvascular*—do not possess a system of specialized cells for carrying water and food through the plants
- frequently found near water, and they remain small
- over 23,000 species of bryophytes

Class Musci

- consists of about 14,000 species of mosses
- tiny plants that grow in clumps
- leaflike parts above ground and rootlike structures, called rhizoids, that absorb water and nutrients
- among the first species to inhabit harsh environments
- on rocky surfaces, mosses break up the rocks and create fertile soil where other plants can survive



Pine trees are conifers.

Phylum Tracheophyta. *Tracheophytes* are any of about 260,000 species of *vascular* plants. Vascular systems allow plants to grow very tall.

- have a system of roots, stems, and leaves
- roots absorb water and minerals from the ground, transport them to the rest of the plant
- stems transport nutrients, provide sturdiness and support for the plant's leaves and branches
- leaves manufacture food through photosynthesis

Class Pteridopsida

- consists of about 11,000 species of seedless plants called *ferns*

Class Gymnospermopsida

- consists of the *gymnosperms*, one of two large groups of seed-bearing plants
- gymnosperm means "naked" or "exposed" seed—the seed is not enclosed by a fruit
- includes about 550 species of cycad trees, over 500 species of conifers, and one species of ginkgo

Division Anthophyta

- consists of the *angiosperms*, or flowering plants
- flowers contain plants' reproductive organs
- flowering plants are divided into two classes: the *monocots*, with one seed leaf; and the *dicots*, with two seed leaves
- monocots include grasses, lilies, orchids, and palms
- dicots include oaks, maples, beeches, willows, mustards, roses, poppies, and mints

Ginkgo. Only one species of ginkgo exists today.

Praying Mantis. Scientists have classified about 1 million insects.

Kingdom Animalia

Animals are multicelled organisms that do not produce their own food and must obtain it by ingesting organic materials. Most animals move. Members of this kingdom fall into two large groups: invertebrates, animals without backbones; and vertebrates, animals with backbones.

Phylum Porifera (sponges)

- consists of over 4,000 species of sponges, which live on the floors of oceans and lakes

Phylum Cnidaria (cnidarians or coelenterates)

- includes hydras, jellyfish, corals, and sea anemones
- animals with definite tissues and a saclike digestive cavity surrounded by special organs such as tentacles

Phylum Platyhelminthes (flatworms)

- have three tissue layers and definite organs
- many flatworms are parasites that live in humans and animals

Phylum Nematoda (roundworms)

- consist of an outer tube enclosing a digestive system
- some are parasites

Phylum Annelida (segmented worms)

- majority of these worms live in marine habitats, but the best known is the earthworm

Phylum Mollusca (mollusks)

- largest group of water animals—110,000 species
- soft body and a foot protruding from it; some mollusks also have a shell
- includes clams, mussels, oysters, snails, slugs, squids, and octopuses

Phylum Echinodermata (echinoderms)

- about 6,000 species of spiny-skinned animals found in marine environment
- includes starfish, sea urchins, sea cucumbers, and sand dollars

Phylum Arthropoda (arthropods)

- the largest phylum in the animal kingdom—around 1 million species
- have jointed legs, segmented bodies, and an exoskeleton—an outside shell
- includes insects, crustaceans, and arachnids

Phylum Chordata (chordates)

- phylum consists primarily of the vertebrates
- includes amphibians, birds, mammals, and reptiles, as well as hagfishes, lampreys, and bony fishes

Gorilla

PLANT TAXONOMY

is the science of naming and classifying plants. Botanists classify plants by grouping them according to their shared similarities. This provides a logical way to organize information about plants and to show how different plants are related to each other.

Most botanists group plants by their overall appearance, their internal structure, and the form of their reproductive organs. One frequently used classification system classifies plants into two large *groups* and several *divisions*.

Classification of Plants

Plant classification and form provides the framework for almost all fields of botany. In studying a plant, a botanist must first know what type of plant it is. Botanists who specialize in *systematics* identify and name plant species.

Given the complexity of their task, it is not surprising that continued revision of earlier taxonomic divisions goes on. Plants can now be examined for differences and similarities at their molecular level, rather than solely on the basis of external appearance, form, and structure. This new information is also altering some taxonomic presumptions about family lineage.

Fortunately, the system of plant classification that has evolved over the centuries is flexible, and its organizational framework is now capable of incorporating not only the 350,000 or so known plant species, but the estimated 100,000 plants that will be discovered in the future. (See pages 90–91.)

Two Main Groups

The plant kingdom is separated into two large groups: those that do not have *vascular tissue* and those that do. Vascular tissue is tissue that carries water and nutrients throughout the plant.

Nonvascular Plants. The nonvascular plants lack *xylem* and *phloem* tissues. These are the vascular tissues that carry water and food from one part of the plant body to another. Nonvascular plants do not have true roots, stems, or leaves. They are usually low to the ground and live in moist, shady places.

Vascular Plants. All other divisions of the plant kingdom are made up of vascular plants that contain xylem and phloem tissues.

- **xylem:** the tissue that carries water throughout a vascular plant
- **phloem:** the tissue that carries food throughout a vascular plant

◀ **Elm trees,** like all trees, are vascular plants. They have true roots, stems, and leaves. Their water and nutrients are transported by xylem and phloem tissues.

DIVISIONS OF THE PLANT KINGDOM

The divisions of Kingdom Plantae are mostly organized in ascending order of complexity.

Chlorophyta	Single celled or multicelled, chlorophyll present, mostly freshwater
Rhodophyta	Multicelled, red pigments cover green of chlorophyll, mostly marine
Phaeophyta	Multicelled, brown pigments cover green of chlorophyll, mostly marine
Bryophyta	Multicellular reproductive structures, no true phloem or xylem
Lycophyta	True leaves have a single vein and are photosynthetic, variously branched stems
Sphenophyta	Nonphotosynthetic, leaves are reduced to scales, jointed stems
Psilophyta	Vascular, without true leaves, stems branch by forking
Pterophyta	Leaves have more than one vein, spores are produced on leaves
Cycadophyta	Gymnosperms, leaves are pinnate and palmlike, sperm are motile
Ginkgophyta	Gymnosperms, strong cambial growth, small fan-shaped leaves, motile sperm
Gnetophyta	Gymnosperms, seeds are produced in cones, wood contains vessels, nonmotile sperm
Pinophyta	Gymnosperms, wood contains no vessels, strong cambial growth, nonmotile sperm
Anthophyta	Flowering, seeds borne within fruits

Modern Classification

Each plant is described in a series of rankings, going from the most general to the most specific, and reflecting the progressive diversification of life from certain common ancestors to ever-greater specialization.

The evolutionary pathways by which plants have progressed over millions of years becomes an integral part of the description.

- Starting at the largest and most inclusive category, every living organism is assigned membership in a *kingdom*.
- The plant kingdom is subdivided into *divisions* (called *phyla* in the animal kingdom) according to certain perceived similarities and differences among their populations.
- Divisions are further divided into distinct *classes*.
- Classes are divided into *orders*.
- Orders are divided into *families*.
- Families are divided into *genera* (singular *genus*).
- Genera are divided into *species*.

Naming Species

Each individual species is officially known by its two-part species name. The first name is the species' Latin or Greek generic, or genus, which is capitalized, and the second, its species epithet or descriptor, which is uncapitalized. The species name usually describes some unique characteristic such as color, form, habit of growth, native origin, or the individual who first introduced it to the Western world.

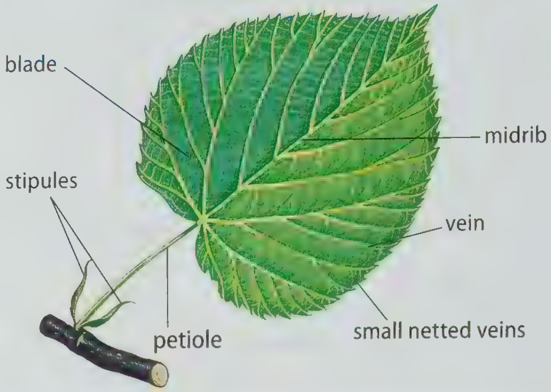
RULES OF BOTANICAL NOMENCLATURE

Consistency is important in science. It allows scientists around the world to share information. In 1930, the International Rules of Botanical Nomenclature were accepted by botanists of virtually all nations, making it possible for scientists to communicate accurately with each other as new species are added or known species are reevaluated and their taxonomic descriptions modified.

Leaves

Leaves are the plant structures that are specialized primarily for the manufacture of food through photosynthesis. Most leaves are constructed to provide the greatest possible surface area. They are arranged around the stem to expose this surface to the maximum amount of sunlight with the least amount of interference from neighboring leaves. Leaves also exchange gases in the process of respiration, store food, and may reproduce new plants vegetatively.

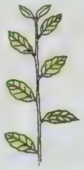
The Parts of a Leaf



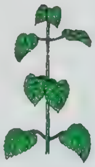
Leaf Arrangement

Leaves appear at nodes, spaced at regular intervals along the stem. Nodes and their leaves are arranged on the stem in one of three patterns.

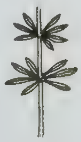
1. **Alternate arrangement.** The most common pattern, a single leaf appears at each node and the leaves form a continuous ascending spiral on the stem.
2. **Opposite arrangement.** Two leaves are attached at the same node and directly across the stem from each other.
3. **Whorled pattern.** Three or more leaves develop at the same node, but at well-spaced positions around the stem.



Alternate



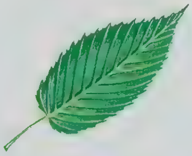
Opposite



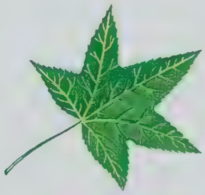
Whorl

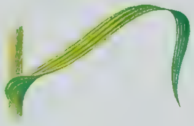
Vein Patterns

pinnately veined
(lower side of blue beech)

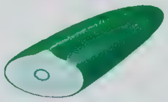


palmately veined
(lower side of sweet gum)





parallel veined
(upper and lower sides of rye)



center veined
(cross section of white pine)

Leaf Form

Most leaves consist of three main parts.

1. A stalk, or petiole
2. A broad, expanded blade, or lamina
3. An arrangement of veins

The petiole joins the leaf blade to the stem. It is usually, but not always, fairly slender, short, and inconspicuous.

The blade is the broad, flat part of the leaf. Photosynthesis occurs in the blade. Blades come in many shapes and sizes, from broad fronds to tiny scales and needles. They can have smooth edges or small, jagged teeth.

The leaf blade may be simple, meaning it consists of a single blade, or compound, meaning it consists of several leaflets.

The veins distribute water and nutrients throughout the leaf. The veins of most broad leaves form a netlike pattern. There are two main types of net-vein patterns: *pinnate* and *palmate.*

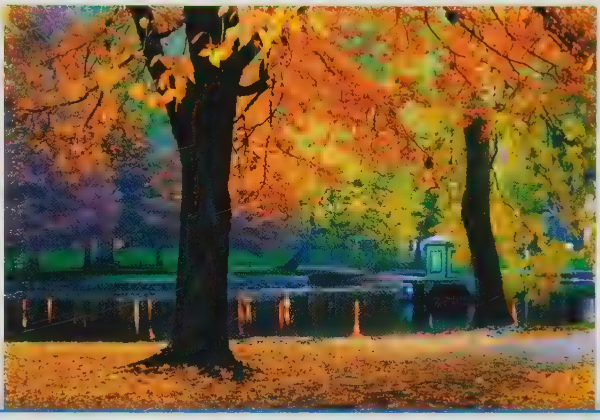
- Pinnately net-veined leaves have one large central vein, called the *midrib*, which extends from the base of the blade to its tip. Other large veins branch off on each side of the midrib.
- Palmately net-veined leaves have several main veins of about equal size, all of which extend from a common point at the base of the blade.

HOW DOES THAT WORK

FALLING LEAVES

The falling of deciduous leaves occurs as the result of chemical changes. In autumn, the level of the hormone *auxin* in the leaf naturally decreases. A special layer of cells, called the *abscission layer,* forms at the base of the petiole and blocks the continued flow of water and nutrients to the leaf. Chlorophyll ceases to be produced, and the green color of the leaf fades, replaced by the brilliant oranges, reds, and yellows of autumn. Meanwhile, the petiole becomes progressively weaker and eventually breaks off the stem, leaving only a leaf scar.

Fall colors ▶



Leaf Structure

Epidermal Layer

- covers top and bottom surface of leaf
- one cell thick
- epidermal cells secrete waterproof waxy cuticle to slow down water evaporation
- openings in cuticle called stomata allow CO_2 to enter the leaf and O_2 to exit the leaf

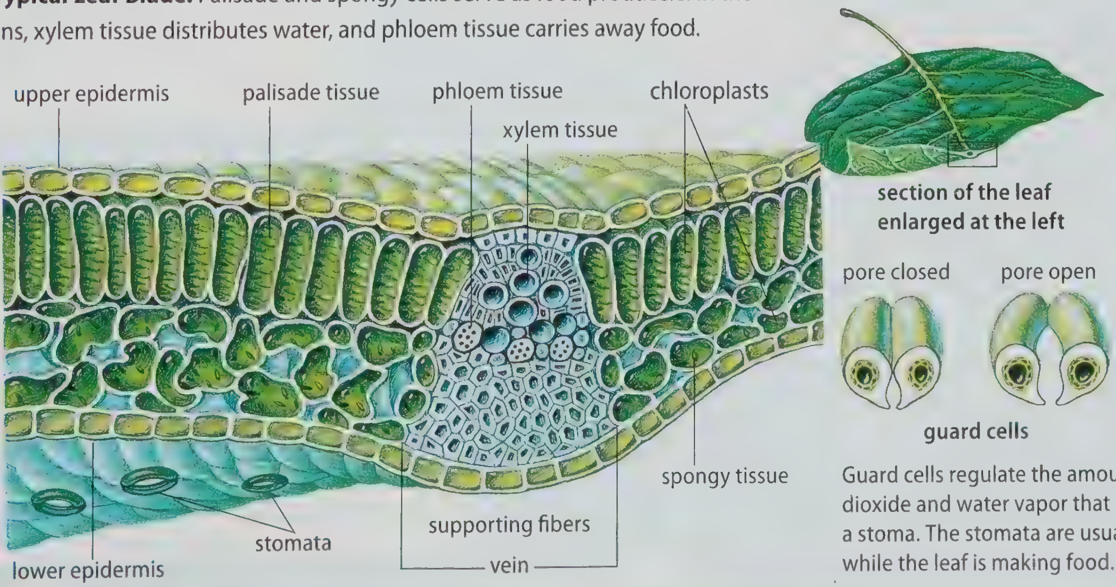
Mesophyll

- multilayered region between the upper and lower epidermis
- composed chiefly of parenchyma cells with numerous chloroplasts
- site of most, if not all, photosynthesis
- upper tier populated with cells arranged in tightly ranked palisades
- lower tier designated the spongy mesophyll because of irregularly fitting, loosely assembled parenchyma cells

Vascular Tissue

- vascular tissue, or veins, is interwoven through the mesophyllic cell and very conspicuous on the leaf's surface.

A Typical Leaf Blade. Palisade and spongy cells serve as food producers. In the veins, xylem tissue distributes water, and phloem tissue carries away food.


upper epidermis
palisade tissue
phloem tissue
xylem tissue
chloroplasts
section of the leaf enlarged at the left
pore closed
pore open
guard cells
spongy tissue
supporting fibers
vein
stomata
lower epidermis

Guard cells regulate the amount of carbon dioxide and water vapor that pass through a stoma. The stomata are usually open while the leaf is making food.

Flowers

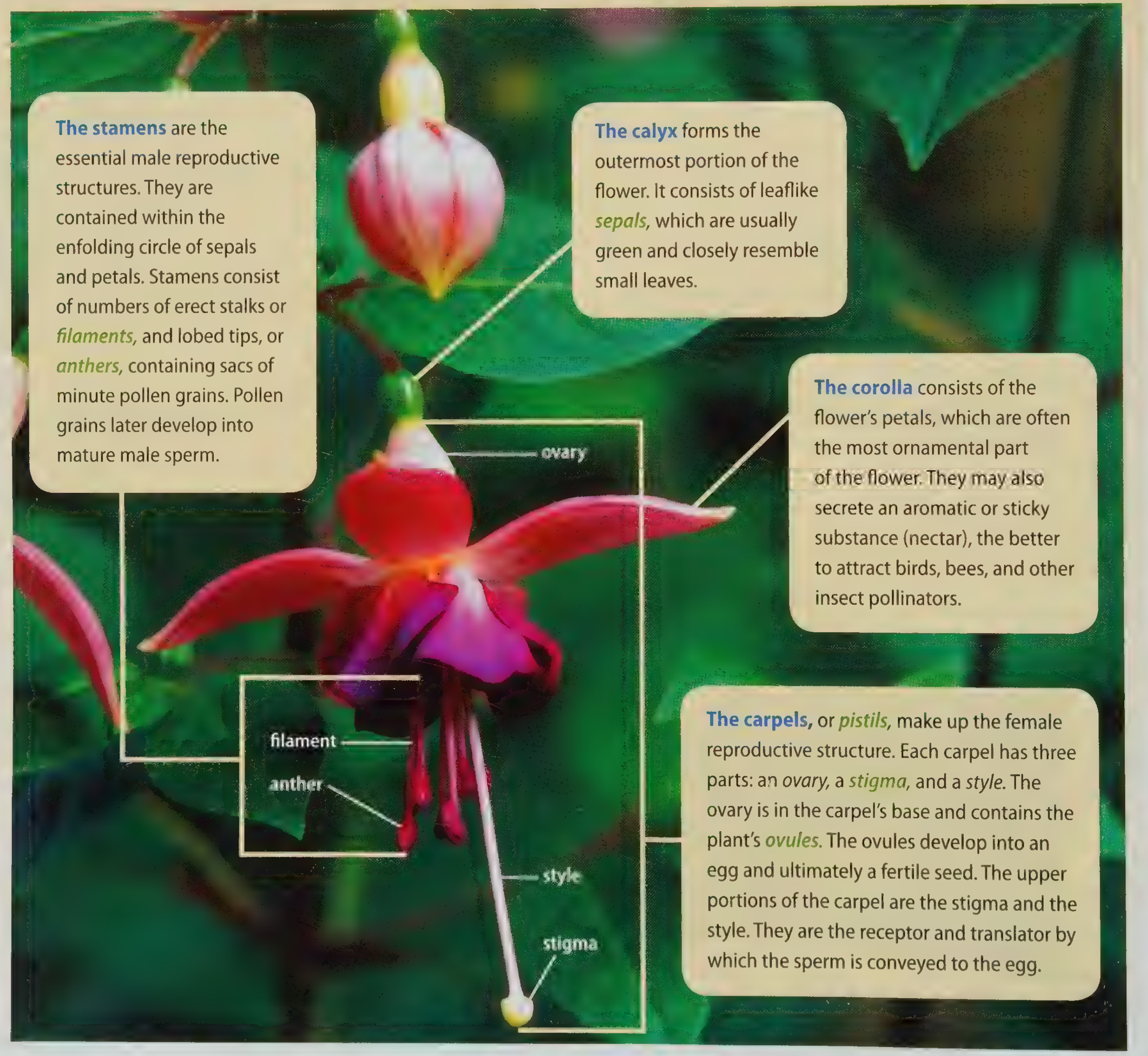
Roots, stems, and leaves are all vegetative organs pertaining to asexual growth processes. By contrast, flowers are organs of sexual reproduction, producing the male and female gametes (pollen and eggs) that will unite to produce the next generation of plants.

Only angiosperms have flowers. The flower and its resulting fruit and seed are the structures that have made angiosperms—the youngest division of the plant kingdom—the most successful in evolutionary terms.

Flower Parts

All flowers are based on the same structural design. The flower stands atop a flower stalk, known as a *pedicel*, which terminates in an enlarged *receptacle*. Attached to the receptacle in orderly succession are four floral organs: the *calyx*, the *corolla*, the *stamens*, and the *carpels*.

PARTS OF A FLOWER

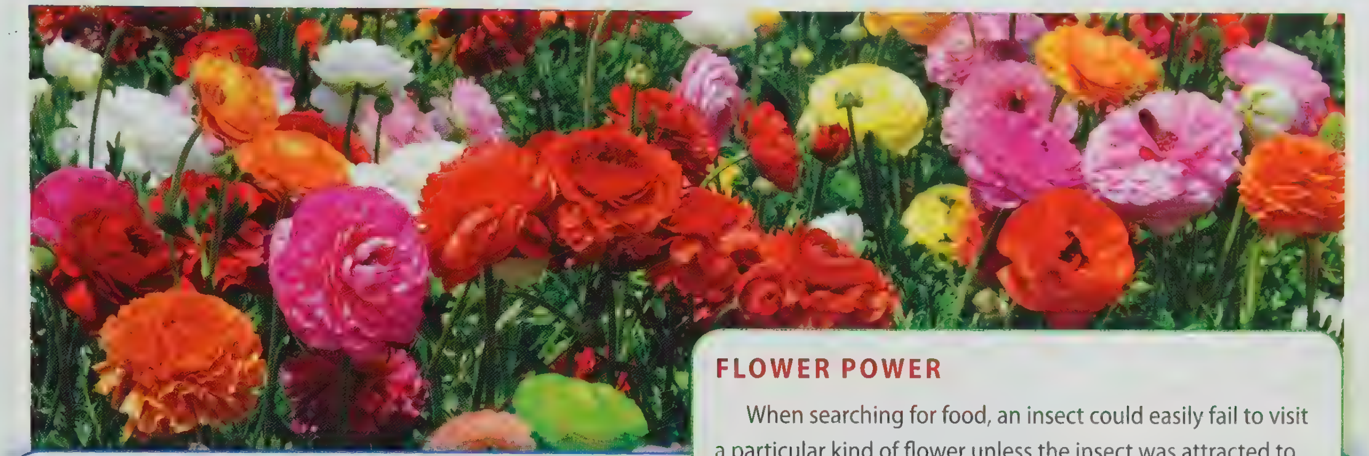


The stamens are the essential male reproductive structures. They are contained within the enfolding circle of sepals and petals. Stamens consist of numbers of erect stalks or *filaments*, and lobed tips, or *anthers*, containing sacs of minute pollen grains. Pollen grains later develop into mature male sperm.

The calyx forms the outermost portion of the flower. It consists of leaflike *sepals*, which are usually green and closely resemble small leaves.

The corolla consists of the flower's petals, which are often the most ornamental part of the flower. They may also secrete an aromatic or sticky substance (nectar), the better to attract birds, bees, and other insect pollinators.

The carpels, or *pistils*, make up the female reproductive structure. Each carpel has three parts: an *ovary*, a *stigma*, and a *style*. The ovary is in the carpel's base and contains the plant's *ovules*. The ovules develop into an egg and ultimately a fertile seed. The upper portions of the carpel are the stigma and the style. They are the receptor and translator by which the sperm is conveyed to the egg.



FLOWER POWER

When searching for food, an insect could easily fail to visit a particular kind of flower unless the insect was attracted to it. Most flowers that depend on insects for pollination are brightly colored or heavily scented. Each kind of pollinating insect is attracted by certain colors or odors and so visits certain flowers rather than others.

Reproduction

Flowering plants reproduce sexually in two main steps: *pollination* and *fertilization.* Pollination is the transfer of pollen from a stamen to a pistil. Fertilization is the union of a sperm with an egg cell.

Pollination

There are two methods of pollination: *self-pollination* and *cross-pollination.* In self-pollination, pollen is transferred from a stamen to a pistil of the same flower or to a pistil of another flower on the same plant. Cross-pollination involves the transfer of pollen from a stamen on one plant to a pistil on another plant.

Self-Pollination. A flower with both stamens and carpel is said to be perfect because it contains male and female parts. A perfect flower is capable of self-pollination. About half of all species of plants normally pollinate themselves.

Cross-Pollination. Most flowering plants are pollinated through cross-pollination. Pollen is transferred from one flower to another mainly by insects, but also by birds, bats, and the wind.

Many insects depend on flowers for food. Bees live on nectar and pollen. Butterflies and moths also live on nectar, and certain beetles and flies feed on both nectar and pollen. As an insect travels from flower to flower in search of food, pollen grains stick to its body. Some or all of these grains may brush off onto the stigmas of some flowers that the insect visits. One or more of these flowers may thus become cross-pollinated.

Fertilization

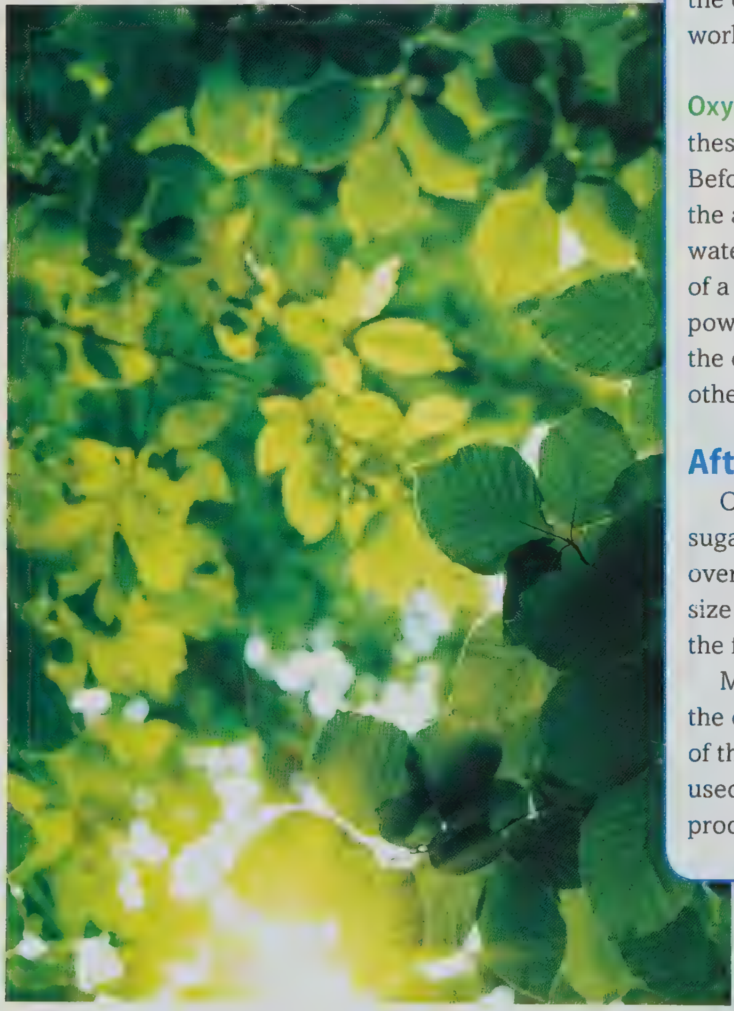
A pollen grain that lands on a stigma may grow a *pollen tube.* The tube pushes its way down the style to an ovule in the ovary. Sperm from the pollen grain travel down the tube to the ovule. Fertilization occurs when a sperm unites with an egg cell in the ovule. A seed then begins to develop. The ovary itself develops into a fruit that encloses the seed.

Busy Bees. More flowers are pollinated by bees than by any other insect. Bees become covered in pollen when they drink nectar from flowers. A bee will leave some of this pollen at other flowers it visits, pollinating them.

Photosynthesis

Photosynthesis is a food-making process that occurs in green plants, algae, and certain microscopic organisms. Photosynthesis is the chief function of leaves. The word *photosynthesis* means *putting together with light.*

Green plants use sunlight to combine carbon dioxide and water to make food. This process converts light energy into the chemical energy of food. Plants use the food to grow, or they store it for later consumption.



Photosynthesis has been called the single most important chemical process in the world. It provides food for plants, and plants in turn support all the animals in the food chain, including humans.

The Importance of Photosynthesis

Photosynthesis provides the critical link between inorganic chemicals and the living world. The food that photosynthesis produces nourishes the plants and the plants in turn support all other organisms higher on the food chain.

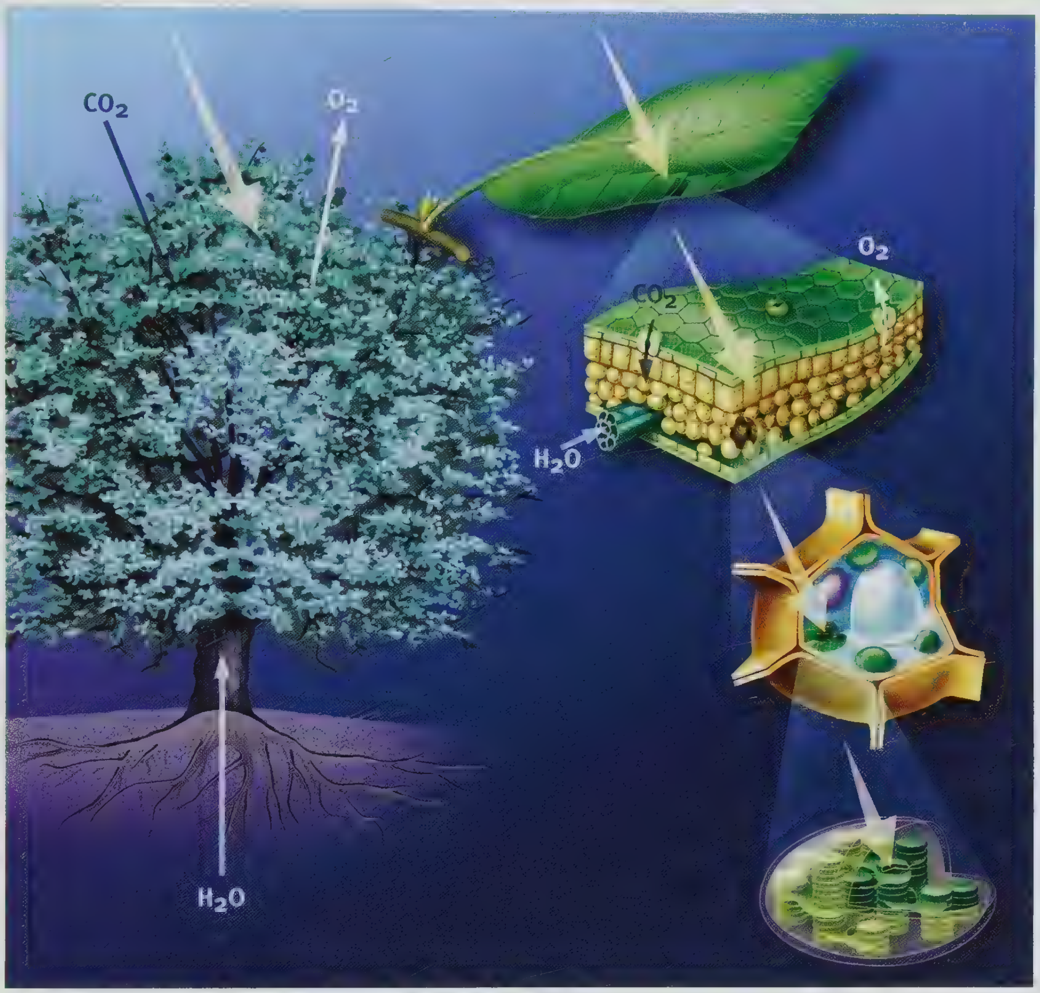
Food and Materials. Photosynthesis is extremely efficient. A plant needs no more than one-sixth of the energy it produces for its own maintenance, leaving as much as five-sixths available to make food, fuel, and the other materials useful to the rest of the natural world, including the human economy.

Oxygen. Equally important to life on Earth, photosynthesis enables the release of free oxygen molecules. Before there were photosynthesizing plants on Earth, the atmosphere consisted largely of ammonia (NH_3), water (H_2O), and methane (CH_4). The development of a vast biomass of green plants with its unique and powerful system of gas exchange has literally created the oxygenated air that supports the great number of other living organisms, including humans.

After Photosynthesis

Once photosynthesis has done its work, the simple sugars and the other organic compounds go to sites all over the plant. Some of the product is used to synthesize proteins within the cell. Some is stored locally in the form of starch.

Most of the sugars, however, are transported from the cell and the leaf to other parts of the plant by way of the vascular system. These sugars will be stored, used to produce new plant parts, or broken down to produce energy for the plant's immediate use.



Green plants capture light energy to first break down water into oxygen molecules and hydrogen. The captured energy is then used to attach the hydrogen to carbon dioxide, thus producing chemical energy in the form of a simple sugar known as glucose.

PHOTOSYNTHESIS

The energy and nourishment created by photosynthesis provide all the food that people and other animals eat. Animals either eat plants directly, or eat animals that eat the plants.

1. The process begins when light strikes the leaf's ***chloroplasts***. Chloroplasts are small green bodies found in the plant's cells. They can occur in the slender cells of the plant's palisade tissue, or in the broad, irregularly-shaped cells of the spongy tissue.
2. Energy from the light is absorbed by ***chlorophyll***, which is the green *pigment*, or coloring matter, in the chloroplasts.
3. Partly surrounding each cell of the palisade and spongy tissues is an air space filled with carbon dioxide, water vapor, and other gases. The energy absorbed by the chlorophyll splits the water molecules in these spaces into molecules of hydrogen and oxygen.
4. The hydrogen then combines with carbon dioxide to produce a simple sugar.
5. The excess oxygen enters the air through the *stomata*, or pores, of the leaf. People, animals, and all other living organisms need oxygen in order to live.
6. The sugar produced by photosynthesis is carried throughout the plant in ***phloem***. Phloem is tissue made up of special tubelike cells.
7. The sugar can be burned in order to release energy for the plant's growth, or it can be combined with various minerals. These minerals enter the plant through the roots.
8. The combination of sugar and minerals produces vital compounds, such as proteins and vitamins. These compounds provide nourishment for the plant.

Muscular System

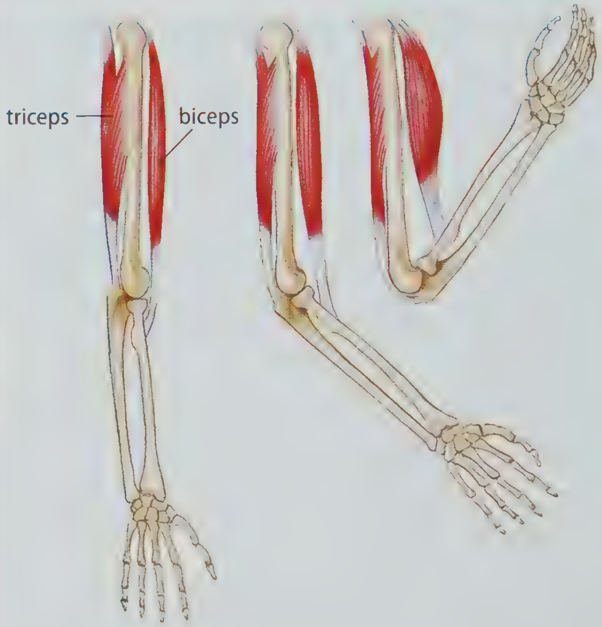
The muscular system moves the body. There are more than 600 muscles throughout the human body, and they occur mostly in pairs. Each muscle consists of special fibers that can contract. When a muscle contracts, it pulls the tissue to which it is attached. This pulling results in movement. There are two main types of muscles.

- skeletal muscles
- smooth muscles

Skeletal Muscles

- the consciously controlled muscles; known as *voluntary muscles*
- attached to the bones
- move the bones of the arms, legs, fingers, and other parts of the skeleton
- made up of fibers called striations that have alternate light and dark crossbands

Attachments. One end of each skeletal muscle is attached to a bone that does not move when the muscle contracts. In most cases, the other end of the muscle is attached to another bone, either directly or by means of cordlike bundles of connective tissue called *tendons.* This second bone moves when the muscle contracts.



PULLING IN PAIRS

Muscles move the body only by pulling. They cannot push the tissues to which they are attached. Two sets of muscles therefore control most skeletal movements, such as the raising and then lowering of the forearm. One set pulls the bones in one direction, and the other set pulls the bones in the opposite direction. Such pairs of muscles are known as antagonistic muscles.

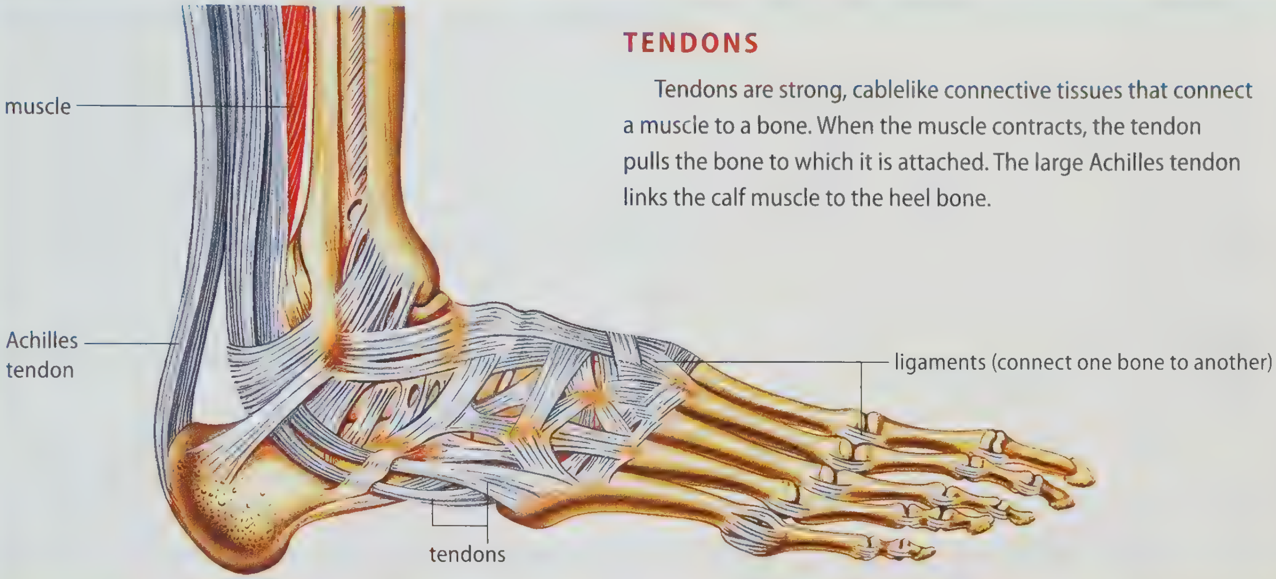
For example, one set of muscles pulls the forearm up, but it cannot push the forearm down. To lower the forearm, a second set of muscles must contract and pull it down.

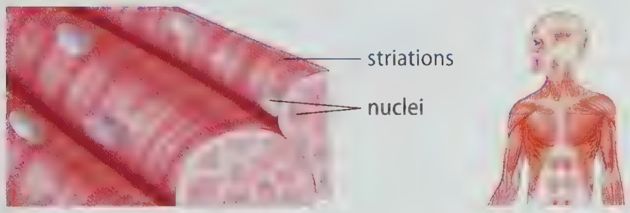
Some Important Skeletal Muscles

Part of Body	Muscle	Action
face	several different muscles	work together to help express emotion—to smile, frown, or raise an eyebrow; make it possible to bite, chew, or speak
shoulder	deltoids	raise the arms
shoulder	trapezius	raises the shoulder; pulls back the head
upper arm	biceps	bend the elbow
upper arm	triceps	straighten the elbow
torso	diaphragm	main muscle involved with breathing; laughing, coughing, sneezing
buttocks	gluteus medius and gluteus maximus	move the body from sitting to standing; help the body to run or walk up stairs
thigh	sartorius and quadriceps	bend knees, move legs, balance when standing
thigh	hamstrings (biceps femoris; semitendinous and membranous tendons)	bend the knee joint; straighten the hip joint

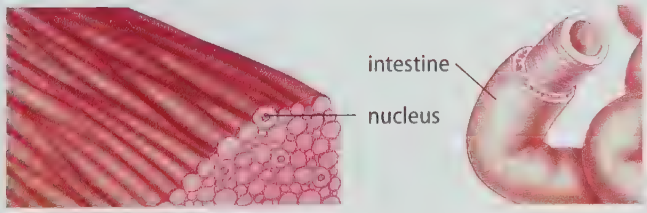
TENDONS

Tendons are strong, cablelike connective tissues that connect a muscle to a bone. When the muscle contracts, the tendon pulls the bone to which it is attached. The large Achilles tendon links the calf muscle to the heel bone.





Skeletal Muscles

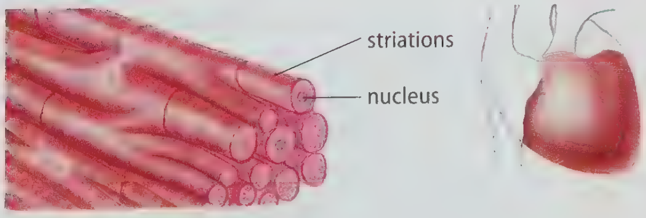


Smooth Muscles

Smooth Muscles

- the *involuntary muscles;* contract and relax automatically—a person has no direct control over them
- found in most of the body's internal organs
- do not have striations
- cannot contract as rapidly as skeletal muscles
- can contract more completely than skeletal muscles and do not tire as quickly
- can produce powerful, rhythmic contractions over long periods

Vital Functions. Smooth muscles in the walls of the stomach and intestines move food through the digestive system. Smooth muscles also control the diameter of the blood vessels and the size of the breathing passages.



Cardiac Muscles

Cardiac Muscle

There is a third kind of muscle that is found only in the heart. Called *cardiac muscle,* it has features of both skeletal muscle and smooth muscle. Cardiac muscle has striations like skeletal muscle, but contracts automatically and rhythmically like smooth muscle.

Cardiac muscle enables the heart to beat an average of 70 times a minute without rest throughout a person's lifetime.

SENSE ORGANS provide people—and other multicelled animals—with information about what is happening in their environment.

The five main sense organs are the eyes, ears, nose, tongue, and skin. However, there are other, *internal senses* that also give information about the position and movement of the body and about the body's needs.

Each sensory system responds to a particular kind of stimulus. Cells called *sensory receptors* alter electrical impulses along their membranes. The brain handles the information provided by the senses. Pathways inside and outside the brain move sensory information throughout the body.

Eye

The eye is the organ of sight. It is our most important organ for finding out about the world around us. The eye is able to see objects by capturing light that either reflects from or is emitted by the objects.

The eye can see in bright light and in dim light, but it cannot see in no light at all. Light rays enter the eye through transparent tissues. The eye changes the rays into electrical signals. The signals are then sent to the brain, which interprets them as visual images.

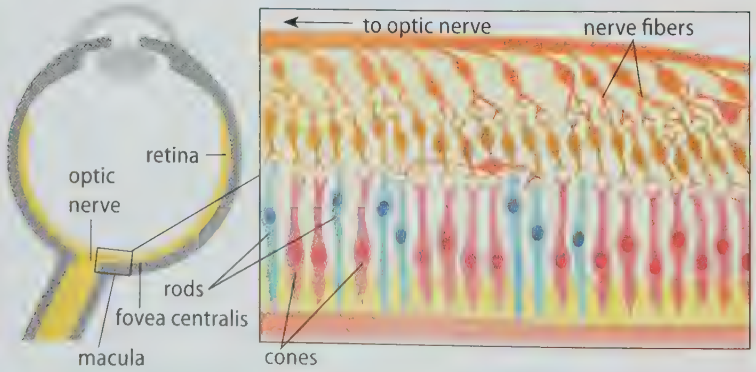
Parts of the Eye

Each eyeball is set in a *socket* in the skull. Its ridges form the brow and the cheekbone. Fatty tissue inside the socket nearly surrounds the eyeball and cushions it. Six muscles move the eyeball in much the same way that strings move the parts of a puppet.

Within the Socket

- The visible parts of the eyeball are the white *sclera* and the colored *iris*.
- A clear membrane called the *conjunctiva* covers the sclera.
- The transparent *cornea* lies in front of the iris.
- The *lens* is connected to the *ciliary body*.
- Inside the eyeball is a clear jellylike substance called *vitreous humor*.
- The *retina*, which underlies the *choroid*, changes light rays into electrical signals.
- The *optic nerve* carries the signals to the brain.
- The *fovea centralis*, a pit in the macula lutea, is the area of sharpest vision.

Rods and cones are named for their distinct shapes. The retina has about 120 million rods and about 6 million cones. ▼



SEEING IN COLOR

The retina has two types of light-sensitive cells: *rods* and *cones*. These cells absorb light rays and change them into electrical signals. Pigment (colored material) in the cells absorbs particles of light.

- **Rods:** The pigment in the rods enables the eye to see shades of gray and to see in dim light.
- **Cones:** The three types of pigment in the cones enable the eye to see colors. The pigments absorb blue, green, or red light, and allow humans to distinguish more than 200 colors.

How We See

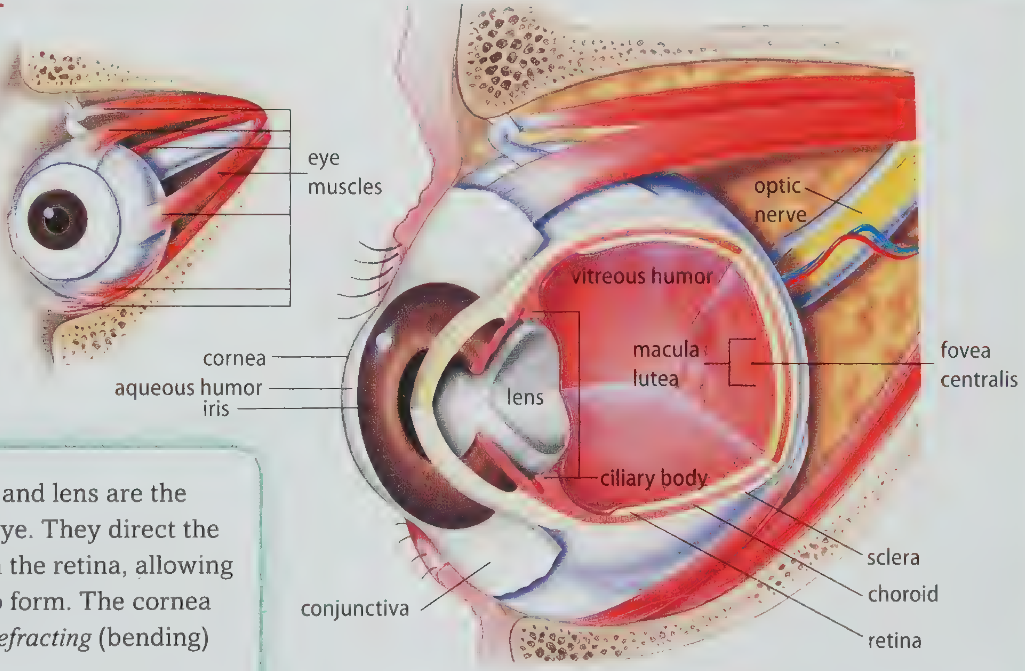
The eyes direct the light rays reflected off objects to a single point on the retina to form a focused image.

Images that form on the retina are upside down, reversed left to right, and flat. Depth perception allows the brain to interpret the image correctly.

Depth perception is the ability to judge distance and to tell the thickness of objects. Because the eyes are set slightly apart, each one sees objects from a slightly different angle. As a result, each eye sends a slightly different message to the brain.

Some of the nerve fibers from each eye cross over at the optic chiasm. Each side of the brain thus receives visual messages from both eyes. The brain puts the images together and so provides depth perception.

PARTS OF THE EYE



Eye muscles rotate the eyeball within its socket. The eye has six of these muscles.

Focusing. The cornea and lens are the focusing parts of the eye. They direct the light rays to a point on the retina, allowing a clear visual image to form. The cornea provides most of the *refracting* (bending) power of the eye.

After light rays pass through the cornea, they travel through the *aqueous humor* and the pupil to the lens. The lens bends the rays even closer together before they go through the vitreous humor and strike the retina.

The eye shifts focus from between nearby objects and distant ones through *accommodation*. When the eye looks at close objects, the muscles around the lens contract. The lens becomes rounder and thicker and thus more powerful. When the eye looks at distant objects, the muscles relax and the lens becomes flatter.

Light and Dark. The eye adjusts to changing levels of light through a process called *adaptation*. In bright light, muscles in the iris contract. The pupil gets smaller so that less light is allowed in. In dim light, the pupil gets larger to let in more light.

Biology Glossary

adenine one of two major purines (nitrogen-containing bases) found in both RNA and DNA. Also found in various free nucleotides, for example, adenosine triphosphate (ATP).

adenosine diphosphate (ADP) a compound of adenosine and two phosphate groups, formed from adenosine triphosphate in the muscles.

adenosine triphosphate (ATP) an enzyme found in all cells, especially muscle cells; a compound of adenosine and three phosphate groups.

allele different forms of the same gene; may be dominant or recessive.

analogous features of living things that have similar functions but do not share a common origin.

angiosperm the flowering plants; seed-bearing plants that are the dominant and most varied division of plants on Earth today.

antibiotics substances produced by some molds that weaken or destroy bacteria and other organisms that cause disease.

antibodies protein substances produced by white blood cells that specifically bind to a single foreign antigen, neutralizing or destroying the invading microorganisms in the bloodstream.

archaea a group of single-celled organisms that make up one of the three basic divisions of life; rank among the oldest forms of life on Earth.

asexual reproduction the formation of new organisms from parts of a single organism; organisms that form through asexual reproduction have the same exact DNA structure as their parent.

auxins the prime growth-promoting hormones in plants.

bacteria single-celled plantlike microorganisms lacking chlorophyll; consist of single cells that are rod-shaped, spherical, or spiral.

bivalve a mollusk with a shell that is hinged together so that it can open and close.

botany the study of plants, including their structure, life processes, historical origins, geographic distribution, economic uses, and roles in the ecosystem.

brain stem the part of the brain that contains vital reflex centers that help to control heart rate, blood pressure, and respiration; connects the spinal cord to the forebrain.

calyx the outermost portion of the flower; consists of leaflike sepals, which are usually green and closely resemble small leaves.

carbohydrates water-soluble sugars; provide the main source of energy for the body.

carpels or pistils, make up the female reproductive structure of a plant; develop the egg and ultimately a fertile seed.

cell the basic unit of all life; cells are self-reproducing, with new cells arising by cell division.

centrioles a pair of small, rodlike bodies in a cell that assist in the distribution of the chromosomes during the process of cell reproduction.

cerebellum the part of the brain that aids in maintaining body balance and coordination of muscle movement; located below the back part of the cerebrum.

cerebrum the largest portion of the brain; consists of two large lobes, on the right and left sides of the brain.

chemical evolution theory that simple organic compounds formed naturally early in Earth's history and combined to form the complex structures associated with living things.

cholesterol a sterol; fatlike substance found in animal fats and oils, nerve tissue, bile, blood, and egg yolk; a normal constituent of bile, also produced in the liver.

chromatin substance found in the cell nucleus, where it forms chromosomes; made up of DNA and proteins.

chromosomes threadlike structures in the cells of organisms that carry the hereditary information of the organism.

cladistic systematics system of animal classification based on how long ago two species shared a common ancestor.

collagen an insoluble protein that accounts for about 30 percent of total body protein; found in the white fibers of connective tissue, cartilage, and bone.

corolla organ of a flower; consists of the flower's petals, which are often the most ornamental part of the flower; may secrete nectar.

cortex botany, plant tissue that forms the bulk of stems, roots, leaves, and fruits; biology, the outer layer of an internal organ.

creationism a set of beliefs based on the idea that a Supreme Being brought into existence Earth and all its life through a direct act of creation.

cytokinesis division of the cytoplasm of a cell which begins during telophase and leads to cell division.

cytoplasm the cell material between the outer membrane and the nucleus; contains the organelles that perform many of the vital functions of the cell.

cytosine a pyrimidine base found in nucleic acids; an essential constituent of both RNA and DNA.

decomposer an organism, such as bacteria or fungi, that feeds on the dead remains of organisms and organic waste.

deoxyribonucleic acid (DNA) the substance of which genes are made; a double-stranded macromolecule, located in the nucleus of eukaryotic (plant and animal) cells and in the organelles, mitochondria, and chloroplasts; often referred to as the genetic code.

diatoms single-celled algae; among the living things that comprise plankton.

dicot group of flowering plants containing two cotyledons, or seed leaves; includes oaks, maples, beeches, willows, mustards, roses, poppies, and mints.

diffusion the movement of a substance from an area of high concentration to an area of low concentration.

digestion the process by which food is broken down, mechanically and chemically, and is converted into absorbable forms, in the gastrointestinal tract.

diploid having a homologous pair of chromosomes for each characteristic except sex, the total number of chromosomes being twice that of a gamete.

dominant a gene which expresses itself when paired with either a recessive or another dominant allele.

dormancy a control mechanism in seed plants that keeps the seed in a state of suspended animation until the time of the year in which its chances for survival are best.

egg a female gamete.

embryo a stage in prenatal development; in humans, it includes the period from the second through the eighth week.

endemic describes infectious diseases that are always present in a particular region.

enzymes proteins that act as biochemical catalysts; speed up the chemical reactions of life; they help digest food, produce energy, and assist in building other proteins.

epidemic an unusually contagious illness that sweeps through a community or an entire continent.

epidermis cuticle, or outer layer, of the skin; consists of four layers of skin, the stratum corneum being the outermost.

eudicot group of flowering plants containing two cotyledons, or seed leaves.

eukaryotic means having a true nucleus. All multicellular animals and plants consist of eukaryotic cells, as do the fungi and some unicellular organisms.

evolution scientific theory that states that all living things developed naturally from simple organisms over millions of years.

fertilization the process by which sperm and egg unite during reproduction.

fixed action patterns unvarying patterns of behavior exhibited by some animals in response to specific environmental conditions.

flavonoids substances in plants that are the basic building blocks of many pigments; play a role in coloring flowers and in attracting insects that carry out pollination.

flower the part of a plant that produces the seeds.

Chemistry

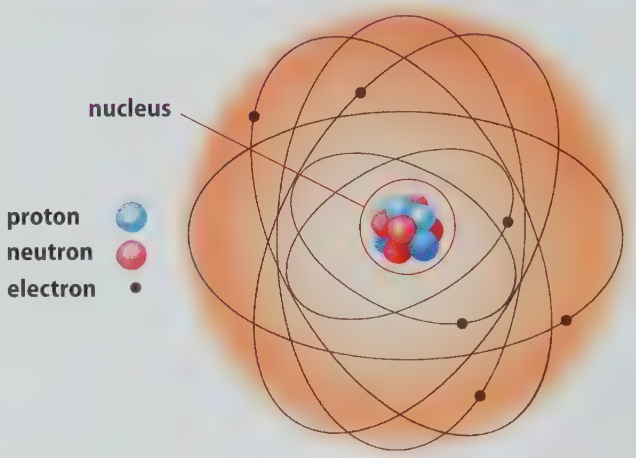
General Chemistry 312

Organic Chemistry 408

Practice Problems and Glossary 426

Atoms

All matter is made up of *atoms*. An atom is the smallest particle of an element that can exhibit the properties of that element. An atom is incredibly tiny—more than a million times smaller than the thickness of a human hair.



▲ **An Atom.** The protons and neutrons are clustered in the nucleus, a tiny region near the center of the atom. The electrons whirl at fantastic speeds through the empty space outside the atom's nucleus. The exact location and speed of electrons is difficult to precisely determine, and they do not move in circular orbits as depicted in the diagram.

Parts of an Atom

A Greek philosopher named Democritus who lived in the fifth century BC pondered what would happen if a grain of sand were split again and again. He believed that ultimately one tiny piece would be left which could not be broken. He called these tiniest pieces of matter atomos, the Greek term for "unbreakable." Although unable to confirm his hypothesis at the time by experimentation, Democritus was correct. Atoms are the smallest bit of a element that can exist. They are not the smallest particles, though.

Atoms consist of *subatomic particles*.

- *protons*
- *neutrons*
- *electrons*

Protons and neutrons have approximately the same mass and constitute the *nucleus* of the atom. Electrons have almost no mass and move around the outside of the nucleus.

The subatomic particles also carry electrical charges: the proton has a single positive charge, the electron has a single negative charge, and the neutron is electrically neutral.

THE NUCLEUS

The protons and neutrons are crowded into the nucleus, which is an exceedingly tiny region at the center of the atom. If a hydrogen atom were 6.5 kilometers (about 4 miles) in diameter, its nucleus would be no bigger than a tennis ball. The rest of an atom outside the nucleus is mostly empty space except for the electrons.

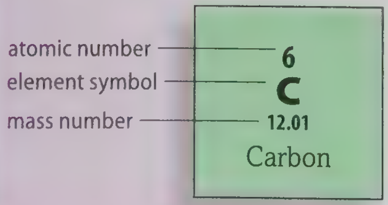
The nucleus makes up nearly all the *mass* of an atom. Mass is the quantity of matter in an atom. Each proton has a mass roughly equal to that of 1,836 electrons. It would take 1,839 electrons to equal a neutron's mass.

Each proton carries one unit of positive electric charge. Each electron carries one unit of negative charge. Neutrons have no charge. Under most conditions, an atom has the same number of protons and electrons, and so the atom is electrically neutral.

Properties of Atoms

All atoms of a given element have the same number of protons. This number is called the *atomic number*. The atomic number determines an element's place in the *periodic table*. This table organizes the elements into groups with similar chemical properties.

The *mass number* of an atom is equal to the average number of protons and neutrons in the nucleus. Sometimes the number of neutrons varies slightly from atom to atom of a particular element, though the number of protons remains identical. This is why the mass number is a decimal for each element. In most lighter elements, the nucleus of each atom contains about an equal number of protons and neutrons. Most heavier elements, however, have more neutrons than protons.



Subatomic Particles

Particle	Symbol	Charge	Mass	Location	Discovered
Electron	e^-	–1	0.00055 u*	Outside nucleus	1897 by Joseph J. Thomson
Proton	p or p^+	+1	1.0073 u	Inside nucleus	1919 by Ernest Rutherford
Neutron	n or n^0	0	1.0087 u	Inside nucleus	1932 by Sir James Chadwick

* u = unified atomic mass unit

Isotopes

Sometimes the number of neutrons in atoms of the same element may vary. Consequently, these atoms have different masses. Atoms of the same element that differ in mass are known as *isotopes*.

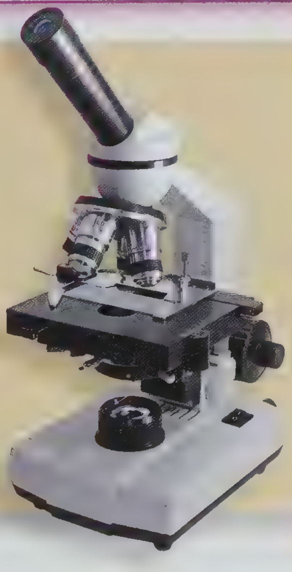
An example of an element with several common isotopes is carbon. Every carbon atom always contains six protons in the nucleus, and most but not all carbon atoms also contain six neutrons. Some carbon atoms contain seven, eight, or possibly more neutrons. Notation for isotopes must represent their differences. Carbon-12, which has six protons and six neutrons is written as $^{12}_{6}C$. The mass number is the sum of protons and neutrons and is written on top. The atomic number is written on bottom.

A very useful isotope of carbon is $^{14}_{6}C$. It's an unstable isotope which is present in organic materials and slowly decays over time. The amount of carbon-14 present in a substance can help us determine its age. This process is known as carbon-14 dating.

HOW SMALL IS IT?

An atom is so small that the smallest speck you can view under an ordinary microscope contains more than 10 billion atoms!

The diameter of an atom ranges from about 0.1 to 0.5 nanometer. A nanometer is a billionth of a meter, or about 1/25,400,000 inch.

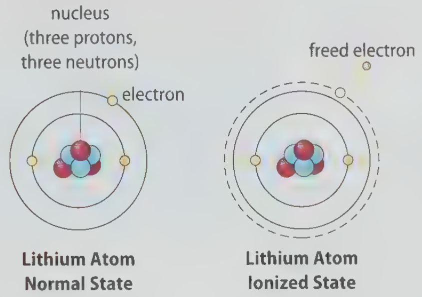


Molecules and Ions

Atoms or groups of atoms called *molecules* sometimes gain or lose electrons through various processes. The atom or molecule is then electrically charged and can either be positively charged (a *cation*) or negatively charged (an *anion*). An *ion* is written to designate how much and what type of charge it has. For example, a chlorine atom which has gained one electron is written with its chemical symbol and a negative sign, or Cl^-. A magnesium atom which has lost two electrons is designated Mg^{2+}.

Diatomic Molecules. The simplest type of molecule is called a *diatomic molecule* and contains two atoms—either two of the same type or two different types. Seven elements exist as diatomic molecules—hydrogen, oxygen, nitrogen, fluorine, chlorine, bromine, and iodine.

Polyatomic Molecules. Molecules with more than two atoms are called *polyatomic molecules*. Sulfur (S_8) and phosphorus (P_4) are examples of polyatomic elements.


nucleus (three protons, three neutrons)
electron
freed electron
Lithium Atom Normal State
Lithium Atom Ionized State

▲ **An atom becomes an ion** when it gains or loses an electron and so acquires an electric charge. A normal atom has an equal number of positive protons and negative electrons. If it loses an electron, it becomes a positively charged ion.

Periodic Table

1^+ — ion charge
6 — atomic number
C — chemical symbol
12.01 — atomic mass
Carbon — name of element

🟧 Alkali metals
🟧 Alkaline earth metals
🟥 Transition metals
🟪 Lanthanide metals
🟦 Actinide metals
🟪 Other metals
🟩 Nonmetals
🟨 Noble gases
☢ Radioactive

	1 **alkali metals** **I A**	2 **alkaline earth metals** **II A**	3 **III B**	4 **IV B**	5 **V B**	6 **VI B**	7 **VII B**	8	9 **VIII**
			transition metals						
Period 1	1^+ 1^- 1 **H** 1.01 Hydrogen								
Period 2	1^+ 3 **Li** 6.94 Lithium	2^+ 4 **Be** 9.01 Beryllium							
Period 3	1^+ 11 **Na** 23.00 Sodium	2^+ 12 **Mg** 24.31 Magnesium							
Period 4	1^+ 19 **K** 39.10 Potassium	2^+ 20 **Ca** 40.08 Calcium	3^+ 21 **Sc** 44.96 Scandium	4^+ 3^+ 22 **Ti** 47.90 Titanium	5^+ 4^+ 23 **V** 50.94 Vanadium	3^+ 2^+ 24 **Cr** 52.00 Chromium	2^+ 4^+ 25 **Mn** 54.94 Manganese	3^+ 2^+ 26 **Fe** 55.85 Iron	2^+ 3^+ 27 **Co** 58.93 Cobalt
Period 5	1^+ 37 **Rb** 85.47 Rubidium	2^+ 38 **Sr** 87.62 Strontium	3^+ 39 **Y** 88.91 Yttrium	4^+ 40 **Zr** 91.22 Zirconium	5^+ 3^+ 41 **Nb** 92.91 Niobium	6^+ 42 **Mo** 95.94 Molybdenum	7^+ 43 **Tc** 98.91 ☢ Technetium	3^+ 4^+ 44 **Ru** 101.07 Ruthenium	3^+ 45 **Rh** 102.91 Rhodium
Period 6	1^+ 55 **Cs** 132.91 Cesium	2^+ 56 **Ba** 137.34 Barium		4^+ 72 **Hf** 178.49 Hafnium	5^+ 73 **Ta** 180.95 Tantalum	6^+ 74 **W** 183.85 Tungsten	7^+ 75 **Re** 186.2 Rhenium	4^+ 76 **Os** 190.2 Osmium	4^+ 77 **Ir** 192.2 Iridium
Period 7	1^+ 87 **Fr** (223) ☢ Francium	2^+ 88 **Ra** (226) ☢ Radium		104 **Rf** (261) ☢ Rutherfordium	105 **Db** (262) ☢ Dubnium	106 **Sg** (263) ☢ Seaborgium	107 **Bh** (262) ☢ Bohrium	108 **Hs** (265) ☢ Hassium	109 **Mt** (266) ☢ Meitnerium

rare earth elements							
Lanthanide series	3^+ 57 **La** 138.9 Lanthanum	3^+ 58 **Ce** 140.12 Cerium	3^+ 59 **Pr** 140.91 Praseodymium	3^+ 60 **Nd** 144.24 Neodymium	3^+ 61 **Pm** (145) ☢ Promethium	3^+ 2^+ 62 **Sm** 150.4 Samarium	3^+ 2^+ 63 **Eu** 151.96 Europium
Actinide series	3^+ 89 **Ac** (227) ☢ Actinium	4^+ 90 **Th** 232.04 ☢ Thorium	5^+ 4^+ 91 **Pa** 231.04 ☢ Protactinium	6^+ 4^+ 92 **U** 238.03 ☢ Uranium	5^+ 93 **Np** 237.05 ☢ Neptunium	4^+ 6^+ 94 **Pu** (244) ☢ Plutonium	3^+ 4^+ 95 **Am** (243) ☢ Americium

A figure in parentheses is the isotope of longest known half-life. No stable isotope is known.

Polyatomic ions—theoretical summary

1^- Ions

• acetate	CH_3COO^-
benzoate	$C_6H_5COO^-$
chlorate*	ClO_3^-
chlorite	ClO_2^-
cyanide	CN^-
dihydrogen phosphate	$H_2PO_4^-$
glutamate	$C_5H_8NO_4^-$
• hydrogen carbonate (bicarbonate)	HCO_3^-
hydrogen oxalate	$HOOCCOO^-$
• hydrogen sulfate (bisulfate)	HSO_4^-
hydrogen sulfide (bisulfide)	HS^-
hydrogen sulfite (bisulfite)	HSO_3^-
• hydroxide	OH^-
• hypochlorite	CLO^-; OCl^-
• nitrate	NO_3^-
nitrite	NO_2^-
perchlorate*	CLO_4^-
• permanganate	MnO_4^-
stearate	$C_{12}H_{35}COO^-$
thiocyanate	SCN^-

*** There are also corresponding ions containing Br and I instead of Cl.** **• Frequently used ions**

10	11	12	13	14	15	16	17	18 noble gases O
			nonmetals					
	I B	II B	III A	IV A	V A	VI A	VII A	
								2 **He** 4.00 Helium
			5 **B** 10.81 Boron	6 **C** 12.01 Carbon	3^{-} 7 **N** 14.01 Nitrogen	2^{-} 8 **O** 16.00 Oxygen	1^{-} 9 **F** 19.00 Fluorine	10 **Ne** 20.18 Neon
			13 **Al** 26.98 Aluminum	14 **Si** 28.09 Silicon	3^{-} 15 **P** 30.97 Phosphorus	2^{-} 16 **S** 32.06 Sulfur	1^{-} 17 **Cl** 35.45 Chlorine	18 **Ar** 39.95 Argon
2^{+} 3^{+} 28 **Ni** 58.71 Nickel	2^{+} 1^{+} 29 **Cu** 63.55 Copper	2^{+} 30 **Zn** 65.37 Zinc	3^{+} 31 **Ga** 69.72 Gallium	4^{+} 32 **Ge** 72.59 Germanium	3^{-} 33 **As** 74.92 Arsenic	2^{-} 34 **Se** 78.96 Selenium	1^{-} 35 **Br** 79.90 Bromine	36 **Kr** 83.80 Krypton
2^{+} 4^{+} 46 **Pd** 106.4 Palladium	1^{+} 47 **Ag** 107.87 Silver	2^{+} 48 **Cd** 112.40 Cadmium	3^{+} 49 **In** 114.82 Indium	4^{+} 2^{+} 50 **Sn** 118.69 Tin	3^{+} 5^{+} 51 **Sb** 121.75 Antimony	2^{-} 52 **Te** 127.60 Tellurium	1^{-} 53 **I** 126.90 Iodine	54 **Xe** 131.30 Xenon
4^{+} 2^{+} 78 **Pt** 195.09 Platinum	3^{+} 1^{+} 79 **Au** 196.97 Gold	2^{+} 1^{+} 80 **Hg** 200.59 Mercury	1^{+} 3^{+} 81 **Tl** 204.37 Thallium	2^{+} 4^{+} 82 **Pb** 207.2 Lead	3^{+} 5^{+} 83 **Bi** 208.98 Bismuth	2^{+} 4^{+} 84 **Po** (209) ☢ Polonium	1^{-} 85 **At** (210) ☢ Astatine	86 **Rn** (222) ☢ Radon
110 **Ds** (269) ☢ Darmstadtium	111 **Rg** (272) ☢ Roentgenium	112 **Cn** (277) ☢ Copernicium	113 **Nh** (284) ☢ Nihonium	114 **Fl** (285) ☢ Flerovium	115 **Mc** (288) ☢ Moscovium	116 **Lv** (289) ☢ Livermorium	117 **Ts** (294) ☢ Tennessine	118 **Og** (294) ☢ Oganesson
			other metals					

3^{+} 64 **Gd** 157.25 Gadolinium	3^{+} 65 **Tb** 158.93 Terbium	3^{+} 66 **Dy** 162.50 Dysprosium	3^{+} 67 **Ho** 164.93 Holmium	3^{+} 68 **Er** 167.26 Erbium	3^{+} 69 **Tm** 168.93 Thulium	3^{+} 2^{+} 70 **Yb** 173.04 Ytterbium	2^{+} 71 **Lu** 174.97 Lutetium
3^{+} 96 **Cm** (247) ☢ Curium	3^{+} 4^{+} 97 **Bk** (247) ☢ Berkelium	3^{+} 98 **Cf** (251) ☢ Californium	3^{+} 99 **Es** (254) ☢ Einsteinium	3^{+} 100 **Fm** (257) ☢ Fermium	2^{+} 3^{+} 101 **Md** (258) ☢ Mendelevium	2^{+} 3^{+} 102 **No** (259) ☢ Nobelium	3^{+} 103 **Lr** (260) ☢ Lawrencium

Polyatomic ions—theoretical summary

2^{-} Ions	
• carbonate	CO_3^{2-}
chromate	CrO_4^{2-}
• dichromate	$Cr_2O_7^{2-}$
hydrogen phosphate	HPO_4^{2-}
oxalate	$OOCCOO^{2-}$
silicate	SiO_3^{2-}
• sulfate	SO_4^{2-}
sulfite	SO_3^{2-}
tetraborate	$B_4O_7^{2-}$
thiosulfate	$S_2O_3^{2-}$
3^{-} Ions	
borate	BO_3^{3-}
citrate	$C_3H_4OH(COO)_3^{3-}$
• phosphate	PO_4^{3-}
5^{-} Ion	
tripolyphosphate	$P_3O_{10}^{5-}$
1^{+} Ions	
• ammonium	NH_4^{+}
hydronium	H_3O^{+}
2^{+} Ion	
mercury(I)	Hg_2^{2+}

• **Frequently used ions**

Chemical Elements

Symbol	Name	Atomic Number
Ac	Actinium	89
Ag	Silver	47
Al	Aluminum	13
Am	Americium	95
Ar	Argon	18
As	Arsenic	33
At	Astatine	85
Au	Gold	79
B	Boron	5
Ba	Barium	56
Be	Beryllium	4
Bk	Berkelium	97
Bh	Bohrium	107
Bi	Bismuth	83
Br	Bromine	35
C	Carbon	6
Ca	Calcium	20
Cd	Cadmium	48
Ce	Cerium	58
Cf	Californium	98
Cl	Chlorine	17
Cm	Curium	96
Cn	Copernicium	112
Co	Cobalt	27
Cr	Chromium	24
Cs	Cesium (Caesium)	55
Cu	Copper	29
Db	Dubnium	105
Ds	Darmstadtium	110
Dy	Dysprosium	66

Symbol	Name	Atomic Number
Er	Erbium	68
Es	Einsteinium	99
Eu	Europium	63
F	Fluorine	9
Fe	Iron	26
Fl	Flerovium	114
Fm	Fermium	100
Fr	Francium	87
Ga	Gallium	31
Gd	Gadolinium	64
Ge	Germanium	32
H	Hydrogen	1
He	Helium	2
Hf	Hafnium	72
Hg	Mercury	80
Ho	Holmium	67
Hs	Hassium	108
I	Iodine	53
In	Indium	49
Ir	Iridium	77
K	Potassium	19
Kr	Krypton	36
La	Lanthanum	57
Li	Lithium	3
Lr	Lawrencium	103
Lu	Lutetium	71
Lv	Livermorium	116
Mc	Moscovium	115
Md	Mendelevium	101
Mg	Magnesium	12

Symbol	Name	Atomic Number
Mn	Manganese	25
Mo	Molybdenum	42
Mt	Meitnerium	109
N	Nitrogen	7
Na	Sodium	11
Nb	Niobium	41
Nd	Neodymium	60
Ne	Neon	10
Nh	Nihonium	113
Ni	Nickel	28
No	Nobelium	102
Np	Neptunium	93
O	Oxygen	8
Og	Oganesson	118
Os	Osmium	76
P	Phosphorus	15
Pa	Protactinium	91
Pb	Lead	82
Pd	Palladium	46
Pm	Promethium	61
Po	Polonium	84
Pr	Praseodymium	59
Pt	Platinum	78
Pu	Plutonium	94
Ra	Radium	88
Rb	Rubidium	37
Re	Rhenium	75
Rf	Rutherfordium	104
Rg	Roentgenium	111

Symbol	Name	Atomic Number
Rh	Rhodium	45
Rn	Radon	86
Ru	Ruthenium	44
S	Sulfur	16
Sb	Antimony	51
Sc	Scandium	21
Se	Selenium	34
Sg	Seaborgium	106
Si	Silicon	14
Sm	Samarium	62
Sn	Tin	50
Sr	Strontium	38
Ta	Tantalum	73
Tb	Terbium	65
Tc	Technetium	43
Te	Tellurium	52
Th	Thorium	90
Ti	Titanium	22
Tl	Thallium	81
Tm	Thulium	69
Ts	Tennessine	117
U	Uranium	92
V	Vanadium	23
W	Tungsten	74
Xe	Xenon	54
Y	Yttrium	39
Yb	Ytterbium	70
Zn	Zinc	30
Zr	Zirconium	40

AN ALPHABETICAL GUIDE TO THE ELEMENTS

Unlike the periodic table of elements, which groups elements according to properties, this is an alphabetical listing of all currently known elements, including general information about the element: when and by whom it was discovered, practical uses (if any), chief compounds built from the element, and a description of the element's properties. Use this table in conjunction with the periodic table on pages 322 and 323.

Table of Elements

Alkali metals
Alkaline earth metals
Transition metals
Lanthanide metals
Actinide metals
Other metals
Nonmetals
Noble gases
Unconfirmed category

At. wt., atomic weight (C=12)
At. no., atomic number
Sp. gr., specific gravity
V., valence
M.P., melting point °C
B.P., boiling point °C
All temperatures in the text are in degrees Celsius (°C.).
C.S., crystal structure
C = cubic, whether body- or face-centered unknown
Cb = cubic, body-centered
Cf = cubic, face-centered
H = hexagonal
T = tetragonal

Element	Occurrence, Preparation, Date of Discovery, and Discoverer	Properties	Chief Compounds and Uses
Actinium **Ac** At. wt. 227 (?) At. no. 89 V. 3 Sp. gr. 10.1 M.P. 1,050° B.P. 3,200°	In uranium ores, resulting from radioactive decay of uranium. Bombardment of bismuth. 1899; Debierne.	Radioactive; half-life ranges from 20 years to 3.7 seconds. It is chemically quite similar to lanthanum.	Used in research.
Aluminum **Al** At. wt. 26.9815 At. no. 13 V. 3 Sp. gr. 2.7 M.P. 660.2° B.P. 2,327° C.S. Cf	Cryolite, bauxite, impure emery, ruby, sapphire (Al_2O_3). Commercially, by electrolysis of Al_2O_3 from bauxite, dissolved in cryolite; water power usual source of electrical energy. 1825; Oersted.	Silver-white, ductile metal; malleable at 120°; tensile strength (wrought) 16 tons per square inch. Better conductor of electricity, weight for weight, than copper. Acted upon by dilute hydrochloric acid, slowly by sulfuric acid, but not by nitric acid or the acids in foods. Soluble in alkaline hydroxides.	Used for cooking utensils, boatbuilding, airplanes, small articles requiring lightness and strength, and electric leads. The powdered metal is used as a body for paint; its mixture with ferric oxide, called thermite, is used for producing very high temperatures (up to 2,700°).
Americium **Am** At. wt. 243.13 At. no. 95 V. 3, 4, 5, 6 Sp. gr. 13.67 M.P. 995°	Does not occur naturally. Bombardment of uranium-238 with very high energy electrons. 1944; Seaborg, James, Morgan, Ghiorso.	Radioactive, transuranium element. Alpha activity amounts to 70 billion alpha disintegrations per minute per milligram.	Its short half-life restricts use of this element to nuclear research.
Antimony **Sb** At. wt. 121.75 At. no. 51 V. 3, 5 Sp. gr. 6,684 M.P. 630.5° B.P. 1,640° C.S. H	Free and as stibnite (Sb_2S_3). Roasting stibnite gives Sb_2O_4, which is then reduced by heating with carbon. 1450; Valentine.	White, brittle, crystalline metal. Its alloys expand on solidification, give very sharp castings for type. Does not tarnish, but may be burned in air; unites directly with the halogens.	Constituent of type metal, Britannia metal, Babbitt metal (used for bearings), and other alloys. Oxide (Sb_2O_3) is both basic and acidic. Trichloride, butter of antimony ($SbCl_3$), is easily hydrolyzed. Tartar emetic is used in medicine and dyeing.
Argon **Ar** At. wt. 39.948 At. no. 18 V. 0 M.P. –189.2° B.P. –185.7°	Present in the air 0.94 percent by volume. To isolate from air, carbon dioxide is removed by soda lime, water by phosphorus pentaoxide, oxygen by red-hot copper, nitrogen by magnesium and calcium; fractional distillation of residue yields argon. 1894; Rayleigh, Ramsay.	Monatomic gas, identified by its characteristic spectrum seen by examining light emitted when the gas is placed in a vacuum tube at low pressure and sparked. More soluble than nitrogen in water; 100 volumes of water dissolves 4 volumes of argon under ordinary conditions.	Forms no compounds, hence its name, argon, meaning "inert."
Arsenic **As** At. wt. 74.9216 At. no. 33 V. 3, 5 Sp. gr. 5.7 M.P. 817° (under pressure) B.P. 613° (sublimes)	Free as arsenical pyrites (FeSAs), or pigment (As_2S_3), and realgar (As_2S_2). By heating arsenical pyrites to change as follows: (FeSAs → FeS + As). 1649; Schröder.	Steel-gray, dull metallic, crystalline element classed as a metalloid because it is between metals and nonmetals. Vapor density corresponds to As_4 at 644°, and to As_2 at 1,700°. Burns in air and unites directly with the halogens, sulfur, and many metals.	Used for hardening lead shot. All compounds are poisonous. White arsenic (As_2O_3) is partly basic, forming a chloride, and partly acidic, forming arsenites. Scheele's green is a dangerous pigment used in wallpaper. Traces of arsenic may be detected by Marsh's test, in which intensely poisonous arsine (AsH_3) is formed.
Astatine **At** At. wt. 210 (?) At. no. 85 V. 1, 3, 5, 7	Does not occur naturally. By bombardment of bismuth with a stream of alpha particles. 1940; Segré, Corson, MacKenzie.	Synthetic element similar to polonium. All the known isotopes are short-lived, the longest-lived isotope having a half-life of 8.3 hours.	Its limited stability restricts its use to nuclear research.

Table of Elements

Element		Occurrence, Preparation, Date of Discovery, and Discoverer	Properties	Chief Compounds and Uses
Francium **Fr**	At. wt. 223 (?) At. no. 87 V. 1	Does not occur naturally. Disintegrates so rapidly that it is almost impossible to obtain in sufficient quantity to enable weighing. 1939; Perey.	Synthetic radioactive element; heaviest of the alkali metals.	Expensive to make; limited to radioactive and nuclear research.
Gadolinium **Gd**	At. wt. 157.25 At. no. 64 V. 3 Sp. gr. 7.94 M.P. 1,312° B.P. 2,800°	In gadolinite and samarskite. By electrolysis of the chloride. 1886; Marignac.	Rare earth. Closely resembles terbium in its compounds.	Salts are colorless and show absorption bands only in the ultraviolet.
Gallium **Ga**	At. wt. 69.72 At. no. 31 V. 2, 3 Sp. gr. 6.095 M.P. 30.150° B.P. 1,983°	In iron ores, zinc blende (ZnS), and bauxite (Al_2O_3). By electrolysis of an alkaline solution of its salts secured from zinc. 1875; Boisbaudran.	Bluish-white, tough metal that can be cut with a knife. Like aluminum, it is soluble in hydrochloric acid and caustic soda, but not in nitric acid.	Forms two chlorides, $GaCl_3$ and $GaCl_2$, that yield very characteristic spark spectra. Alloys with aluminum and cadmium are used for optical mirrors and cathodes.
Germanium **Ge**	At. wt. 72.59 At. no. 32 V. 2, 4 Sp. gr. 5.35 M.P. 947° B.P. 2,700°	In the rare mineral argyrodite. By the reduction of the dioxide (GeO_2) by the element carbon. 1886; Winkler.	Grayish-white, brittle, lustrous metal, insoluble in hydrochloric acid. Combines directly with the halogens.	Close relation of this element to carbon and silicon is shown in the compound germanium chloroform. Mendeleev described it before its discovery, calling it ekasilicon. The oxide is used to treat pernicious anemia.
Gold **Au**	At. wt. 196.967 At. no. 79 V. 1, 3 Sp. gr. 19.4 M.P. 1,063° B.P. 2,600°	Chiefly free, but also a telluride; many specimens of iron pyrites are auriferous. From gold-bearing sands by washing away the lighter materials and dissolving the gold from the residue in mercury, which is subsequently separated from the gold by distillation. Known in antiquity.	Soft, bright-yellow metal, easily scratched by a knife; most ductile and malleable of the metals; excellent conductor of heat and electricity. Chemically, gold is rather inert and is not attacked by the oxygen of the air, by hydrogen sulfide, or by any single acid.	Pure gold is 24-carat gold. Jewelry is made in 18-, 14-, and 9-carat gold because alloying it with copper increases hardness and rigidity. Sodium chloraurate is used for toning in photography, and potassium auricyanide is used in electrogilding.
Hafnium **Hf**	At. wt. 178.49 At. no. 72 V. 4 Sp. gr. 13.3 M.P. 2,230° B.P. 3,200°	Associated with zirconium. By decomposing the tetraiodide. 1922; Coster, Hevesy.	Analogous to zirconium.	Similar to zirconium compounds, from which it is separated by fractional crystallization.
Hassium **Hs**	At. wt. 265 (?) At. no. 108	Does not occur naturally. By bombardment of lead-208 with iron-58 ions. 1984; Armbruster, Münzenberg et al.	Radioactive, transuranium element.	No commercial applications; of research interest only.

Element	Occurrence, Preparation, Date of Discovery, and Discoverer	Properties	Chief Compounds and Uses
Helium **He** At. wt. 4.0026 At. no. 2 V. 0 Sp. gr. (liquid) 0.15 M.P. –272.2° (at 26 atmospheres) B.P. –268.9°	In the air to the extent of 1 to 2 volumes per million; also occluded in certain minerals. First observed in sun's spectrum (1868; Lockyer, Jannsen). Neon and helium are boiled off crude argon, and the neon, when cooled, solidifies. 1895; Ramsay.	Lightest gas except hydrogen; transparent, odorless, and colorless.	Forms no compounds. Used for balloons; not flammable.
Holmium **Ho** At. wt. 164.930 At. no. 67 V. 3 Sp. gr. 8.803 M.P. 1,461° B.P. 2,600°	Occurs with, and is separated from, the erbium subgroup of the rare earths. Has never been isolated. 1879; Cleve.	Rare earth. Salts are orange-yellow and similar to those of dysprosium.	Used in glass to transmit radiant energy for wavelength-calibration instruments.
Hydrogen **H** At. wt. 1.00797 At. no. 1 V. 1 Sp. gr. (liquid) 0.09 M.P. –259.1° B.P. –252.7°	In the air to the extent of 1 volume per 20,000 volumes of air; combined, in water (11.19 percent by weight), natural gas, petroleum, and animal and vegetable bodies. By treating zinc with hydrochloric or sulfuric acid; by electrolysis. 1766; Cavendish.	Lightest gas, transparent, odorless, and colorless. Soluble in water (2 volumes in 100 volumes of water under average conditions), in platinum, in palladium (502 volumes in 1 volume of palladium). Burns in air and in chlorine and unites with many other elements.	Its two oxides are water and hydrogen peroxide, the latter used in solution as a bleaching agent. Every acid contains hydrogen as an essential constituent. Its compounds with carbon and other elements number over 100,000.
Indium **In** At. wt. 114.82 At. no. 49 V. 1, 3 Sp. gr. 7.3 M.P. 156° B.P. 2,000°	In zinc blende (ZnS) in small quantities. Electrolytically from solutions of its salts. 1863; Reich, Richter.	White, malleable metal, softer than lead; about as heavy as tin.	Compounds color the nonluminous gas flame blue and show a characteristic indigo blue line in the spectrum—hence its name. Sometimes used as a coating on bearings.
Iodine **I** At. wt. 126.9044 At. no. 53 V. 1, 3, 5, 7 Sp. gr. 4.93 M.P. 113.7° B.P. 184.35°	In the ocean and certain seaweeds; always in the combined state. From iodides by displacement of their iodine by chlorine. 1811; Courtois.	Dark gray, brittle solid with a metallic luster. Vapor is violet, as are its solutions in chloroform and carbon bisulfide. Requires more than 5,000 parts of water for solution. Combines directly with many elements, but is much less active than chlorine or bromine.	Used in pharmacy as an antiseptic and in prescriptions for treatment of goiter. Potassium iodide and iodoform likewise find application in medicine. The alkyl iodides are much used in synthetic organic chemistry.
Iridium **Ir** At. wt. 192.2 At. no. 77 V. 3, 4 Sp. gr. 22.42 M.P. 2,454° B.P. 4,800° C.S. Cf	With platinum and osmium. Obtained from platinum ores by a complex series of operations. 1804; Tennant.	White metal, brittle when cold, very hard, and one of the densest substances known. Attacked by fused alkalis, but not by aqua regia.	Used for pointing gold pens. Its alloy with 9 parts of platinum is used for standard meter bars because of its unalterability. Used as a black color in china decorations.

Table of Elements

Element		Occurrence, Preparation, Date of Discovery, and Discoverer	Properties	Chief Compounds and Uses
Iron **Fe**	At. wt. 55.847 At. no. 26 V. 2, 3 Sp. gr. 7.86 pig 7.03 – 7.73 M.P. 1,535° wrought 1,600° steel 1,375° gray pig 1,275° white pig 1,075° B.P. 3,000° C.S. Cf, Cb	As magnetite (Fe_3O_4), hematite (Fe_2O_3), limonite ($2Fe_2O_3{\cdot}3H_2O$), siderite ($FeCO_3$), which are important ores; iron pyrites (FeS_2); in rocks as complex silicates; in plants and animals. Pig iron is prepared in blast furnace by reduction of the ore by means of carbon monoxide in presence of suitable flux. From pig iron, wrought iron is obtained by puddling, and steel by the Bessemer, or by open-hearth or other processes. Known in antiquity.	Malleable, ductile, magnetic metal, that is unchanged in dry air, but rusts in water and moist air. Easily attacked by dilute acids, but not by fused alkalis. Cast iron contains 2 to 5 percent carbon and other impurities, and is hard and brittle. Wrought iron contains less than 0.2 percent carbon and is softer and tougher, with tensile strength of 22 to 25 tons per square inch. Steel contains from 0.2 to 1.5 percent carbon, and is permanently magnetic.	The metal is used as a structural material for rails, machinery, tools, etc. Jeweler's rouge and Venetian red consist of the oxide (Fe_2O_3). Rust is chiefly hydrated oxide. Hammer scale and lodestone have the composition Fe_3O_4. Ferric chloride ($FeCl_3$), ferrous iodide (FeI_2), and other iron compounds are used in medicine. Green vitriol ($FeSO_4{\cdot}7H_2O$) is used in making ink and in dyeing.
Krypton **Kr**	At. wt. 83.80 At. no. 36 V. 0, 2, 4 M.P. –156.6° B.P. –152.3°	In minute quantities in the air. From crude argon by fractional distillation. 1898; Ramsay, Travers.	Inert, colorless, and odorless gas resembling, but denser than, argon.	Once thought to be chemically inert; forms series of fluoride compounds, such as KrF_2 and KrF_4.
Lanthanum **La**	At. wt. 138.91 At. no. 57 V. 3 Sp. gr. 6.15	As lanthanite [$La_2(CO_3)_3{\cdot}8H_2O$]. By electrolysis of the fused chloride ($LaCl_3$). 1839; Mosander.	Rare earth; iron-gray metal; tarnishes in air to steel-blue; malleable and ductile. Attacked slowly even by cold water.	When heated in air, it forms a strongly basic oxide (La_2O_3) that is diamagnetic, and a nitride (LaN).
Lawrencium **Lr**	At. wt. 256 (?) At. no. 103 V. 3	Does not occur naturally. By bombardment of californium-252 with boron ions. 1961; Ghiorso, Sikkeland, Larsh, Latimer.	Radioactive, transuranium element.	Expensive to make; used in nuclear research.
Lead **Pb**	At. wt. 207.19 At. no. 82 V. 2, 4 Sp. gr. 11.34 M.P. 327.43° B.P. 1,515° C.S. Cf	End product of certain radioactive decompositions. As galena (PbS) and in silver ores. By calcination of partially roasted galena. Purification is affected by Parkes process. Known in antiquity.	Soft, gray metal; malleable and of low tensile strength, relatively impermeable to X-rays and atomic radiation. In presence of air, water acts on lead to produce the hydroxide which, being slightly soluble, may cause lead poisoning. When heated in air, lead is oxidized to litharge and, under suitable conditions, to minium.	Used for water pipes, roofs and gutters, and storage batteries. For shot, it is alloyed with 0.4 percent arsenic. Type metal contains 80 percent lead. Babbitt metal, for bearings, contains over 70 percent lead. Solder and pewter are alloys of lead and tin. The basic carbonate, white lead, is the basis of most oil paints.
Lithium **Li**	At. wt. 6,939 At. no. 3 V. 1 Sp. gr. 0.53 M.P. 179° B.P. 1,317° C.S. C	In amblygonite [$Li(AlF)PO_4$]. By electrolysis of the fused chloride (LiCl). 1817; Arfwedson.	Lightest metal; silver-white, softer than lead, tarnishes quickly in air, and easily reacts with water. When heated, it unites vigorously with nitrogen.	The carbonate (Li_2CO_3) is used in medicine as a solvent for uric acid, lithium urate being soluble. The salts give a carmine flame coloration.
Livermorium **Lv**	At. wt. 293 (?) At. no. 116	Does not occur naturally. By bombardment of curium with calcium ions. 2000; Joint Institute for Nuclear Research.	Radioactive	No commercial applications; of research interest only.

Element		Occurrence, Preparation, Date of Discovery, and Discoverer	Properties	Chief Compounds and Uses
Lutetium **Lu**	At. wt. 174.97 At. no. 71 V. 3 Sp. gr. 9.84 M.P. 1,652° B.P. 3,330°	In euxenite. By electrolysis of the chloride. 1907; Urbain, Welsbach	Rare earth. Like ytterbium but has lower magnetic susceptibility.	Its compounds resemble those of ytterbium. It was once known as Casseopium.
Magnesium **Mg**	At. wt. 24.312 At. no. 12 V. 2 Sp. gr. 1.74 M.P. 651° B.P. 1,107° C.S. H	Occurs as magnesite ($MgCO_3$), dolomite ($MgCO_3 \cdot CaCO_3$), carnallite ($MgCl_2 \cdot KCl \cdot 6H_2O$), and in very many complex silicates. By electrolysis of dried, fused carnallite. 1808; Davy.	Silver-white, very lightweight metal, ductile when hot, and malleable. It tarnishes in air and reacts slowly with water, rapidly with steam. Burns in air to the oxide (MgO), emitting a very bright light. Unites directly with nitrogen.	Used as a reducing agent. Sulfate, known as Epsom salts, is used in medicine, as are the oxide (magnesia), the carbonates, and the citrate. The bright light emitted when the metal is burned in air is used in photography.
Manganese **Mn**	At. wt. 54.9380 At. no. 25 V. 2, 3, 4, 6, 7 Sp. gr. 7.2 M.P. 1,220° B.P. 2,152° C.S. CT	As pyrolusite (MnO_2), braunite (Mn_2O_3), hausmannite (Mn_3O_4), and manganese spar ($MnCO_3$). By heating Mn_3O_4 with aluminum filings. 1774; Gahn.	Steel-gray, hard, brittle metal with a pinkish tinge. Rusts in moist air and is attacked by dilute acids.	Ferromanganese and spiegeleisen are alloys with iron, used in making steel tougher. With copper, it forms the tough, hard manganese bronzes, with tensile strength up to 30 tons per square inch.
Meitnerium **Mt**	At. wt. 266 (?) At. no. 109	Does not occur naturally. By bombardment of bismuth-209 with iron-58 ions. 1982; Armbruster, Münzenberg et al.	Radioactive, transuranium element.	No commercial applications, of research interest only.
Mendelevium **Md**	At. wt. 258 (?) At. no. 101 V. 3	Does not occur naturally. By bombardment of einsteinium-253 with alpha particles. 1955; Thompson, Harvey, Choppin, Ghiorso, Seaborg.	Radioactive, transuranium element.	Expensive to make; limited to nuclear and radioactive research.
Mercury **Hg**	At. wt. 200.59 At. no. 80 V. 1, 2 Sp. gr. 13.6 M.P. –38.87° B.P. 356.58° C.S. H	Free and as cinnabar (HgS). By roasting cinnabar: $HgS + O_2 \rightarrow Hg + SO_2$. Known in antiquity.	Silver-white, mobile liquid, 20 percent heavier than lead. Has vapor pressure of 0.0002 millimeter at 0°. Tarnishes slowly in air and is attacked only by nitric among the dilute acids. Vapor is monatomic.	Used in thermometers and barometers. Alloys, some of which are used in dentistry, are called amalgams. Calomel ($HgCl$) is administered internally in medicine; corrosive sublimate ($HgCl_2$) forms a solution with very powerful germicidal properties.
Molybdenum **Mo**	At. wt. 95.94 At. no. 42 V. 3, 4, 5, 6 Sp. gr. 10.2 M.P. 2,620° B.P. 4,507° C.S. Cb	As molybdenite (MoS_2) and wulfenite ($PbMoO_4$). By reducing the oxides with aluminum powder. 1778; Scheele.	White metal as malleable as iron; will not scratch glass. Insoluble in hydrochloric or dilute sulfuric acid.	Ferromolybdenum alloys are used in the manufacture of special steels.
Moscovium **Mc**	At. wt. 289 (?) At. no. 115	Does not occur naturally. By bombardment of americium with calcium ions. 2003; Joint Institute for Nuclear Research and Lawrence Livermore National Laboratory	Radioactive	No commercial applications; of research interest only.

Gas Laws

Experiments with a large number of gases reveal that the state, or condition, of many gaseous substances can be defined using four variables: temperature (T), pressure (P), volume (V), and the quantity of gas measured in a unit called a *mole* (n). Three laws explain the relationships between these variables.

- *Boyle's law*
- *Charles's law*
- *Avogadro's law*

The three gas laws have been combined into a single statement known as the *ideal gas law.*

MOLE

A mole is a unit used to measure the amount of a substance. One mole of any substance contains about 6.022×10^{23} (602.2 billion trillion) of the elementary entities that make up the substance. These entities can be molecules, atoms, ions, electrons or other subatomic particles, or groups of particles.

The number is known as *Avogadro's constant.* For example, a mole of water contains 6.022×10^{23} molecules of H_2O.

In addition, one mole of a substance has a mass in grams that is numerically equal to its weight in *atomic mass units* (amu's). The term *molar mass* is used to describe the mass in grams of one mole of a substance. The molar mass of water is determined by adding the amu's of each atom in the molecule.

H_2O

$2\ \text{H} = 2 \times 1.01\ \text{amu} = 2.02$

$1\ \text{O} = 1 \times 16.00\ \text{amu} = 16.00$

Molar mass of water: 18.02 g/mol (grams per mole)

Boyle's Law

Boyle's law states that for a fixed quantity of gas maintained at constant temperature, the volume of the gas is inversely proportional to the pressure. Therefore, pressure increases as the volume of gas decreases.

According to Boyle's law, the product of the pressure (P) multiplied by the volume (V) remains constant if there is no change in the temperature or in the number of particles inside the container. The law is written this way.

$$PV = \text{constant}$$

Boyle's law says that the pressure doubles when a gas is compressed to half its volume at constant temperature.

Robert Boyle first published Boyle's law in 1662. Boyle (1627–1691) was an Irish scientist who is considered one of the first modern chemists. He helped establish the experimental method in chemistry and physics.

Charles's Law

Charles's law states that for a fixed quantity of gas at constant pressure, the volume of the gas is directly proportional to its temperature. Therefore, a gas expands by the same fraction of its original volume with each degree that its temperature rises.

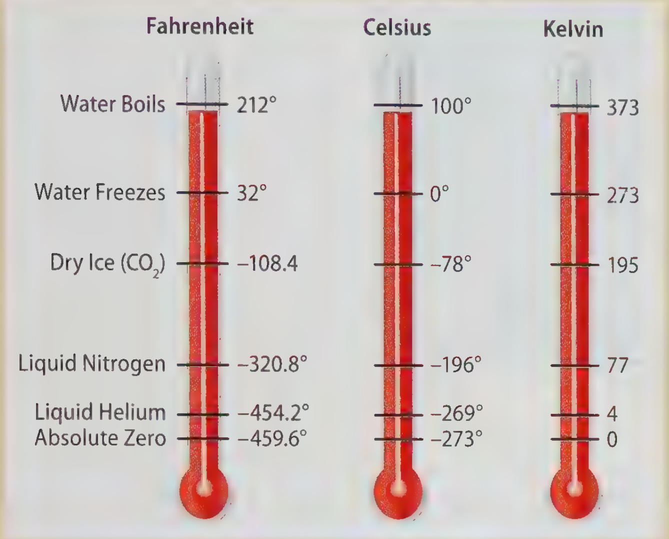
According to Charles's law, the ratio between the volume (V) of a gas and its temperature (T) remains constant if the pressure does not change. The law is written this way.

$$\frac{V}{T} = \text{constant}$$

In this equation, T is the *absolute temperature* of the gas. It is usually measured in *kelvins* (Celsius degrees plus 273.15). Kelvin is abbreviated K.

For example, when a gas is heated from 300 K (room temperature) to 600 K, its absolute temperature doubles. Doubling the temperature doubles the gas's volume if the pressure does not change.

Jacques Alexandre Cesar Charles, a French chemist, stated Charles's law in 1787.



ABSOLUTE TEMPERATURE SCALE

Temperature in gas law equations is measured in kelvins, on the *absolute temperature* scale. The unit size of one kelvin is the same as one °C, but no degree symbol is used for kelvins. The Kelvin scale is defined in terms of two points: 1. *absolute zero* and 2. the *triple point* temperature of water.

Absolute zero is the lowest possible temperature. It is assigned a value of 0 K. The triple point of a substance is the one temperature and pressure at which its solid, liquid, and gas phases can coexist in equilibrium. The triple point of water occurs at a temperature of 273.16 K.

Avogadro's Law

Avogadro's law states that at constant temperature and pressure, the volume of the gas is directly proportional to the moles of gas. It says that equal volumes of different gases all contain the same number of particles if they all have the same pressure and temperature.

$$\frac{V}{n} = \text{constant}$$

Amedeo Avogadro, an Italian scientist and philosopher, proposed Avogadro's law in 1811. It was later discovered that a volume of 22.4 liters of gas at 0°C and atmospheric pressure contains about 602 billion trillion particles. This number is called Avogadro's constant. (See Mole, on the facing page.) This number of particles of any substance is called one mole of the substance.

Atomic Mass. Avogadro distinguished between gases composed of complex units (molecules) and gases made up of simple units (atoms). He was able to use gas densities to calculate the amount of matter in atoms and molecules.

Ideal Gas Law

The three laws have been combined together in one equation, known as the ideal gas law. The law is written:

$$PV = nRT$$

In this equation, P represents the pressure of the gas, V represents its volume, n represents the number of moles of gas, and T represents its absolute temperature. R is a constant called the *universal gas constant.* Its value is 8.314 *joules* per kelvin per mole.

A joule is a unit used to measure work or energy. One joule is the amount of work done when a force of one newton acts on an object that moves 1 meter in the direction of the force. (See pages 770–771.)

According to the ideal gas law, the pressure of a gas can be doubled in three ways.

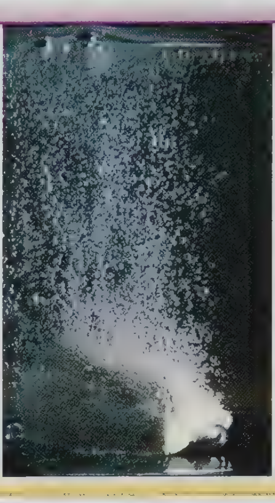
1. The gas can be squeezed into one-half its original volume.
2. Twice as much gas can be forced into the original volume.
3. The absolute temperature can be doubled.

Balancing Equations

Chemists balance chemical equations according to a fundamental principle of chemistry known as the *law of conservation of mass.* (See page 312.) To satisfy the law, a chemical equation must have the same number and types of atoms in both the reactants and the products.

Consider the reaction that occurs when hydrochloric acid is added to solid calcium carbonate. It produces carbon dioxide. A skeletal description of this reaction is

$$HCl + CaCO_3 \rightarrow CO_2 + CaCl_2$$



◀ **Adding hydrochloric acid to calcium carbonate** causes a chemical reaction that yields carbon dioxide, calcium chloride, and water.

Represent All Atoms. The equation at left may properly identify the important reactants and products, but it is not a balanced chemical equation. To balance the equation, we must see that all the atoms that are on the left side are properly represented on the right side and vice versa.

To make $CaCl_2$, we must use two molecules of HCl. In addition, the hydrogen from the HCl must go somewhere. We note that CO_2 accounts for only two oxygen atoms, and so the solution to the two extra hydrogens and the one extra oxygen is the formation of water, H_2O. These considerations give the following balanced chemical equation:

$$2HCl + CaCO_3 \rightarrow CO_2 + CaCl_2 + H_2O$$

Indicating Phases. We should also indicate the state of the reactants and products by writing:

$$2HCl_{(aq)} + CaCO_{3(s)} \rightarrow CO_{2(g)} + CaCl_{2(aq)} + H_2O_{(l)}$$

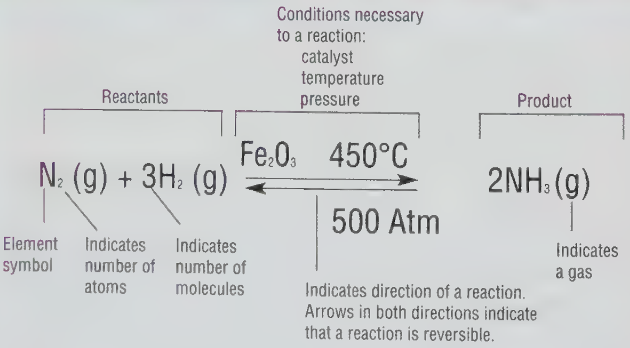
where the (aq) indicates an aqueous solution and the (s) and (g) refer to solid and gas phases.

For many purposes, this equation is still not the best description of the reaction. The acid HCl is fully separated into hydrogen and chloride ions, as is the salt $CaCl_2$. To represent the separate ions, a complete ionic equation is written.

$$2H^+ + 2Cl + CaCO_{3(s)} \rightarrow CO_{2(g)} + Ca^{2+} + 2Cl + H_2O_{(l)}$$

The chloride ion, Cl^-, is present on both sides of the equation and is not really a reactant. It is called a spectator ion. Therefore, we may eliminate the Cl^- to write the net ionic equation.

$$2H^+ + CaCO_{3(s)} \rightarrow CO_{2(g)} + Ca^{2+} + H_2O_{(l)}$$


Conditions necessary to a reaction:
catalyst
temperature
pressure
Reactants
Product
N₂ (g) + 3H₂ (g)
Fe₂0₃ 450°C
500 Atm
2NH₃ (g)
Element symbol
Indicates number of atoms
Indicates number of molecules
Indicates direction of a reaction. Arrows in both directions indicate that a reaction is reversible.
Indicates a gas

The coefficient values in an equation do not indicate amounts in any particular system of measure. The coefficients in an equation indicate a proportion of one reactant to another. For example, if the following values were assigned to the above equation, differing amounts of the same product would result.

$$1 \text{ molecule } N_2 + 3 \text{ molecules } H_2 \rightarrow 2 \text{ molecules } NH_3$$

$$1 \text{ mole } N_2 + 3 \text{ moles } H_2 \rightarrow 2 \text{ moles } NH_3$$

$$28 \text{ metric tons } N_2 + 6 \text{ metric tons } H_2 \rightarrow 34 \text{ metric tons } NH_3$$

How to Balance Equations

1. Write down the unbalanced equation.

$CH_4 + O_2 \rightarrow CO_2 + H_2O$

2. Create a table to identify the amount of each atom in the reactants and the products in the original, unbalanced equation.

Reactants		Atom	Products	
Original	Balanced		Original	Balanced
1		C	1	
4		H	2	
2		O	3	

3. Pick an atom and utilize coefficients, numbers in front of the reactants and products, to balance the number of each atom. Start with an atom that appears in only one substance on each side of the equation.

$CH_4 + O_2 \rightarrow CO_2 + H_2O$

4 hydrogens exist in CH_4, but only 2 hydrogens exist in H_2O. If a coefficient of 2 is placed in front of H_2O, the hydrogens will be balanced.

$CH_4 + O_2 \rightarrow CO_2 + \mathbf{2}H_2O$

4. Continue with each atom to use coefficients to equalize the number of each atom in the reactants and products. Update the table to keep track of the changes and progress towards achieving a balanced equation.

Reactants		Atom	Products	
Original	Balanced		Original	Balanced
1		C	1	
4	4	H	2	4
2		O	3	

When the coefficient of 2 was added in front of H_2O, the 2 applies to both the hydrogen and oxygen. There are now 2 oxygens counted in H_2O. Combining those 2 oxygens with the 2 from the other product, CO_2, now produces 4 oxygens on the product side.

Reactants		Atom	Products	
Original	Balanced		Original	Balanced
1		C	1	
4	4	H	2	4
2		O	3	4

If a 2 is added in front of O_2 in the reactants, then the oxygens will be balanced.

$CH_4 + \mathbf{2}O_2 \rightarrow CO_2 + \mathbf{2}H_2O$

Only one carbon appears in both the reactants and products, so now the entire equation is balanced.

Reactants		Atom	Products	
Original	Balanced		Original	Balanced
1	1	C	1	1
4	4	H	2	4
2	4	O	3	4

Balancing Equations

ON YOUR OWN

Practice balancing the following reactions:

1. During electrolysis, water is decomposed into hydrogen and oxygen gas.

$\_\_\_ H_2O_{(l)} \rightarrow \_\_\_ H_{2(g)} + \_\_\_ O_{2(g)}$

2. Ammonia breaks down readily into nitrogen and hydrogen gas.

$\_\_\_ NH_{3(g)} \rightarrow \_\_\_ N_{2(g)} + \_\_\_ H_{2(g)}$

3. Hydrochloric acid reacts with sodium hydroxide in a neutralization reaction to form sodium chloride and water.

$\_\_\_ HCl_{(aq)} + \_\_\_ NaOH_{(s)} \rightarrow \_\_\_ NaCl_{(aq)} + \_\_\_ HOH_{(l)}$

Note: HOH may also be used to represent water or H_2O.

4. Methane burns in the presence of oxygen to form carbon dioxide and water and is an example of a combustion reaction.

$\_\_\_ CH_{4(g)} + \_\_\_ O_{2(g)} \rightarrow \_\_\_ CO_{2(g)} + \_\_\_ H_2O_{(g)}$

5. In a double replacement reaction, lead nitrate reacts with potassium iodide to form lead iodide and potassium nitrate.

$\_\_\_ Pb(NO_3)_{2(aq)} + \_\_\_ KI_{(aq)} \rightarrow \_\_\_ PbI_{2(s)} + \_\_\_ KNO_{3(aq)}$

Note: In most reactions, the bonds within the polyatomic ion (NO_3) will not be broken.

6. Butane from a lighter and oxygen combine to form carbon dioxide and water.

$\_\_\_ C_4H_{10(l)} + \_\_\_ O_{2(g)} \rightarrow \_\_\_ CO_{2(g)} + \_\_\_ H_2O_{(g)}$

Note: When balancing hydrocarbon combustion reactions, first balance the C atoms on each side of the equation followed by H and then O. If an odd number of O atoms results, go back and double the Cs and Hs respectively.

7. In a single replacement reaction, aluminum reacts with nickel bromide to form aluminum bromide and solid nickel.

$\_\_\_ Al_{(s)} + \_\_\_ NiBr_{2(aq)} \rightarrow \_\_\_ AlBr_{3(aq)} + \_\_\_ Ni_{(s)}$

Note: Balance bromide ions first.

8. Rust or iron (III) oxide forms when iron combines with oxygen in the air.

$\_\_\_ Fe_{(s)} + \_\_\_ O_{2(g)} \rightarrow \_\_\_ Fe_2O_{3(s)}$

	Answers	Explanation*
1.	2, 2, 1	Since there are two atoms of oxygen in oxygen gas, there must be two atoms of oxygen in the reactant. Therefore, you need to add a two in front of H_2O. This leaves two molecules of hydrogen gas and one molecule of oxygen gas in the products.
2.	2, 1, 3	Ammonia has three hydrogen atoms. The chemical reaction must occur with an even number of ammonia molecules in order to yield hydrogen gas, H_2.
3.	1, 1, 1, 1	Since each element needs only one atom to yield the molecules in the reactants, the coefficient is one for each molecule. On the products side, the coefficient remains one since there are two hydrogen atoms available to form the water molecule.
4.	1, 2, 1, 2	There are four atoms of hydrogen in a methane molecule, which means that H_2O in the products needs a coefficient of two. That gives you four oxygen atoms in the products, so you need to give O_2 in the reactants a coefficient of two as well.
5.	1, 2, 1, 2	Two molecules of potassium iodide in the reactants give you enough atoms to balance out the products. You are left with one molecule of lead iodide (PbI_2) and two molecules of potassium nitrate (KNO_3).
6.	2, 13, 8, 10	You need at least four carbon atoms on the products side to balance the equation. But that leaves you with five water molecules and an uneven number of oxygen atoms. Doubling the coefficient for butane allows you to balance the oxygen atoms, giving you eight molecules of carbon dioxide and 10 molecules of water.
7.	1, 3, 2, 3	Since there are three atoms of bromine in an aluminum bromide molecule, you need at least two $AlBr_3$ molecules to balance out the bromine in the reactants. Balancing the rest of the equation, you are left with three atoms of solid nickel.
8.	4, 3, 2	Start by balancing the oxygen, since there are three atoms in the products. You are left with four iron atoms and three oxygen molecules, which gives you two molecules of Fe_2O_3, or rust.

**To ease understanding in the explanations above, the units used will be individual molecules and atoms. Remember, however, that in reality the coefficients indicate only a proportion of one reactant to another; they do not indicate amounts in any particular system of measure.*

Stoichiometry

When working with chemical reactions, it is important for chemists to know what measured quantities of the reactants are needed to produce a specific amount of product. *Stoichiometry* is the branch of chemistry that allows chemists to determine these quantities.

Relationships and Ratios

Stoichiometry deals with the mathematical relationships between the elements making up substances and the properties of the substances. The mole concept is important in making stoichiometric calculations. (See page 374.)

For example, look at the following reaction.

$$2H_{2(g)} + 1O_{2(g)} \rightarrow 2H_2O_{(l)}$$

Two hydrogen molecules react with one oxygen molecule to form two water molecules. To yield any significant amount of water, this reaction will involve trillions of hydrogen and oxygen molecules. We can use the coefficients of the balanced reaction to provide us with a *mole ratio.*

Mole ratio for the above reaction forming water comes from the coefficients in front of each molecule:

2 : 1 : 2

So, if 3.5 mol of H_2O are produced from this reaction, how many moles of hydrogen and oxygen gas were consumed during the reaction?

$$2H_2 : 2H_2O = 1 : 1 = 3.5 \text{ mol of } H_2$$

$$1O_2 : 2H_2O = 1 : 2 = 1.75 \text{ mol of } O_2$$

The mole ratio and the molecular mass can then be used to determine the mass of the reactants needed to produce a specific amount of a substance.

$$3.5 \text{ mol } H_2 = 3.5 \cancel{\text{mol } H_2} \times \frac{2.02 \text{ g } H_2}{1 \cancel{\text{mol } H_2}} = 7.04 \text{ g } H_2$$

$$1.75 \text{ mol } O_2 = 1.75 \cancel{\text{mol } O_2} \times \frac{32.00 \text{ g } O_2}{1 \cancel{\text{mol } O_2}} = 56.00 \text{ g } O_2$$

Utilizing all of the above information, you can solve for the various unknowns in a reaction.

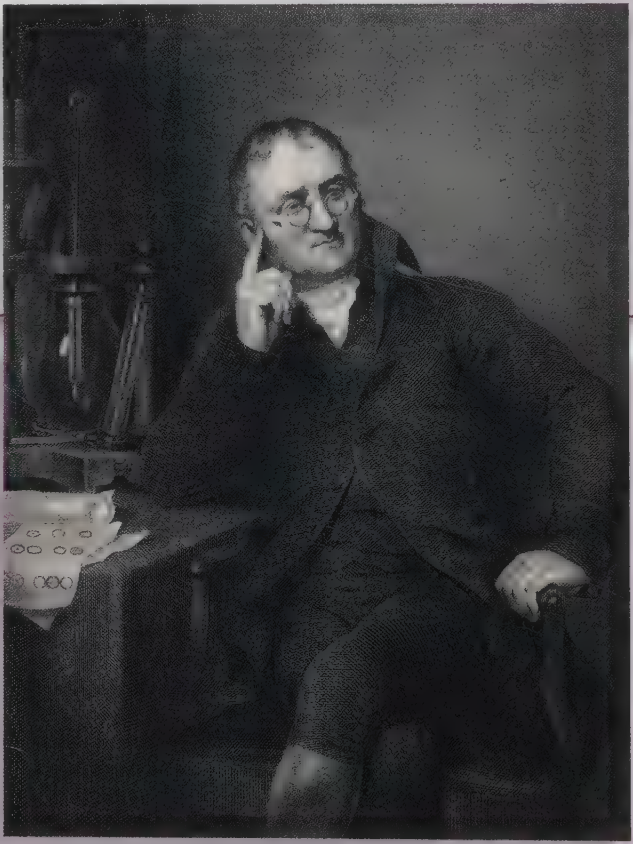
LAWS OF PROPORTION

There are two laws that form the basis for stoichiometry. They allow chemists to accurately calculate the relationships between the reactants and products in chemical reactions.

- the law of definite proportions
- the law of multiple proportions

The law of definite proportions deals with chemical *compounds,* or substances that consist of two or more elements. The law states that the elements of every pure compound exist in the same ratio by mass.

The law of multiple proportions states that when chemical elements combine in a chemical reaction to form a compound, they combine in a ratio of small whole numbers.



▲ **John Dalton,** an English chemist, discovered the law of multiple proportions in 1804. The law helped support Dalton's theory that matter is made up of atoms.

Stoichiometry Problems

1. How many moles of hydrogen gas are needed to react with 12.1 g chlorine gas to produce hydrogen chloride gas?

Balanced equation:

$H_2 + Cl_2 \rightarrow 2HCl$

? mol 12.1 g

2. How many grams of oxygen gas are needed to react completely with 17.5 g of hydrogen gas to produce water?

Balanced equation:

$O_2 + 2H_2 \rightarrow 2H_2O$

? g 17.5g

3. How many grams of aluminum are required to produce 515 g of aluminum oxide in a reaction with oxygen gas?

Balanced equation:

$4Al + 3O_2 \rightarrow 2Al_2O_3$

? g 515g

4. How many grams of zinc chloride will be produced in a reaction between zinc and 61 g of hydrochloric acid?

Balanced equation:

$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

61 g ? g

Solutions

1. a) Determine number of moles in 12.1 grams of Cl_2.

$$12.1\ \cancel{\text{g } Cl_2} \times \frac{1\text{ mol } Cl_2}{70.90\ \cancel{\text{g } Cl_2}} = 0.171\text{ mol } Cl_2$$

b) Convert number of moles of Cl_2 into moles of H_2.

$$0.171\ \cancel{\text{mol } Cl_2} \times \frac{1\text{ mol } H_2}{1\ \cancel{\text{mol } Cl_2}} = 0.171\text{ mol } H_2$$

2. a) Determine number of moles of H_2 in 17.5 g of H_2.

$$17.5\ \cancel{\text{g } H_2} \times \frac{1\text{ mol } H_2}{2.02\ \cancel{\text{g } H_2}} = 8.66\text{ mol } H_2$$

b) Convert moles of H_2 into moles of O_2.

$$8.66\ \cancel{\text{mol } H_2} \times \frac{1\text{ mol } O_2}{2\ \cancel{\text{mol } H_2}} = 4.33\text{ mol } O_2$$

c) Convert moles of O_2 into grams of O_2.

$$4.33\ \cancel{\text{mol } O_2} \times \frac{32.00\text{ g } O_2}{1\ \cancel{\text{mol } O_2}} = 138\text{ g } O_2$$

3. a) Determine number of moles of Al_2O_3 in 515 g of Al_2O_3.

$$515\ \cancel{\text{g } Al_2O_3} \times \frac{1\text{ mol } Al_2O_3}{101.86\ \cancel{\text{g } Al_2O_3}} = 5.05\text{ mol } Al_2O_3$$

b) Convert moles of Al_2O_3 into moles of Al.

$$5.05\ \cancel{\text{mol } Al_2O_3} \times \frac{4\text{ mol Al}}{2\ \cancel{\text{mol } Al_2O_3}} = 10.4\text{ mol Al}$$

c) Convert moles of Al into grams of Al.

$$10.1\ \cancel{\text{mol Al}} \times \frac{26.98\text{ g Al}}{1\ \cancel{\text{mol Al}}} = 273\text{ g Al}$$

4. a) Determine number of moles of HCl in 61 g of HCl.

$$61\ \cancel{\text{g HCl}} \times \frac{1\text{ mol HCl}}{36.5\ \cancel{\text{g HCl}}} = 1.67\text{ mol HCl}$$

b) Convert moles of HCl into moles of $ZnCl_2$.

$$1.67\ \cancel{\text{mol HCl}} \times \frac{1\text{ mol } ZnCl_2}{2\ \cancel{\text{mol HCl}}} = 0.84\text{ mol } ZnCl_2$$

c) Convert moles of $ZnCl_2$ into grams of $ZnCl_2$.

$$0.84\ \cancel{\text{mol } ZnCl_2} \times \frac{135.4\text{ g } ZnCl_2}{1\ \cancel{\text{mol } ZnCl_2}} = 114.58\text{ g } ZnCl_2$$

ACIDS AND BASES are two types of chemical compounds characterized by their opposite effects in certain physical and chemical properties and their ability to neutralize each other in a chemical reaction.

Many acids occur naturally and some are essential for life. For example, hydrochloric acid (HCl) is produced in the stomach of most people and helps digestion. But many acids are poisonous, and strong acids can cause severe burns.

Bases have many practical uses. For example, magnesium hydroxide ($Mg(OH)_2$) is often used as an ingredient in antacids.

▲ **Carbonic acid** (H_2CO_3) is an acid that forms when rainwater absorbs carbon dioxide in the soil. Over long periods of time, the acid removes calcium carbonate from limestone underground, forming magnificent caves.

Properties of Acids and Bases

Over the years, scientists have tried to explain the fact that all acids and bases show certain characteristic properties. Two important theories that explain these properties are

- the Arrhenius theory
- the Brønsted-Lowry theory

The Arrhenius Theory

In 1884, the Swedish chemist Svante Arrhenius observed that all substances classified as acids contain hydrogen ions, H^+, and that all substances known as bases contain the hydroxide ion, OH^-.

An *Arrhenius acid* was thus identified as a substance whose water solution contains a high concentration of hydrogen ions.

An *Arrhenius base* was identified as a substance whose water solution contains a high concentration of hydroxide ions.

Properties of:	
Acid	Base
have a sour taste and produce a prickling or burning sensation if they come into contact with the skin	feel slippery and taste bitter when dissolved in water
dissolve many metals	are also called *alkalis*
turn blue litmus paper red	turn red litmus paper blue
are neutralized by bases	react with an acid to decrease or neutralize its acidic properties

USEFUL AND CORROSIVE

One of the most important commercial chemicals is sulfuric acid (H_2SO_4). It is a colorless, dense, oily liquid that is extremely corrosive.

Sulfuric acid is one of the strongest acids. It can burn the skin and irritate the lining of the nose, trachea, and lungs.

The Brønsted-Lowry Theory

In 1923, chemists Johannes Brønsted and Thomas Lowry independently announced a new acid-base theory. They theorized that an acid-base reaction is a proton-transfer reaction in which a proton (a hydrogen ion) is transferred from the acid to the base.

Receivers and Donors. According to this theory, a *Brønsted acid* can be defined as a proton donor, and a *Brønsted base* is a proton acceptor. The general equation for a Brønsted-Lowry reaction is

$$B + HA \rightleftarrows HB^+ + A^-$$

- In this equation, B is the base proton receiver and HA is the acid proton donor.
- Notice that if this reaction is reversed, then A^- is the base and HB^+ is the acid.
- Combinations such as the acid HA and the base A^- that result from an acid losing a proton or a base gaining one are called *conjugate acid-base pairs*.

Common Acids and Their Conjugate Bases

Acid	Base
Hydrochloric acid (HCl)	Chloride ion (Cl^-)
Nitric acid (HNO_3)	Nitrate ion (NO_3^-)
Hydrocyanic acid (HCN)	Cyanide ion (CN^-)
Perchloric acid ($HClO_4$)	Perchlorate ion (ClO_4^-)
Sulfuric acid (H_2SO_4)	Hydrogen sulfate ion (HSO_4^-)

Sulfuric acid is used

- in the manufacture of fertilizer.
- in the refining of petroleum.
- in the production of such items as automobile batteries, explosives, pigments, iron and other metals, and paper pulp.
- to make alcohol from ethylene.
- to make sulfonates, which are used in powerful detergents.
- in making some dyes and medicines.

Strengths of Acids and Bases

According to the Brønsted-Lowry theory, the strength or weakness of different acids and bases is a measure of their tendency to lose or gain protons.

Acid Strength

- A strong acid donates protons easily.
- A weak acid clings to its protons.

Base Strength

- A strong base has a strong attraction for protons.
- A weak base has a weak attraction for protons.

Conjugate Base Strength. In addition, the strengths of conjugate acid-base pairs are related.

- the stronger the acid, the weaker its conjugate base
- the weaker the acid, the stronger its conjugate base

SALT FORMATION

Salts are formed from the reaction of acids and bases. At one time, salts were considered to be neither acidic nor basic, but it turns out that salts formed by the reaction of a strong base and a weak acid are basic. An example is Na_3PO_4, or trisodium phosphate (TSP), which is such a weak basic that it is used to clean and break down the gloss on walls before they are repainted.

▲ **Trisodium Phosphate.** Poultry processors dunk chickens into water containing TSP to lessen the risk of salmonella.

The pH Scale

Chemists use a scale called the pH scale to measure the acidity and alkalinity (quality of being a base) of a substance. Knowing the pH measurement of a substance is important in many fields, such as farming. It is essential for farmers to know the pH value of the soil and what can be done to control or modify that value. It determines the type of crops that can be grown.

CALCULATING pH

If enough base is added to a solution to make the concentration of the hydroxide ion equal 0.020 M, what would be the pH of this solution?

Using the equilbrium constant:

$$K_w = [H^+][OH^-] = 1.0 \times 10^{-14}$$

then, substitute 0.020 M for $[OH^-]$:

$$[H^+] = \frac{1.0 \times 10^{-14}}{0.020\ M} = 5.0 \times 10^{-1}$$

$$pH = -\log[H^+] = -\log(5.0 \times 10^{-13}) = 12.3$$

From the pH scale, you can see that this solution is highly basic.

Equilibrium Constant and pH

Water spontaneously ionizes into H^+ and OH^- ions. The product of the concentrations or molarities of these ions is called the *equilibrium constant* for the ionization of water. This constant is given the symbol K_w. At 25°C:

$$K_w = [H^+][OH^-] = 1.0 \times 10^{-14}$$

In this formula, the $[H^+]$ and $[OH^-]$ are the concentrations of the hydrogen ion and the hydroxide ion, respectively.

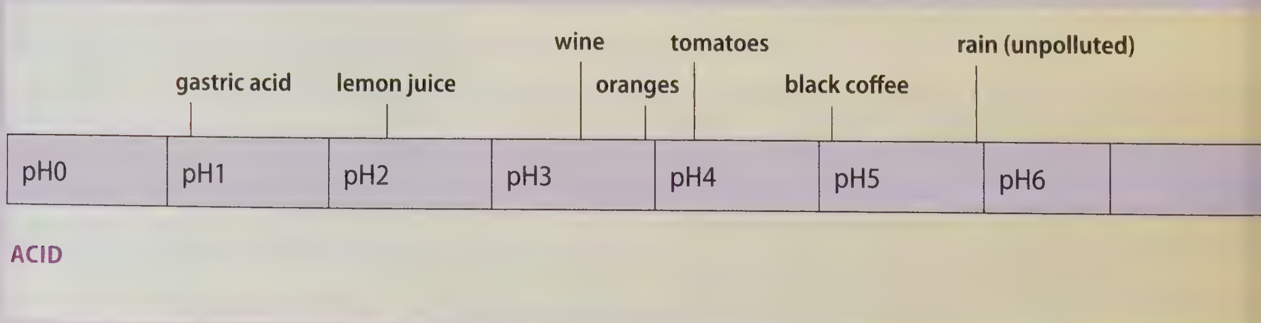
- Theoretically, the concentrations of H^+ and OH^- are equal in pure water.
- If the concentrations are equal, the water or aqueous solution is said to be *neutral*.
- If $[H^+]$ is greater than $[OH^-]$, the solution is *acidic*.
- If $[H^+]$ is less than $[OH^-]$, the solution is *basic*.

The pH of a solution is equal to the negative logarithm of the hydrogen ion concentration:

$$pH = -\log[H^+]$$

- Most solutions have a pH range of 0 to 14.
- Solutions with pH less than 7 are acidic.
- Solutions with pH greater than 7 are basic.
- Solutions with pH equal to 7 are neutral.

THE pH SCALE



Acid Dissociation Constant

Each acid has its own constant, called the *acid dissociation constant,* which is a measure of the acid's strength. This constant is given the symbol K_a. For acid *HA,* made up of the ions H^+ and A^-, you can determine the strength using the following equilibrium constant expression:

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

The values for these dissociation constants can vary a great deal, but for sulfuric acid ($H_2SO_4^-$), it is close to 10^{-2}; for a very weak acid such as $H_3PO_4^{2-}$ it is 10^{-12}. For the common organic acid, acetic acid (CH_3COOH), $K_a = 1.8 \times 10^{-5}$.

Weak Acids and Buffers. In many cases, it is important to keep the pH independent of dilution or small additions of acids or bases. This is done by using a *buffer* to keep the $[H^+]$ nearly constant. A buffer is a solution with similar amounts of a weak acid, HA, and its conjugate base, A^-, both present in solution. The hydrogen ion concentration can be solved using the following equation:

$$[H^+] = K_a \frac{[HA]}{[A^-]} = K_a \frac{[\text{weak acid}]}{[\text{conjugate base}]}$$

PH OF BLOOD

The weak acid that determines the pH of blood is carbonic acid (H_2CO_3), formed by dissolved CO_2 gas. The conjugate base of the CO_2 in blood plasma is a hydrogen carbonate ion (HCO_3^-) that comes from the kidneys.

Direct chemical analysis determined that the concentrations in the blood plasma of a person at rest were:

$$[H_2CO_3] = 1.3 \times 10^{-3}\text{ M}$$

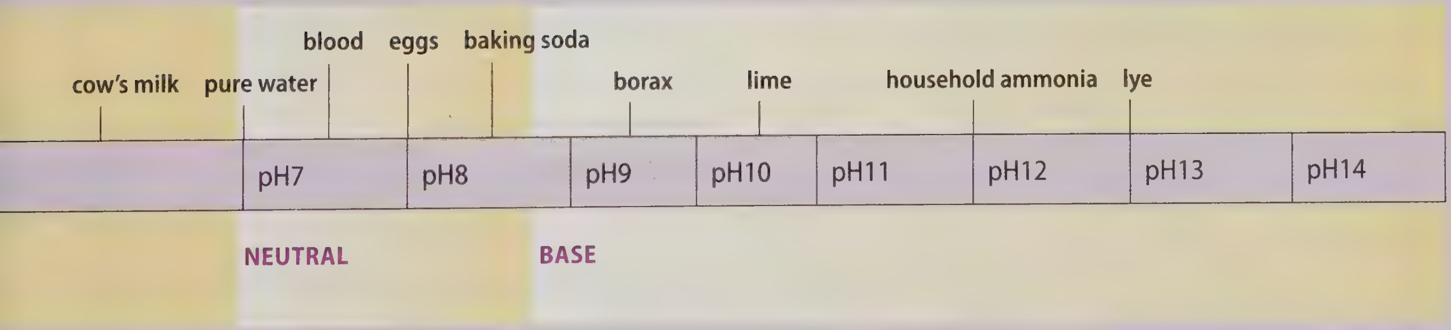
$$[HCO_3^-] = 26 \times 10^{-3}\text{ M}$$

If $K_a = 8.1 \times 10^{-7}$ for carbonic acid in blood plasma at 37°C, what is the normal pH of blood?

$$[H^+] = 8.1 \times 10^{-7} \times \frac{1.3 \times 10^{-3}}{26 \times 10^{-3}} = 4.1 \times 10^{-8}$$

$$pH = -\log[H^+] = -\log[4.1 \times 10^{-8}] = 7.4$$

If a person hyperventilates (that is, quickly breathes too hard while at rest), too much CO_2 gas is removed, and before the kidneys can respond, the pH may increase by 0.2 units. This results in fainting.



Types of Nuclear Reactions

Some Types of Nuclear Reactions

Type	Example	Equation	Occurrence or Use
Alpha decay	Emission of alpha particle by uranium-238	${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$ uranium-238 (radioactive) thorium-234 (radioactive) alpha particle (helium-4)	Natural radioactivity
Beta decay	Emission of beta particles by thorium-234	${}^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{1}{-\mathrm{e}}$ thorium-234 (radioactive) protactinium-234 (radioactive) beta particle (electron)	Natural radioactivity
Beta decay	Emission of beta particle by carbon-14	${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}_{1}{-\mathrm{e}}$ carbon-14 (radioactive) nitrogen-14 (stable) beta particle (electron)	Radiocarbon dating
Artificial transmutation	Bombardment of nitrogen-14 with alpha particle	${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$ nitrogen-14 (stable) alpha particle (helium-4) oxygen-17 (stable) proton (hydrogen-1)	Rutherford experiment
Artificial transmutation	Bombardment of aluminum-27 with alpha particle	${}^{27}_{13}\mathrm{Al} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{30}_{15}\mathrm{P} + {}^{1}_{0}\mathrm{n}$ aluminum-27 (stable) alpha particle (helium-4) phosphorous-30 (radioactive) neutron	Joliot-Curie experiment (step 1)
Decay of synthetic radioisotope	Emission of positron by phosphorous-30	${}^{30}_{15}\mathrm{P} \rightarrow {}^{30}_{14}\mathrm{Si} + {}^{0}_{1}{+\mathrm{e}}$ phosphorous-30 (radioactive) silicon-30 (stable) positron	Joliot-Curie experiment (step 2)
Production of radioisotope of same element	Bombardment of sodium-23 with neutron	${}^{23}_{11}\mathrm{Na} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{24}_{11}\mathrm{Na}$ sodium-23 (stable) neutron sodium-24 (radioactive)	Nuclear reactor
Nuclear fission	Capture of neutron by uranium-235, with splitting	${}^{235}_{93}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n} + \text{energy}$ uranium-235 (radioactive) neutron barium-141 (radioactive) krypton-92 neutrons (radioactive)	Nuclear reactor (controlled) or atomic fission bomb (uncontrolled)
Synthesis of new element	Capture of neutron by uranium-238, with synthesis of plutonium-239	${}^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{239}_{94}\mathrm{Pu} + 2\,{}^{0}_{1}{-\mathrm{e}}$ uranium-238 (radioactive) neutron plutonium-239 (radioactive) beta particles (electrons)	Breeder reactor for production of nuclear fuels
Nuclear fusion	Reaction of deuterium with tritium	${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He}\ \ {}^{1}_{0}\mathrm{n} + \text{energy}$ deuterium (hydrogen-2) tritium (hydrogen-3) proton (helium-4) neutron	Thermonuclear or hydrogen-bomb reaction

Mushroom Cloud. When a nuclear weapon explodes, it releases a fireball of dust and extremely hot gases. The fireball releases neutrons and gamma rays. More gamma rays are released by the huge mushroom-shaped cloud that forms from the explosion. ▶

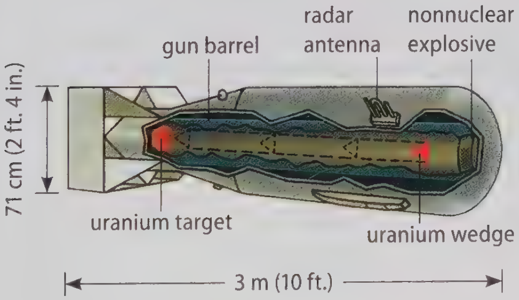
NUCLEAR WEAPONS

The first nuclear weapons were two fission bombs dropped by the United States on the Japanese cities of Hiroshima and Nagasaki in 1945, during World War II. The bombs killed from 110,000 to 140,000 people and destroyed large areas of both cities. The bombs are the only two nuclear weapons that have ever been used.

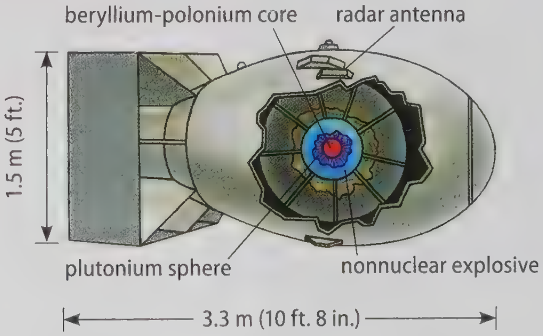
Hiroshima, Japan, was largely destroyed by a gun-type fission bomb called the Little Boy bomb on August 6, 1945. The bomb was dropped from a B-29 bomber. When the bomb reached 564 meters (1,850 feet), a radar echo set off an explosive inside. This explosive drove a wedge of U-235 into a larger piece of U-235, setting off the nuclear blast.

Nagasaki, Japan, was struck by an implosion-type fission bomb called the Fat Man bomb on August 9, 1945. In this bomb, an explosive crushed a hollow sphere of plutonium into a core made up of the chemical elements beryllium and polonium. This core then released neutrons, which triggered a fission chain reaction in the plutonium.

Gun-Type Fission Bomb (Little Boy)


radar antenna
nonnuclear explosive
gun barrel
71 cm (2 ft. 4 in.)
uranium target
uranium wedge
3 m (10 ft.)

Implosion-Type Fission Bomb (Fat Man)


beryllium-polonium core
radar antenna
1.5 m (5 ft.)
plutonium sphere
nonnuclear explosive
3.3 m (10 ft. 8 in.)

ORGANIC CHEMISTRY is the study of compounds that contain carbon atoms. It is considered a separate branch of chemistry because of the unique ways in which carbon atoms can link to form long molecular chains.

It is called organic because until 1828 chemists believed that familiar carbon compounds, such as ethyl alcohol, acetic acid, and sugars, could only be produced by living things. In 1828, however, the German chemist Friedrich Wöhler succeeded in making urea, a typical organic compound, from ammonium chloride and silver cyanate, typical inorganic compounds.

Spotlight on...

FRIEDRICH WÖHLER

1800–1882

German chemist Friedrich Wöhler revolutionized organic chemistry in 1828 when he synthesized the organic substance urea from inorganic substances. Urea is a substance found in the urine of mammals.

Wöhler was the first person to make an organic substance from inorganic chemicals. His experiments helped disprove the belief that organic substances could be formed only in the living bodies of animals or plants.

Hydrocarbons

Carbon is unique because its atoms have the ability to link together to form chains of great size and complexity. Carbon has four electrons in its outer shell, and these are available for covalent sharing with other elements but especially with other carbon atoms.

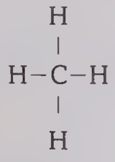
The most important class of organic compounds consists solely of the elements carbon and hydrogen. They are called *hydrocarbons.* The number and variety of compounds that carbon forms with hydrogen is quite large.

Hydrocarbons can be divided into three classes.

- *aliphatics*
- *alicyclics*
- *aromatics*

Aliphatics

Aliphatics consist of compounds with their carbon atoms arranged in an open-chain structure. The simplest of these compounds, methane (CH_4), can be represented by a structural formula in which each dash represents a pair of shared electrons between the atoms indicated by their symbols.



$$\begin{array}{c} \mathrm{H} \\ | \\ \mathrm{H-C-H} \\ | \\ \mathrm{H} \end{array}$$

A carbon atom may share one, two, or even three of its electrons with another carbon. Aliphatics can be further subdivided into *alkanes, alkenes,* and *alkynes,* based on the bonds between their carbon atoms.

Alkanes are a series of hydrocarbons characterized by *single bonds* between carbon atoms (written C−C).

- A single bond exists when each of the atoms that share electrons contributes only one electron to the bond.
- Methane, the main component of natural gas, is the simplest of the alkanes. (See pages 670–671.)
- The alkanes are *saturated* hydrocarbons. This means that they contain only single bonds.
- The *–ane* in *methane* indicates that it is an alkane.

Alkenes are a series of hydrocarbons that have a double bond between carbon atoms (written C = C).

- The first member of the alkenes is ethylene.
- Alkenes and alkynes are *unsaturated* hydrocarbons—they may undergo reactions in which hydrogen atoms are added at the double or triple bond.

Alkynes are a series of hydrocarbons that have a triple bond in their structure (written C ≡ C).

- Acetylene is the first member of the alkynes.
- Alkynes have reactions similar to the alkenes.

Alicyclics

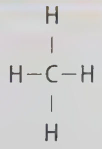
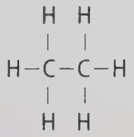
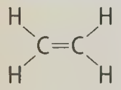
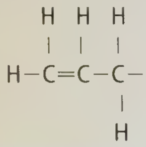
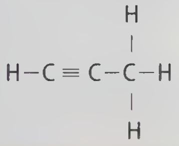
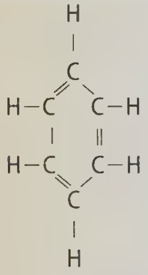
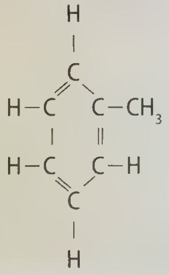
Alicyclics are hydrocarbons that have their carbon atoms arranged in a ring. These atoms may have single or double bonds between them. Triple bonds in alicyclics are rare, but they can occur in sufficiently large rings. Many naturally occurring organic compounds fall into this class.

Aromatics

Aromatics are highly unsaturated ring compounds. They are so named because many members have a distinctive odor. Each of the aromatics has a ring of six carbons that contains three double bonds.

Aromatics are extremely stable compared with ordinary unsaturated compounds. The most common of the aromatics is benzene.

Structures of Some Hydrocarbons

Series	Hydrocarbon	Molecular Formula	Structural Formula
Alkane	Methane	CH_4	 $\mathrm{H-\underset{H}{\overset{H}{C}}-H}$
	Ethane	C_2H_6	 $\mathrm{H-\underset{H}{\overset{H}{C}}-\underset{H}{\overset{H}{C}}-H}$
Alkene	Ethene	C_2H_4	
	Propene	C_3H_6	 $\mathrm{H-\overset{H}{C}=\overset{H}{C}-\underset{H}{\overset{H}{C}}-H}$
Alkyne	Ethyne	C_2H_2	$\mathrm{H-C\equiv C-H}$
	Propyne	C_3H_4	 $\mathrm{H-C\equiv C-\underset{H}{\overset{H}{C}}-H}$
Aromatic	Benzene	C_6H_6	
	Toluene	C_7H_8	

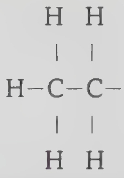
◀ **Hydrocarbons.** Petrochemical companies use hydrocarbons from crude oil and natural gas to manufacture solvents, plastics, and synthetic fibers and rubber.

Hydrocarbon Structure and Names

Hydrocarbons are systematically named according to the number of carbon and hydrogen atoms and how those atoms are connected. The simplest organic molecule is methane (CH_4). The four hydrogens are arranged three-dimensionally to form a regular tetrahedron with the carbon using *sp*^{3} hybrid orbitals. Hybrid orbitals are created by combining atomic orbitals in the same atom. Each C–H bond is covalent and made by the sharing of two electrons between the atoms. In total, the carbon atom donates four electrons, its valence electrons, one to each of the four bonds.

Naming Aliphatics

After methane, the next largest alkane is the two-carbon ethane, represented by C_2H_6 or, better yet, by CH_3CH_3. This second representation better reflects ethane's structure. A visual representation is given by the structural formula

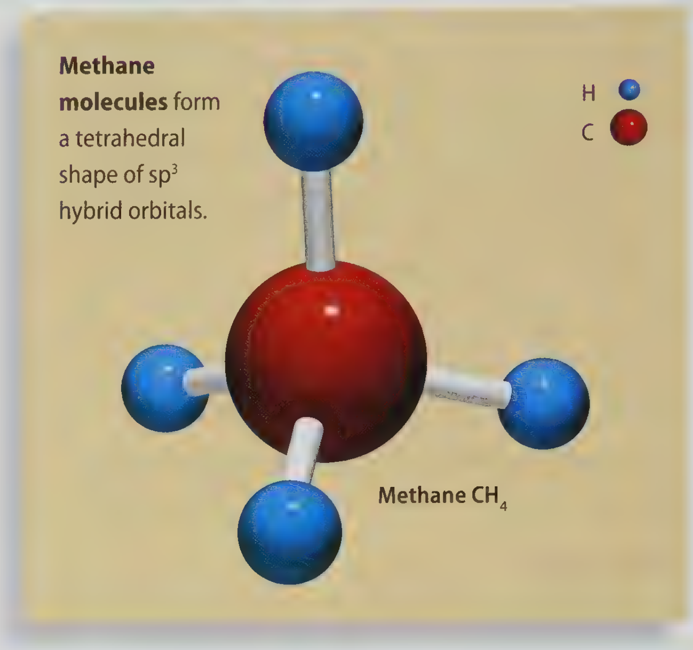


$$\begin{array}{ccccc} & H & & H & \\ & | & & | & \\ H- & C & - & C & -H \\ & | & & | & \\ & H & & H & \end{array}$$

Chemical Formulas. The next larger alkane is propane (C_3H_8), followed by butane (C_4H_{10}), pentane (C_5H_{12}), and hexane (C_6H_{14}). In general, the chemical formula for an alkane is given by the expression C_nH_{2n+2}, where *n* is the number of carbons in the compound.

Other alkanes include heptane (C_7H_{16}), octane (C_8H_{18}), nonane (C_9H_{20}), and decane ($C_{10}H_{22}$), which finishes the list of the first 10 alkanes.

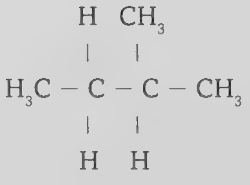
Alkane Structure. Although there is only one possible structure for ethane or propane, an alkane with four or more carbon atoms will have more than one possible structural isomer. Isomers are molecules with the same chemical formula but different arrangements of the atoms which leads to differing chemical properties.



Methane molecules form a tetrahedral shape of sp^3 hybrid orbitals.

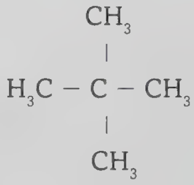
Methane CH_4

Pentane has three isomers, including the linear form called normal pentane. The other two forms are listed below.



$$\begin{array}{ccccccc} & & H & & CH_3 & & \\ & & | & & | & & \\ H_3C & - & C & - & C & - & CH_3 \\ & & | & & | & & \\ & & H & & H & & \end{array}$$

isopentane



$$\begin{array}{ccccc} & & CH_3 & & \\ & & | & & \\ H_3C & - & C & - & CH_3 \\ & & | & & \\ & & CH_3 & & \end{array}$$

neopentane

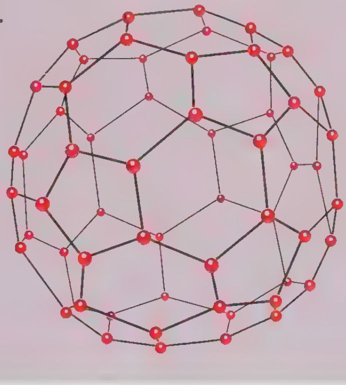
As the number of carbons in an alkane increases, so does the number of possible isomers. For instance, hexane, which has six carbons, has five possible structural isomers. Octane, which has eight carbons, has 18 possible structural isomers. As the number of carbons gets quite large, the number of isomers increases significantly.

Naming Alkenes. The smallest alkene is ethylene, or ethene (C_2H_4). The next in size is propene (C_3H_6), followed by butene (C_4H_8), pentene (C_5H_{10}), and hexene (C_6H_{12}). In general, the chemical formula for an open-chain monounsaturated (that is, having one double bond) molecule is C_nH_{2n}, where *n* is the number of carbons.

BUCKYBALLS

In 1985, three chemists produced a molecule consisting of 60 carbon atoms. The *buckminsterfullerene* molecule is shaped like a geodesic dome and named after R. Buckminster Fuller, the engineer who designed the geodesic dome.

Buckminsterfullerenes, nicknamed "buckyballs," can be chemically modified to block a key step in the reproduction of the human immunodeficiency virus (HIV), which causes AIDS. Similar molecules can be filled with metal atoms to form the smallest wire imaginable, or made into superconductors, which conduct electric current with no resistance at extremely low temperatures.



Multiple Double-Bonds. There is the possibility of multiple double-bonds in alkenes having four or more carbons. The simplest example is butadiene (C_2H_6), but its proper name is 1,3-butadiene to indicate the location of the double bonds.

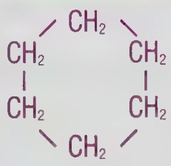
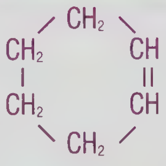
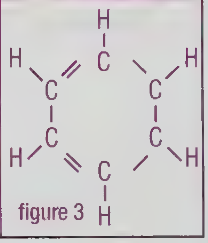
The *di* in *butadiene* indicates that there are two double bonds in the structure. In the same manner, *triene* indicates there are three double bonds present.

Alkynes containing a single triple bond have the general formula of C_nH_{2n-2}, where n is again the number of carbons. The simplest and most common of these compounds is ethyne, better known as acetylene (C_2H_2).

The next in size is propyne (C_3H_4), followed by butyne (C_4H_6), pentyne (C_5H_8), and hexyne (C_6H_{10}). Alkynes are not as common as other aliphatics except for acetylene, which is used as a fuel for torches in welding.

Naming Alicyclics

A good example of an unsaturated alicyclic is cyclohexane (C_6H_{12}). The *cyclo* indicates it is a ring system, and *hexane* indicates it contains six carbons and has no double bonds, as discussed in the alkane section. Of course, as with the open-chained compounds, there is the possibility of having a double bond, as in the case of cyclohexene (C_6H_{10}).

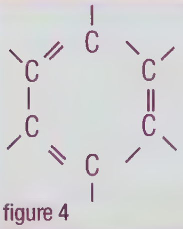
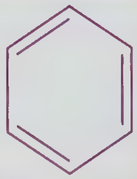
CYCLOHEXANE	CYCLOHEXENE	BENZENE
 figure 1	 figure 2	 figure 3

As one can now see, the naming of alicylics is analogous to the naming of aliphatic compounds except for the addition of the prefix *cyclo-*.

SIMPLIFYING MOLECULAR STRUCTURE DIAGRAMS

The naming and drawing of organic molecules becomes very complex as the size of the molecule becomes larger.

- One method of simplifying the drawing of organic structures is to omit the hydrogen atoms from the drawing. For example, figure 4 depicts benzene without showing the hydrogens.
- We can take this simplification even further by removing the C–H bonds and the letter C (figure 5).
- These structural formulas are easier to draw and are commonly used.

 figure 4	 figure 5
BENZENE	BENZENE

CHEMISTRY PRACTICE

In solving these problems, we will show the units with modern scientific abbreviations. We will also try to keep the same number of figures in the final answers as are in the original data. The solutions have been separated from the problems. Solutions begin on page 430.

Problems

Moles, Molecules, Gases, and Stoichiometry

1. a) Suppose that you have 25.0 g of SeO_2, how many moles and molecules would you have?

b) If it was treated to make H_2Se and this gas was collected in a 2.00 L bulb, what would be its pressure in atmospheres at 25˚C?

2. Assume that the chemical reaction, $3H_2(g) + N_2(g) \rightarrow 2NH_3(g)$, goes to completion.

a) If the gases were measured by volume, all at the same temperature and pressure, what would be the final volume if 3 volumes of H_2 were allowed to react with 1 volume of N_2?

b) To make 1.00 mol of NH_3, how many moles of H_2 and N_2 would have to be mixed?

c) If 2.00 mol of H_2 were mixed with 1.00 mol of N_2, how many moles of NH_3 would be produced?

Molecular and Electronic Structures

3. Write valence and electron dot diagrams for: Cl_2, HCl, CH_4, C_2H_2, and C_2H_2O.

4. Molecules with a central atom which is surrounded by four atoms and has a closed octet are found to have perfect tetrahedral structures. Examples are CH_4, NH_4^+, BH_4^-, ClO_4^-, and many organic molecules. The molecule SF_4 does not have a perfect tetrahedral structure. Give a logical reason why this might be true.

5. What is the simplest electron configuration for the carbon atom which has four unpaired electrons suitable to form four single bonds? Is this the normal configuration for carbon?

Bond Energy, Spectroscopy, and Thermochemistry

6. Use bonding arguments to explain why the bond-dissociation energy for the first oxygen from CO_2 giving CO and an O-atom is much less than the bond-dissociation energy of CO.

7. When the atomic weights of the elements were first being established, an important tool was the law of Dulong and Petit. It stated that the heat capacity of metals was equal to $6.00 \frac{cal}{mol \bullet °C}$,and it was used to establish approximate atomic weights. As an example, use the following data to determine an approximate atomic weight for a metal and identify it in the periodic table. A 278 g sample of shiny metal was heated to 98.7°C in a dry beaker immersed in boiling water. The heated metal was dropped into 200 g of water, previously cooled to 8.7°C, in an insulated cup and the final temperature reached was 29.5°C. The specific heat capacity of water is $1.00 \frac{cal}{mol \bullet °C}$.

8. An important chemical question about the interaction of light and matter is whether ordinary visible light can break chemical bonds and so take molecules apart. Its energy is high enough to make excited electronic states in many molecules, but is it large enough to break chemical bonds? We have not given you Planck's constant, but from it one can calculate that the energy of a photon of visible light, and at 500 nm wavelength it is very close to $4.0 \bullet 10^{-19}$ J. This seems a small number, but when we talk about molecules, chemists use moles. Compare this photon energy to the bond-dissociation energies of typical molecules. If it can break bonds, what kind of bonds could it break?

Acids, Bases, and Chemical Equilibrium

9. Every acid has a conjugate base. The stronger the acid, the weaker its conjugate base. The weaker the acid, the stronger the base. For each of the following acids, identify their conjugate base.

H_2S, HS^-, H_3PO_4, $H_2PO_4^-$, HPO_4^{2-}, NH_4^+ , H_2SO_4 , HSO_4^-

10. Ammonium ion is a typical weak acid, and its acid dissociation constant, $K_a = 5.7 \bullet 10^{-10}$. Assume that one started with a solution which had $[NH_4^+] = 0.10$ M and initially had the $[NH_3] = 0$. Calculate the equilibrium $[H^+]$ and pH.

11. Balance the following chemical equations using the principle that the same number of atoms of each type must be present on both sides of the equation. Some may involve oxidation and reduction, but they can be easily balanced using atoms alone. For aqueous reactions omit spectator ions.

a) $H_{2(g)} + Fe_2O_{3(s)} \rightarrow Fe_{(s)} + H_2O_{(g)}$

b) $HCl_{(aq)} + Fe_2O_{3(s)} \rightarrow Fe^{3+}_{(aq)} + H_2O$

c) $NaOH_{(aq)} + H_2S_{(g)} \rightarrow$ two possible products $+ H_2O$

Solutions

Moles, Molecules, Gases, and Stoichiometry

1. a) The symbol Se represents Selenium. Its atomic weight is 78.96, or $78.96\ \frac{g}{mol}^{-1}$ with units, and the molecular weight of the compound is

$$76.96\ \frac{g}{mol} + 2(15.999\ \frac{g}{mol}) = 110.96\ \frac{g}{mol}^{-1}$$

Then for the moles,

$$25.0\ \cancel{g\ SeO_2} \times \frac{1\ mol\ SeO_2}{110.96\ \cancel{g\ SeO_2}} = \mathbf{0.225\ mol\ SeO_2}$$

For molecules,

$$0.225\ \cancel{mol\ SeO_2} \times \frac{6.02 \times 10^{23}\ molecules\ SeO_2}{1\ \cancel{mol\ SeO_2}} =$$

$$\mathbf{1.35 \times 10^{23}\ molecules\ SeO_2}$$

b) Since SeO_2 and H_2Se both contain one Se atom, we can conclude that the moles of gaseous H_2Se would also be 0.225 mol. To solve for the pressure, we need to use the ideal gas relations. At 1 atm and 0°C, it would occupy

$$0.225\ \cancel{mol} \times \frac{22.4\ L}{1\ \cancel{mol}} = 5.04\ L$$

with $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$P1 = P2\ (\frac{V_1}{V_2})\ (\frac{T_1}{T_2})$$

$$= (1\ atm)\ (\frac{5.04L}{2.00L})\ (\frac{(273 + 25)°C}{273°C})$$

$$= \mathbf{2.75\ atm}$$

2. a) Since the number of moles is proportional to the volume, we can see that these volumes follow the balanced chemical reaction. As a result, the reaction will produce 2 volumes of NH_3.

b) If we divide all the coefficients in the equation by 2, the equation will then produce 1.00 mole of NH_3. It will then show that 3/2 moles of H_2 and 1/2 moles of N_2 will give 1.00 mole of NH_3.

c) This is known as a limiting reagent problem. One of the two reactants is too small compared with the other to follow the balanced equation. This means that one reactant will be used up completely, the limiting reagent, and some of the other will be left over after the reaction takes place. Since the reaction requires three times more moles of H_2 than of N_2, and we only start with twice as much, it is clear that the H_2 would be the limiting reagent.

To determine how much NH_3 is produced, we need to determine how many moles of NH_3 can be produced by 2.00 moles of H_2 since the hydrogen will be entirely used in the reaction. Using the mole ratio of H_2 to NH_3 in the balanced chemical equation will allow conversion from moles of H_2 to moles of NH_3.

$$2\ mol\ H_2 \times \frac{2\ mol\ NH_3}{3\ mol\ H_2} = \mathbf{1.33\ mol\ NH_3}$$

This is the same result as multiplying all the coefficients in the balanced chemical equation by $\frac{2}{3}$. By doing that one can see that $\frac{2}{3}$ of the moles of N_2 will be turned into NH_3, and $\frac{1}{3}$ of the moles of N_2 will be left unreacted.

Molecular and Electronic Structures

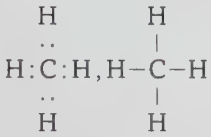
3. For Cl_2, its valence diagram is Cl—Cl, and its electron dot diagram is

$$:\underset{\cdot\cdot}{\overset{\cdot\cdot}{\mathrm{Cl}}}:\underset{\cdot\cdot}{\overset{\cdot\cdot}{\mathrm{Cl}}}:$$

For HCl, the covalent valence diagram is H−Cl, and its ionic diagram is H^+ and Cl^-. The two corresponding electron dot diagrams are

$$\text{ionic } H^+:\underset{\cdot\cdot}{\overset{\cdot\cdot}{\mathrm{Cl}}}:^-\text{, covalent } H:\underset{\cdot\cdot}{\overset{\cdot\cdot}{\mathrm{Cl}}}:$$

For CH_4, they are

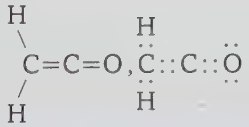


$$\begin{array}{ccc} & \mathrm{H} & \\ & \cdot\cdot & \\ \mathrm{H}: & \mathrm{C} & :\mathrm{H} \\ & \cdot\cdot & \\ & \mathrm{H} & \end{array},\quad \begin{array}{ccc} & \mathrm{H} & \\ & | & \\ \mathrm{H}- & \mathrm{C} & -\mathrm{H} \\ & | & \\ & \mathrm{H} & \end{array}$$

For C_2H_2, they are

$$\mathrm{H{-}C{\equiv}C{-}H}\;,\; \mathrm{H:C:::C:H}$$

For C_2H_2O, there are three possible valence diagrams, but the one with an O–H bond and the one with a three-membered ring are not stable molecules. The only stable molecule with the formula of C_2H_2O is called ketene and its diagrams are



$$\begin{array}{l} \mathrm{H} \\ \;\backslash \\ \;\;\mathrm{C{=}C{=}O} \\ \;/ \\ \mathrm{H} \end{array},\quad \begin{array}{ccccc} \mathrm{H} & & & & \\ \cdot\cdot & & & & \cdot\cdot \\ \mathrm{C} & :: & \mathrm{C} & :: & \mathrm{O} \\ \cdot\cdot & & & & \cdot\cdot \\ \mathrm{H} & & & & \end{array}$$

4. The sulfur atom has 6 valence electrons and each fluorine has 7 valence electrons. This is a total of 34 valence electrons. If we assume that each fluorine has a closed octet, this will require 32 electrons, and if four single bonds are formed to the sulfur that would complete its octet, we have a pair of electrons left over. The simplest reasoning is that the sulfur keeps this extra pair and is surrounded by five pairs of electrons. This is called an expanded octet. A similar expanded octet is found in PF_5, with the phosphorous surrounded by five pairs of electrons. The structure of PF_5 is that of a trigonal bipyramid, that is, two three-sided pyramids back to back with shared bases. The two 180° bonds along the axis of the pyramids are longer, and weaker than, the three 120° bonds along the base. The structure found for SF_4 looks a little like two 180° bonds and two 120° bonds, and the extra electron pair is assumed to take up the space of the missing 120° bond. This is explained by the repulsion of the electron pairs, both bonding and lone pair, by a very useful method called valence shell electron pair repulsion (VSEPR) theory.

5. Carbon has six electrons with the first two filling the 1s. For the remaining four electrons to be unpaired, they would occupy the next four orbitals which are the 2s, $2p_x$, $2p_y$, and $2p_z$. This electron configuration would be $(1s)^2(2s)^1(2p_x)^1(2p_y)^1(2p_z)^1$. The lowest energy configuration has the 2s orbital filled with paired electrons, and the last two electrons in any two of the 2p orbitals. As a result, in forming its four chemical bonds, carbon utilizes an excited electronic configuration, but this extra energy is more than made up by the extra energy released in forming four bonds instead of just two.

Chemistry Glossary

acid compound that produces hydrogen ions in water and that is able to donate hydrogen ions to other compounds.

alcohol class of organic compounds in which the hydroxyl group ($-OH$) is added on to a hydrocarbon group.

alicyclics hydrocarbons that have their carbon atoms arranged in a ring.

aliphatics hydrocarbon consisting of compounds with their carbon atoms arranged in an open-chain structure.

alkali metal metallic element that belongs to Group IA of the periodic table and that forms a strong base when combined with the hydroxide group.

alkane hydrocarbon in which there are only single bonds between carbon atoms.

alkene hydrocarbon in which there is one double bond between two carbon atoms, and the rest of the carbon bonds are single.

alkyne hydrocarbon in which there is one triple bond between two carbon atoms, and the rest of the carbon-carbon bonds are single.

alpha particle particle consisting of two protons and two neutrons; a helium nucleus.

anion a negatively charged ion formed by acquiring an additional electron or electrons.

aromatic hydrocarbon in which the benzene ring structure is present. They are so named because many members have a distinctive odor.

atmospheric pressure the pressure produced by the weight of the air from the top of the atmosphere as it presses down upon the layers of air below it.

atom the smallest particle of an element that can exhibit the properties of that element.

atomic mass the mass of an atom compared with carbon-12, which has been assigned a mass of 12 atomic mass units.

atomic number number of protons that are in the nucleus of an atom.

Avogadro's number 6.022×10^{23} (602.2 billion trillion), or the number of elementary entities that make up 1 mole of any substance. It is equal to the number of atoms found in 12 grams of the isotope carbon-12. Scientists chose the number as a standard reference number for the International System of Units.

balanced equation an equation that has an equal number of atoms of each element on each side of the arrow.

base compound that produces hydroxide ions in water, or that is capable of receiving a hydrogen ion from an acid.

beta particle electron that is emitted by the nucleus of a radioactive atom.

Bose-Einstein condensate state of matter that results when certain types of atoms are cooled to temperatures close to absolute zero; the atoms clump together and behave identically.

breeder reactor nuclear reactor used to produce nuclear fuel (often in the form of synthetic elements) as well as nuclear energy.

catalyst a substance that increases the rate of a chemical reaction by lowering the activation energy for the reaction.

chain reaction process in which the splitting of one atomic nucleus gives off neutrons that cause the splitting of other atomic nuclei.

chemical change a change in which one substance is converted to another.

chemical element any substance that contains only one kind of atom.

chemical equation a statement, consisting of symbols and formulas, that summarizes the changes that occur in a chemical reaction.

chemical equilibrium the state of balance achieved when the rate of the forward reaction is equal to the rate of the reverse reaction.

chemical properties properties that describe the way a substance may change to form other substances.

chemical reaction the process by which one or more substances are converted into one or more different substances.

coefficients the numbers in front of chemical formulas in a chemical equation which indicate how many units of the substances are used and produced in the reaction.

compound substance consisting of two or more elements joined by chemical bonds.

covalent bond chemical bond between two atoms in which electrons are shared.

crystalline solids solids whose atoms are arranged in an orderly geometric pattern that repeats itself. Most solids are crystalline solids.

diatomic elements elements that always appear in bonded pairs which include hydrogen, nitrogen, oxygen, fluorine, chlorine, bromine and iodine.

distillation a separation process in which a liquid's components are separately converted to a vapor by applying heat (a vacuum may be applied simultaneously to lower the boiling points of the components involved), then drawn off, cooled or condensed, and collected.

electrolyte a substance that conducts electricity.

electron fundamental negatively charged particle of matter.

electronegativity the ability of an atom to attract a bonded pair of electrons.

element substance that cannot be broken down into simpler substances by ordinary chemical means.

endothermic process the conversion of a solid to a liquid (melting), a liquid to a gas (vaporization), or a solid to a gas (sublimation).

endothermic reactions changes in which energy is absorbed.

energy the ability to do work or transfer heat.

enzymes molecules (usually proteins) that speed up chemical reactions in all living things.

equilibrium in a chemical reaction, the state of balance achieved when the rate of the forward reaction is equal to rate of the reverse reaction.

exothermic process the conversion of a liquid to a solid (freezing), a gas to a liquid (condensation), or a gas to a solid (deposition).

exothermic reactions changes in which energy is released.

formula weight the sum of the atomic mass of each atom in a chemical formula.

gamma rays high-frequency radiation, similar to X-rays, emitted by substances that are radioactive.

gas matter that has no fixed volume or shape—it takes the volume and shape of its container.

group column on the periodic table containing elemens with the same number of valence electrons.

half-life period of time it takes for half the radioactive atoms in a given mass to decay.

heat the energy that results from the vibrations, motions, reactions, and collisions of nuclear particles, atoms, and molecules.

heat capacity the amount of heat (calories) necessary to raise the temperature of 1 gram of a given substance by 1°C. The heat of fusion is the amount of heat needed to convert 1 gram of a solid into a liquid at its melting point. The heat of vaporization is the amount of heat required to convert 1 gram of a liquid into a vapor at its boiling point.

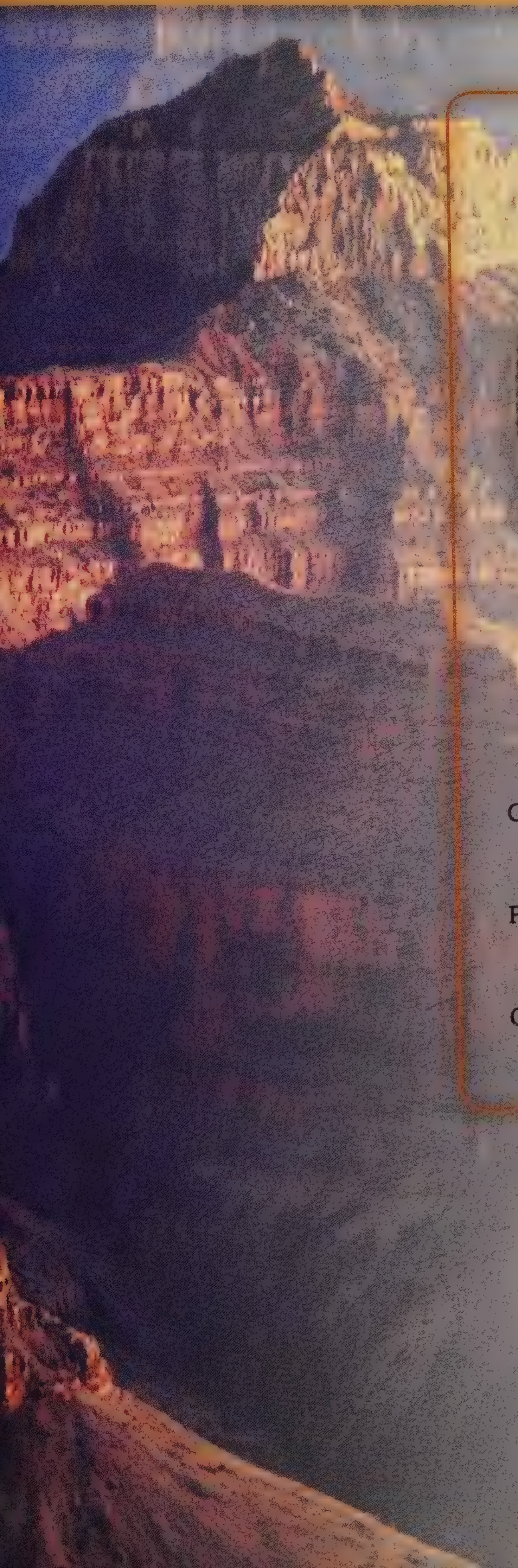
hydrocarbon compound consisting only of hydrogen and carbon.

inorganic chemistry this is the branch of chemistry that deals with the reactions of elements other than carbon.

intermolecular forces attractive forces that exist between individual molecules which include hydrogen bonds, dipole forces, and dispersion forces.

ion atom or group of atoms having a negative or positive electric charge due to the number of protons and electrons being unequal.

Earth Science



Geology 442

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Plate Movement

Today, Earth's surface continues to be in motion. Tectonic plates are made up of Earth's *lithosphere*—the crust and the outermost layer of its mantle. Earth's entire surface—all the dry land, all the ocean floors, and the beds of all the other bodies of water—are part of the crust.

A tectonic plate may consist of only ocean floor, only continent, or some combination of both. These plates move around on the asthenosphere much as a boat floats on water. As the tectonic plates move, they interact with one another at their boundaries. There are three types of boundaries:

Divergent Plate Boundaries. Divergent plate boundaries occur mostly on ocean floors, where two tectonic plates move apart in a process called *seafloor spreading*. As the plates move apart, molten rock wells up from beneath the surface to form new ocean floor.

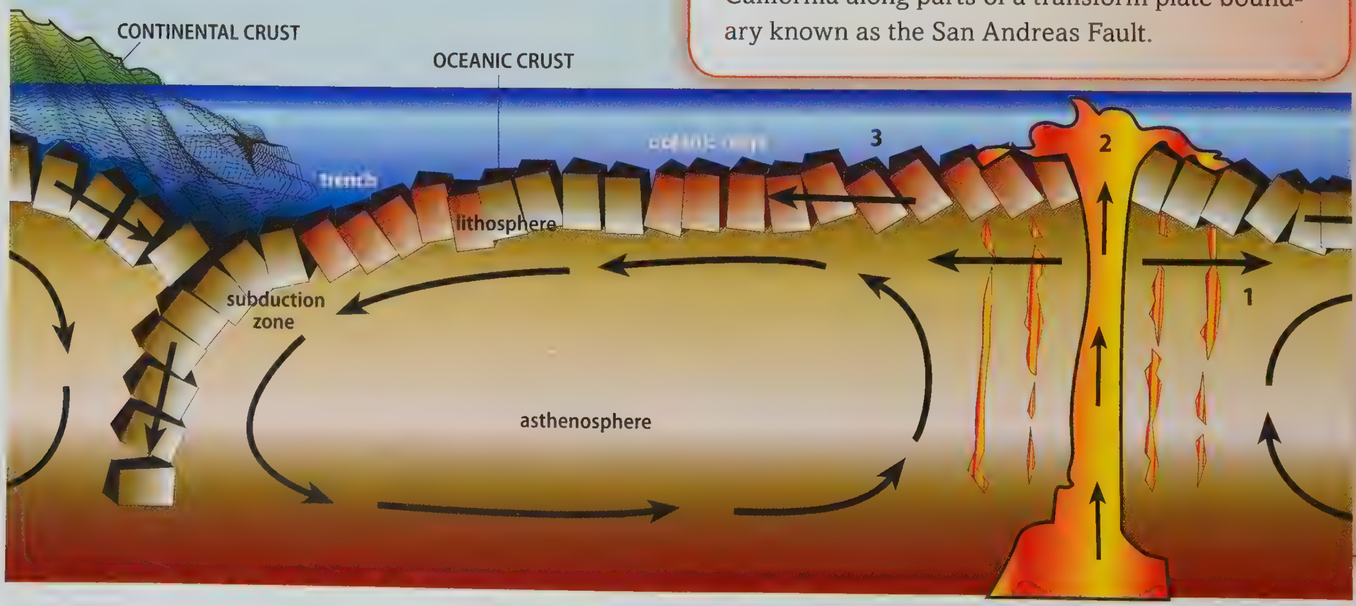
Based on the hypothesis of seafloor spreading, geologists now believe that the entire seafloor has formed within the past 100 to 200 million years. Before seafloor spreading was discovered, geologists believed that the ocean floor was the site of Earth's oldest rocks, billions of years old.

Convergent Plate Boundaries. In some parts of the world, tectonic plates converge, or collide with one another. When two plates collide, *subduction* occurs, or one plate slides beneath the other. During subduction, crust is destroyed. This offsets the new crust being formed by seafloor spreading. Otherwise, Earth's surface would be getting larger each year.

Transform Plate Boundaries. Instead of separating or coming together, two plates may simply slide against each other. These boundaries are called transform plate boundaries, or transform faults. Crust is neither created nor destroyed along transform faults, but powerful earthquakes can occur there. Devastating earthquakes have occurred in California along parts of a transform plate boundary known as the San Andreas Fault.

SEAFLOOR SPREADING originates in the long undersea mountain system often called the mid-ocean ridge. The process, repeated for millions of years, widens the ocean basins and carries along the continental plates like luggage on a conveyor belt.

1. The spines of the mid-ocean ridge contain active fissures that are constantly pulled apart.
2. Each time a fissure opens, lava from the upper part of the mantle rises to fill the gap.
3. The lava quickly hardens into rock after coming into contact with cold seawater.



The Big Picture. The overall pattern of movement of the tectonic plates is a widening of the Atlantic Ocean and a shrinkage of the Pacific Ocean. The Atlantic is widening because seafloor spreading at the mid-Atlantic ridge continues to create lithosphere. The Pacific is shrinking because much of it is ringed by convergent plate boundaries that are consuming its lithosphere.

WHEN PLATES COLLIDE!

Oceanic Plate Collides with a Continental Plate

- the heavier seafloor sinks beneath the continent
- forms a deep ocean trench
- volcanoes common in these areas

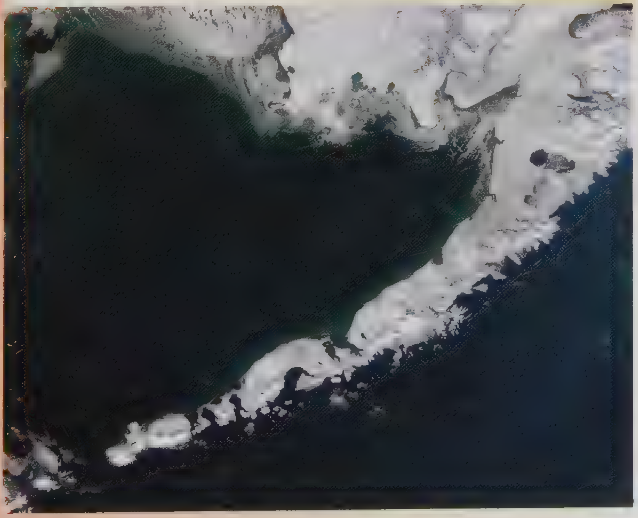
Two Oceanic Plates Collide

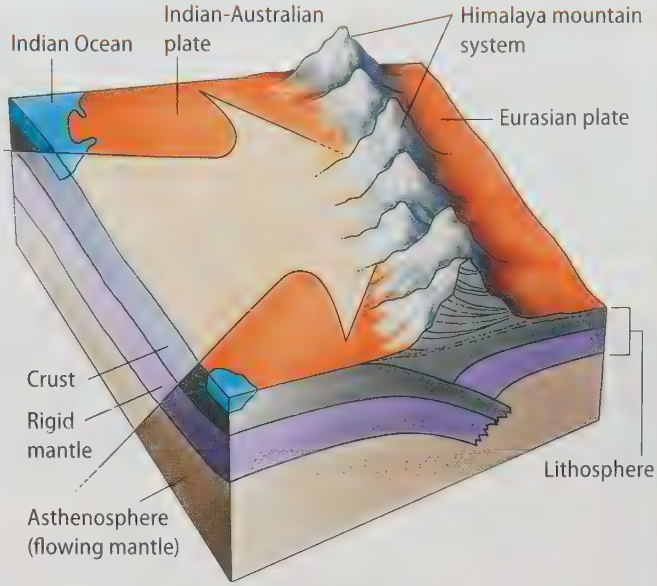
- one forced beneath the other
- forms an ocean trench
- undersea volcanic activity and earthquakes

Two Continental Plates Collide

- neither plate subducts
- layers of rock in the overriding plate crumple and fold
- creates a mountain range

The Aleutian Islands —a volcanic island chain off the tip of Alaska—formed when an oceanic plate collided with a continental plate. ▼





▲ **The Himalayas,** the world's highest mountain system, formed when the Indian-Australian plate collided with the Eurasian plate about 50 million years ago.

HOW DOES THAT WORK ?

SEAFLOOR SPREADING

During World War II, a geologist named Harry Hess commanded a submarine for the Navy. He used instruments on the sub to map the ocean floor. Hess's findings led him to develop the hypothesis of seafloor spreading.

In 1968, a research vessel gathered samples of oceanic crust from the mid-Atlantic ridge. They found that the rocks nearest the ridge were the youngest, indicating that they had been pushed up through the ridge.

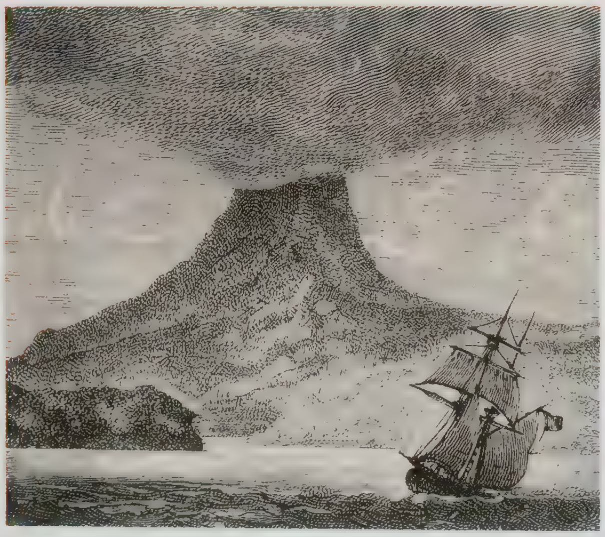
Photographs taken from a deep-sea diving vessel in 1974 showed newly erupted lava flows from the mid-ocean ridge, providing the first visual evidence of seafloor spreading.

Harry Hess

Volcanoes

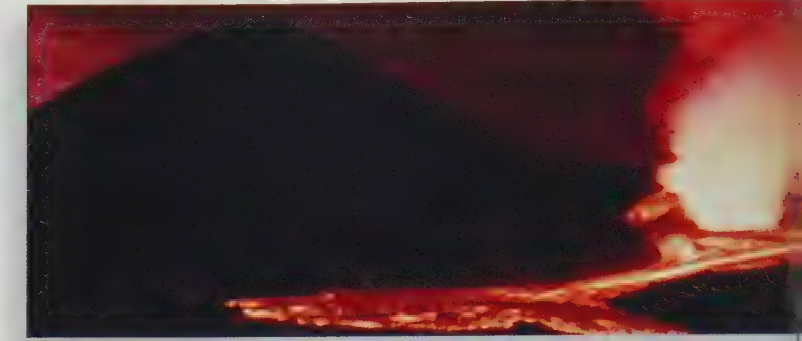
A volcano is a fissure, or vent, in Earth's crust where ash, gases, and molten rock from deep underground erupt to the surface. The word volcano also refers to the type of mountain that forms as the erupted rock and ash build up around the fissure. As molten rock accumulates and hardens around a fissure, a variety of volcanic cones are produced. (See also pages 526–527.)

Where Volcanoes Form. There are about 500 active volcanoes on Earth. Volcanoes form above regions where *magma*, molten rock beneath Earth's surface, is produced in the mantle, the rocky layer beneath Earth's crust. Nearly all volcanoes form along the tectonic plate boundaries. Some appear above locations called hot spots that can be far from plate boundaries.



KRAKATOA ERUPTS!

In 1883, Mount Krakatoa in Indonesia erupted with tremendous force, killing 36,000 people. The eruption caused abnormal sea waves, weather phenomena, and extraordinary sunsets throughout the world. It was also notable as the first large volcanic eruption that occurred after the invention of the telegraph. Consequently, the news of Krakatoa's eruption was sent around the world within hours, enabling its worldwide effects to be related to a single cause.



▲ **Surtsey,** an island off the coast of Iceland, appeared in 1963 when a volcanic cone built up over the mid-ocean ridge and broke the water's surface.

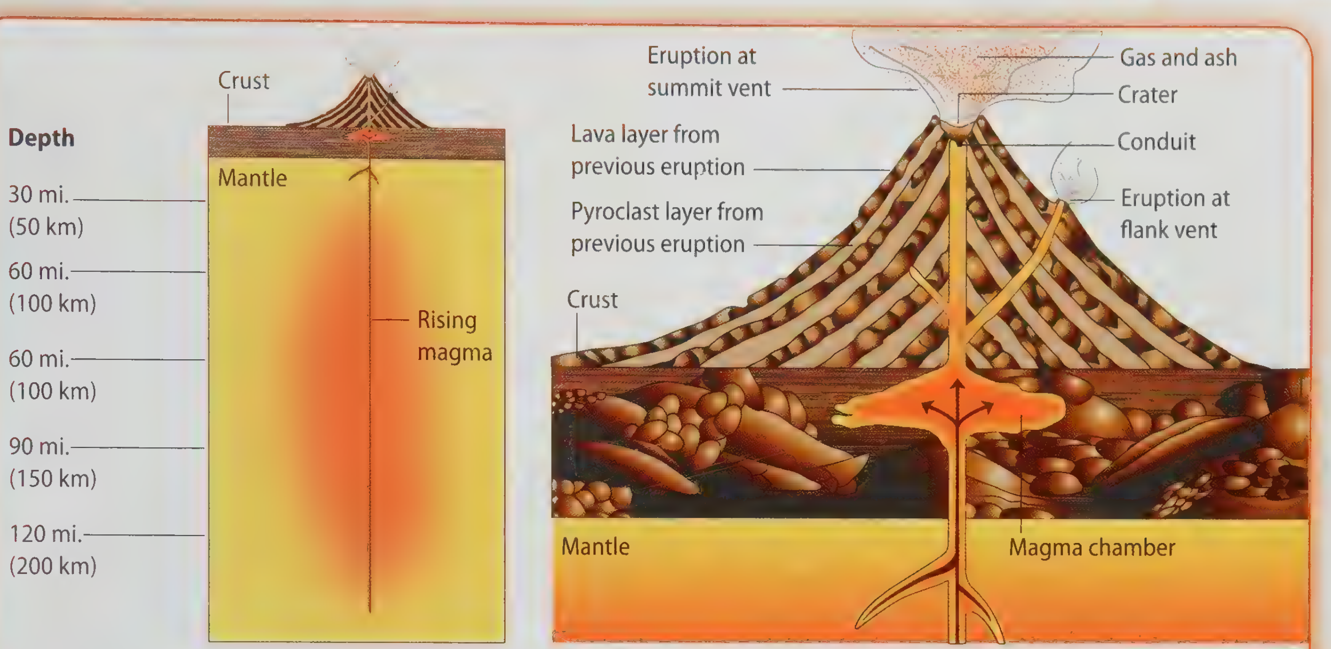
Eruptions. A volcanic eruption takes place when magma rises through a vent leading from a magma chamber deep within Earth's crust. The main products of an erupting volcano are lava (molten rock), gases (steam, carbon dioxide, nitrogen, sulfur dioxide, and other gases), and pyroclastic material. Pyroclastic fragments include pumice, rock, cinders, and ash propelled into the air by superheated volcanic gases and ejected by the force of the eruption.

The violence of an eruption is determined by the magma's chemical composition and the quantity and content of the gases.

Magma Composition

	Silica	Gases and Fluid	Eruption
Molten rhyolite	High percentage of silica results in high viscosity (thickness)	Large quantity of dissolved gases and fluids	Violent
Molten basalt	Lower percentage of silica; magma is thinner	Lower quantity of dissolved gases	Gentler

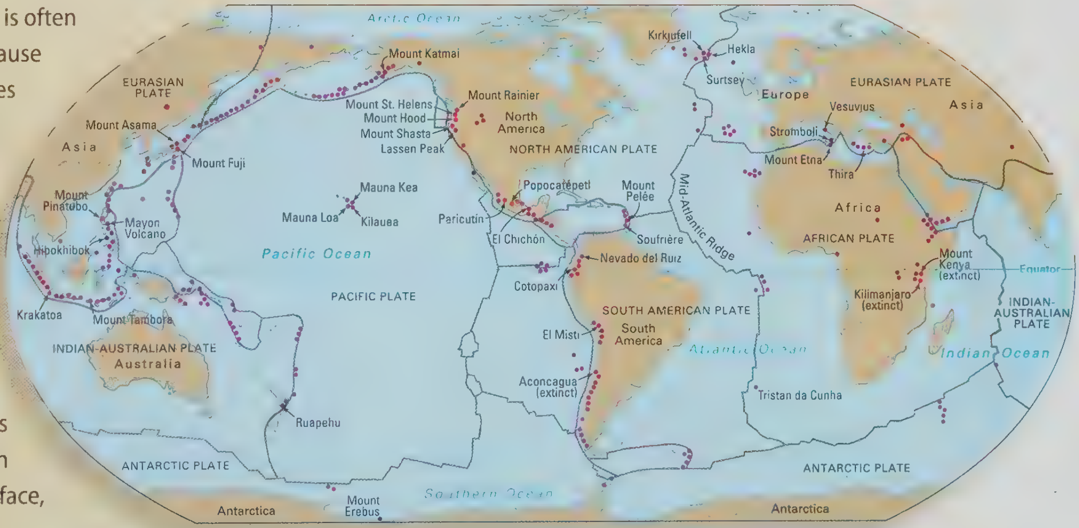
Violent Eruptions. The most violent eruptions occur at convergent plate boundaries. When two plates collide, subduction occurs (one plate slides beneath the other). The melting of one of the plates often results in molten rock being forced violently upward through cracks in the ocean floor or on land.


Depth
30 mi. (50 km)
60 mi. (100 km)
60 mi. (100 km)
90 mi. (150 km)
120 mi. (200 km)
Crust
Mantle
Rising magma
Eruption at summit vent
Lava layer from previous eruption
Pyroclast layer from previous eruption
Crust
Mantle
Gas and ash
Crater
Conduit
Eruption at flank vent
Magma chamber

How a Volcano Erupts. Magma forms deep underground and rises toward the surface, *left*, collecting in a magma chamber. As pressure builds, the chamber breaks open and magma rises through a conduit, *right*. At openings called vents, the magma erupts as gas, lava, and pyroclasts (rock and ash).

VOLCANOES OCCUR AT ...

- **convergent boundaries,** where tectonic plates collide. The most powerful eruptions occur at these boundaries. The Pacific Rim is often called the "Ring of Fire" because of the many active volcanoes around the Pacific plate.
- **divergent boundaries,** where tectonic plates separate, such as at the mid-ocean ridge. As the lava pours out through openings, it forms new seafloor and may build up into huge cones. Sometimes a cone above the mid-ocean ridge breaks the water's surface, thereby forming an island.
- **hot spots,** which are located in the middle of a tectonic plate. Perhaps the most famous examples are the volcanoes in the Pacific plate that form the Hawaiian Islands.


Arctic Ocean
EURASIAN PLATE
Mount Asama
Asia
Mount Fuji
Mount Pinatubo
Mayon Volcano
Hibokhibok
Krakatoa
Mount Tambora
INDIAN-AUSTRALIAN PLATE
Australia
ANTARCTIC PLATE
Antarctica
Mount Erebus
Ruapehu
Mount Katmai
Mount St. Helens
Mount Hood
Mount Shasta
Lassen Peak
Mauna Kea
Mauna Loa
Kilauea
Pacific Ocean
PACIFIC PLATE
Southern Ocean
Mount Rainier
North America
NORTH AMERICAN PLATE
Popocatépetl
Paricutín
El Chichón
Mount Pelée
Soufrière
Nevado del Ruiz
Cotopaxi
SOUTH AMERICAN PLATE
South America
El Misti
Aconcagua (extinct)
Mid-Atlantic Ridge
Atlantic Ocean
Tristan da Cunha
Kirkjufell
Hekla
Surtsey
Europe
EURASIAN PLATE
Asia
Vesuvius
Stromboli
Mount Etna
Thira
Africa
AFRICAN PLATE
Mount Kenya (extinct)
Equator
Kilimanjaro (extinct)
INDIAN-AUSTRALIAN PLATE
Indian Ocean
ANTARCTIC PLATE
Antarctica

▲ **Volcanoes** occur where plates collide, where plates separate, and at hot spots in the middle of a plate.

Earthquakes

Earthquake Zones. Earthquakes usually occur along faults, at the boundary between two tectonic plates. More than 80 percent of the world's earthquakes occur in a narrow zone, or belt, which encircles the rim of the Pacific Ocean. This belt is sometimes called the Ring of Fire because it has many volcanoes, earthquakes, and other geologic activity.

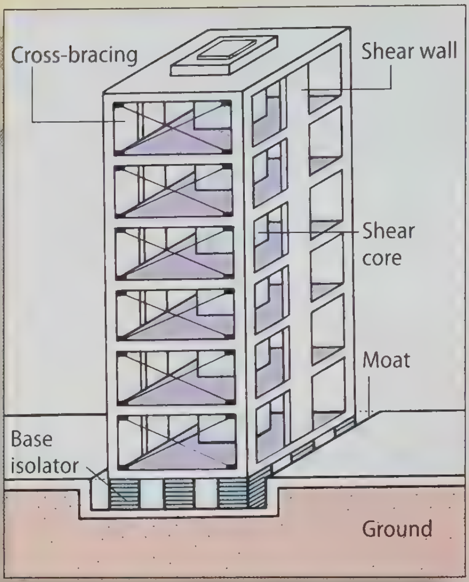
Another major earthquake zone, the Mediterranean-Himalayan zone, cuts across the Mediterranean Sea, crosses the Mideast and the Himalayas, and passes through the East Indies before meeting the Ring of Fire north of Australia.

Japan experiences frequent earthquakes because of its location near two colliding plates. In fact, Tokyo has experienced at least one earthquake each century for the past 2,000 years. In China, severe earthquakes occur as the Asian plate is pushed upward by the descending Indian plate. In the United States, earthquakes occur in California, along the San Andreas Fault. However, earthquakes also occur along the Eastern seaboard and in the Midwest.

HOW TO RESIST AN EARTHQUAKE

Quakeproof Buildings

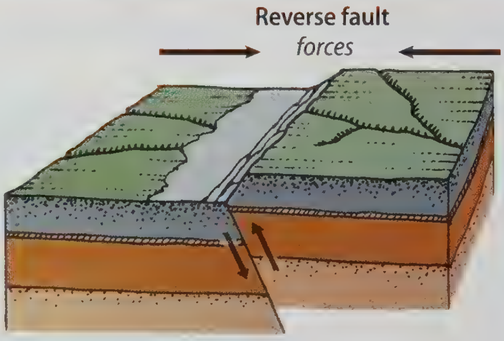
An earthquake-resistant building includes such structures as *shear walls,* a *shear core,* and *cross-bracing. Base isolators* act as shock absorbers. A *moat* allows the building to sway.

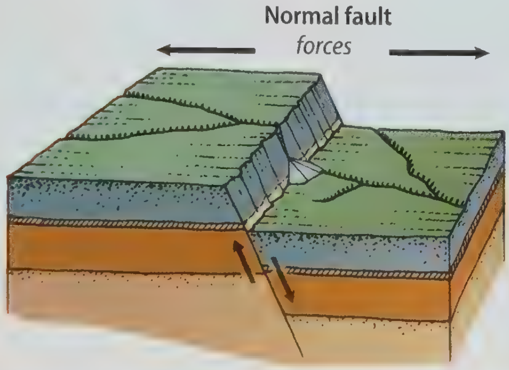


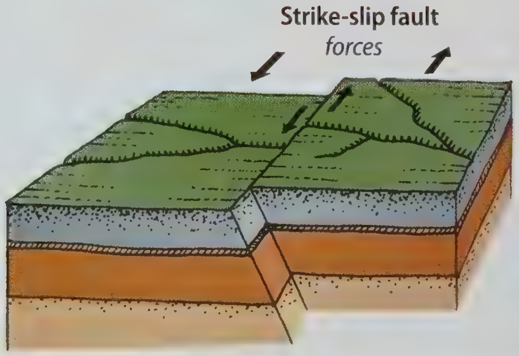
TYPES OF FAULTS

There are three main types of faults.

- **normal fault:** the upper block of crust is displaced downward along the dip of the fault plane. Movement along a normal fault broadens the area of crust.
- **reverse fault:** the block of crust above the fault is displaced upward. Movement along a reverse fault shortens the crustal area.
- **strike-slip fault:** also called a transform fault, movement along a strike-slip fault is lateral rather than vertical.

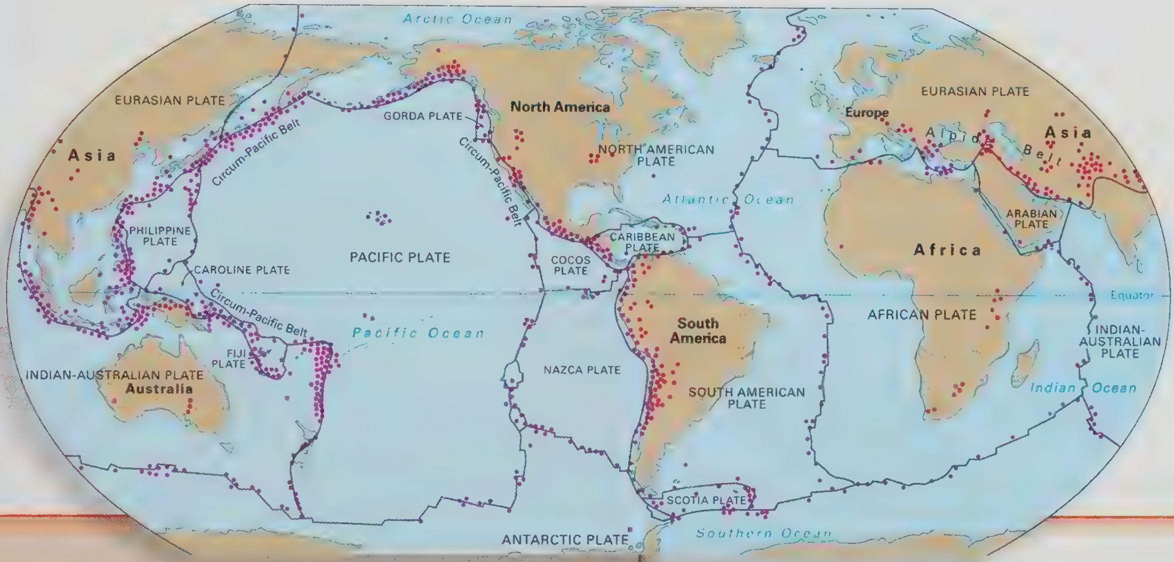


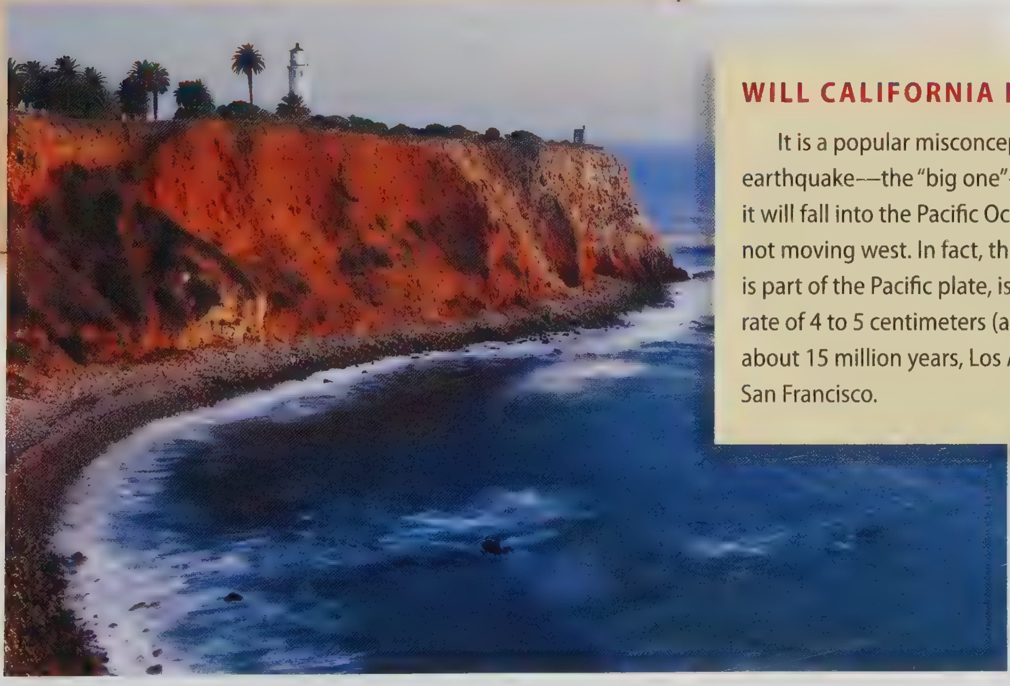




Predicting Earthquakes. Seismologists still cannot predict precisely when an earthquake will occur. By studying the incidence of earthquakes in a given area over thousands of years (paleoseismology), however, predictions can be made regarding the general area where an earthquake is most likely to occur. Seismologists also can predict where the chances are greatest for a large earthquake to occur soon, but those predictions cannot be accurately pinpointed.

Where Earthquakes Occur. This map shows where many major earthquakes have occurred. Each dot on the map represents a major earthquake. Most earthquakes occur near and along the boundaries of the rocky plates that cover Earth's surface. ▼


Arctic Ocean
EURASIAN PLATE
Asia
Circum-Pacific Belt
GORDA PLATE
North America
Circum-Pacific Belt
NORTH AMERICAN PLATE
Europe
EURASIAN PLATE
Alpide Belt
Asia
PHILIPPINE PLATE
CAROLINE PLATE
PACIFIC PLATE
Atlantic Ocean
CARIBBEAN PLATE
COCOS PLATE
ARABIAN PLATE
Africa
Equator
Circum-Pacific Belt
Pacific Ocean
South America
AFRICAN PLATE
INDIAN-AUSTRALIAN PLATE
FIJI PLATE
INDIAN-AUSTRALIAN PLATE
Australia
NAZCA PLATE
SOUTH AMERICAN PLATE
Indian Ocean
SCOTIA PLATE
ANTARCTIC PLATE
Southern Ocean



WILL CALIFORNIA FALL INTO THE OCEAN?

It is a popular misconception that someday a powerful earthquake—the "big one"—will strike southern California and it will fall into the Pacific Ocean. In reality, southern California is not moving west. In fact, the southern half of California, which is part of the Pacific plate, is actually moving northward at a rate of 4 to 5 centimeters (about 2 inches) a year. At this rate, in about 15 million years, Los Angeles could become a suburb of San Francisco.

◀ **The Southern California coast** is moving slowly northward, but it will probably not fall into the ocean from an earthquake.

MINERALS are solid, inorganic elements and compounds that occur naturally in rocks. They have a distinctive crystal form and usually have a definite chemical composition. Mineralogists, geologists who study minerals, have described and named about 3,000 different minerals.

Mineral Facets

Without exception, every mineral has a chemical composition and an arrangement of atoms that give it unique properties. These properties can be useful in identifying a mineral.

Mineral Properties	
Color	• some minerals can be reliably identified by their color • color alone may not be a dependable identifier, as the same mineral may have different colors
Streak	• determined by grinding a mineral into a fine powder • reveals a mineral's true color
Luster	• the quality and intensity of light reflected from the surface of a mineral • two main kinds of luster: metallic and nonmetallic
Cleavage and Fracture	• cleavage: tendency of some minerals to break along smooth, flat planes • fracture: tendency of most minerals to break along widely spaced irregular, smooth, or curved surfaces
Hardness	• the ability of a mineral to withstand scratching • determined mostly by the atomic structure of the mineral
Specific Gravity	• describes the density of a mineral compared to the density of water

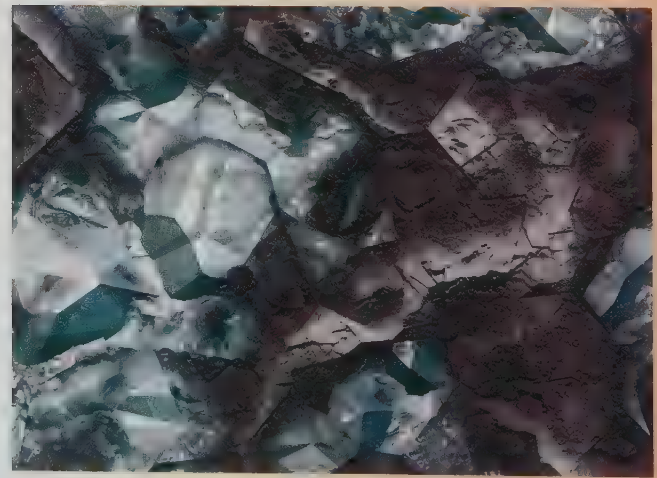
ONE ELEMENT, TWO MINERALS

The element carbon can exist as graphite or diamond. Both are minerals that are composed entirely of carbon atoms. The difference is in their crystal form. (See Carbon, page 329.)

- **graphite:** carbon atoms occur in regularly spaced, thin parallel sheets
- **diamond:** each carbon atom is in direct contact and bound tightly with four other carbon atoms

The result? Graphite is a very soft, slippery mineral that splits easily, while diamonds are the hardest naturally occurring substance.

Graphite

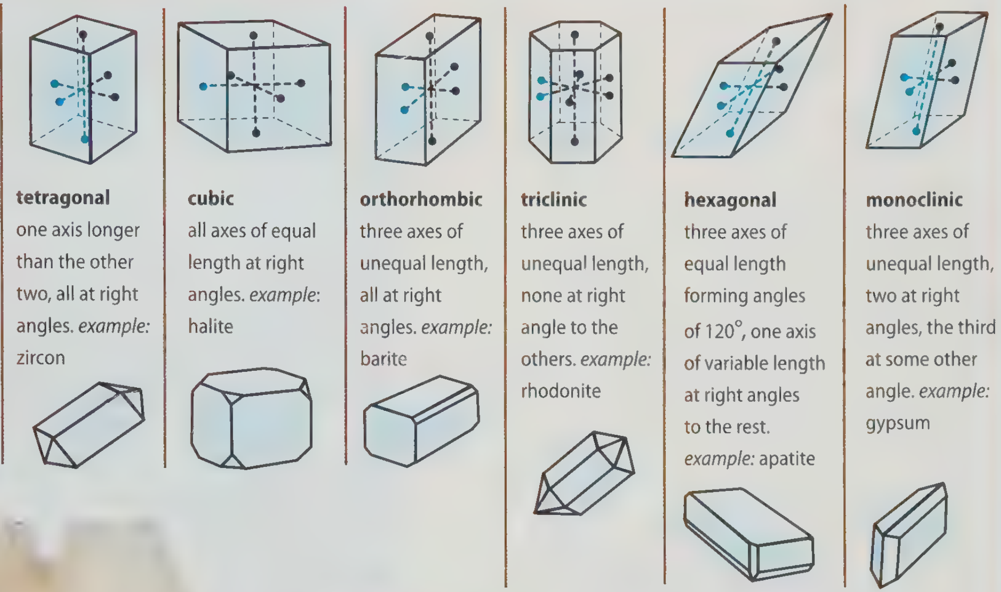


Diamond

Crystal Form

The distinctive geometric form of a mineral is known as its *crystal form*. The crystal form of a mineral is caused by the atomic structure, or the way that the atoms of its elements are bound together. All minerals are classified into six crystal systems. ▶

CRYSTAL STRUCTURES



tetragonal one axis longer than the other two, all at right angles. *example:* zircon

cubic all axes of equal length at right angles. *example:* halite

orthorhombic three axes of unequal length, all at right angles. *example:* barite

triclinic three axes of unequal length, none at right angle to the others. *example:* rhodonite

hexagonal three axes of equal length forming angles of 120°, one axis of variable length at right angles to the rest. *example:* apatite

monoclinic three axes of unequal length, two at right angles, the third at some other angle. *example:* gypsum

◀ **Crystals** grow over a period of time as atoms and molecules form complex three-dimensional shapes in an orderly manner.

Color

Some minerals can be reliably identified by their color. For example, *native metals*, metallic minerals of only one element, such as gold and copper, can be identified by color.

In most cases, though, color alone is not a dependable way to identify minerals. This is because the same mineral may have different colors. For example, pure quartz is a glasslike, colorless mineral, but there are some varieties of quartz that are dark gray, milky white, or pink owing to the impurities of other minerals.

Streak

A more accurate way to identify minerals by color is to grind the minerals into fine powder. When a mineral has been powdered, the true color, or *streak*, of the mineral shows. The colors of the impurities do not appear. The streak of a mineral usually is more reliable than its external color. For example, the external color of pyrite (fool's gold) is brassy yellow, but its streak is black. Real gold, on the other hand, does leave a golden streak.

The Rock Cycle

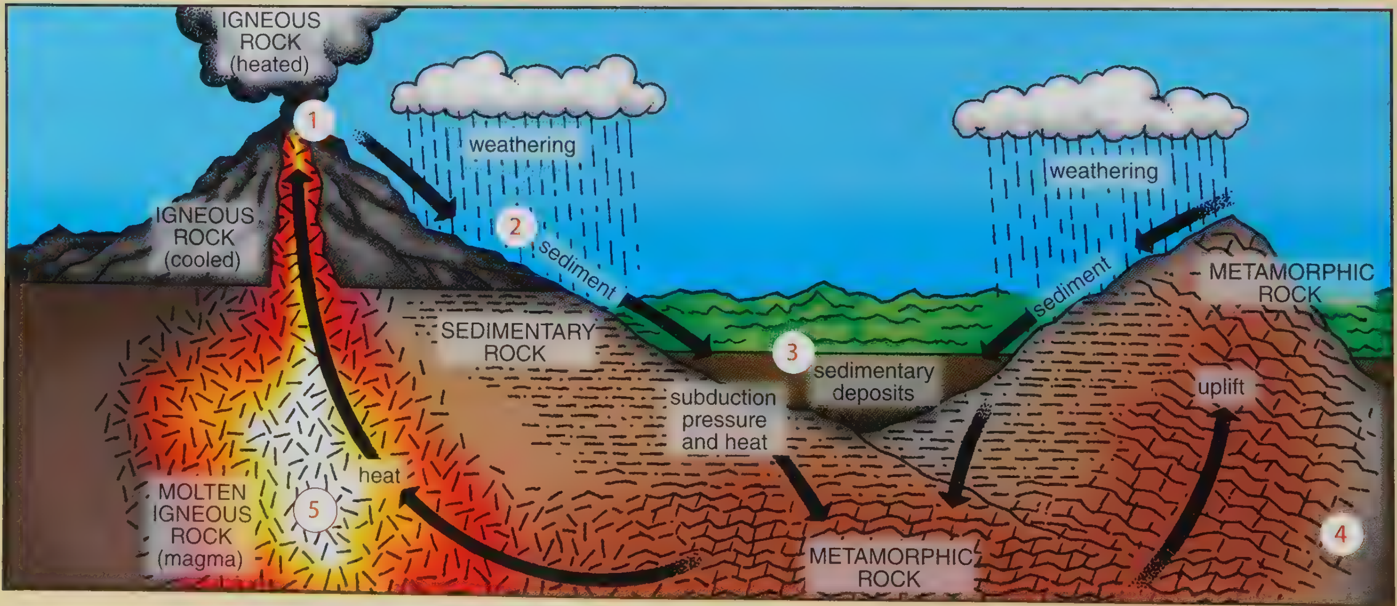
HOW ROCKS FORM

Each type of rock develops from other types of rock. Geologists use the rock cycle to explain how rocks form.

1. Igneous rock created by magma flows to Earth's surface.
2. Weathering alters the rock chemically and breaks it into particles.
3. The particles accumulate and harden to form sedimentary rock.
4. As the sedimentary rock becomes buried, heat and pressure transform it into metamorphic rock.
5. With enough heating, metamorphic rocks melt into magma that can form igneous rock, completing the cycle.

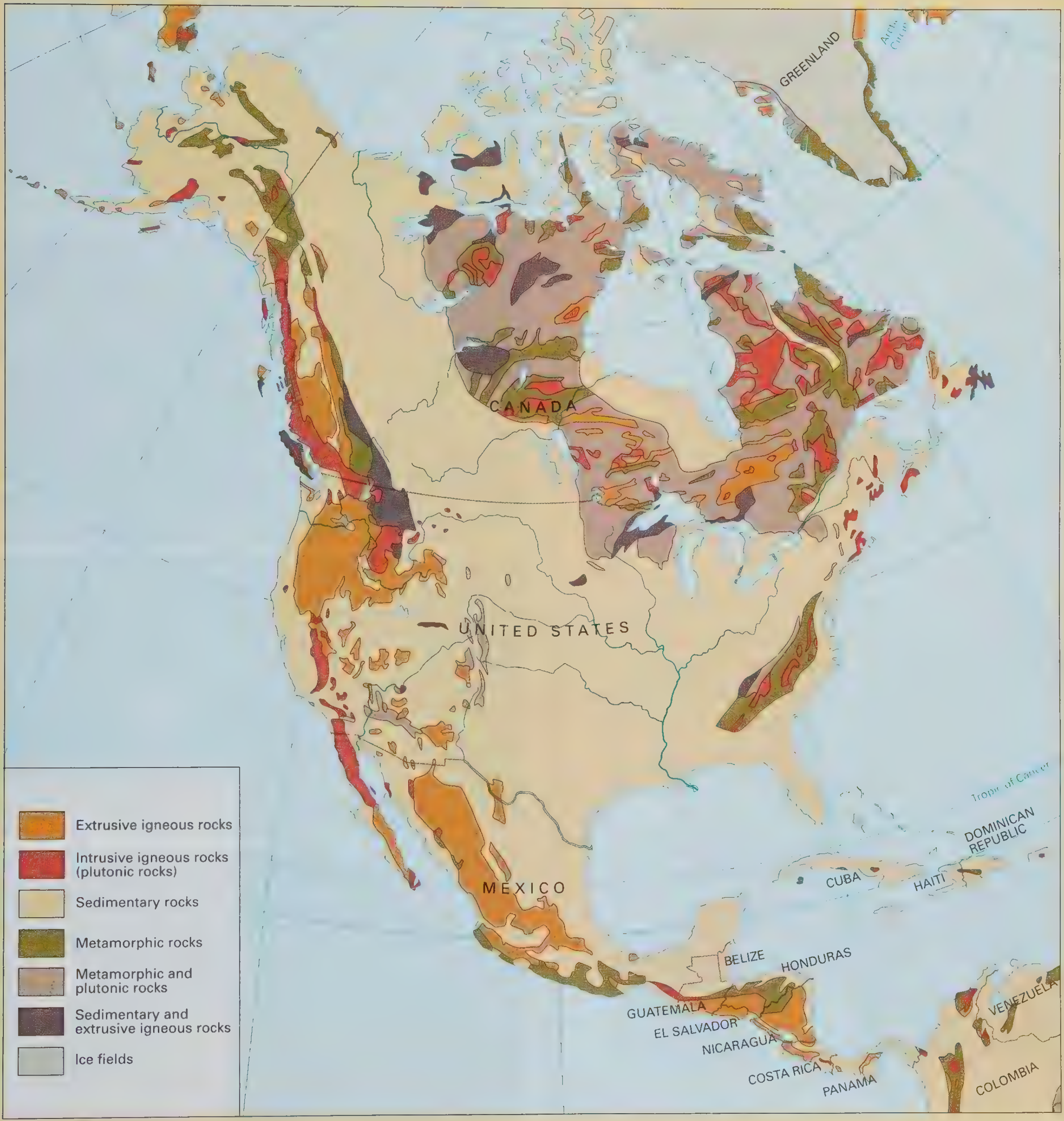
Rock rarely proceeds through the entire rock cycle. The cycle may be halted, or steps can be skipped, repeated, or reversed. For example, some igneous rocks transform directly into metamorphic rock, and sedimentary rock can be broken down by weathering and then form other kinds of sedimentary rock.

▲ **Balanced Rock,** in the Garden of the Gods near Colorado Springs, Colorado, is an enormous block of sandstone delicately balanced on a small base.


IGNEOUS ROCK (heated)
1
weathering
2
sediment
IGNEOUS ROCK (cooled)
SEDIMENTARY ROCK
3
sedimentary deposits
subduction pressure and heat
weathering
sediment
METAMORPHIC ROCK
uplift
heat
5
MOLTEN IGNEOUS ROCK (magma)
METAMORPHIC ROCK
4

ROCKS OF NORTH AMERICA

This map shows the types of rock found at or near the surface in different areas of North America. Sedimentary rock underlies most of North America. Pockets of igneous and metamorphic rock appear in the Allegheny Mountains in the east and in the Rocky Mountains and other regions along the continent's west coast. A vast region of igneous and metamorphic rock called the Canadian Shield covers the eastern half of Canada.


GREENLAND
Arctic Circle
CANADA
UNITED STATES
MEXICO
Tropic of Cancer
CUBA
DOMINICAN REPUBLIC
HAITI
BELIZE
HONDURAS
GUATEMALA
EL SALVADOR
NICARAGUA
COSTA RICA
PANAMA
VENEZUELA
COLOMBIA
Extrusive igneous rocks
Intrusive igneous rocks (plutonic rocks)
Sedimentary rocks
Metamorphic rocks
Metamorphic and plutonic rocks
Sedimentary and extrusive igneous rocks
Ice fields

ATMOSPHERE

The *atmosphere* is an envelope of gases that surrounds Earth. Three of the gases—oxygen, nitrogen, and carbon dioxide—are essential for the survival of all life forms on our planet.

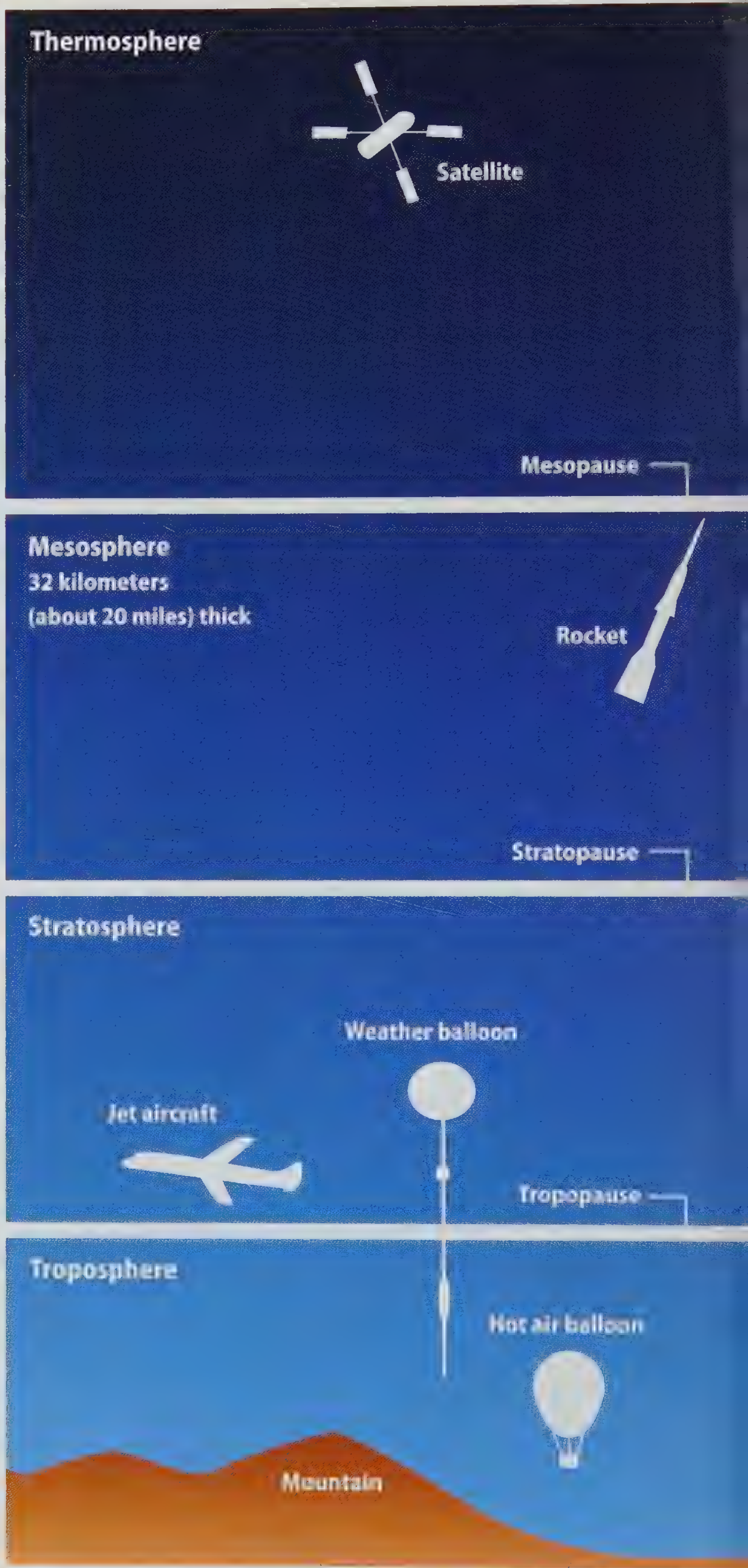
The atmosphere contains the oxygen required in respiration by animals and humans as well as the nitrogen and carbon dioxide necessary in plant growth. These three gases make up about 99 percent of the atmosphere.

With increasing altitude, the atmosphere changes in temperature, pressure, and composition. Research balloons and satellites have shown that the atmosphere consists of a series of distinct layers.

WHY IS THE SKY BLUE?

The colors of the sky result from the scattering of sunlight by the gas molecules and dust particles in the atmosphere. Sunlight consists of light waves of varying wavelengths, each of which is seen as a different color. When the sky is clear, the waves of blue light are scattered much more than those of any other color. As a result, the sky appears blue. When the sky is full of dense clouds or smoke, the light waves of all colors are scattered, causing the sky to turn gray. At sunrise or sunset, sunlight must travel farther through the atmosphere than when the sun is overhead. Light waves of most colors are scattered. However, red light waves are undisturbed and give the sun and sky near the horizon a red or orange appearance.

Layers of Air


Thermosphere
Satellite
Mesopause
Mesosphere
32 kilometers
(about 20 miles) thick
Rocket
Stratopause
Stratosphere
Weather balloon
Jet aircraft
Tropopause
Troposphere
Hot air balloon
Mountain

LAYERS OF THE ATMOSPHERE

Thermosphere

- uppermost layer of Earth's atmosphere; begins at an average altitude of about 80 kilometers (50 miles) and extends into space.
- air is extremely thin; nitrogen and oxygen make up most of the lower parts
- above about 1,000 kilometers (620 miles), contains mainly hydrogen and helium
- temperature rises from about –90°C (–130°F) at an altitude of 80 kilometers (50 miles) to more than 1,200°C (2,200°F) at about 350 kilometers (220 miles)
- solar radiation and cosmic rays ionize (electrically charge) atoms in the upper atmosphere
- region of ionized particles, known as the ionosphere, extends through parts of the upper mesosphere and lower thermosphere
- atoms in the outermost region of the thermosphere, called the exosphere, can gain enough speed to escape the atmosphere

Mesosphere

- from an altitude of about 48 kilometers (30 miles) to about 80 to 100 kilometers (50 to 60 miles)
- temperature drops with rising altitude due to a falling concentration of ozone
- averages 0°C (32°F) at the base of the mesosphere and –85°C (–120°F) at the mesopause, the upper limit of the mesosphere
- lowest temperatures in Earth's atmosphere occur at the mesopause; can drop below –130°C (–200°F)
- too high for weather balloons and airplanes; scientists can only directly measure the layer by rocket

Stratosphere

- from an altitude of about 10 kilometers (6 miles) to about 48 kilometers (30 miles)
- air temperature fairly uniform and very cold; average temperature of about –55°C (–67°F)
- in upper reaches, ultraviolet rays from the sun split molecules of oxygen
- oxygen molecules then recombine to form ozone
- ozone makes life possible on Earth by absorbing much of the ultraviolet radiation that strikes the upper atmosphere

Troposphere

- contains water vapor, dust particles from wind erosion, and smoke
- altitude varies, averages about 19 kilometers (12 miles) above the equator and 10 kilometers (6 miles) above the poles
- contains most of the atmosphere's mass
- all weather phenomena occur in the troposphere
- air temperature decreases with increasing altitude; between sea level and top of troposphere, drops a total of 28° to 56°C (98° to 196°F)

EARTH'S GREENHOUSE

The atmosphere functions like a giant greenhouse. (See pages 670–671.) By trapping the heat from solar energy in its lowest layer, the atmosphere keeps the temperature in most places on Earth within the narrow range necessary for life. The lower part of Earth's atmosphere contains sufficient oxygen, water vapor, and carbon dioxide to support a wide variety of plants and animals. Most living things inhabit the relatively narrow region where the atmosphere blends with the lithosphere and the hydrosphere.

EARTH'S SHIELD

Earth's atmosphere also acts as a shield. It protects the planet from certain forms of solar radiation and from impact by meteorites. For example, the atmosphere blocks most of the harmful X-rays and ultraviolet radiation from the sun.

Small rocky meteors continually plummet toward Earth. Most of these meteors disintegrate before reaching the ground because of the heat generated by friction as they pass through the atmosphere. When a meteor survives passage through the atmosphere and actually strikes Earth, it is called a meteorite.

Water Cycle

Water on Earth moves constantly, from the oceans to the air, from the air to the land, and then back to the seas again, changing from liquid, to gas, to solid, and back again. This continuous cycle is called the *hydrologic cycle,* or the water cycle.

Evaporation and Transpiration

Evaporation is the changing of water from a liquid state to a gaseous state. The sun's heat evaporates water from land, lakes, rivers, and oceans. About 85 percent of the vapor in the air comes from the oceans.

Plants also add moisture. After plants have drawn water from the ground through their roots, they pass it out through their leaves as vapor in a process called *transpiration.* (See page 133.)

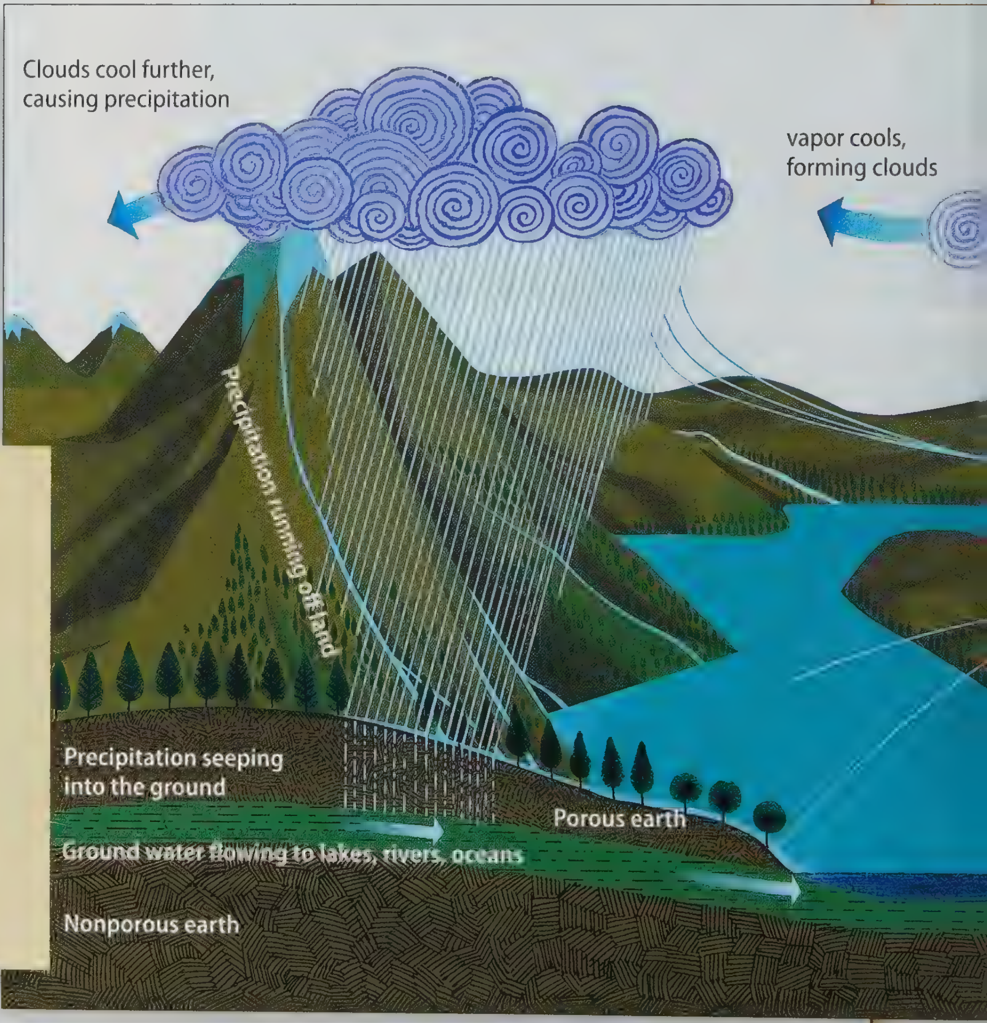
Condensation

Condensation is the process in which water changes from a vapor to a liquid or a solid. Water condenses to form clouds, dew, fog, rain, and snow. When air cools to its saturation or dew point (the lowest temperature at which it can hold water vapor), it condenses, or contracts, into visible clouds or water droplets. When air temperatures are below freezing, snow forms.

Humidity

Humidity is the amount of water vapor, or moisture, that is present in the air. Because the molecules of gases in warm air are spread farther apart than the molecules in cool air, warm air can hold much more moisture than cool air.

Measurements of humidity are normally stated in terms of relative humidity. Relative humidity is a ratio of the amount of water vapor in the air to the greatest amount of water vapor the air can hold at a given air temperature. Relative humidity is usually converted into a percent; it is 100 percent when the air is saturated with water vapor.


Clouds cool further, causing precipitation
vapor cools, forming clouds
Precipitation running off land
Precipitation seeping into the ground
Porous earth
Ground water flowing to lakes, rivers, oceans
Nonporous earth

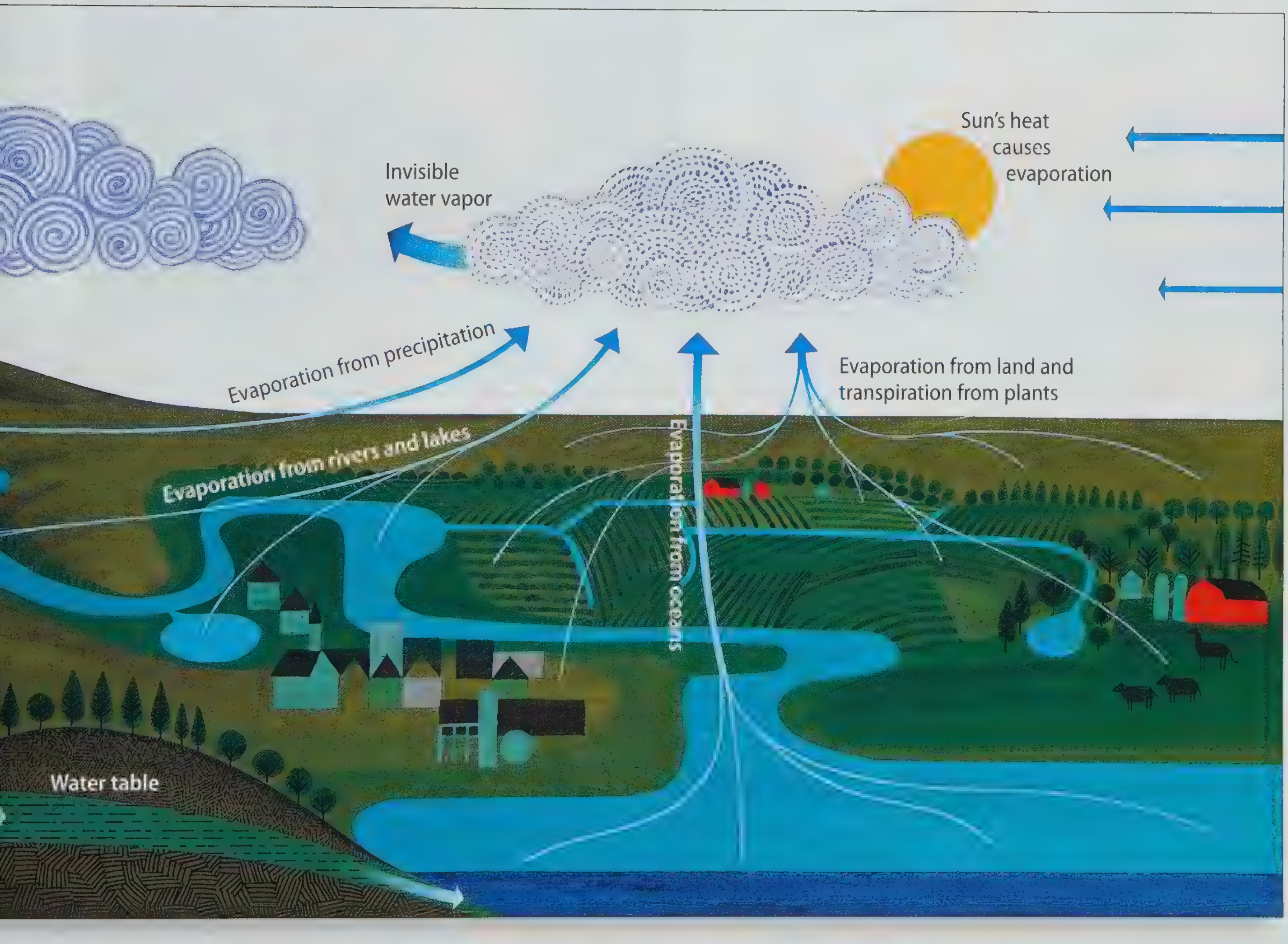
THE HYDROLOGIC CYCLE

- The sun's heat evaporates water from the oceans.
- The water rises as invisible vapor and falls back to Earth as rain, snow, or some other form of moisture.
- This moisture is called precipitation. Most precipitation drops back directly into the oceans.
- The remainder falls on the rest of Earth. In time, this water also returns to the sea, and the cycle starts again.

ANCIENT WATERS

Because of nature's water cycle, there is as much water on Earth today as there ever was, or ever will be—1.4 billion cubic kilometers (326 million cubic miles) to be exact. Water changes only from one form to another, and moves from one place to another. The water you drank last night might have flowed in Russia's Volga River last year. Or perhaps Alexander the Great bathed in it more than 2,000 years ago.

The same water that flowed through this ancient Roman aqueduct still exists somewhere on Earth today.


Invisible water vapor
Sun's heat causes evaporation
Evaporation from precipitation
Evaporation from rivers and lakes
Evaporation from oceans
Evaporation from land and transpiration from plants
Water table

Water in Motion

The water of the ocean is in constant motion. Vast currents move across the oceans of the world in complex parts. Some of the factors that cause ocean currents are

- wind.
- Earth's rotation.
- seawater density and temperature.

Wind. Currents close to the ocean's surface are caused almost entirely by the wind. The wind pushes water into currents that flow parallel to Earth's surface.

Earth's Rotation. The spinning of the planet causes the path of a moving object to veer from a straight line. (See page 547.) Ocean currents shift to the right in the Northern Hemisphere and to the left in the Southern Hemisphere.

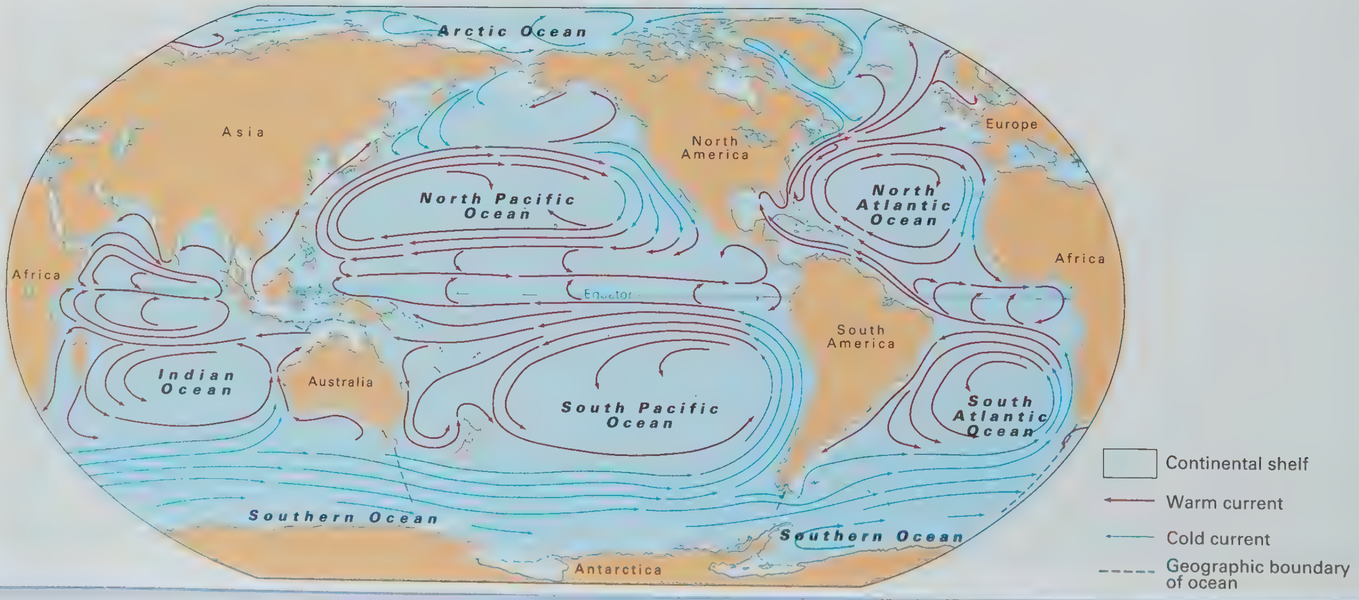
TRAVELING TURTLES

The Gulf Stream is a major ocean current that flows from the Caribbean Sea, through the Gulf of Mexico, then northeast along the North American coast. Loggerhead sea turtles, like the one at right, use the Gulf Stream to guide them on a migration that eventually takes them across the Atlantic Ocean.

Density and Temperature. Heating of the oceans in the warmer climates near the equator, combined with cooling near the poles, also produces ocean currents. Since water becomes slightly heavier when cooled, the cold water from the polar regions tends to sink.

Conversely, the warmer water near the equator tends to remain near the surface of the sea, so there is a slow but steady movement of very cold water along the seafloor toward the equator. Water near the seafloor is always near freezing, even in tropical climates.

Gyres flow clockwise in the subtropics of the Northern Hemisphere and counterclockwise in the subtropics of the Southern Hemisphere. ▼


Arctic Ocean
Asia
North America
Europe
North Pacific Ocean
North Atlantic Ocean
Africa
Africa
Equator
South America
Indian Ocean
Australia
South Pacific Ocean
South Atlantic Ocean
Southern Ocean
Southern Ocean
Antarctica
Continental shelf
Warm current
Cold current
Geographic boundary of ocean

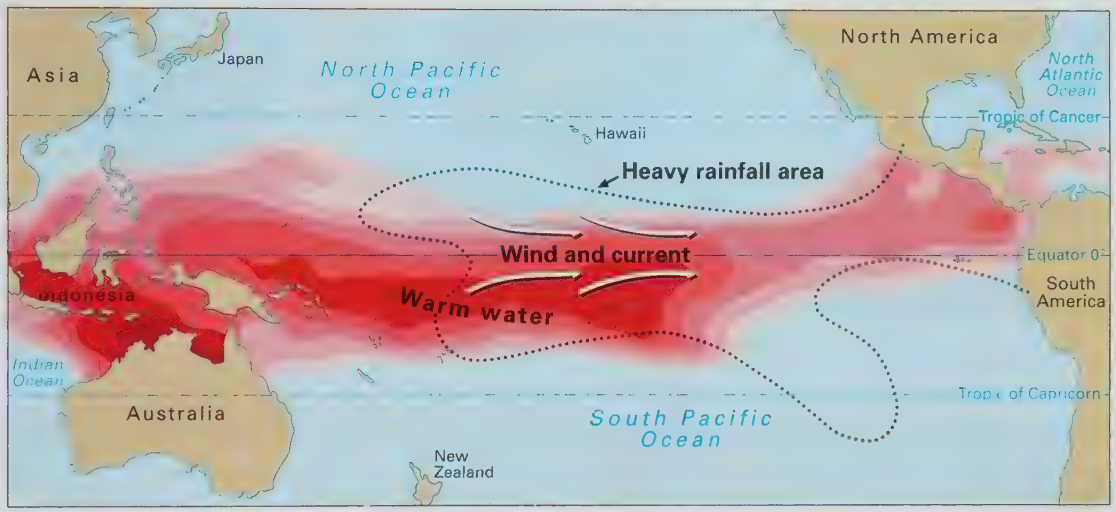
Surface Currents

Earth's wind systems produce the surface currents of the ocean. The force of the wind mainly affects the water within the upper 300 meters (1,000 feet) of the ocean.

Gyres. Wind-driven currents move in enormous circular patterns called *gyres*. Several conditions influence the direction of wind-driven currents and make them form gyres. Earth's wind systems drive the currents in an easterly or westerly direction. Continents direct the flow toward the north or south.

Upwelling. In some areas, *upwelling* occurs when winds cause surface waters near the coast to move offshore. Colder, deeper waters, which are rich in nutrients, then rise to the surface near the coast. The upwelling of deeper waters provides nutrients for the growth of phytoplankton, which fish and other sea animals eat.

Upwelling areas have great numbers of fish and, in fact, yield half of the world's fish catch. Important upwelling regions include the coasts of Peru and northwestern Africa, as well as along the equator and around Antarctica.


Asia
Japan
North Pacific Ocean
North America
North Atlantic Ocean
Tropic of Cancer
Hawaii
Heavy rainfall area
Wind and current
Equator 0°
South America
Indonesia
Warm water
Indian Ocean
Australia
Tropic of Capricorn
South Pacific Ocean
New Zealand

El Niño. The east-to-west winds and currents over the tropical Pacific weaken or even reverse when an El Niño is present, and the pattern of heavy rainfall shifts eastward.

EL NIÑO

One of the best examples of how an ocean current can affect the global climate and the environment can be seen in *El Niño*, a phenomenon caused by a warm ocean current flowing off the coast of western South America.

During an El Niño, winds cause a current of warm water to flow south from Ecuador down the coastlines of Peru and Chile. An El Niño occurs about every 2 to 7 years and lasts about 18 months.

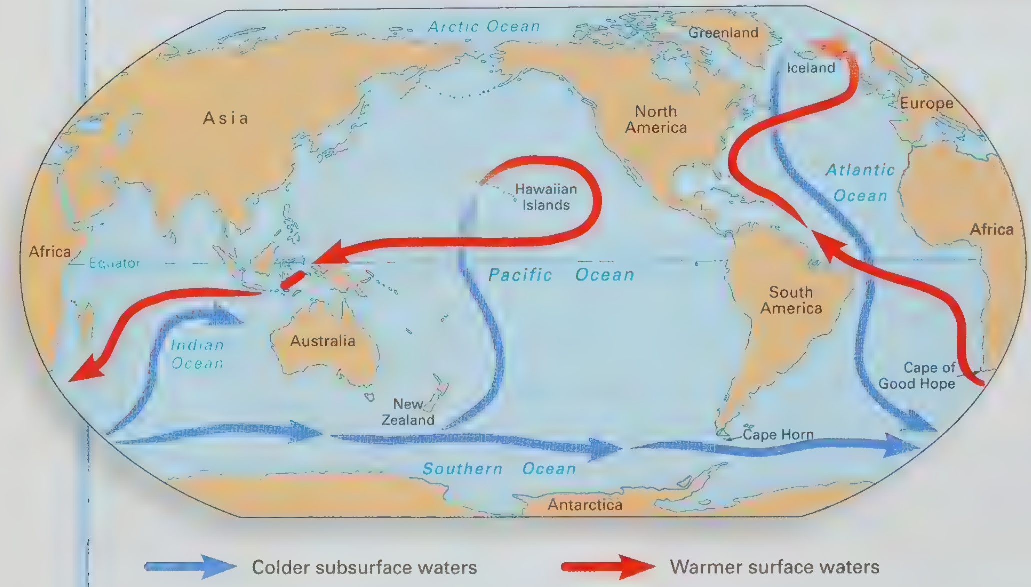
The warm waters result in a dramatic decline of the amount of plankton—small floating plants and animals—as well as of the fish that eat plankton, and, in turn, of the sea birds that eat the fish.

An El Niño is caused by a buildup of westward trade winds. The buildup results in an increase in the amount of warm water carried westward. When the winds weaken, the warm water sloshes back eastward toward the western coast of South America, producing an El Niño.

Currents, Tides, and Waves

In addition to the wind-driven surface currents, strong currents also flow deep beneath the ocean surface. The most powerful deep currents, known as *density currents*, are caused by the sinking of water as it becomes denser from cooling or from an increase in salt content.

fyi! *The world ocean would rise about 60 meters (200 feet) if the Greenland and Antarctic ice sheets should suddenly melt. New York City would be submerged, with only the tops of the tallest buildings above water.*


Arctic Ocean
Greenland
Iceland
Europe
Asia
North America
Hawaiian Islands
Atlantic Ocean
Africa
Africa
Equator
Pacific Ocean
South America
Indian Ocean
Australia
Cape of Good Hope
New Zealand
Cape Horn
Southern Ocean
Antarctica
Colder subsurface waters
Warmer surface waters

◀ **Thermohaline circulation** occurs when cold, dense water in the polar regions sinks and spreads toward the equator. There, the water warms up and rises to the surface, then spreads back toward the polar regions. Water in thermohaline circulation takes about 1,000 years to complete a global circuit.

Deep Currents

One type of density current is called *thermohaline circulation*. Thermohaline circulation is a convection process in which dense cold water from high latitudes sinks and flows slowly toward the equator. As the water moves toward the equator, it warms, rises, and flows back again toward the poles. In this way, the ocean water of the entire world is mixed.

Thermohaline circulation occurs primarily in two places: the North Atlantic Ocean and the Antarctic.

In the North Atlantic, cold, heavy water sinks and moves southward across the equator.

In the Antarctic region, cold, heavy water sinks and moves northward across the equator.

Tides and Waves

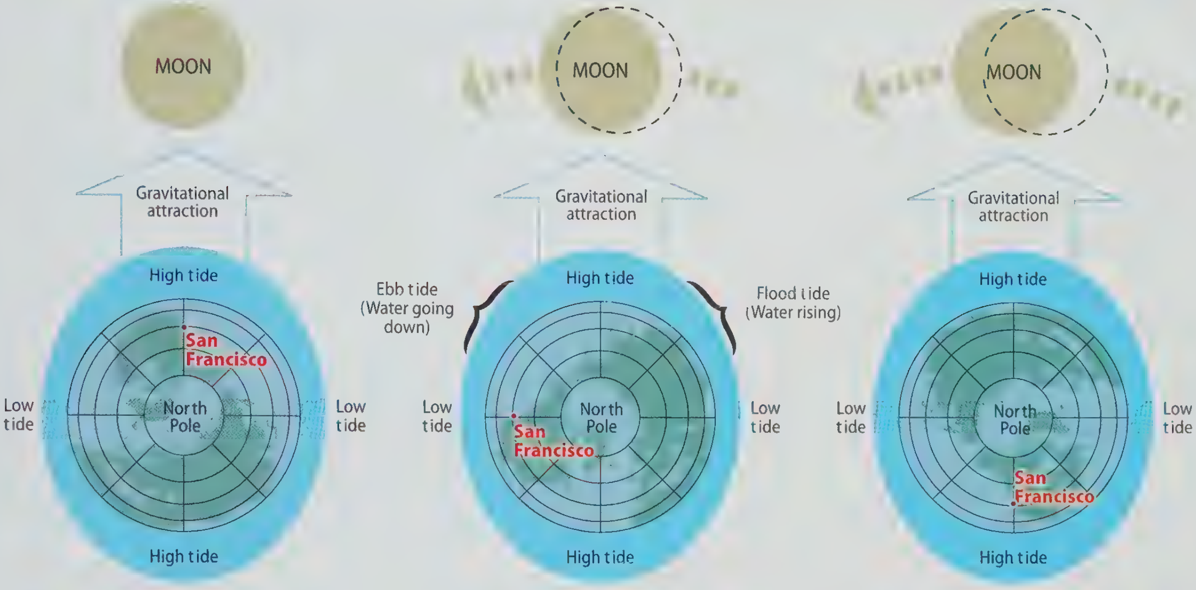
In addition to the circulation of ocean water by surface and deep-sea currents, there are other, more familiar, motions of seawater, such as the periodic movements of the sea called tides and waves.

Tides are the daily rising and falling of sea level. They are related to the gravitational pull of the moon and sun on Earth.

Waves are produced by the action of winds on the surface of the ocean. (See pages 576–577.)

HOW DOES THAT WORK

OCEAN TIDES


MOON
Gravitational attraction
High tide
San Francisco
Low tide
North Pole
Low tide
High tide
MOON
Gravitational attraction
Ebb tide (Water going down)
High tide
Flood tide (Water rising)
Low tide
North Pole
San Francisco
Low tide
High tide
MOON
Gravitational attraction
High tide
Low tide
North Pole
San Francisco
Low tide
High tide

High tide occurs directly below the moon and on the opposite side of Earth. When Earth is in the position above, San Francisco has a high tide.

As Earth turns, the tides rise and fall at each place on the ocean. About 6 hours and 13 minutes after high tide, San Francisco has a low tide, above.

The next high tide at San Francisco occurs about 12 hours and 25 minutes after the first. Earth has turned 186° in this time. The moon has moved 6°.

Tides

Tides are primarily caused by the gravitational pull of the moon on Earth. The moon's gravitation causes the oceans to bulge (high tide) on one side of Earth and depress (low tide) on the other.

Earth's spin results in every meridian of longitude having two high tides and two low tides each day. The time interval between high tides is about 12 hours and 25 minutes, or half of a lunar day. Therefore, high tide occurs about 50 minutes later each day.

Twice each month, there are unusually high and low tides, called *spring tides* and *neap tides*.

Spring tides, which are very high tides, occur at the times of the new and full moons. The term "spring" has no seasonal significance. It is used to denote the extreme range of the tide.

Neap tides, which are very gentle low tides, occur twice monthly, near the times of the first- and the last-quarter phases of the moon.

Earth Science Glossary

aa flowing lava that is cool enough to have partially solidified; and moves as a slow, pasty mass.

air mass a large body of air with uniform temperature and humidity.

alluvial fan fan-shaped accumulation of sediment that forms where a stream emerges from a narrow mountain canyon onto a flat plain.

anticline a fold or folds of rock layers that slope upward to form a crest.

aquifer a body of saturated rock or sediment through which water can move easily.

asthenosphere the thick layer of Earth's mantle that lies under the lithosphere, thought to consist of a thick slush of partially melted rocks and minerals.

atmosphere the envelope of gases that surrounds Earth; three gases essential to life on Earth—oxygen, nitrogen, and carbon dioxide—make up 99 percent of the atmosphere.

avalanche a powerful, rapid rock slide.

bar a visible ridge of sand or gravel deposited in the middle or along the banks of a stream.

bed load the heavy sediment that travels on or near the bottom of a stream.

caldera a volcanic crater larger than 0.6 kilometers (about 0.4 miles) in diameter, produced when a violent volcanic eruption completely destroys the upper part of a volcanic cone.

caliche a layer of calcium carbonate that forms in soil in dry regions over 100,000 to 500,000 years.

Cambrian Explosion the dramatic increase in the variety of life found on Earth during the Cambrian period.

chemical weathering the process by which rocks decay and decompose, and are eventually changed into substances with different chemical compositions and physical properties.

clastic rock sedimentary rock formed from grains of sediment that have been pressed and cemented together.

cleavage the ability of some minerals to break along smooth, flat planes.

climate the average weather in a specific place over a long period of time.

condensation the process in which water changes from a vapor to a liquid or a solid.

conifer any of a group of trees and shrubs that bear cones.

continental drift the slow movement of large landmasses over Earth's surface, caused by pressure that shifts them over the asthenosphere.

convection the movement of air from areas of high pressure to areas of low pressure.

crest the highest part of a wave.

crust the relatively thin, brittle outer layer of Earth.

crystal form the distinctive geometric form of a mineral.

cyclone a storm or winds that blow around in a spiral toward a calm center of low pressure, which also moves.

deformation the process by which plate movements cause horizontal layers of sedimentary rock to become tilted, folded, or cracked.

delta body of accumulating sediment that forms at the mouth of a stream or river, where it flows into a lake or ocean.

dendrochronology the study of tree rings to establish the ages of fossils and environmental conditions in the past.

density a physical quality that reflects how tightly the atoms in a substance are packed.

dew point the air temperature at which water condenses from vapor into liquid.

dissolved load mineral particles that dissolve within the water of a stream.

doldrums the belt of light winds around the equator characterized by frequent thunderstorms or squalls. The doldrums are located between the trade winds of the Northern and Southern hemispheres.

earthquake a sudden, strong trembling, or shaking, of the ground, usually occurring as a result of the movement of tectonic plates.

electromagnetic radiation energy from space that consists of waves of electricity and magnetism.

elevation height above sea level.

El Niño a periodic ocean current of warm water that flows south from Ecuador down the coasts of Peru and Chile, causing a decline in plankton, fish, and birds.

equator an imaginary circle around the center of Earth, halfway between the North Pole and the South Pole.

erosion the removal of rock particles from exposed bedrock or topsoil by the impact of water, wind, or ice. Erosion is also the result of the impact on the landscape of rock particles carried by water, wind, or ice.

evaporation the process by which water changes from a liquid into a vapor.

evaporite a mineral that is deposited when water evaporates.

fault a crack in Earth's crust that occurs at the boundary between two lithospheric plates.

floodplain a broad strip of land built up by sediment deposited on both sides of a stream channel.

folds wavelike features in rock layers that resemble ocean waves.

foliation a parallel arrangement of flaky and needlelike minerals characteristic of some metamorphic rock.

fossil the remains or traces of prehistoric plants and animals, usually preserved in sedimentary rock.

front the boundary that forms when a warm air mass and a cool air mass meet.

frost wedging the process by which a rock splits after water freezes and expands in a joint, prying the rock apart.

fumarole a vent in Earth's continental or oceanic crust emitting volcanic gases and steam at high temperatures.

geology the study of Earth's structure, composition, and history.

geosyncline an elongated, downward curve of Earth's crust.

geothermal gradient the regular pattern of temperature increase from Earth's crust inward toward the core.

geyser spring that erupts columns of water into the air at certain times.

glaciers huge masses of ice that flow due to gravity and currently cover about 10 percent of Earth's total land area.

GPS the Global Positioning System, a worldwide navigation system that uses radio signals broadcast by satellites to pinpoint locations.

gravity the force by which objects are attracted to one another.

groundwater water that lies beneath Earth's surface, filling the cracks, crevices, and pores of rocks.

gyres enormous circular patterns made by wind-driven ocean currents.

half-life the amount of time it takes for half the atoms of an element to decay.

historical geology the study of changes that have occurred in Earth's structure and appearance.

Ecology

General Ecology 590

The Environment 630

Energy and Glossary 734

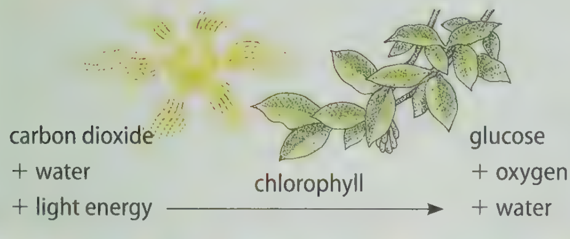
Energy's One-Way Path

Initial Energy Flow

In ecosystems, the source of almost all energy is sunlight, and only green plants, algae, and certain microscopic organisms are equipped to utilize it.

Photosynthesis. The mechanism by which green plants use solar energy is known as photosynthesis. The presence of the green pigment chlorophyll permits capture of energy from the sun and storage of that energy in the chemical bonds of glucose. Through further use of sunlight energy, molecules of glucose are broken down and linked with other chemicals. This results in the formation of the various carbohydrates, proteins, vitamins, and other substances that constitute the body of a plant. (See also pages 134–135.)

PHOTOSYNTHESIS



▲ **In herbivores,** some energy remains in the undigested plant residue, some is lost as heat generated in the process of digestion, and some is lost during various other metabolic processes.

Oxygen Restoration. During photosynthesis, two chemical compounds—carbon dioxide from the air and water from the soil—are combined into simple sugars. In the process, oxygen is released back into the atmosphere. Without green plants or some other means of restoring atmospheric oxygen, the continued respiration by animals would eventually exhaust the supply of oxygen.

GOT TO KNOW

ONE-WAY ENERGY FLOW

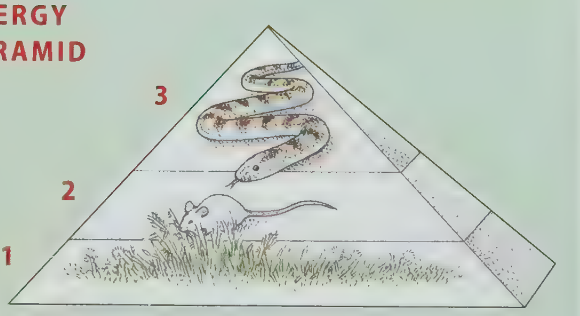
Energy follows a one-way path through the ecosystem, with the initial supply rapidly decreasing as it passes from one organism to the next. In order for the system to function, energy must be supplied continually at the green plant end of the chain.

Energy Storage

Green plants are capable of storing large amounts of solar energy. However, only about 1 percent of the total solar energy reaching Earth is actually fixed and stored by plants. The rest is lost because it is in unusable wavelengths of light, because it is reflected away, or because it is dissipated in the form of heat. Nevertheless, the 1 percent remaining is more than adequate to maintain life on Earth.

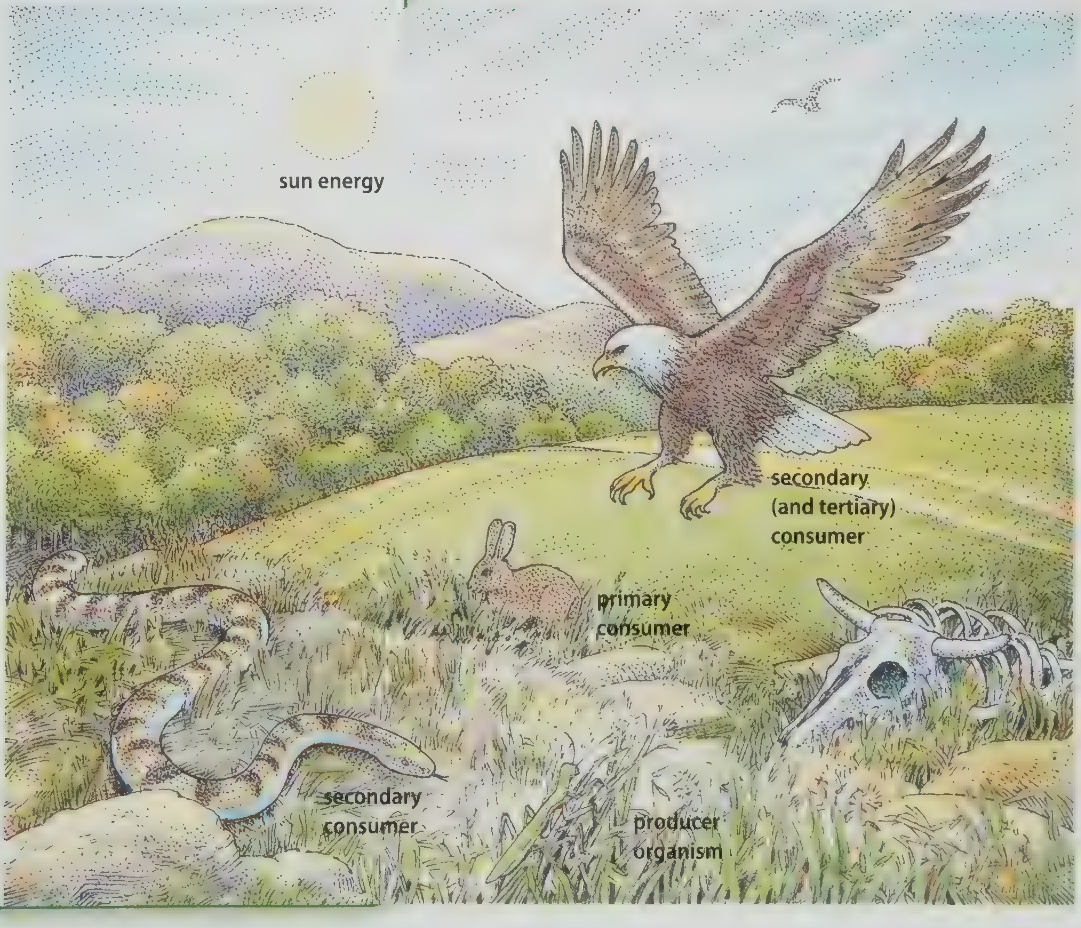
The energy stored within plant bodies cannot be transferred to animal tissues without further loss. At most, 20 percent of the energy is stored in the body tissues of herbivores. A diminished amount of energy is thus available to carnivores. Further energy is lost in eating, digesting, and metabolizing the energy stored in the body of the herbivore, resulting in only a quarter or less of that energy being stored in the body of a carnivore. Further energy is also lost when one carnivore feeds on another.

ENERGY PYRAMID

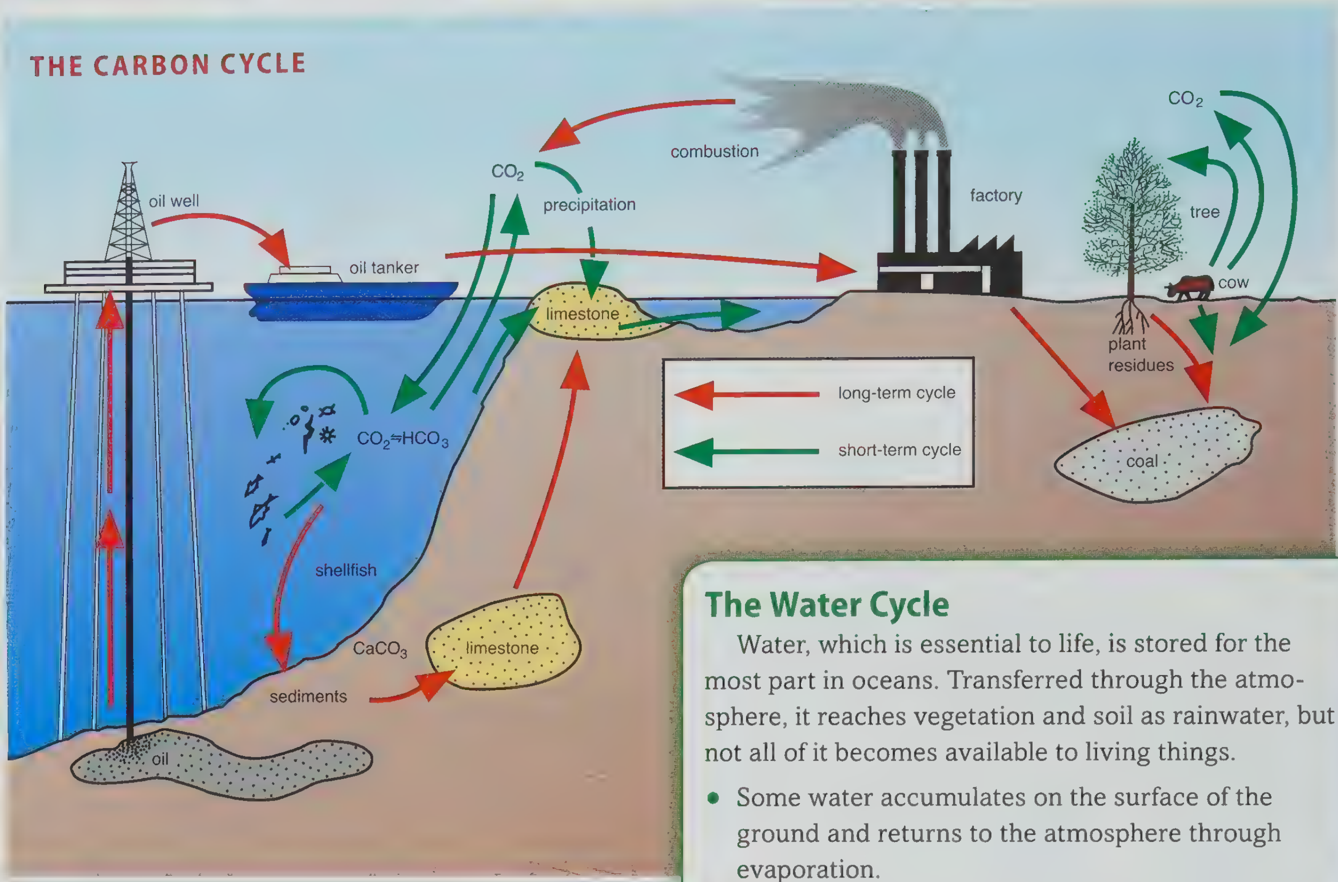


1. energy stored and used by green plants
2. energy available to herbivores
3. energy available to primary carnivores

The sun's energy flows to the producer organisms, which are then eaten by primary consumers. As the food chain lengthens, from primary to secondary to tertiary consumers, energy is lost. Reducer organisms can break down chemical compounds and return them to the soil, but they cannot return energy to the system. Energy needs to be constantly fed into an ecosystem via the producer organisms (plants). ▶



Water, Carbon, and Nitrogen


THE CARBON CYCLE
combustion
CO_2
precipitation
oil well
oil tanker
factory
CO_2
tree
cow
limestone
plant residues
long-term cycle
short-term cycle
coal
$CO_2 \rightleftharpoons HCO_3$
shellfish
$CaCO_3$
limestone
sediments
oil

▲ **The flow of certain nutrients,** such as water and carbon, is circular, the nutrients being used again and again. ▼

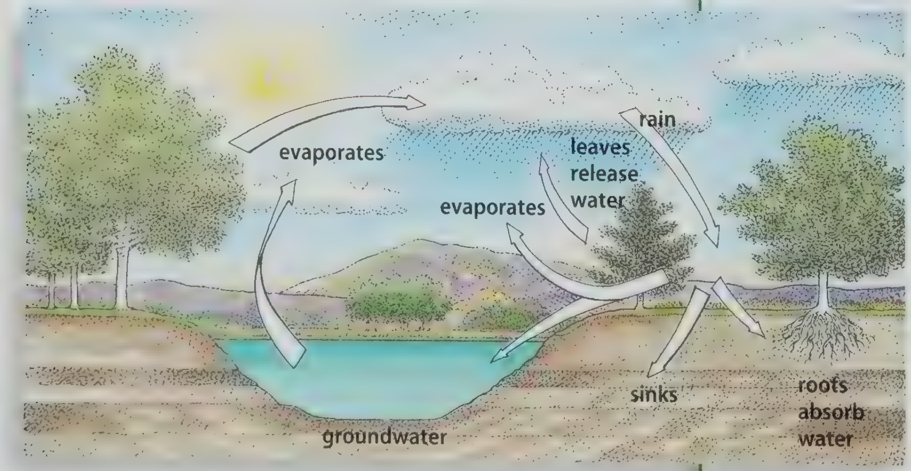
The Water Cycle

Water, which is essential to life, is stored for the most part in oceans. Transferred through the atmosphere, it reaches vegetation and soil as rainwater, but not all of it becomes available to living things.

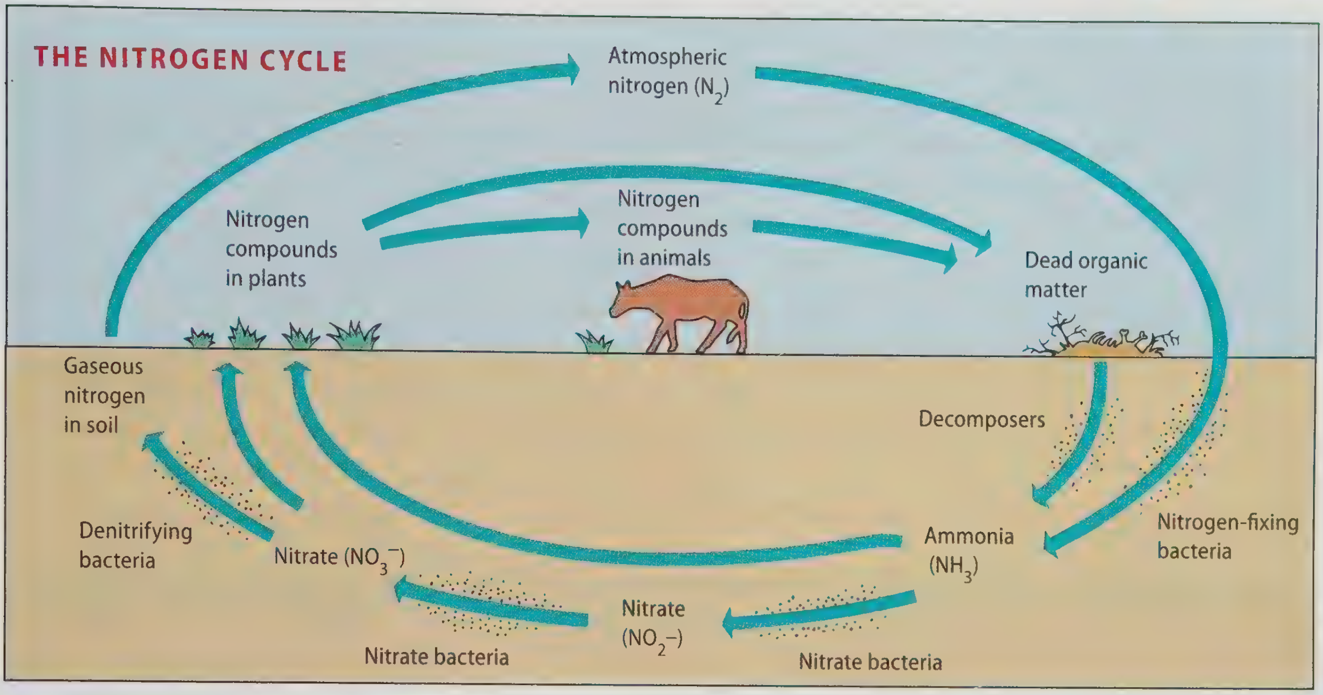
- Some water accumulates on the surface of the ground and returns to the atmosphere through evaporation.
- Much water moves through the soil and runs into underground channels.
- In heavy rains, or when the soil is soaked, water may run off the surface and again be lost.
- Some water that enters the soil becomes bound to soil compounds and unavailable to plants.

The water held in the soil provides not only the water but also the dissolved chemicals necessary for plant life. Only part of the solution that enters the plant roots is taken into the plant cells. The rest is lost through the leaves in the process of transpiration. From plants, the water is transferred to animals. Eventually, all of the water used by plants and animals returns to the soil or goes directly into the atmosphere to begin another cycle. (See also pages 538–539.)

THE WATER CYCLE


evaporates
rain
leaves release water
evaporates
sinks
roots absorb water
groundwater

THE NITROGEN CYCLE


Atmospheric nitrogen (N_2)
Nitrogen compounds in plants
Nitrogen compounds in animals
Dead organic matter
Gaseous nitrogen in soil
Decomposers
Denitrifying bacteria
Nitrate (NO_3^-)
Ammonia (NH_3)
Nitrogen-fixing bacteria
Nitrate (NO_2^-)
Nitrate bacteria
Nitrate bacteria

▲ **Nitrogen** makes up about 78 percent of Earth's atmosphere, but most organisms cannot use nitrogen in its gaseous form. Nitrogen-fixing bacteria convert atmospheric nitrogen into a form that other living things can use. After nitrogen has been fixed by the bacteria, it circulates repeatedly between organisms and the soil. Denitrifying bacteria help regulate the amount of nitrogen in biological circulation by changing some fixed nitrogen back into a gas.

Mineral Nutrient Cycle

Food chains provide pathways for mineral nutrients. From soil, fresh water, or saltwater, minerals can be taken up by green plants and introduced into food chains. Mineral nutrient flow is cyclical; the same atom or molecule may move from plant to animal, from animal to microorganism, and from microbe back to plant. When returned to the soil, it is taken up once again by some other plant.

The supply of mineral nutrients in the soil is not limitless. There must be a continuing turnover of these materials if an area is to continue to support life. When soil nutrients are scarce, new growth depends on the decay of dead plants and animals. Organisms such as earthworms process great amounts of plant litter through their bodies. Their actions accelerate decomposition and make available the materials required for new growth. When the cycle is interrupted for any reason, the soil may become infertile.

IMPORTANT MINERAL NUTRIENTS

The following elements are commonly found in fertile soil.

- nitrogen
- sulfur
- phosphorus
- calcium
- carbon
- oxygen
- hydrogen
- magnesium
- potassium
- boron
- chlorine
- copper
- iron
- manganese
- molybdenum
- zinc

Indoor Pollution

The air in homes, schools, offices, and other buildings often contains pollutants that can cause or aggravate illnesses. People exposed to these pollutants may suffer from such problems as irritation of the eyes, nose, and throat; headaches; dizziness; and fatigue. This is sometimes called "sick-building syndrome."

Other health effects might not show up until years after exposure has occurred, or only after long or repeated periods of exposure. These include emphysema and other respiratory diseases, heart disease, and cancer.

Who Is at Risk?

Research indicates that people spend approximately 90 percent of their time indoors. Thus, for many people, indoor air pollution may cause greater risks than outdoor pollution. Also, the people who are exposed to indoor air pollutants for the longest periods of time are frequently those who are most susceptible to the adverse effects of such pollution. They include

- young children.
- the elderly.
- people who are chronically ill, particularly those with respiratory or cardiovascular diseases.

THE USUAL SUSPECTS

The many potential sources of indoor air pollution can be grouped into six major categories.

- combustion sources, such as oil and kerosene burners
- central heating and cooling systems and humidification devices
- building materials and furnishings, including insulation, carpeting, furniture, and cabinets made of certain pressed wood products
- products used for cleaning, personal care, and hobbies
- tobacco smoke from cigarettes, pipes, and cigars
- outside sources, including air pollution, pesticides, and radon

Many of these pollutants are present in minute quantities. This makes it difficult to prove that they are threats to health. It can also be difficult to determine the effects caused by interactions among the various pollutants.

◄ **Chemicals** in household cleaning products can pose health risks.

◄ **Indoor pollution** is not limited solely to homes. Many office buildings have significant sources of air pollution, often made worse because they lack ventilation.

In addition to the indoor pollutants found in homes, offices can also contain chemicals from certain carpeting and cleaning materials, rest room air fresheners, adhesives, and copy machines. They may also contain biological pollutants from dirty ventilation systems.

Solutions

It can be difficult to remove indoor air pollution. Some pollutants are trapped by carpets, drapes, and other surfaces. These particles remain in the home and are gradually rereleased into the air.

Reduction. The best way to improve indoor air quality is to eliminate sources of pollution or reduce their emissions. For example, converting gas appliances from pilot lights to spark ignitions reduces carbon monoxide emissions.

Ventilation. Another approach is to improve ventilation. Ventilation gradually lowers the offending substances' concentration indoors. Circulate fresh air by opening windows and doors and by using fans or heat-recovery ventilators (also known as air-to-air heat exchangers).

Plants. Another method of reducing pollution is to keep indoor plants. Many houseplants absorb carbon monoxide, nitrogen dioxide, and formaldehyde.

Common Indoor Pollutants

Pollutant	Sources	Effects
Biological (bacteria, fungi, pollen, viruses, animal dander)	Wet or moist walls, ceiling, and carpets; air conditioners; humidifiers; bedding; household pets	Allergies, asthma, Legionnaire's disease, digestive problems, humidifier fever, dizziness, lethargy
Tobacco smoke	Cigarettes, pipes, cigars	Respiratory diseases, including lung cancer
Carbon monoxide, nitrogen oxides, sulfur dioxide	Stoves, heating systems, clothes dryers, water heaters	Headaches, drowsiness, nausea, chronic bronchitis, death (in high concentrations of carbon monoxide)
Benzene	Solvent cleaners, cigarette smoke	Cancer
Formaldehyde	Foam insulation, particleboard, plywood, draperies, carpets, tobacco, smoke, fire	Irritation of eyes, skin, lungs; flulike symptoms; shortness of breath; fatigue; cancer
Asbestos	Insulation, ceiling and floor tiles	Lung disease, cancer
Lead	Sanding or burning of lead paint, soldering	Impaired mental and physical development of children; damage to kidneys, nervous system, red blood cells
Pesticides	Products used to kill household pests; outdoor products that drift or are tracked inside	Eye, nose, and throat irritation; damage to nervous system and kidneys; cancer
Styrene	Carpets, plastics	Kidney and liver damage
Radon	Uranium in soil	Lung cancer

Slowing Climate Change

The potential impact of global changes in climate is far too disastrous to ignore. Researchers have developed a number of ways to limit global warming. But because the warming is a global problem, many strategies require the cooperation of a diversity of nations, each with its own interests.

Nevertheless, many countries are taking action individually and by international agreement to limit future warming.

Limiting Future Warming

Two areas of action are considered to be particularly important in the fight to reduce global warming: fuel conservation and reforestation.

Fuel Conservation. Decreasing consumption of fossil fuels is essential if global warming is to be slowed. One conservation method is improved efficiency.

- Increasing the average fuel efficiency of automobiles has already slowed carbon dioxide emissions—and helped extend diminishing oil reserves.
- Buildings can be made more fuel efficient by adding extra insulation, installing storm or thermal windows and heat pumps, caulking around windows and doors, and so on.
- Appliances can be improved with better heat-transfer techniques and more efficient motors.

Fossil fuels can also be conserved by switching to alternate energy sources and technologies. For example, solar energy—used to produce heat and electricity—is nonpolluting and inexhaustible.

National Forest
REFORESTATION

▲ **After the Fire.** In 1999, a forest fire destroyed a pine plantation in Florida's Apalachicola National Forest. The area has since been replanted with slash pine.

Reforestation. A fast-growing tree absorbs 48 pounds of carbon dioxide per year. An acre of trees uses 10 tons per year! Increased tree planting in large areas of once-forested land, as well as the planting of trees in urban areas, would significantly reduce carbon dioxide buildup.

- Trees placed around buildings offer the added benefit of reduced energy needs for winter heating and summer cooling.
- Three trees placed around a house can cut home air conditioning energy needs by as much as 10 to 50 percent—a significant savings.
- Studies show that each tree that provides summer shade and cooling indirectly causes reductions in carbon dioxide emissions equivalent to 15 times the amount of carbon dioxide the tree alone can absorb.

At the same time, it is important to halt the rapid destruction of many of the world's forests. Consumers can contribute to this effort by recycling paper. If Americans recycled all their Sunday newspapers, they would save more than 500,000 trees a week!

◀ **Solar plants,** like the ones in California's Mojave Desert, have played an important role in meeting the electrical needs of Los Angeles.

◀ **Genetically modified plants** may be one way that humans will adapt to the changes resulting from global warming; however, the use of such crops remains controversial.

THE KYOTO PROTOCOL

The Kyoto Protocol entered into force in 2005.

- The protocol restricts emissions of carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and three other greenhouse gases.
- Each nation's, or party's, target is a certain percentage of its emission levels in 1990. As a whole, the developed parties (wealthier nations) must cut their emissions to an average of 5 percent below 1990 levels.
- Most of the world's countries have agreed to the protocol. But a few nations, including the United States, have refused.
- In late 2005, delegates from the parties agreed to extend the Kyoto Protocol beyond 2012.

ADAPTING

Even if immediate action is taken to severely limit greenhouse gas emissions, some warming will still occur because of the great volume of gases already emitted into the atmosphere by human activities. A variety of steps may enable people to prepare for the changes ahead.

- building bulkheads and levees to protect buildings and roads in coastal areas
- restricting building in coastal areas to enable wetlands and beaches to migrate inland as sea levels rise
- growing more heat- and drought-resistant crops, such as semitropical crops
- genetically engineering new varieties of crops and plants

HOW DO WE KNOW THAT ?

GLOBAL WARMING

Climatologists (scientists who study climate) use information from many sources to analyze global warming. Since the late 1800s, the most reliable climate information has come from standardized measurements using weather instruments. To look further back, climatologists analyze other types of evidence. Such evidence includes:

- growth rings in trees.
- cores (cylindrical samples) of ice drilled from Antarctica and Greenland.
- cores of sediment from ocean or lake beds.

Water Pollution

Pollution occurs when people contaminate water with wastes, toxic chemicals, metals, and oils.

Healthy Systems

Rivers, lakes, and other bodies of water have a remarkable ability to cleanse themselves. Special alkaline substances in the water neutralize acid from acid rain. Bacteria in the water break down, or oxidize, organic wastes. Sunlight breaks down other compounds. (See pages 564–567.)

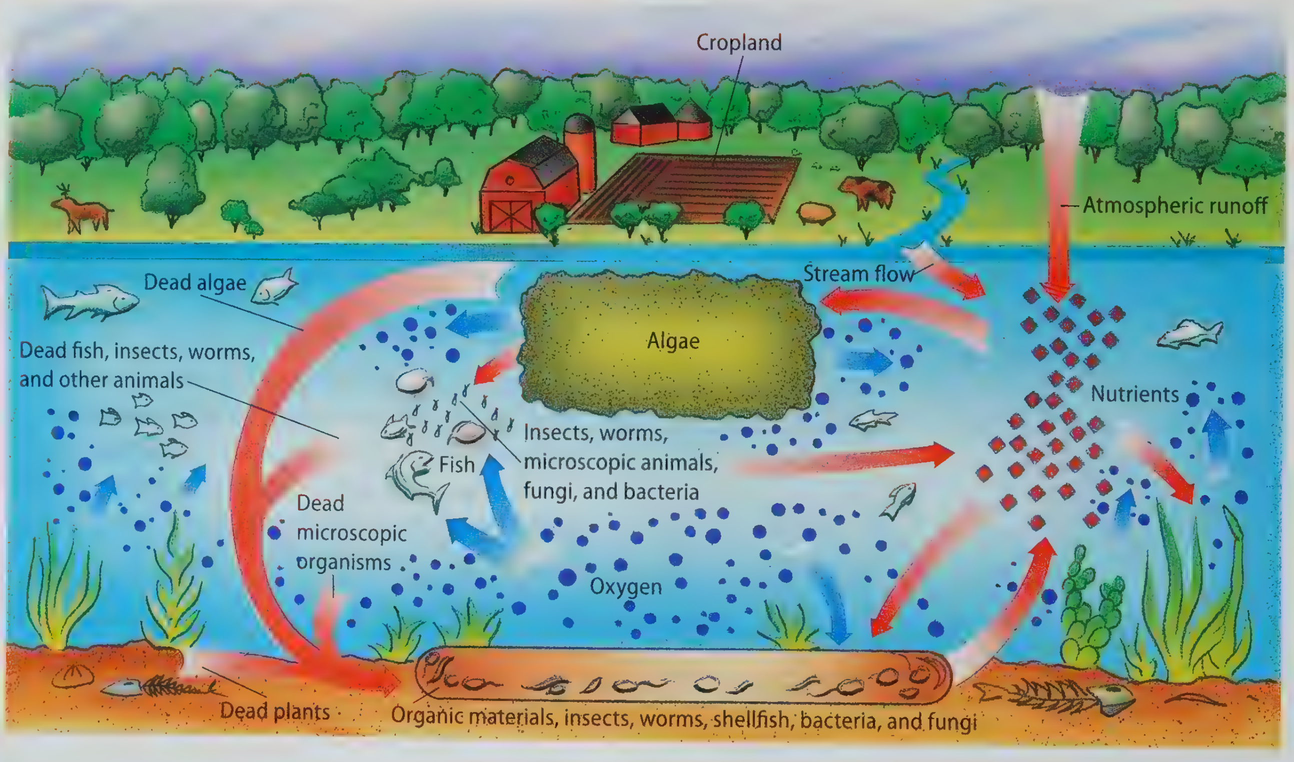
Unhealthy Systems

But there are limits to a water system's purifying ability. If too many wastes are dumped into the water, the natural systems become overloaded. They cannot break down the wastes fast enough.

Oxygen Levels. As bacteria multiply to feed on increasing amounts of organic wastes, they consume more and more oxygen. Oxygen levels fall. This makes breathing difficult or impossible for fish and other organisms in the water.

If the water loses all its oxygen, it becomes "dead," which means it contains few (if any) living things.

Accumulation. Another problem is that many of the pollutants produced by modern industries are not biodegradable. They cannot be broken down by natural systems. For example, plastics dumped into water will remain unchanged for hundreds of years. Other pollutants, such as pesticides, become concentrated in the bodies of organisms. This buildup poses a threat not only to the organisms, but also to those who eat them.


Cropland
Atmospheric runoff
Stream flow
Dead algae
Algae
Dead fish, insects, worms, and other animals
Nutrients
Insects, worms, microscopic animals, fungi, and bacteria
Fish
Dead microscopic organisms
Oxygen
Dead plants
Organic materials, insects, worms, shellfish, bacteria, and fungi

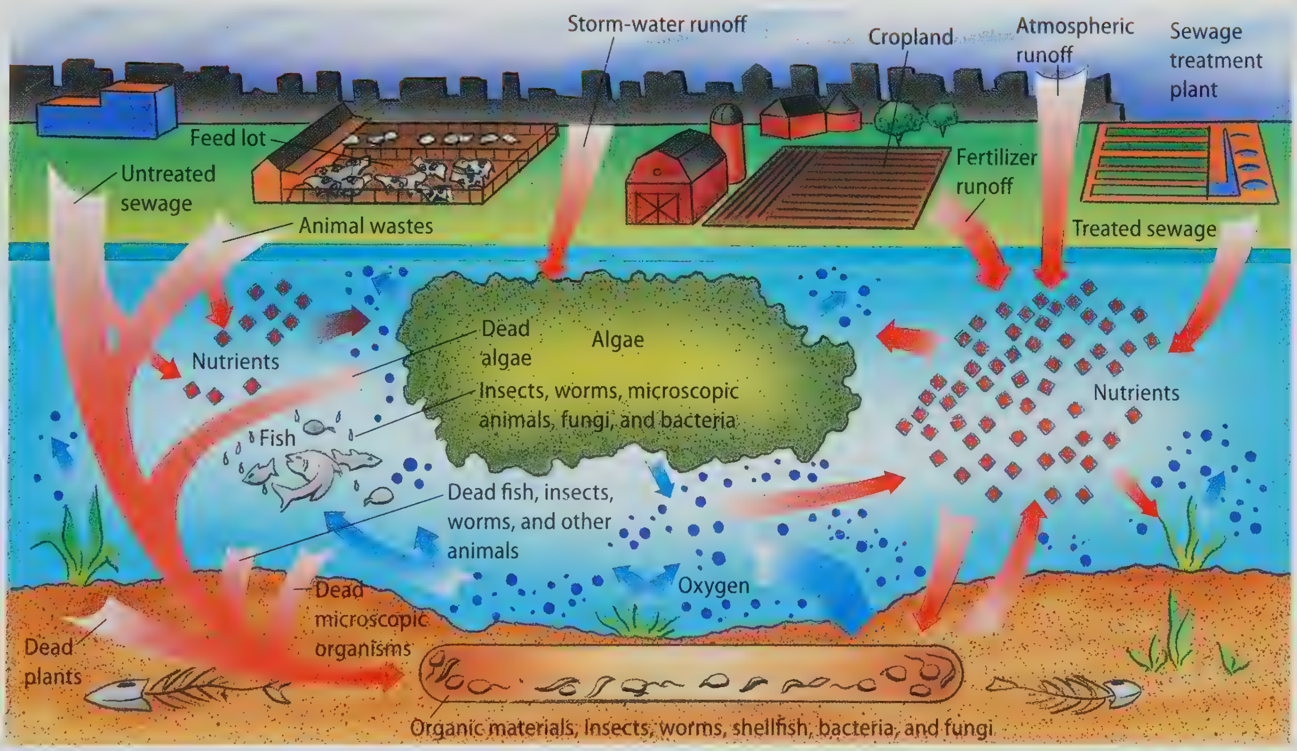
A healthy water system relies on a cycle of natural processes that turns waste into useful substances. Aerobic (oxygen-using) bacteria break down dead plants and animals and animal wastes, releasing nitrates, phosphates, and other nutrients (chemicals needed for growth). Nutrients also enter the water from streams. Algae absorb the nutrients. Aquatic plants release oxygen. Microscopic animals called zooplankton eat the algae, and fish eat the zooplankton. The fish produce wastes and eventually die. Bacteria break down the wastes and dead fish, and the cycle continues.

POINT AND NONPOINT SOURCES

Point-source pollutants come directly from identifiable sources. For example, wastes carried from a factory or sewage treatment plant into a waterway by discharge pipes are a point-source pollutant.

Nonpoint-source pollutants cannot be traced easily to a single source. They include pollutants that are washed into waterways or seep into groundwater from fields, streets, landfills, mine tailings, and building sites. Acid rain and other forms of polluted precipitation that fall on waterways are also nonpoint-source pollutants.

- Nonpoint sources are difficult to locate and control because they have no single source. Therefore, no individual or organization can be held responsible.
- To date, most water cleanup efforts have focused on regulating point-source pollutants. With the exception of toxic chemicals, these pollution sources have been controlled to a large extent in industrialized countries.
- Until recently, there has been a lax attitude toward pollution from nonpoint sources, largely because such contamination has not been recognized or well understood by the general public.


Storm-water runoff
Cropland
Atmospheric runoff
Sewage treatment plant
Feed lot
Untreated sewage
Fertilizer runoff
Animal wastes
Treated sewage
Dead algae
Algae
Nutrients
Insects, worms, microscopic animals, fungi, and bacteria
Nutrients
Fish
Dead fish, insects, worms, and other animals
Oxygen
Dead microscopic organisms
Dead plants
Organic materials, insects, worms, shellfish, bacteria, and fungi

Water pollution occurs when human activities upset the balance, often by introducing excess nutrients from fertilizers and untreated sewage. The algae grow faster than the fish can eat them. As more algae grow, more die and block light needed by aquatic plants to make oxygen. Bacteria use up much oxygen while consuming the excess dead algae. As the oxygen levels drop, many aquatic plants and animals die and decay, using more oxygen. Without oxygen, the aerobic bacteria work at a slower rate. Dead fish and other wastes sink to the bottom.

Nuclear Energy

Nuclear energy, also called atomic energy, is the powerful energy released by changes in the core of atoms. The heat and light of the sun result from nuclear energy. Scientists and engineers have found many uses for this energy, including the production of electric power, the explosion of nuclear weapons, and the diagnosis and treatment of medical conditions. (See pages 404–407.)

Nuclear Power in the Production Matrix

Yesterday. Some nuclear power plants have had accidents that have killed people, although not as many people as in other types of industrial accidents. The worst nuclear accident in the United States, at the Three Mile Island power plant in Pennsylvania, killed no one.

The worst nuclear accident ever, the Chernobyl explosion in Ukraine in 1986, killed dozens of people, but it may have affected many more in ways that will not be known for years.

Today. Almost all the world's electric energy is produced by hydroelectric and thermal power plants. Hydroelectric plants use the force of rushing water from a dam or waterfall to generate electric power. Thermal plants use the force of steam from boiling water. The great majority of thermal plants burn fossil fuels—coal, oil, and natural gas—to produce heat to boil water. The remaining thermal plants fission uranium.

Tomorrow. Few countries have enough water power to generate large amounts of hydroelectric power, and many regions have already fully exploited their hydroelectric capacity. Most countries depend mainly on fossil fuels. But fossil fuels are a nonrenewable resource, and burning them produces gases that can damage the environment. Therefore, many experts predict that nuclear power will become increasingly important.

Chain Reactions

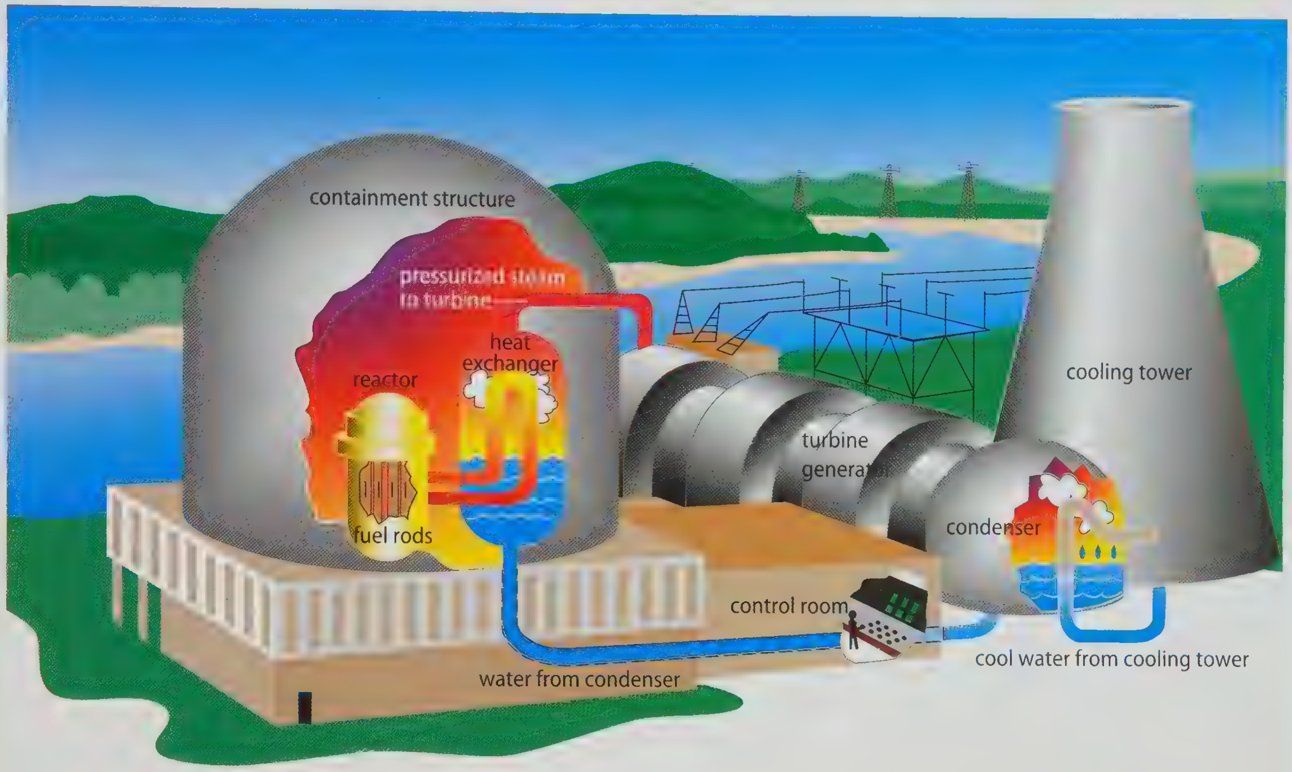
In a nuclear reactor, chain reactions produce energy. The process happens when a mass of uranium or plutonium releases nuclear energy. That energy is created by a succession of nuclear fissions that continues automatically once it has been started. A fission is a splitting of an atomic nucleus.

Process. In a chain reaction, neutrons from the regular decay of uranium bump into other atoms of uranium and speed up the decay process. Each atom that decays produces more neutrons, so the number of atoms decaying increases with lightening speed. To produce nuclear power, the chain reaction is controlled by slowing down the neutrons.

Result. Like fire, a chain reaction produces heat. Unlike fire, it can do so without releasing chemicals into the air. When the heat from a chain reaction is used to produce electricity, the central heat-producing source is called a nuclear reactor. The entire complex is called a nuclear power plant.

fyi!

Scientists first released nuclear energy on a large scale at the University of Chicago in 1942, 3 years after World War II began. This achievement led to the development of the atomic bomb. The first atomic bomb was exploded in the desert near Alamogordo, New Mexico, on July 16, 1945. In August, United States planes dropped bombs on Hiroshima and Nagasaki, Japan. The bombs largely destroyed both cities and helped end World War II.


containment structure
pressurized steam to turbine
heat exchanger
reactor
fuel rods
turbine generat
cooling tower
condenser
control room
cool water from cooling tower
water from condenser

The nuclear power plant shown here is typical of a pressurized water reactor. It has three separate water systems. Water used to cool the reactor passes through a heat exchanger, where water in the second system is changed into steam. This steam drives a turbine and then is changed back into water in a condenser that is cooled by water from a nearby water source.

Pros and Cons of Nuclear Power Plants

Advantages. Nuclear power plants have two main advantages over fossil fuel plants.

1. Nuclear plants are less expensive to operate than a fossil fuel plant. While a nuclear facility costs more to build, the fuel it uses is much cheaper, which leads to savings over the long term.
2. Uranium, unlike fossil fuels, releases no chemical or solid pollutants into the air during use.
3. It takes only a small amount of uranium to produce a large amount of energy. A chunk of uranium the size of a softball can release more energy than a trainload of coal that weighs 3 million times as much.

Disadvantages. Nuclear power plants have three major disadvantages. These disadvantages have slowed the development of nuclear energy in some countries.

1. Nuclear plants cost more to build than fossil-fuel plants.
2. Because of the need to assure that hazardous amounts of radioactive materials are not released, nuclear plants must meet certain government regulations that fossil-fuel plants do not have to meet.
3. Used nuclear fuel produces dangerous radiation long after it has been removed from the reactor. As a result, safe disposal of nuclear waste presents a challenge.

Ecology Glossary

abiotic factors the nonliving aspects of an environment, such as water, soil minerals, and atmospheric gases.

acid rain rain and other precipitation that is polluted by acids, particularly sulfuric acid and nitric acid.

aquifers underground formations filled with groundwater.

biome a major type of ecosystem; each type of biome shares common characteristics; biomes are scattered worldwide.

biotic factors the living aspects of an environment, such as producer organisms, consumer organisms, and reducer organisms.

biotic potential the inherent capacity or maximum rate at which a population could increase under the best of conditions.

carbon dioxide a colorless, odorless gas. It occurs in the atmospheres of many planets, including that of Earth. On Earth, all green plants must get carbon dioxide from the atmosphere to live and grow. Carbon dioxide is also created by the burning of any substance that contains carbon. Such substances include coal, gasoline, and wood.

carbon monoxide a toxic gas produced by any process that involves the incomplete combustion of carbon-containing compounds; it is primarily emitted in the exhausts of gasoline-powered vehicles.

carnivore an animal that feeds mainly on meat.

chlorofluorocarbons (CFCs) a family of synthetic, nontoxic chemicals used as refrigerants, solvents, aerosol propellants, and so on. Highly volatile, they are not destroyed in the lower atmosphere but drift into the upper atmosphere, where their chlorine atoms destroy ozone.

climate the weather of a place averaged over time.

commercial energy power that is produced by businesses and governments to be sold to the public.

community in ecology, a group of animals or plants living together; any group of mutually related organisms.

competition the simultaneous demand by different organisms for food, places for habitation, and other vital factors.

coniferous a type of tree that bears cones.

conservation the protection and improvement of natural resources such as forests and fossil fuels.

consumer in ecology, an organism in an ecosystem that does not obtain its energy directly, but acquires it secondhand from energy stored in green plants. Primary consumers feed directly on plants. Secondary consumers feed on other animals and thus receive their energy after it has been processed by two or more organisms.

DDT (dichlorodiphenyltrichloroethane) the first chlorinated hydrocarbon insecticide. It is quite persistent in the environment and causes harm as it accumulates in food chains. Its use in the United States has been restricted since 1972.

deciduous a type of tree that sheds its leaves each autumn.

decomposer organisms that decay and decompose organic matter by feeding on the remains of dead plants and animals; reducer organisms. Mainly bacteria and fungi.

desalinization the process of extracting fresh water from seawater.

desert a dry region where evaporation exceeds rainfall.

deep-well injection the practice of disposing of hazardous fluid wastes by injecting them into rock formations hundreds of feet below Earth's surface.

dispersal in ecology, the process by which members of a population spread out into unknown territory; helps to maintain a stable population.

domestication the transformation of animals or plants from wild to tame.

ecosystem a system made up of a group of living organisms and its physical environment, and the relationship between them.

endangered species living creatures threatened with extinction.

epiphytes plants that grow attached to larger plants without deriving nourishment from the hosts.

estuary a water passage where the tide meets a river current.

ethics the study of the standards of right and wrong.

extinction a state that occurs when every member of a species has died. Mass extinctions, or events in which large numbers of species suddenly become extinct, have occurred several times in Earth's history.

fallout radioactive material or other polluting particles that fall out of the atmosphere in dust or precipitation.

flocculants chemicals used in water treatment processes to improve the sedimentation or filterability of small particles.

food chain the pathways along which energy is transferred from one organism to another; a group of interrelated organisms; each member of the group feeds upon the one below it and is in turn eaten by the organism above it.

food web a group of interrelated food chains in an ecosystem.

fossil fuels coal, oil, and natural gas formed from the remains of organisms that lived millions of years ago.

fuel cell a device that converts chemical energy to electrical energy.

genotoxins pollutants that interfere with cell reproduction.

global warming an increase in the average temperature at Earth's surface. People often use the term global warming to refer specifically to the warming observed since the mid-1800s. Global surface temperatures have risen chiefly because of a process called the greenhouse effect.

greenhouse effect the absorption by gases in the atmosphere of heat energy radiated from Earth's surface, causing the atmosphere to become warmer.

greenhouse gases gases in Earth's atmosphere that absorb heat from the surface. The primary greenhouse gases are carbon dioxide, methane, nitrous oxide, and chlorofluorocarbons, or CFCs.

groundwater fresh water under the surface of Earth that fills aquifers and moves between soil particles and rock, supplying wells and springs. It is susceptible to contamination by leaching agricultural and industrial pollutants, by substances from leaking underground storage tanks, and so on.

habitat the kind of place in which an organism usually lives. A single habitat may satisfy the needs of many different types of organisms.

half-life a measure of radioactivity, different for each element or isotope, defined as the average time it takes for half the atoms of a sample to undergo radioactive decay. Half-lives vary from trillions of years to fractions of seconds. For example, the half-life of samarium-152 is 1,012 years; that of polonium-212 is $3 \times 10_{-7}$ seconds.

hazardous wastes wastes that may cause illness or death or pose other threats to human health or that may harm the environment.

herbivore an animal that feeds on plants.

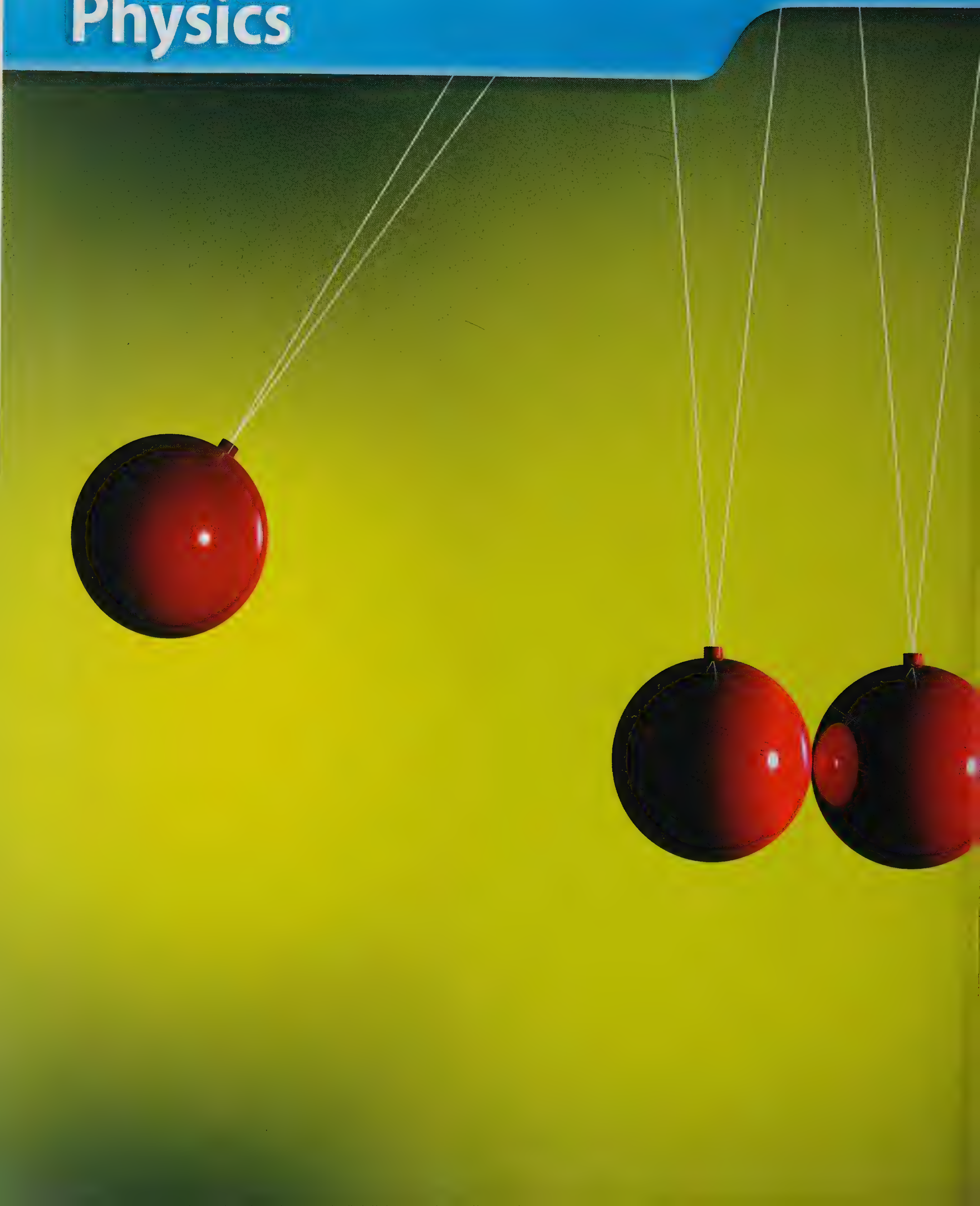
hydrocarbons compounds in fossil fuels such as petroleum, natural gas, and coal that contain carbon and hydrogen; may be carcinogenic.

hydroponics the growing of plants without soil, by the use of water containing the necessary mineral nutrients.

incineration the destruction of waste products by fire in furnaces.

intercropping an agricultural technique in which two or more crops are grown side by side in alternating rows in combinations that increase soil fertility and crop yields.

Physics



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Light, Nuclear Physics 798

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ENERGY, in physics, is a quantity related to work. Energy has many forms. For example, you use mechanical energy to toss a ball. Heat energy warms a room. Electric energy makes a lightbulb glow. And chemical energy provides the driving force for an automobile.

Forms of Energy

Energy, the capacity for doing work, can exist in many forms. Two forms of energy are fundamental.

- **potential:** the energy an object or particle has due to its position
- **kinetic:** energy of motion

An object can posses both potential and kinetic energy at the same time. (See also pages 312–313.)

Potential Energy

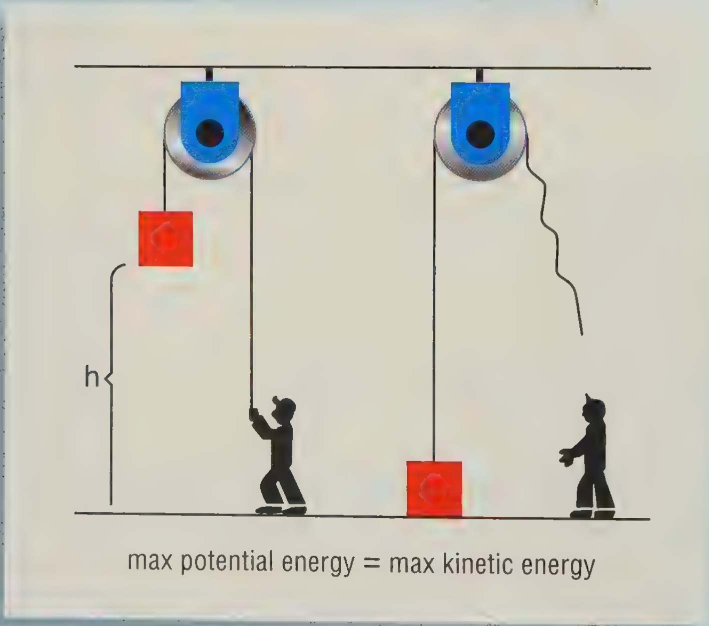
Potential energy is the energy a body possesses by virtue of its location. Water in a reservoir, or a weight lifted to an elevated position, has potential energy that can be converted by a machine into a more desirable or convenient form.

Gravitational potential energy is the most common form. Since Earth's gravity attracts all bodies, work is required to move an object farther away from the center of the earth. The physical work done to move the object represents energy that is then "stored" in the object due to its farther distance from the center of gravity of Earth.

Some other forms of potential energy include chemical potential energy which is energy contained in chemical bonds, and electrical potential energy. which is energy due to the relative position of two electrically charged objects. Frequently the symbol *U* is used to represent forms of potential energy.

Kinetic Energy

Kinetic energy involves motion. A bird in flight, a spinning wheel, and a speeding car all possess kinetic energy. The amount of work done to bring a moving object to rest, or the work required to produce the velocity at which the body moves, is a measure of its kinetic energy. Often the symbol *KE* is used to represent kinetic energy.



The potential energy of the mass decreases and the kinetic energy increases as the mass falls.

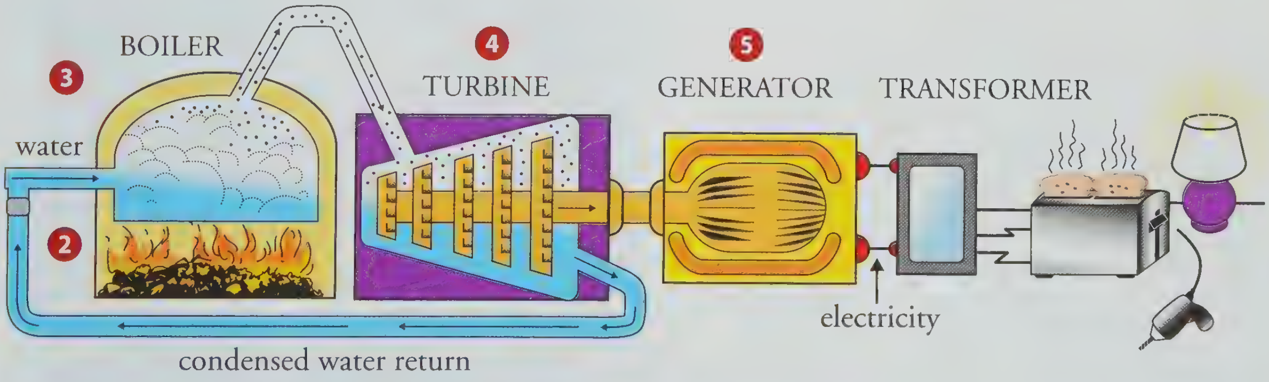
Energy Conversion

Energy can be converted from one form into another form. For example, a coal-burning power plant converts chemical energy to electric energy. The following description of that process will help you understand various forms of energy and how they can be converted.

1. The coal has potential energy in the form of chemical energy. That energy is stored in electrons of atoms that make up molecules in the coal. (Not pictured.)
2. When the coal burns, its molecules change as bonds break and other bonds form. As the molecules change, electrons release energy, which is immediately converted to heat energy. The hotter an object, the greater the kinetic energy of its atoms and molecules.
3. In the power plant, the molecular changes that occur when the coal burns create hot gases. In the next stage of the process, heat energy in the gases changes to heat energy in the metal that makes up a boiler, then to heat energy in molecules of water inside the boiler. As the water molecules absorb heat energy, they move more and more rapidly. Eventually, they move so rapidly that water turns to steam.
4. The steam flows from the boiler through pipes to a device known as a steam turbine. The turbine has several wheels with fanlike blades. The steam rushes through the blades, pushing against them and thereby spinning the wheels. In this stage, heat energy of the steam is converted to mechanical energy of the turbine.
5. The turbine is connected to a machine called an electric generator. The generator converts mechanical energy of the spinning turbine to electric energy. That energy is partly kinetic and partly potential.

THE LAW OF CONSERVATION OF ENERGY

This law is fundamental to our understanding of energy. It states that in energy conversions, the total quantity of energy is conserved. The quantity of energy that exists at the end of a process is the same as the quantity that existed when the process began. That means that energy can never be created or destroyed.



Motion

Motion is a change of position in space. Moving things surround us. When we catch a ball or safely cross a busy street, we use our understanding of motion. Physicists study motion to better understand the world.

Relativity

All motion is relative. That means that an object can only be described as moving or stationary in relation to another object. On an airplane in flight, for example, two passengers sitting in their seats are moving rapidly relative to the ground. But they are stationary relative to each other. This concept, called *relativity*, presents special challenges to our understanding of motion.

SIR ISAAC NEWTON

Sir Isaac Newton (1642–1727), an English scientist, astronomer, and mathematician, invented a new kind of mathematics, discovered the secrets of light and color, and showed how the universe is held together. He is sometimes described as one of the greatest names in the history of human thought because of his great contributions to mathematics, physics, and astronomy.

Newton did not enjoy the scientific arguments that arose from his discoveries. Many new scientific theories are opposed violently when they are first announced, and Newton's did not escape criticism. He was so sensitive to such criticism that his friends had to plead with him to publish his most valuable discoveries.

Newton was a bachelor who spent only part of his time on mathematics, physics, and astronomy. He studied and experimented with alchemy. He also spent a great deal of his time on questions of theology and Biblical chronology.

Although Newton's achievements were extremely important, he spoke modestly of himself shortly before his death, saying, "I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

Newton's Laws

In the 1600s, the relationship of force to motion was described in three laws formulated by Sir Isaac Newton. These laws can help us understand the kinds of motion we see every day.

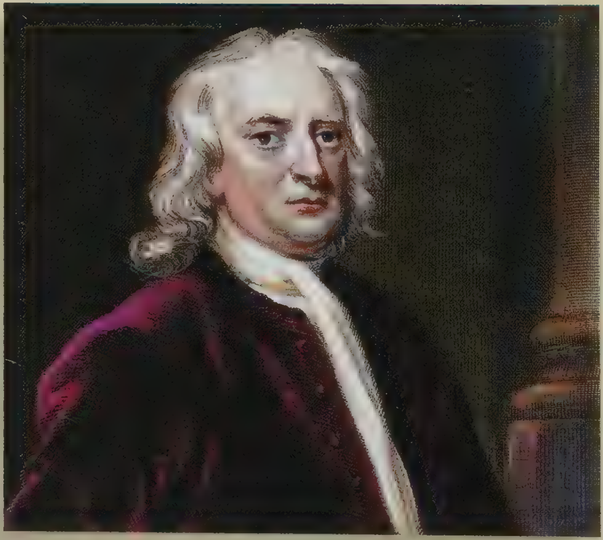
First Law of Motion. Newton's first law is known as the principle of inertia. *Inertia* is the tendency of an object to continue moving if it is moving and to remain motionless if it is at rest.

Newton's first law has two important parts.

- A body at rest remains at rest unless acted on by an outside force.
- A body in motion continues to move at constant speed along a straight line unless acted on by an outside force.

Second Law of Motion. The second law states that a force acting on an object produces an acceleration equal to the force divided by the mass of the object. This relationship is usually written as the equation $F = ma$, where F is the force, m is the object's mass, and a is the acceleration.

- **Acceleration increases with force.** Imagine two people pulling two identical wagons. If one person exerts more force, that person's wagon will accelerate more. (See pages 764–765.)



- **Acceleration decreases with mass.** Imagine two people using the same amount of force to pull two identical wagons. One wagon is empty. The other is loaded with rocks. The full wagon will accelerate less than the empty wagon because the full wagon has more mass.

Third Law of Motion. The third law states that for every action or force, there is an equal and opposite reaction or force.

For example, rockets take off by expelling gases. The downward motion of the gases creates a reaction of the rocket upward. The reaction helps it overcome gravity and fly into space.

In the case of Earth, the planet tugs at the sun in reaction to the sun's pulling on it. But because the sun has much more mass than Earth does, the sun accelerates little in response.

Friction

Many kinds of motion we encounter every day are more complicated. Imagine, for example, rolling a ball across the ground. Newton's first law states that an object in motion will continue moving unless acted on by an outside force. However, we know from experience that the ball will eventually slow down and stop.

According to Newton's laws, some force must have acted on the ball. Physicists call this force *friction*. Friction occurs when one surface moves over another. As the ball rolls, its outer surface rubs against the ground. This rubbing generates a force of friction that slows the ball's movement.

The effects of friction complicated early efforts to understand motion. Newton developed his laws by studying planets. The planets experience almost no friction as they move through nearly empty space.

Friction is the reason why

- the tires of a car grip the road.
- a conveyor belt turns on pulleys without slipping.
- ice is more slippery than concrete.

EQUATIONS FOR LAWS OF MOTION AND GRAVITY

Second Law of Motion

$F = ma$ where F is force
m is mass
a is acceleration

$F = mv/t$ where F is force
m is mass
v is velocity
t is time

Third Law of Motion

$m_1v_1 = m_2v_2$ where m_1 is mass
m_2 is another mass
v_1 is the velocity of the first mass
v_2 is the velocity of the second mass

Law of Gravity

$f = G\frac{m_1m_2}{r^2}$ where f is the attractive force
G is the gravitational constant (6.673×10^{-8} dyne-cm^2/g^2 in the cgs system)
m_1 is one mass
m_2 is another mass
r is the distance between masses

Motion of body falling to Earth

$v = v_0 + gt$ where v is velocity
v_0 is initial velocity
g is acceleration because of gravity (980 cm/s^2)
t is time

$s = v_0t + \frac{1}{2}gt^2$ where s is distance fallen
v_0 is initial velocity
t is time
g is acceleration because of gravity (980 cm/s^2)

Gravitation

Gravitation is the force of attraction that acts between all objects because of their mass. Gravitational force causes many phenomena that we observe around us.

- An object that is dropped near Earth falls toward the surface of the planet and accelerates as it falls.
- An object that is already on the surface experiences a downward force due to gravitation.
- Our bodies have weight because the Earth exerts a gravitational force on us.

Gravitation also holds together the hot gases that make up the sun, and it keeps the planets in their orbits around the sun.

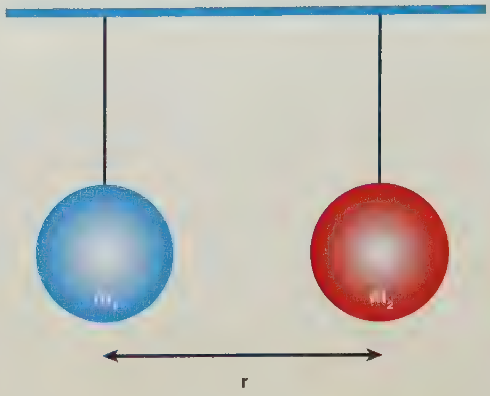
Newton's Universal Law of Gravitation

Ancient astronomers measured the movement of the moon and planets across the sky, but they didn't understand those motions. In the late 1600s, Sir Isaac Newton described a connection between the movements of the celestial bodies and the gravitation that attracts objects to Earth.

In 1665, when Newton was 23 years old, a falling apple caused him to question how far the force of gravity reaches. Using the laws of planetary motion, Newton showed how the sun's force of gravity must decrease with the distance from the sun. He then assumed that Earth's gravitational force decreases in the same way with the distance from Earth.

Newton knew that Earth's gravitation holds the moon in its orbit around Earth, and he calculated the strength of Earth's gravitation at the distance of the moon. Using his assumption, he calculated what the strength of that gravitation would be at Earth's surface.

Each particle of matter attracts every other particle with a force, F. Imagine that there are two balls attached to strings hanging down from the ceiling. Because each has mass, they attract each other with a gravitational force. The strength of the force between them depends on two properties of the balls: the mass of the balls and the distance between them. The greater the mass of the balls, the stronger the force is between them. Alternatively, the larger the distance between the balls, the weaker the force is that attracts them.

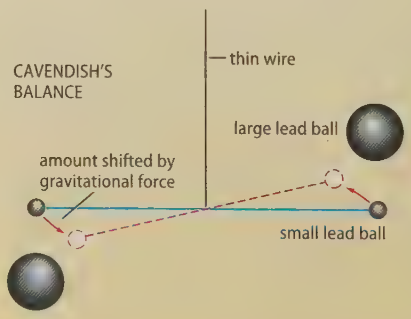


The *force of gravity* between two balls depends on the mass of the balls and the distance between their centers.

The gravitational force is directly proportional to the product of the masses, m_1 and m_2, and inversely proportional to the square of the distance, r, between them.

$$F = G\frac{m_1 m_2}{r^2}$$

The equation is referred to as Newton's Universal Law of Gravitation. In the equation, G is the gravitational constant with a value of about $6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$. The existence of this constant was known by the scientists who determined the equation, but the exact value of it was not identified until British scientist Henry Cavendish performed a critical experiment in the late eighteenth century. The diagram shows his experimental design.

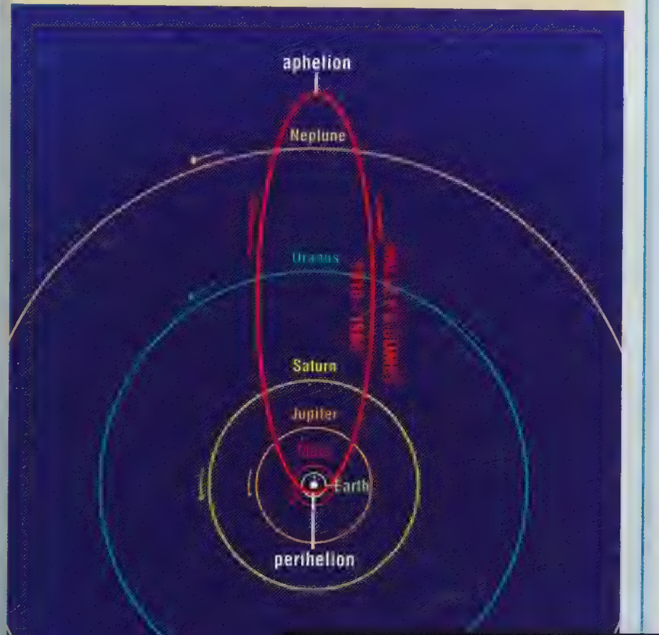


Henry Cavendish used a sensitive torsion balance to find the gravitational constant by measuring the attraction between two equal small masses and two equal large masses.

HALLEY'S COMET

Edmund Halley saw Halley's comet in 1682. It had been seen earlier, but he was the first to suggest it traveled a regular path. He said the comet would return in 1758 or 1759. It arrived on schedule, reinforcing the validity of Newton's theory of gravitation. (See pages 934–935.)

Newton's Universal Law of Gravitation is among the most tested and verified scientific laws. The motion of every planet, moon, star and comet in our universe can be predicted by calculations using this law.



Einstein's Theory

In 1915, the German-born physicist Albert Einstein announced his general theory of relativity, his theory of space, time, and gravitation. By expanding on Newton's law, it completely changed scientists' way of thinking about gravitation. (See pages 852–853.)

Einstein based his theory on two assumptions: space-time and the principle of equivalence.

Space-time. In the complex mathematics of relativity, time and space are not absolutely separate. Instead, physicists refer to space-time, a combination of time and the three dimensions of space—length, width, and height. Einstein assumed that matter and energy can change the shape of space-time, curving it. Gravitation is an effect of the curvature.

The principle of equivalence states that the effects of gravity are equivalent to the effects of acceleration. To understand this principle, suppose you were in a rocket ship so far from any planet, star, or other celestial object that the ship experienced virtually no gravitation. Imagine that the ship is traveling at a constant speed in a constant direction. If you held out a ball and released it, the ball would not fall. Instead, it would hover beside you.

BLACK HOLES

Einstein's theory predicts the existence of objects called black holes. A black hole is a region of space with a gravitational force that is so strong that not even light can escape from it. Researchers have found strong evidence that most very massive stars eventually evolve into black holes, and that most large galaxies have a gigantic black hole at their centers.

Work

Work, in physics, is a result of a force moving an object through a distance against a resistance.

Work is a word used to describe many things in everyday life. People go to work, they do yard work, or they even work through problems. In physics, though, two specific factors define work:

- the amount of force applied and
- the displacement an object achieves as it moves.

When Is It Work?

Work occurs only when the force is sufficient to move the object. (See pages 764–765.) If the object does not move, no work is done. In other words, work is a measure of what is done, not the effort applied in attempting to move the object.

For example:

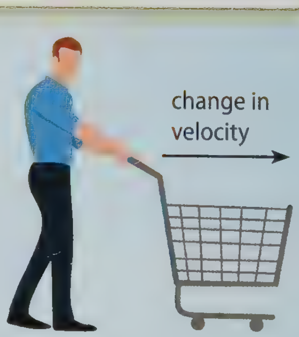
- A truck does not perform work when it holds a load of bricks.
- A truck does perform work when it moves the same load up a hill.
- People do work when lifting, pushing, or sliding an object from one place to another.
- They do no work when holding an object without moving it, even though they may become tired.

Work is done when an object moves in response to force.

Work is NOT done when an object doesn't move, even if force is applied.

Work-Energy Theorem

When someone goes shopping at a grocery store, they often grab a shopping cart in which to put their groceries. The shopping cart is initially sitting at rest with a velocity of 0 m/s, then as a force is applied the cart begins to move. It accelerates from rest, so its velocity is no longer zero. Work is definitely done on the cart because it achieves a displacement while force is applied. When work is done, then the kinetic energy of the cart must change as it accelerates. This relationship between work and kinetic energy is called the Work-Energy Theorem.



Work is done on the shopping cart as the man changes the cart's velocity which changes its kinetic energy.

$W = \Delta KE$, where W refers to the total work done by all forces acting on an object, and ΔKE represents the change in kinetic energy of the object.

Power

Sometimes work is done quickly, like when an elevator is rapidly tugged upwards in a tall building. Work can also be done slowly, such as pushing a large piece of furniture across a room. The rate at which work is done is called power. Power is how much work is done or how much energy changes per unit of time.

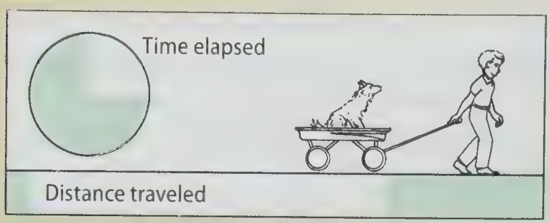
$$P = \frac{W}{t}$$

where P stands for power, W for work, and t for time. Power is typically measured in watts but can also be measured in horsepower.

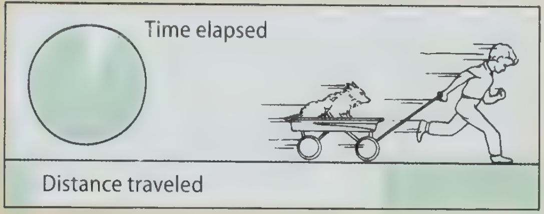
A task requires the same amount of work or energy whether it is done quickly or slowly. But the faster the task is done, the greater the power required. For example, to push a load of bricks 10 feet in 10 seconds takes twice as much power as does pushing the same load the same distance in 20 seconds.

POWER

The power needed to pull a wagon depends on its weight, and how far and how fast it is pulled.



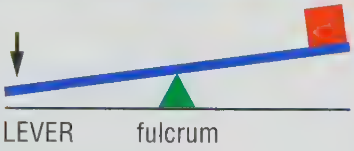
A boy uses a certain amount of power to pull a dog in a wagon over a given distance in a set time.



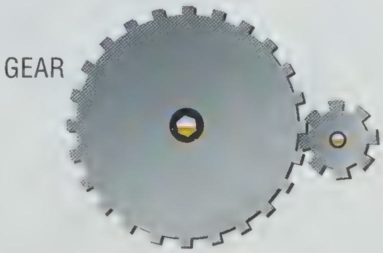
He must use twice as much power to pull the dog and wagon over the distance in half the time.

MECHANICAL ADVANTAGE

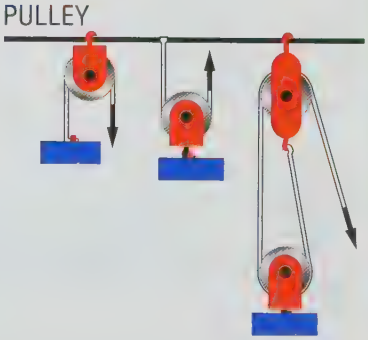
Work may sometimes be difficult to perform if it involves moving something heavy. Machines function to reduce the amount of force or displacement necessary to produce work. *Mechanical advantage* is defined as the ratio of the force exerted by a machine to the force applied to the machine. This equation describes the ideal mechanical advantage of a simple machine, but in reality the mechanical advantage must be smaller due to friction and environmental factors.



With a lever, the mechanical advantage is the ratio between the lengths of the lever on either side of the fulcrum.



With gears, it is determined by the proportion of the teeth on each gear.



With pulleys, it is equal to the number of pulleys.

Machines

Machines are devices that we use to do work. Over time, we have constructed a wide variety of machines to gain greater control over our surroundings. For example, early people made stone axes that served as weapons and tools. Today, engineers design complex machines to be safe and efficient.

Since the purpose of machines is to make life easier, we use them all the time.

- Industries use drill presses, lathes, and grinding machines to make products.
- Businesses depend on computers, copy machines, and fax machines.
- People use automobiles, buses, and airplanes to travel swiftly over great distances.
- Farmers and other vendors use trucks, railroads, and ships to haul goods to and from markets.

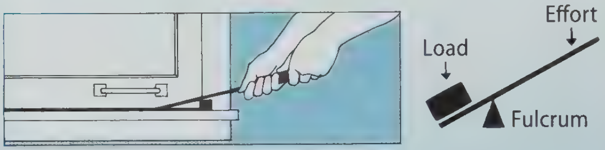
Types of Simple Machines

There are six types of simple machines: levers, pulleys, wheels and axles, inclined planes, wedges, and screws. These machines help us do work by multiplying the force we put into them or by changing the direction of that force.

Levers. A lever consists of a bar that is supported at one position while a force is applied at another position. The place where the lever is supported is the fulcrum.

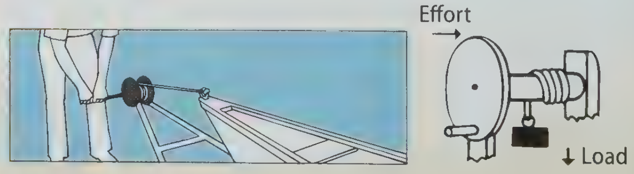
There are three basic types of levers, depending on where the effort is applied, the position of the load, and the position of the fulcrum.

- **First-class levers** have the fulcrum placed between the load and the effort, as in seesaws, crowbars, and balance scales.
- **Second-class levers** have the load between the effort and the fulcrum, as in wheelbarrows.
- **Third-class levers** have the effort placed between the load and the fulcrum, as in tweezers.



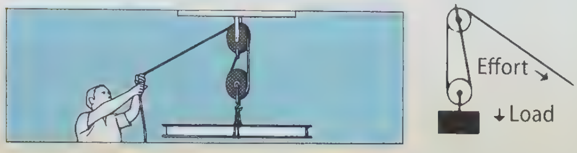
The lever is one of the earliest and simplest machines. Its advantage lies in the short distance between the fulcrum (pivotal point) and load, and in the long distance between the fulcrum and the point where effort is applied.

Wheels and Axles. The wheel and axle is a machine made of a large wheel attached to a smaller wheel (the axle) so that both wheels turn together. It is essentially a modified lever. It can move a load farther than a lever can, but the effort must move through a greater distance.



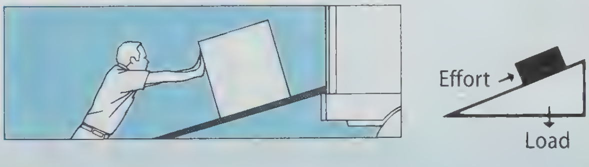
The wheel and axle has a rope attached to the axle to lift the load. The crank handle is the point where effort is applied. The effort is smaller than the load because it is at a greater distance from the axle, which is the fulcrum.

Pulleys. A pulley is a wheel over which a rope or belt is passed. It is a form of the wheel and axle. The simplest pulley is one that is fixed. This machine does not increase the force applied, but does allow a change in the direction of the force.



The pulley consists of a wheel with a grooved rim over which a rope is passed. It is used to change the direction of the effort applied to the rope. A block and tackle uses two or more pulleys to reduce the amount of effort needed to lift a load.

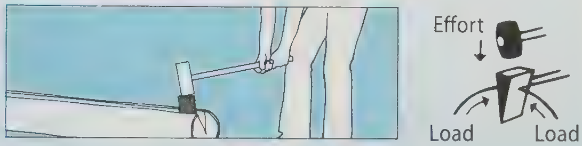
Inclined Planes. An inclined plane is a structure used to raise heavy loads with relatively small forces. For example, pushing a load up a ramp onto a platform requires less force than lifting the load onto the platform. In return, however, the load must be pushed farther.



The inclined plane makes it easier to slide or skip a load upward than to lift it directly. The longer the slope, the smaller the effort required. The amount of work, however, is no less than if the load were lifted directly upward.

Wedges. The wedge is an adaptation of the inclined plane. The effectiveness of the wedge depends on the angle of the thin end. The smaller the angle, the less the force required to raise a given load.

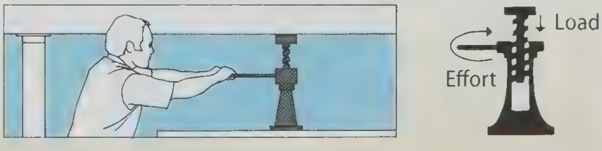
- Wedges are used to raise loads like furniture and appliances over a short distance.
- A small wedge can be used to split a large log.
- Thin wedges called shims are used by carpenters to fasten wood pieces together.



The wedge, when struck with a mallet or sledge, exerts a large force on its sides. A thin wedge is more effective than a thick one. The mechanical advantage of the wedge is of great importance.

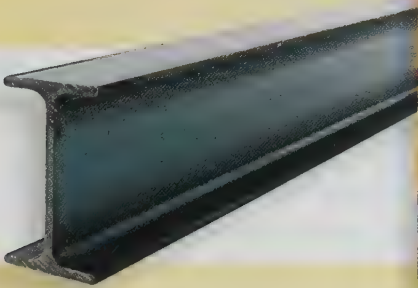
Screws. A screw is an inclined plane wrapped around a post. The ridges of the screw are called the threads. The distance between threads is the pitch. The smaller the pitch of a screw, the more times it must be turned to fasten it in place. The number of turns equals the effort distance.

Screws have many practical applications, especially as fasteners. They also open and close vises, produce motion, and perform important operations in delicate and complex machinery.



The screw is a spiral inclined plane. The jackscrew is a combination of the lever and the screw. It can lift a heavy load with relatively small effort. Therefore, it has a high mechanical advantage for practical purposes.

MATTER is the substance of which all objects are made. Since ancient times, matter has been known to exist in three states: solids, liquids, and gases. Matter is classified according to the spacing and movement of the molecules that make up the substance, which is related to the density of the substance. Matter is classified according to the spacing and movement of the molecules that make up the substance, which is related to the density of the substance. On a larger scale, solids, liquids, and gases differ in their ability to fill a container and how much they can be compressed.



EXTREME STATES OF MATTER

Scientists have now identified two additional states of matter that exist at extreme temperatures: plasmas and Bose-Einstein condensates.

Plasmas generally exist at very high temperatures and are made up of charged particles called ions. We don't often observe matter in this state on a daily basis, but it's a common state for much of the matter in the universe, such as the matter contained in stars.

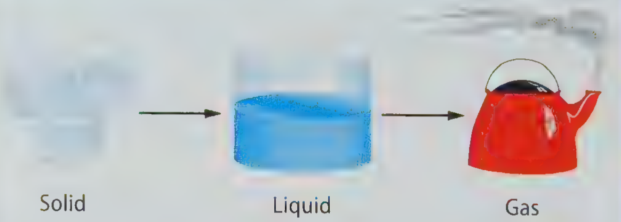
Advances in laser and magnetic technology in the mid 1990s allowed experimental physicists to dramatically cool down and trap molecules to observe their behavior. When small groups of molecules are cooled to within a few degrees of absolute zero, they tend to clump together and act as if they are one giant super-molecule. This phase of matter called Bose-Einstein condensates is useful for probing quantum mechanics and the nature of light.

Properties

Solids

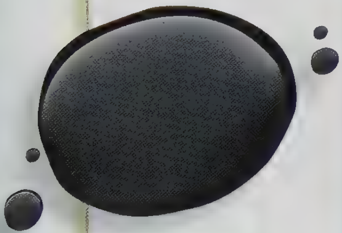
- Shape is fixed.
- Molecules cannot move freely.
- Difficult to compress
- Two types: amorphous and crystalline

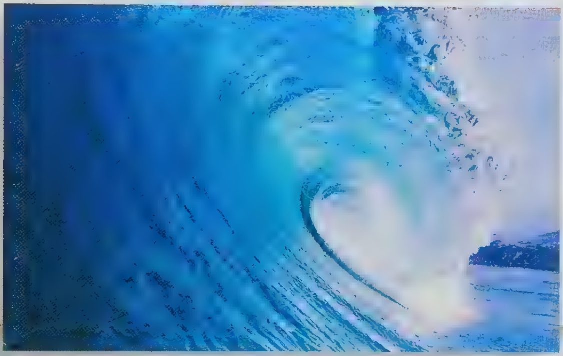
Solids have a fixed shape because the molecules can't move freely.

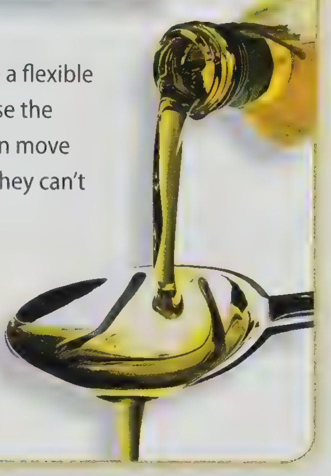


Liquids

- Assume the shapes of their containers
- No definite shape
- Molecules move freely, but don't separate
- Difficult to compress
- Have surface tension and viscosity







Liquids have a flexible shape because the molecules can move around, but they can't separate.

Gases

- Expand to fill their containers
- No definite shape
- Molecules that move freely and separately
- Easy to compress
- Particles move faster with heat

Gases have no shape or volume because the molecules are loose and active.

Thermodynamics

Thermodynamics is the study of how heat is produced, how it is transmitted from one place to another, how it changes matter, and how it is stored. It is based on three laws.

First Law

The first law of thermodynamics is the law of conservation of energy. It states that energy is never created or destroyed. (See page 381.)

Energy can change form. For example, internal energy can become mechanical motion. But the total quantity of energy in any system (group of things) remains the same.

Second Law

The second law of thermodynamics states that heat flows spontaneously from a hot object to a cool object, but never from a cool object to a hot object.

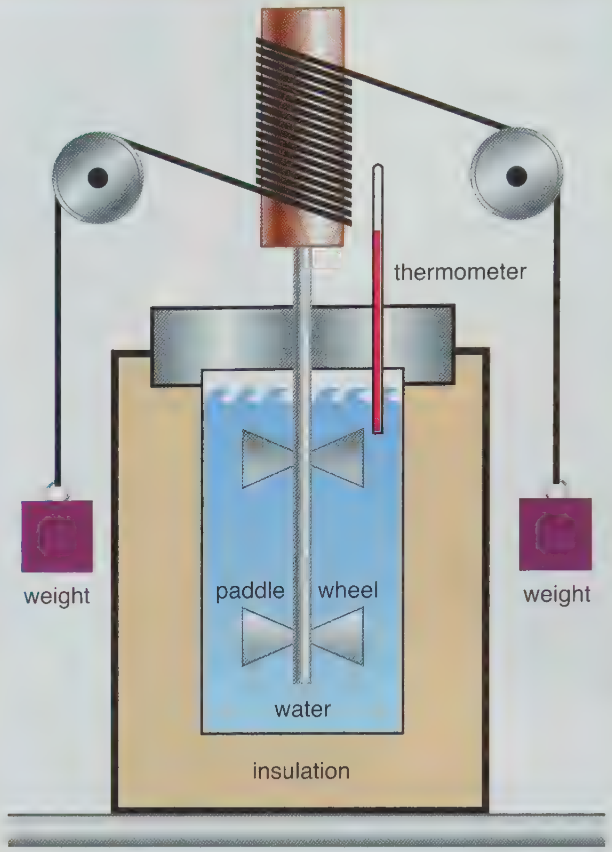
This law is important for engines that transform heat into mechanical work. They can produce only work if heat can flow from a hot reservoir to a cool reservoir, or heat sink. An automobile engine is a good example. It can work only when heat resulting from combustion is allowed to flow to a heat sink, the radiator.

Some of the heat from the radiator is used to warm the passenger compartment. Most of it is lost to the environment, so it can't be converted to mechanical work. No machine can convert heat energy completely into mechanical energy, because part of the heat energy is always lost.

Third Law

The third law of thermodynamics states that at absolute zero, the entropy of a system is zero. *Entropy* is a measure of the amount of disorder or randomness in a system.

While it is impossible for a substance to actually reach absolute zero, scientists have come extremely close.



Joule's apparatus measures the amount of heat produced when the falling weights turn the paddles. It is named after the physicist who found that a fixed amount of mechanical work always produces a fixed amount of heat. The evidence produced by this experiment led to the First Law of Thermodynamics.

THERMAL EFFICIENCIES

The thermal efficiency of an engine relates the amount of heat that is converted to work to the amount that is released as waste heat. When an engine is said to be 40 percent efficient, it means that 40 percent of the heat created by combustion, for instance, has been used, and 60 percent of the heat has been wasted.

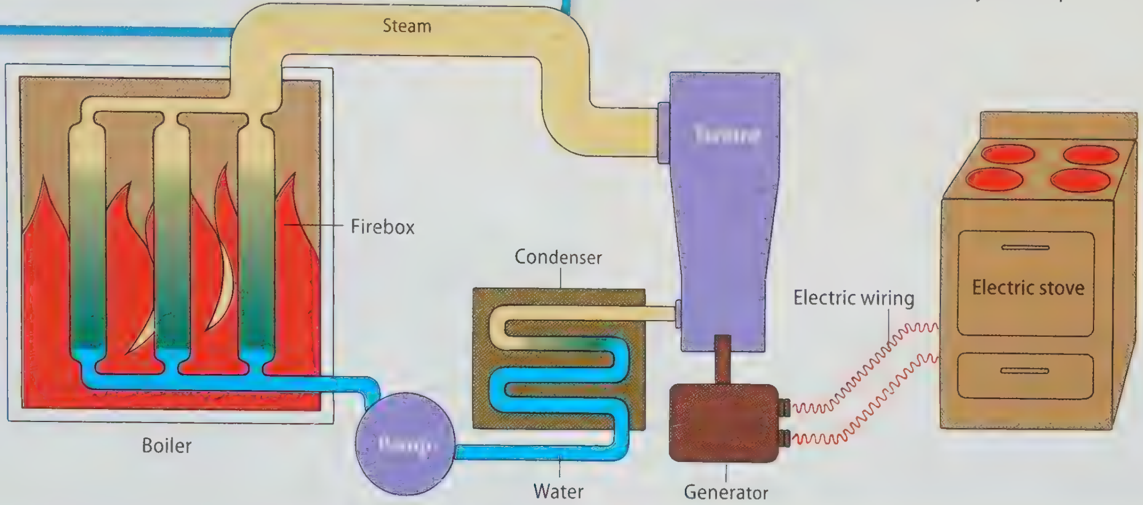
steam turbine	40%
diesel engine	38%
gasoline engine	25%
steam locomotive	8–12%

fyi!

These laws are important not only in the context of physics but also virtually every type of science and engineering. They provide a fundamental framework for understanding the behavior of systems on both microscopic and macroscopic scales. There are no known exceptions in our universe to the Laws of Thermodynamics.

Laws of Thermodynamics in Action

Heat energy can be changed into other forms of energy. This diagram shows how heat is changed into electricity. Heat in a boiler creates steam that turns a turbine. The turbine drives an electric generator. A condenser changes the steam back to water, and the cycle is repeated.



Changing Heat into Motion

Mechanical energy and heat energy are related. For example, mechanical energy is changed into heat by friction between the moving parts of a machine. Heat energy, in turn, can be changed into mechanical energy by heat engines.

There are two types of combustion engines: external and internal.

External-combustion engines produce hot gases that transfer heat energy to another fluid. The heat energy in this fluid, in turn, is changed into mechanical energy. For example, the heat from a steam turbine turns water into steam, which pushes the blades of a turbine.

Internal-combustion engines produce hot gases in which heat energy is changed directly into mechanical energy. An example is the gasoline engine in a car. Burning gasoline produces hot gases that expand and push down the pistons in the cylinders. The pistons move other parts of the car that turn the wheels.

Refrigeration

The temperature of an object can be lowered by bringing it into contact with a colder object. The temperature difference causes heat to flow from the warmer object into the colder one. For example, ice put in an insulated chest keeps food cold by removing heat from it.

A good way to remove heat from an object without using a colder object is mechanical refrigeration. It works by changing a substance called a refrigerant from a gas to a liquid back to a gas again.

In a refrigerator, for example, a compressor squeezes a gaseous refrigerant into a small space. The compression reduces the refrigerant's disorder so it becomes a liquid. The compressed liquid refrigerant then expands at a valve leading to pipes in the insulated part of the refrigerator. As the pressure falls, so does the temperature. The refrigerant absorbs heat from the foods in the refrigerator. As heat flows out of the foods, their temperatures fall. The warmed refrigerant becomes a gas and flows through pipes back to the compressor.

LIGHT is a form of electromagnetic radiation that can travel freely through space. This energy travels in patterns of electric and magnetic influence called electromagnetic waves. (See pages 810–811.)

There are many kinds of electromagnetic energy, including infrared rays, radio waves, ultraviolet rays, and X-rays. Human beings can see only a tiny part of all the different kinds of electromagnetic energy. This part is called visible light.

The Nature of Light

Light is dualistic in nature—it behaves both like a wave and like a particle traveling along a ray. Therefore, its properties must be explained by two separate and distinct theories.

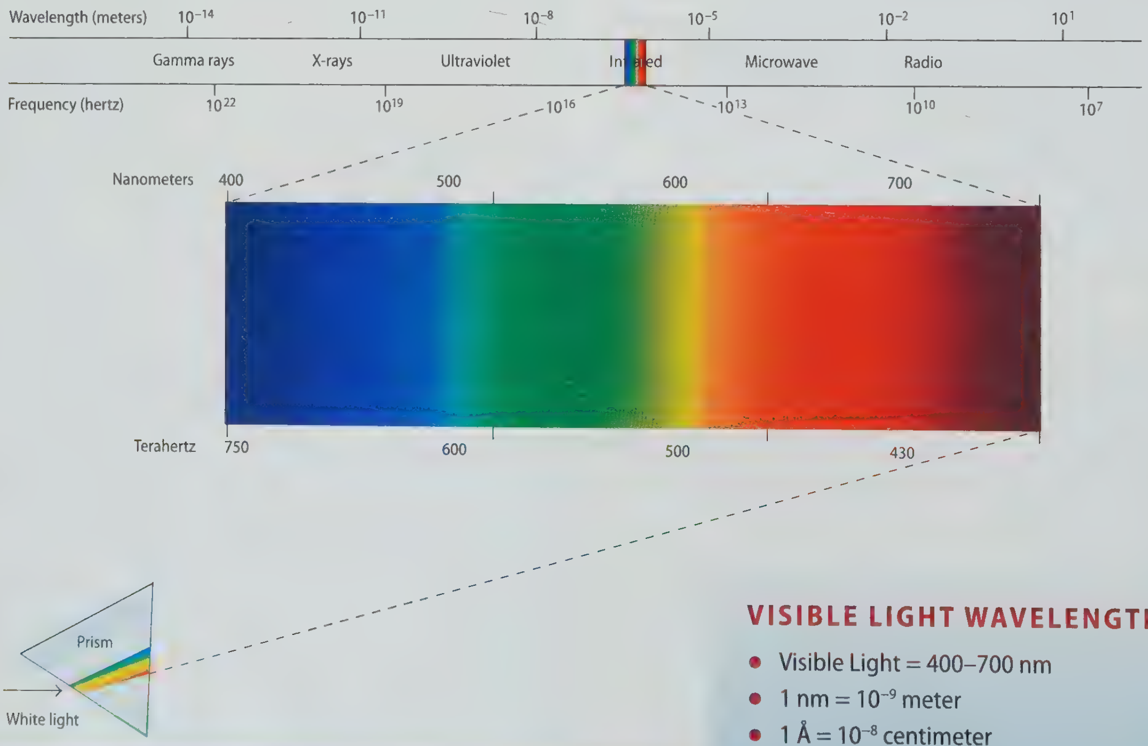
- wave theory
- quantum theory

VISIBLE LIGHT AND THE ELECTROMAGNETIC SPECTRUM

A beam of white light passing through a prism is spread into a rainbowlike band of colors called the visible spectrum. Each color has a particular wavelength and frequency. The spreading occurs because the prism bends light of different wavelengths to varying degrees.

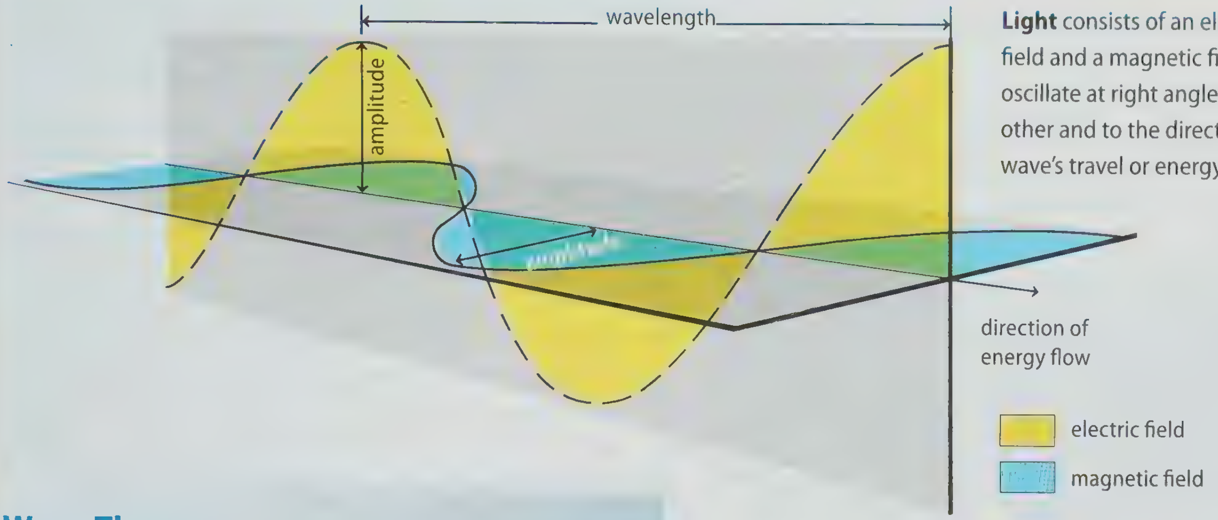
Light with shorter, or bluer, wavelengths bends the most. Light with longer, or redder, wavelengths bends the least.

All of the light that we can see makes up only a tiny portion of the full electromagnetic spectrum, as seen in the illustration.


Wavelength (meters)
10^{-14}
10^{-11}
10^{-8}
10^{-5}
10^{-2}
10^{1}
Gamma rays
X-rays
Ultraviolet
In ed
Microwave
Radio
Frequency (hertz)
10^{22}
10^{19}
10^{16}
10^{13}
10^{10}
10^{7}
Nanometers
400
500
600
700
Terahertz
750
600
500
430
Prism
White light

VISIBLE LIGHT WAVELENGTHS

- Visible Light = 400–700 nm
- 1 nm = 10^{-9} meter
- 1 Å = 10^{-8} centimeter
- 1 nm = 10 Å



Light consists of an electric field and a magnetic field that oscillate at right angles to each other and to the direction of the wave's travel or energy flow.

Wave Theory

Light is a form of energy that travels in electromagnetic waves. These waves are *transverse waves,* meaning that light energy travels at right angles to the direction in which the waves travel.

Spherical Waves. In a luminous body, such as an incandescent bulb, the light waves travel out spherically, in three dimensions, from the bulb.

Electromagnetic waves can be described as follows.

- A represents the *amplitude* of the transverse wave—the maximum intensity. In visible light wavelengths, this corresponds to the brightness of the light.
- The period, T, is the time it takes the wave to travel one wavelength, λ, or to complete one cycle.
- Thus, λ equals velocity times period, or $\lambda = vT$. Alternatively, $v = \frac{\lambda}{T}$.
- Frequency is equal to $\frac{1}{T}$, so also $v = f\lambda$..

The Electromagnetic Spectrum. Electromagnetic waves are in the electromagnetic spectrum, which is divided into several categories based on the frequency and wavelength of the waves. These categories, from long wavelengths to short, are radio waves, microwaves, infrared, visible light, ultraviolet rays, X-rays, and gamma rays. Scientists study everything from astronomical phenomena to tiny particle interactions by building detectors that are sensitive to the whole range of wavelengths.

The Visible Spectrum. In the visible light region, or *visible spectrum,* the waves are classified by the colors the human eye senses on receiving certain wavelengths. The wavelength is usually given in *nanometers* (nm). Wavelengths, especially in the visible region, are also denoted in *angstrom* (Å) units where 1 Å = 10^{-10} m. Frequency is measured in *terahertz* (trillion hertz). One *hertz* equals one cycle per second.

Visible light has wavelengths from about 400 to 700 nm (4000 to 7000 Å). Its frequency ranges from about 430 to 750 terahertz.

Quantum Theory

Quantum theory explains light effects left unexplained by electromagnetic theory. It regards electromagnetic energy as traveling in small packets. A single packet is called a *quantum,* or *photon.*

The energy in a quantum is given by the expression $e = hv$.

- e is the energy in the photon
- h is a constant called Planck's constant
- v is the frequency

The existence of these discrete photons has been shown by the work of Niels Bohr, Albert Einstein, and Max Planck.

How Light Behaves

Scientists study light by observing how it interacts with itself and with matter. They use their knowledge of how light behaves to develop new uses of light in technology. The study of light has led to the invention of such instruments as microscopes and telescopes.

SPEED OF LIGHT

Light travels at a velocity of approximately 299,793 km per second (186,283 miles per second) in a vacuum and at practically the same speed in the atmosphere. The velocity is lower in all other transparent media.

- In water, the velocity of light is approximately 225,500 km per second (140,000 miles per second).
- The distance light travels in 1 second is roughly equivalent to seven times around Earth at the equator.

Reflection and Absorption

Some materials do not let light pass through them. They are said to either *reflect* the light or *absorb* it.

Reflection. Light is reflected when most of its energy is radiated back, as, for example, by a mirror. When light strikes such shiny metal surfaces as silver, gold, or aluminum, nearly all of the energy is reflected.

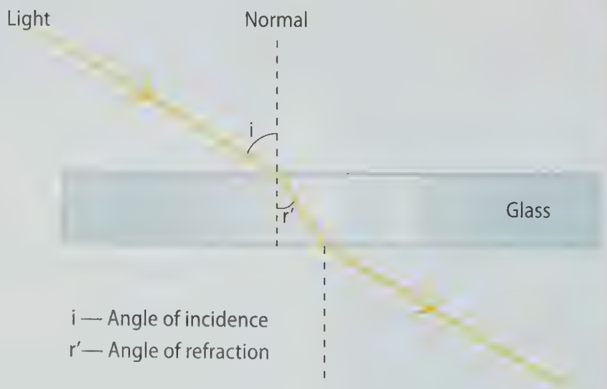
Absorption. When light strikes a surface, the amount of light that is not reflected is absorbed by the surface. A sheet of black paper does not reflect light, but absorbs it. Carbon black absorbs almost all the wavelengths of light, so it appears to us as black.

Refraction

When light passes from one substance into another and there is a difference in density of the substances, the path of the light bends. This change in the direction of a light ray is called *refraction.* (See page 800.)

The velocity of light varies in different media, being greater in a less dense medium than in a denser medium. For example, the velocity of light is greater in air than in glass. Light is slowed down more in a more dense medium as it collides with more molecules, so it will change direction more significantly.

Refraction. Light slows as it moves from air into glass, causing it to bend toward the normal. The *angle of refraction,* measured between the normal and the refracted ray, is thus smaller than the angle of incidence. As light exits the glass, it travels faster, bending away from the normal.

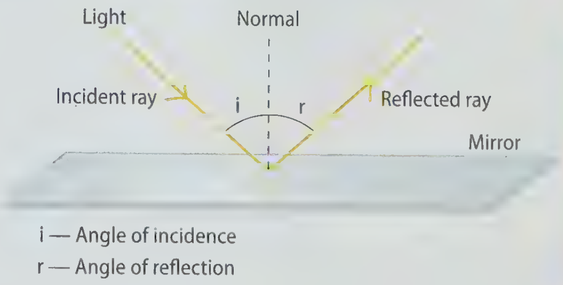


Transmission. Glass is said to *transmit* light almost perfectly. That is, it allows significant amounts of light to pass through it. Only a small part of the transmitted light is absorbed.

Creating Color. Some materials absorb only a fraction of wavelengths and reflect light at other wavelengths.

A surface will appear as red if it absorbs all wavelengths but red light. A white surface does not absorb light in any specific wavelength range, so it reflects all the wavelengths it receives.

Reflection. A ray of light will reflect from a smooth surface, such as a mirror. The *angle of reflection* is measured in relation to the *normal,* an imaginary line perpendicular to the surface. The angle of reflection equals the *angle of incidence,* the angle formed by the normal and the incident (incoming) ray.



Index of Refraction. Every material that transmits light is said to have an *index of refraction:* It is a ratio of the speed of light in vacuum to the speed of light in a material.

The index of refraction, *n,* may be stated in the form of an equation:

$$n = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in material}}$$

In the equation, the two velocities are for the same wavelength of light. The index of refraction depends, therefore, on the material and on the wavelength. Typically more dense materials have greater indices of refraction.

Snell's Law. An equation exists linking indices of refraction of two materials and angles of incidence and refraction. Many scientists over several centuries identified the equation but did not formally publish it, and Dutch scientist and mathematician Willebrord Snellius eventually received the credit for his 1621 derivation of the law.

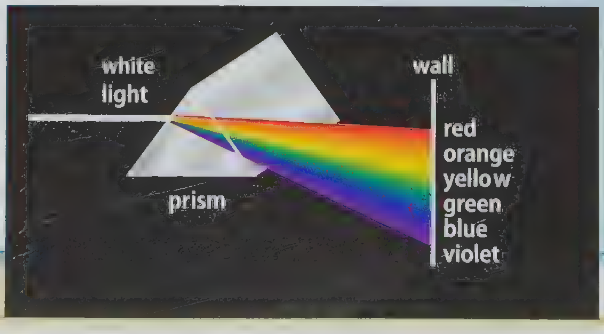
$$n_i \sin\Theta_i = n_r \sin\Theta_r$$

Each *n* stands for the index of refraction of either the incident or refracted medium. The sine of the incident and refracted angles must also be calculated.

Dispersion

Since the index of refraction of a medium is different for different wavelengths of light, the component wavelengths of white light are bent by varying amounts when the light passes through a prism.

The light is separated into its component colors: red, orange, yellow, green, blue, indigo, and violet. This process is called *dispersion.*



THEORIES OF RELATIVITY

Einstein's theories of relativity explain the behavior of matter, energy, and even time and space. They are two of the "foundation blocks" upon which modern physics is built. The theories describe events so strange that people find it difficult to understand how they could possibly occur. For example, one person can observe that two events happen at the same time, while another person observes that they occur at different times. Matter can turn into energy, and energy can turn into matter.

Special and General Relativi

Until the beginning of the 20th century, scientists believed that Newton's laws of motion and of gravity offered a satisfactory basis for explaining the physical world. (See pages 762–763.) Then a series of experiments began to raise questions about motion and about the behavior of objects moving at nearly the speed of light.

Special Relativity

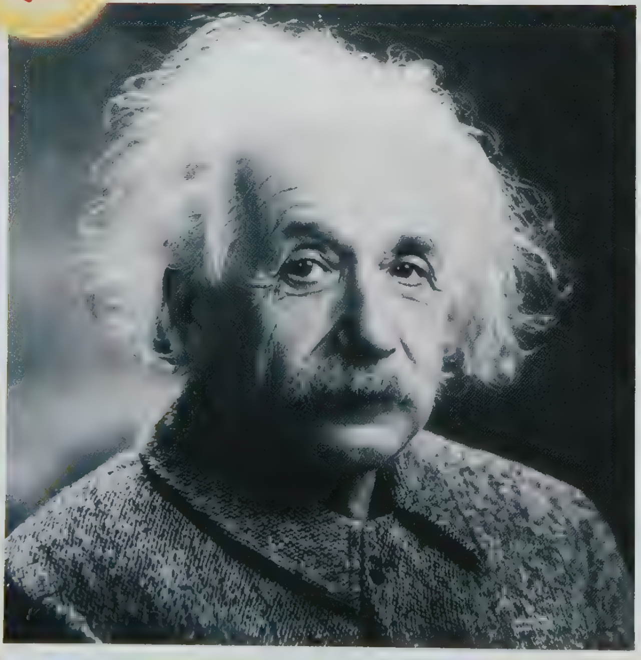
In 1905, Albert Einstein published a theory based on the notion that it is impossible to determine the absolute motion of a moving object.

Assumptions. Einstein's theory is based on the following assumptions.

- Absolute speed cannot be measured; only speed relative to some other object can be measured.
- The measured value of the speed of light in a vacuum is always the same no matter how fast the observer or light source is moving.
- The maximum velocity possible in the universe is that of light.

Spotlight on...

ALBERT EINSTEIN



1879–1955

Einstein was the most important physicist of the 1900s and one of the greatest and most famous scientists of all time. He was a theoretical physicist, a scientist who creates and develops theories of matter and energy.

- He was working as a patent clerk when he wrote three of his most famous theories.
- He left his native Germany due to anti-Semitic threats.
- In 1939, he warned the U.S. that Germany might be developing an atomic weapon. This led to the U.S. development of the atomic bomb.
- He became a United States citizen in 1940.
- A pacifist, he worked for international controls on nuclear energy and weapons after World War II.

Conclusions

From these assumptions, Einstein could show that when the velocity of a body increases, three things happen.

1. Its mass increases.
2. Its length changes: a body moving at high speed is shorter than a body at rest. However, these changes in mass and length will be perceived differently by observers moving at different speeds relative to each other. This phenomenon is called *length contraction*.
3. Time slows at high speeds. This phenomenon is called *time dilation*.

The Twin Paradox

The slowing of time can be demonstrated by the twin paradox: One twin takes off in a spaceship and makes a trip through space at a speed close to the velocity of light; the other twin stays on Earth. When the first twin comes back from his trip, he will be younger than the twin who stayed behind.

$E=mc^2$

One of the most important consequences of Einstein's theory is the equivalency of mass and energy, $E = mc^2$. This equation shows that whenever energy is created it is associated with a decrease in mass. In ordinary chemical reactions, the decrease in mass is not measurable, but in nuclear reactions, a sizable fraction of the mass of atoms is converted into energy.

◀ **Mass-to-energy conversion** is responsible for the tremendous destructive force of nuclear weapons.

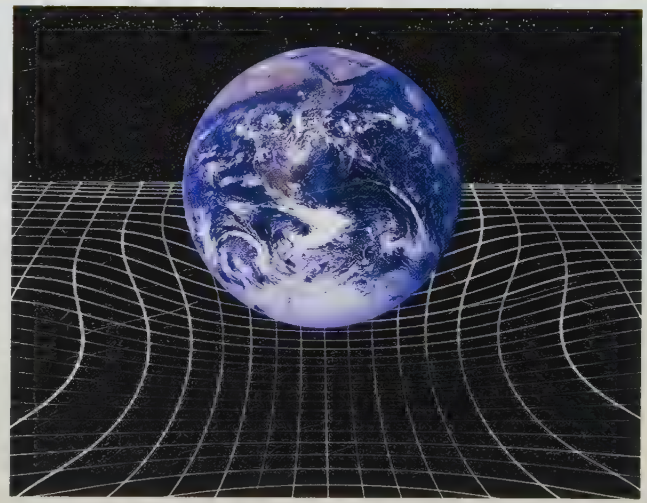
General Relativity

Einstein developed the general theory of relativity to modify Newton's law of gravitation so that it would agree with special relativity.

Testing the Theory. Einstein suggested that astronomers could make certain observations to test the general theory of relativity. The most dramatic of these would be a bending of light rays by the sun's gravitation.

- In relativity, mass and energy are equivalent.
- Because light carries energy, it is also affected by gravity.
- The light-bending effect is small, but Einstein calculated that it could be observed during a solar eclipse.

In 1919, the British astronomer Arthur S. Eddington observed it, thereby making Einstein world-famous.



▲ **In general relativity theory,** matter and energy *distort* (change the shape of) space-time, and the distortion is experienced as gravity. A more common—but less precise—way of explaining the distortion is "mass curves space." This diagram shows how space curves around a mass.

Forces and Grand Theories

Four fundamental forces exist in nature: gravity, electromagnetic force, the strong nuclear force (or strong interaction), and the weak nuclear force (or weak interaction).

- Gravity is the force of attraction between masses.
- Electromagnetic force refers to the attraction and repulsion forces between electric charges.
- The strong nuclear force holds the atomic nucleus together.
- The weak nuclear force is responsible for certain forms of radioactive decay.

How They Act. Subatomic particles interact with one another mainly through the last three forces. Strong and weak forces act only over very short distances, typically the distance between protons and neutrons in the nucleus. Over these short distances, however, the strong force is much stronger than gravitational or electromagnetic forces.

Even though gravity seems like a mighty force sinc it governs the motion of all celestial bodies and also keeps our feet planted on the earth, it is the overall weakest of the four fundamental forces compared by force strength per unit of distance.

fyi!

An experiment to test GUTs directly would require an energy about a trillion times higher than what is now available.

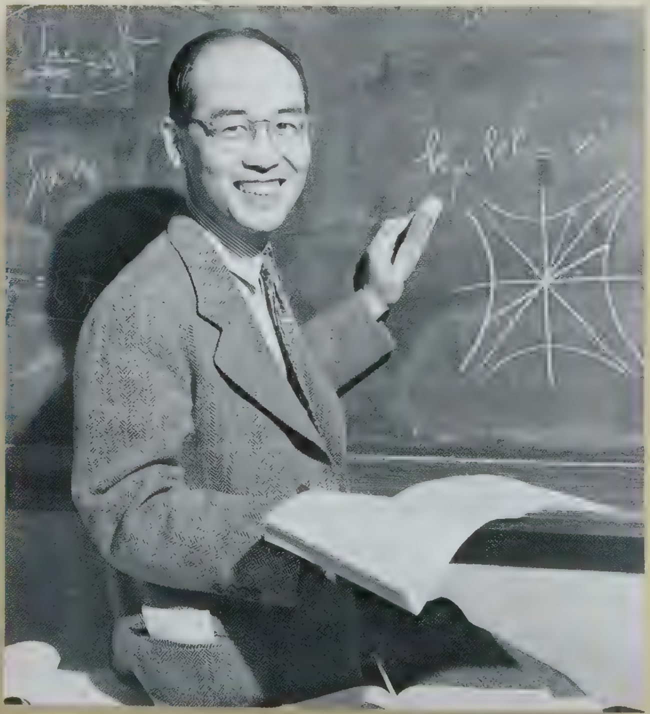
Grand Unification Theories

Several attempts have been made in recent years to explain the four fundamental forces by a single theory. These theories are called *unification schemes.* A theory that would unify the strong, weak, and electromagnetic forces is called a grand unified theory, or GUT.

Gravitation, the fourth fundamental force, is not included in GUTs. The effect of gravity on a single atom or nucleus is so weak that scientists have never been able to detect it.

Exchange Particles. One basic concept in these theories is that forces between particles are mediated by exchange particles. The mass of these particles should be inversely proportional to the range of the force. For example:

- Electromagnetic forces are transmitted by virtual photons—photons with a very short lifetime.
- Photons have no mass, which agrees with the fact that the electromagnetic force has an infinite range.
- The exchange particle for gravity, the hypothetical graviton, would also have zero mass, because gravitation is also a long-range force.



Hideki Yukawa first predicted the existence of the exchange particle for the strong force in 1935. The particle was found by a group at the University of Bristol, England, in 1947. Since the strong force is a short-range force, this exchange particle had to have a mass, which was confirmed to be 15 percent of that of the proton. ▶

The Electroweak Theory. In the early 1970s, Steven Weinberg, Abdus Salam, and Sheldon Lee Glashow formulated a theory that would unify the electromagnetic and the weak force. They called it the *electroweak theory.*

- They predicted that three particles, the W+, W–, and Z^0, would transmit the weak force in atomic nuclei.
- Since the weak force has a shorter range than the strong force, the masses of these particles had to be greater than the mass of the pi-meson, the Yukawa particle.
- They predicted that the two W particles would have a mass of 81 GeV, which is 80 times the mass of the proton, and that the mass of the Z particle would be 93.8 GeV.
- All three particles were discovered by Carlo Rubbia in 1983.

CERN. The W+, W–, and Z^0 particles were formed in collisions of protons and antiprotons in the huge proton-antiproton collider at the *Centre Européen de Recherche Nucléaire* (CERN), near Geneva, Switzerland. CERN is now home to the Large Hadron Collider. (See page 858.)

SUPERSYMMETRY AND SUPERSTRINGS

Some scientists are now trying to find a theory that would combine gravitation with GUTs. Because such a theory would explain all behavior of all forces and matter, it is termed a "theory of everything (TOE)." The concept of supersymmetry is one step in the direction of a TOE.

- Scientists divide subatomic particles into two major classes, fermions and bosons.
- Fermions are the particles that make up matter, such as quarks and electrons.
- Bosons are the particles that mediate forces, such as photons, pions, and the W and Z particles.
- Supersymmetry assumes that each fermion has a counterpart boson and each boson has a counterpart fermion.

The supersymmetric counterparts are denoted with an "s" or with the ending "-ino." For example, sleptons are the corresponding supersymmetric particles of leptons, squarks correspond with quarks, and photinos with photons.

Superstrings. By assuming that the most elementary building blocks of matter are not infinitely small points but extremely small strings—typically 10^{-35} long—scientists have been able to formulate theories that account for gravity. The combination of string theory with supersymmetry resulted in the superstring theory, which is today the most promising theory for giving a unified picture of all four fundamental forces.

M-theory. The most recent attempt at a TOE is called M-theory and involves the unifications of many versions of superstring theory. M-theory differs from superstring theory in that it conceptualizes the building blocks of matter not as strings but as two dimensional membranes. M-theory is still purely theoretical, meaning that it is only supported by mathematics and not by experimental data.

Formulas and Constants

Basic Physics Formulas

The following twenty equations are some of the basic mathematical tools that physicists use to model motion, forces, energy, and electricity.

Acceleration	$a = \frac{\Delta v}{t}$	a = acceleration (s^2) Δv = change in velocity ($\frac{m}{s}$) t = time (s)
Centripetal force	$F = \frac{mv^2}{r}$	F = force (N) m = mass (kg) v = velocity ($\frac{m}{s}$) r = radius of the circular orbit (m)
Coulomb's law	$F_E = \frac{kQ_1Q_2}{r^2}$	F_E = electrostatic force (N) k = Coulomb's constant Q_1 & Q_2 = charges (C) r = distance between charges
Density	$d = \frac{m}{V}$	d = density ($\frac{kg}{m^3}$) m = mass (kg) V = volume (m^3)
Electrical power in a circuit	$P = IV$	P = power (W) I = electric current (A) V = voltage (V)
Energy-matter relationship	$E = mc^2$	E = energy (J) m = mass (kg) c = speed of light ($\frac{m}{s}$)
Gravitational potential energy	$PE_g = mgh$	PE_g = gravitational potential energy (J) m = mass (kg) h = height (m)
Kinetic energy	$KE = \frac{1}{2}mv^2$	KE = kinetic energy (J) m = mass (kg) v = velocity ($\frac{m}{s}$)
Momentum	$p = mv$	p = momentum ($\frac{kg \cdot m}{s}$) m = mass (kg) v = velocity ($\frac{m}{s}$)
Newton's 2nd law	$\Sigma F = ma$	ΣF = sum of forces (N) m = mass (kg) a = acceleration ($\frac{m}{s^2}$)
Ohm's law	$V = IR$	V = voltage (V) I = current (rA) R = resistance (Ω)
Power	$P = \frac{E}{t} = \frac{W}{t}$	P = power (W) E = energy (J) W = work (J) t = time (s)

Snell's Law of Refraction	$n_i sin\Theta_i = n_r sin\Theta_r$	n_i = index of refraction incident medium (no units) n_r = index of refraction incident medium (no units) Θ_i = incident angle (degrees) Θ_r = refracted angle (degrees)
Torque	$\tau = Fr$	τ = torque (Nm) F = force (N) r = moment arm (m)
Universal Law of Gravitation	$F_g = \frac{GM_1M_2}{r^2}$	F_g = force of gravity (N) G = gravitation constant M_1, M_2 = masses (kg) r = distance between centers of masses (m)
Velocity	$v = \frac{\Delta x}{t}$	v = velocity ($\frac{m}{s}$) Δx = displacement (m) t = time (s)
Wave energy	$E = hf$	E = energy (J) h = Planck's constant f = frequency (Hz)
Wave speed	$v = f\lambda$	v = velocity ($\frac{m}{s}$) f = frequency (Hz) λ = wavelength (m)
Weight	$F_g = mg$	F_g = weight, or force of gravity (N) m = mass (kg) g = constant gravitational acceleration ($\frac{m}{s^2}$)
Work	$W = F\Delta x$	W = work (J) F = force (N) Δx = displacement (m)

Fundamental Constants

A constant is a numerical quantity that remains unchanged under different conditions. A constant can express a value, as of a physical substance, or a relation. In physics formulas, constants are represented by particular symbols. The symbols and values of some fundamental constants are given here.

Fundamental Constants

Quantity	Symbol	Value
Boltzmann's constant	k	$1.3806 \times 10^{-23}\ \frac{J}{K}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\ C$
Gravitational acceleration	g	$9.81\ \frac{m}{s^2}$
Planck's constant	h	$6.626 \times 10^{-34}\ Js$
Mass of electron	m_e	$9.109 \times 10^{-31}\ kg$
Mass of neutron	m_n	$1.675 \times 10^{-27}\ kg$
Mass of proton	m_p	$1.673 \times 10^{-27}\ kg$
Speed of light in a vacuum	c	$2.999 \times 10^{8}\ \frac{m}{s}$
Universal gravitational constant	G	$6.673 \times 10^{-11}\ \frac{Nm^2}{kg^2}$

PRACTICE PROBLEMS

These problems are designed to give supplementary information and practice in applying the basic concepts of physics. *Some of the information needed to solve some of these problems was not included in the section, so don't feel frustrated if you need to read the solutions on pages 878–891 before figuring it out.* Be careful—many of the problems contain information that you won't need to solve the problem. Don't let unnecessary information confuse you as you consider the questions.

Physics Practice

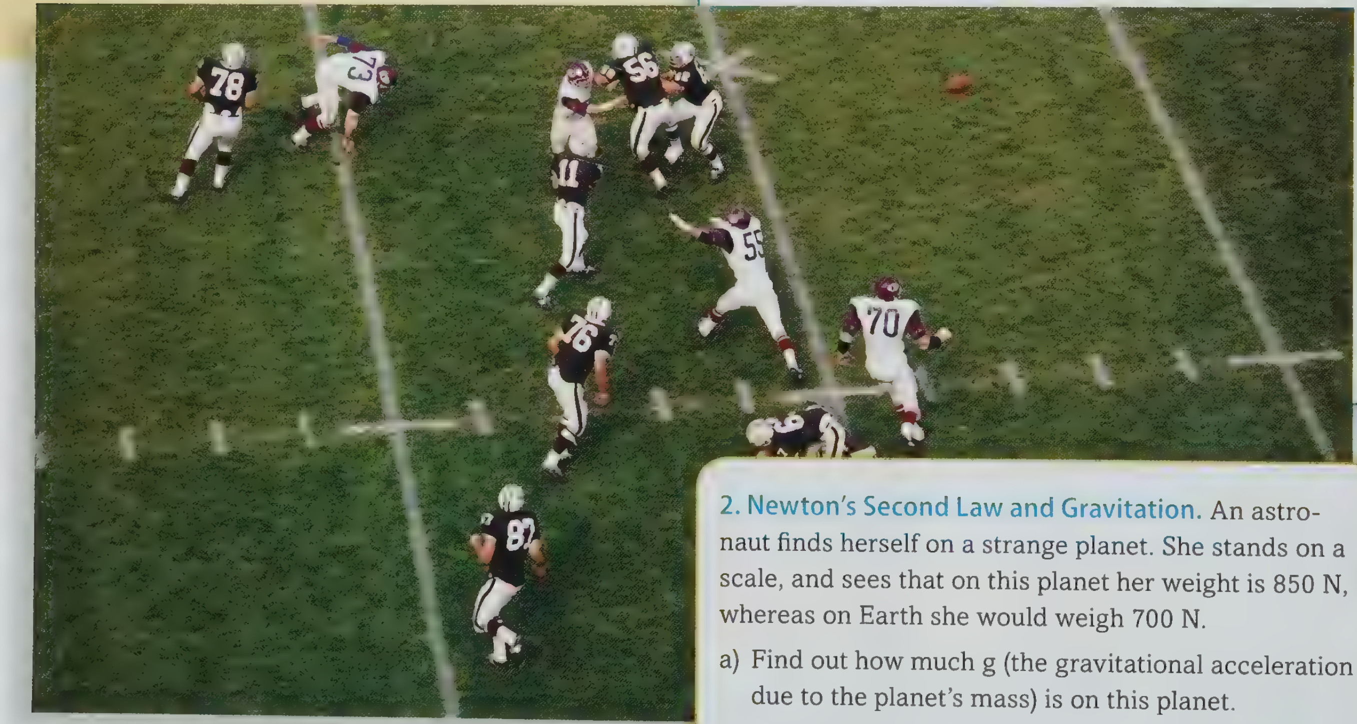
1. Vectors. The Tennessee Titans have possession of the football, and they have a few new star players.

- Lee snaps the ball to the quarterback, Bob, south 2 yards from the line of scrimmage.
- Bob runs east with the ball for 8 yards, and then throws it northwest 15 yards to Dan, for a completed pass.

a) Draw the vector diagrams and give the magnitude and direction of the net displacement (i.e., from Lee to Dan) of the football.

b) Was this a first down (i.e., 10 yards north of the line of scrimmage)?

The movements of both a football and the football players across the playing field can be described using vectors. ▼



2. Newton's Second Law and Gravitation. An astronaut finds herself on a strange planet. She stands on a scale, and sees that on this planet her weight is 850 N, whereas on Earth she would weigh 700 N.

a) Find out how much g (the gravitational acceleration due to the planet's mass) is on this planet.

b) If, on her way to this planet, she had measured that the planet's radius was $7 \cdot 10^7$ m, find out the mass of this planet.

3. Kinematics and Friction. Susan and Elizabeth go skiing at Lake Louise and find a cool jump that they think they can use to clear a small tree. But first, they want to make sure that they will build up enough speed to fly that high.

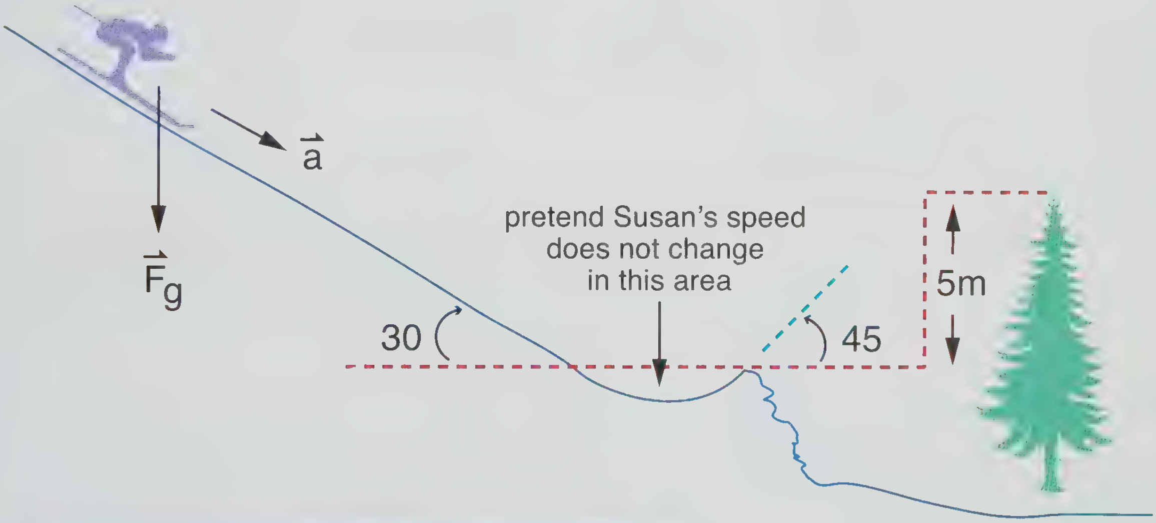
Using the information and assumptions provided, find out the minimum distance up the hill from which each girl will need to start her descent.

Information:

- Decline of hill = 30°
- Incline of jump = 45°
- Coefficient of friction between skis and snow is $u_d = 0.2$
- Height of tree above base of jump = 5 m
- Air resistance = Negligible

Assumptions:

- Assume that Susan does not use any extra lift from jumping with her knees, as she takes off from the jump.
- Assume that Elizabeth jumps with her knees at the right time, giving her an additional 2.4 m/s of vertical velocity.
- Assume that the jumper's speed after going up the jump is the same as her max downhill speed.
- Assume that if Susan and Elizabeth can gain enough height to clear the tree, then they will make it (in other words, don't worry about horizontal displacement).



4. Friction and Circular Motion. James Bond is driving his BMW, pursued by his enemy's henchman. He takes an unbanked curve so fast that his car slides at 80 miles/hr. His pursuer's car flips instead of sliding, at the same speed, because it wasn't built as well.

What was the radius of the turn he took for the car to flip over, if the coefficient of friction between the tires and the road is 0.91?

Physics Practice—Answers

1. Vectors

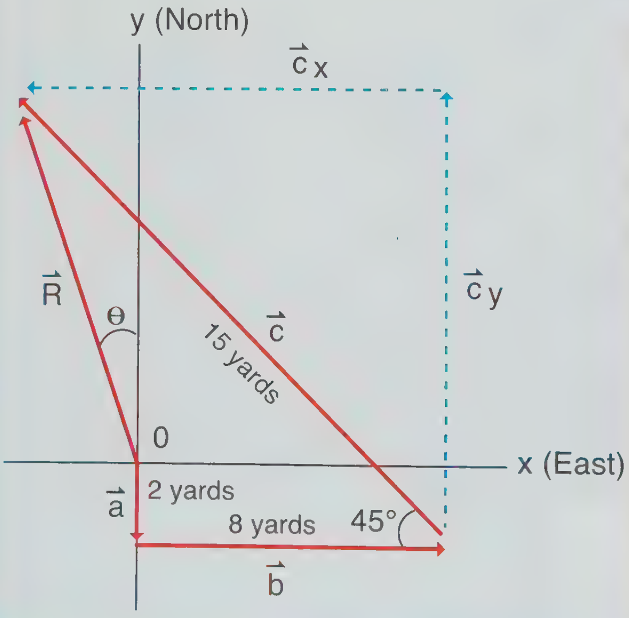
a) If Dan was tackled right at the spot where he caught the pass from Bob, then the final displacement of the ball from Lee to Dan is 9.0 yards, 29.4° west of north.
b) There was no first down, since there was a gain of only 8.6 yards.

Explanation: Vectors are a very important concept in physics, and your ability to work with many other types of problems will depend on your understanding of vectors.

Remember that vectors have both magnitude and direction. In this case, the magnitude is how far the ball gets carried, and the direction is which way. (By convention, we'll use North, South, East, West, instead of forward, back, etc.) If a direction does not directly match N, S, NW, etc., then we can state the direction by saying how many degrees west of north it is, for example. NE can also be stated as 45° east of north.

For our example, let's assign North/South to be the y-axis, and East/West to be the x-axis. Then Lee snaps the ball from the origin (0,0) yards.

The first step is to draw the vector diagram:



If R is the resultant displacement, and a, b, c, are the first, second, and third carries, then
$R = a + b + c$

To add vectors, it is necessary to break them down into their components. This means separating the vectors into the horizontal and vertical components, or x and y components. Some vectors are only either horizontal or vertical, but slanted vectors can be imagined as the hypotenuse of a triangle with horizontal and vertical sides which serve as the component vectors.

Or put another way:
$R_x = a_x + b_x + c_x$
$R_y = a_y + b_y + c_y$

Texts usually denote a vector by writing in boldface the letter that is used to represent it (e.g. **A**). Handwritten, they are usually denoted by an arrow above or a line underneath (e.g. **A**, $\vec{A}$, $\bar{A}$, $\underline{A}$). You can use whatever you like.

2. Newton's Second Law and Gravitation

a) $11.91\ m/s^2$
b) $8.74 \bullet 10^{26}$ kg

Explanation: From NII (Newton's 2nd law), we know $F = ma$. We know that a body's mass does not change, so $F' = ma'$ (where ′ is called *prime*, meaning new or different).

In this case,

$a = g =$
gravitational attraction of Earth =
$9.81\ m/s^2$

We also know that the weight is given by the force that the planet's gravitational pull exerts on the body, so

$Fg = mg$ (= weight)

Two vertical lines around a vector, | |, are used to indicate magnitude only. |a| means the distance that *a* carries the ball, and we don't care about direction. Now, to find *c*,

- we can use trigonometry relations, such as

$$\sin 45 = \frac{|c_x|}{|c|}$$

and

$$c_x = \sin 45 \cdot |c|$$

- or we can use Pythagoras:

$$|c|^2 = |c_x|^2 + |c_y|^2 = 2|c_x|^2$$

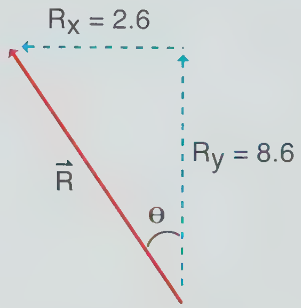
(since $|c_x| = |c_y|$)

to find that $c_x = (-10.6, 0)$ yards and $c_y = (0, 10.6)$. You can look at the diagram to make sure of the +/− sign. So now we know that

$R_x = +8 - 10.6 = -2.6$ yards and

$R_y = -2 + 10.6 = 8.6$ yards

Let's redraw our diagram to fill in the values of R.



We can use Pythagoras again to find the magnitude of R, since

$R = R_x + R_y$, so

$|R|^2 = |R_x|^2 + |R_y|^2 =$

$2.61^2 + 8.61^2 = 80.86$, and so:

$|R| = \sqrt{80.86} = 8.99$ yards.

Right away, you can see that this was not a first down, since R_y is less than 10 yards.

The direction can be found with the relation

$$\tan(\theta) = \left|\frac{R_y}{R_x}\right| = \frac{2.6}{8.6} \text{ and}$$

$$\theta = \tan^{-1}\left(\frac{2.6}{8.6}\right) = 29.4°$$

By isolating m in the two equations F = ma and F′ = ma′, we can see that

$$m = \frac{F}{a} = \frac{F'}{a'}$$

Then

$$a' = g \cdot \frac{F'}{F} = 9.81\ \text{m/s}^2 \cdot \frac{850\text{N}}{700\text{N}} = 11.91\ \text{m/s}^2.$$

So the acceleration due to gravity on this planet, g′, is 11.91 m/s^2.

Note: Remember not to confuse G (the gravitational constant, which doesn't change, and equals

$$6.67 \cdot 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

with g (the acceleration on a body due to the gravitational pull of the planet, which changes with the planet's mass and radius).

To find the mass of this planet, we can use Newton's law of gravitation.

$$F = G \cdot \frac{m_1 m_2}{r^2}$$

We want to find the mass of this planet, which we will call m_2. The astronaut will be the mass m_1. Then we have

$$m_2 = \frac{F_g \cdot r^2}{(G \cdot m_1)}$$

From part (a), we know that

$$\frac{F_g}{m_1} = a' = 11.91\ \text{m/s}^2$$

So:

$$m_2 = \frac{F_g}{m_1} \cdot \frac{r^2}{G}$$

$$= 11.91\ \text{m/s}^2 \cdot \frac{(7 \cdot 107\text{m})^2\ \text{kg}^2}{6.67 \cdot 10^{11}\ \text{Nm}^2}$$

$$= 8.74 \cdot 10^{26}\ \text{kg}$$

Physics Glossary

absolute zero the temperature at which substances have no heat whatever, and all molecules stop moving. Theoretically, absolute zero is the lowest temperature, –273.15 degrees Celsius or –459.67 degrees Fahrenheit.

absorption in optics, the act of taking in, as opposed to reflecting back. A sheet of black paper does not reflect light but absorbs it.

acceleration a change in velocity over a period of time.

acoustics the branch of physics that deals with sound. It includes the study of how sound is produced, transmitted, detected, measured, and used.

aerodynamics the branch of physics and engineering that deals with the movement of air and the ways in which forces of air act upon objects in motion (such as airplanes).

alpha particle the nucleus of a helium atom, consisting of two protons and two neutrons. Alpha particles are emitted by radioactive materials.

amplitude the vertical distance from the crest or trough to the equilibrium position of a wave.

antiparticle elementary particle corresponding to a normal particle but opposite to it in electric and magnetic properties. When brought together with its counterpart, the result is their mutual annihilation.

atom the smallest unit of matter having the characteristics of an element. The atom is composed of a nucleus which contains neutrons with no electrical charge as well as positively charged protons and surrounded by revolving negatively charged electrons. Most of the mass of an atom is in its nucleus.

beta particle an electron emitted during radioactive decay of such substances as thorium and uranium. Positive beta particles are positrons; they are emitted during the radioactive decay of certain elements.

capacitor a device that stores and releases electric charge.

center of gravity the point at which the entire weight of a body is concentrated.

composite particle a particle made up of smaller elementary particles, such as quarks.

concave curved in, like the inside of a sphere or circle.

concurrent forces forces acting simultaneously on a mass.

conduction the movement of heat through a material.

conductor a material through which electric charge moves easily.

constant a measurement or quantity that never changes.

convection the transfer of heat by the movement of a heated material.

convex curved out, like the outside of a sphere or circle.

critical point the temperature at which the liquid and gaseous phases of a pure stable substance become identical.

cryogenics the study of extremely low temperatures.

density the mass per unit of volume. The density of an object once determined is used to calculate specific gravity.

diffraction the spreading out of waves as they pass through an opening or by the edge of an obstacle.

diffusion the scattering of light in all directions during transmission or reflection. During transmission, diffusion is caused by light waves striking minute particles. During reflection, diffusion is caused by irregularities in the reflecting surface.

dispersion in optics, the separation of light into its component colors: red, orange, yellow, green, blue, indigo, and violet.

displacement the difference between the final and initial positions of a moving object.

distance the total length that an object travels regardless of direction.

Doppler effect the change in frequency of sound or light waves caused by the relative motion of the source of the waves and their observer.

dynamics the study of objects that change their speed or the direction of their motion because of forces acting on them.

electric circuit the closed loop path followed by an electric current.

electric current the flow of electric charge through a conductor. The current is measured in amperes; 1 ampere equals the flow of 1 coulomb (the unit of electric charge) per second.

electric field the space in which a charged particle experiences an electrostatic force exerted by another charge.

electric signal an electric current or voltage modified in some way as to represent information, such as sounds, pictures, numbers, or letters.

electric voltage the difference in electric potential energy per unit charge in an electric field; a voltage causes charges to move in an electric field.

electric potential energy the energy possessed by charged particles due to their relative position in an electric field; doing work on a charge changes its electric potential energy

electrode a terminal, pole, or conductor that conveys a flow of electrons through a solid, a molten solution, a gas, or a liquid. Electrodes are either negatively or positively charged.

electrostatic force the attraction between positive and negative charges that binds protons to electrons in an atom.

electrolysis the application of a direct electric current through the positive electrode (anode), electrolytes, and negative electrode (cathode) of an electrolytic cell to produce a chemical reaction.

electrolyte in electrolysis, any substance (usually an acid, base, or soluble or molten salt) that dissociates into its respective cations (positive ions) and anions (negative ions) by the application of a direct electric current.

electromagnet a temporary magnet produced by electric currents.

electromagnetic force a force that consists of two parts: (1) the electric force and (2) the magnetic force. The electric force is an attraction or repulsion between objects that carry an electric charge. The magnetic force arises from two sources: (1) the combined magnetic forces of "miniature magnets" and (2) the movement of electrons through wires as electric current.

electromagnetic spectrum the entire range of wavelengths or frequencies according to which electromagnetic radiation is classified; the range is from about 10^{-14} meter (gamma rays) to about 10^7 meters (radio waves).

electromagnetic wave a wave of energy made up of an electric and a magnetic field, generated when an electric charge oscillates or is accelerated. The chief kinds of electromagnetic waves, ranging from longest to shortest wavelength, are long radio waves, short radio waves, infrared rays, visible light, ultraviolet light, X-rays, and gamma rays.

electromagnetism the branch of physics that studies the relationship between electricity and magnetism. Electromagnetism is based on two facts: (1) An electric current or a changing electric field produces a magnetic field and (2) A changing magnetic field produces an electric field.

electron a subatomic particle with negative charge that commonly occupies the outer region of the atom. An electron has a mass of about 1/1,800 that of a proton or about 9.11×10^{-31} kg.

Space Science

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Making Progress

The influence of Aristotle on scientific thinking throughout the Middle Ages was so great that most scholars contented themselves with teaching his theories and elaborating on them without attempting much innovation. Most of the teaching was in the hands of the Church, and religious thinkers believed that Aristotle's ideas were in accordance with the Holy Scriptures.

Galileo's telescopes were larger and more powerful than the telescopes that had been made previously.

Galileo

Galileo Galilei (1564–1642), an Italian physicist and astronomer, was a contemporary of astronomer Johannes Kepler. Galileo has been called the founder of modern experimental science. He discovered several new scientific laws and designed a variety of scientific instruments.

Galileo's Telescope. In 1609, while Galileo was staying in Venice, he heard about a system of lenses, invented in Holland, that magnifies distant objects. Galileo set about assembling his own instrument.

Galileo's Discoveries. When Galileo directed his telescope to the night sky, he made more discoveries than had been made by all the astronomers who had lived before him, discovering the craters on the moon, the moons of Jupiter, and the phases of Venus.

Copernicus

Around 1507, Nicolaus Copernicus (1473–1543), himself a church official, became convinced that Ptolemy's system was wrong. He proposed the idea that Earth, instead of being the fixed center of the universe, is a planet circling the sun.

Copernicus completed his famous book *De Revolutionibus Orbium Coelestium (Concerning the Revolutions of the Celestial Orb)* around 1533, but he postponed publication of it until 1543, shortly before his death, since he feared being accused of heresy.

Copernicus's System. Similar to Ptolemy's system, Copernicus's system uses perfect circles and deferents to describe the motions of the planets, with the difference that Earth revolves around the sun.

Discrepancies. Copernicus explained that the discrepancies between his model and actual observations were caused by observational errors. He tried to account for the errors by adding circles.

Brahe's Splendid Data

A great advance in observational astronomy came from the experimental skills of Danish astronomer Tycho Brahe (1546–1601). Brahe not only developed instruments of novel design that greatly increased observational accuracy, he also introduced sound observational procedures.

Brahe's Assistant. The improved determinations of stellar and planetary positions served as ideal material for Johannes Kepler (1571–1630), a German astronomer and mathematician whom Brahe appointed as his assistant not long before Brahe's death.

Elliptical Orbits. Kepler used Brahe's splendid data to devise a heliocentric system that would agree with the observations. His outstanding finding was that the true shape of the orbit of a planet around the sun is not a circle, but an ellipse with the sun at one of the foci of the ellipse, not at its center.

Kepler's Laws

Kepler used Brahe's observations of Mars to figure out three laws of planetary motion. Kepler developed his laws for the planets of our solar system, but astronomers have since realized that Kepler's laws are valid for all orbiting heavenly bodies. With these three laws, a planet's path and orbital velocity can be determined.

Kepler's first law states that the orbit of every planet is an ellipse with the sun at one focus.

Kepler's second law states that the *radius vector,* an imaginary line joining the sun to a planet, sweeps over equal areas in equal times.

Kepler's third law states that the square of the time any planet takes to complete its orbit is proportional to the cube of its distance from the sun. When a planet is closer to the sun, it moves faster. When it is farther from the sun, it moves more slowly.

UNIVERSAL GRAVITATION

Sir Isaac Newton developed his revolutionary theory of universal gravitation while he was staying in the country to avoid a plague outbreak.

Newton realized that the same force that pulls an object toward the Earth also keeps the moon in its orbit. He found that the force of universal gravitation makes every pair of bodies in the universe attract each other. (See pages 766–767.)

The force depends on

1. the amount of matter in the bodies being attracted.
2. the distance between the bodies.

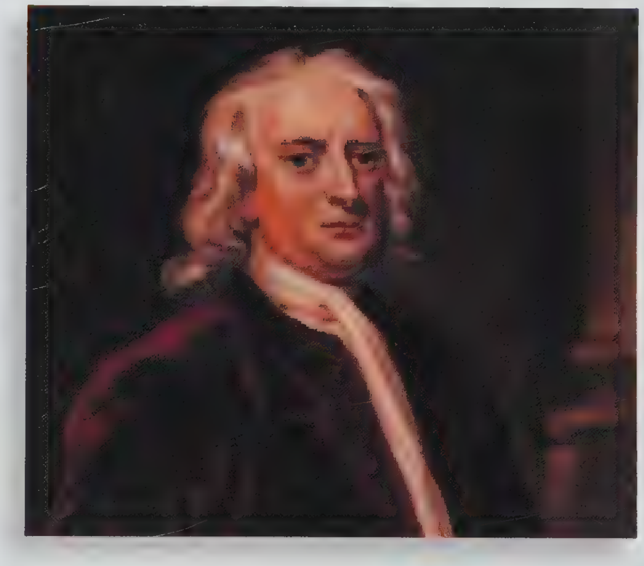
Newton

If Kepler showed *how* the planets move, it was Isaac Newton (1642–1727), an English scientist, astronomer, and mathematician, who showed *why* they move in elliptical orbits.

Delaying Gravity. Before he was 24, Newton had deduced from Kepler's laws that the force keeping a planet in its orbit must be inversely proportional to the square of the distance between the planet and the center about which it revolves.

However, Newton did little with his result for almost 20 years. In 1684, the physicist Edmund Halley (1656–1742) urged him to publish his law of gravity.

Sir Isaac Newton is sometimes described as "one of the greatest names in the history of human thought" because of his great contributions to mathematics, physics, and astronomy. ▼



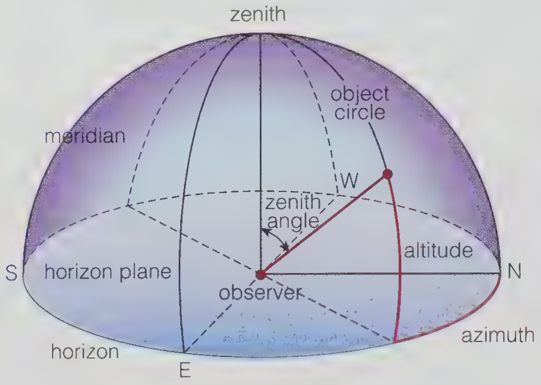
The *Principia*. The resulting manuscript, *Philosophiae Naturalis Principia Mathematica* (the *Principia*), published in 1687, sets forth the physical principles for the motion of bodies under the influence of gravitational forces. Newton's work demonstrates mathematically that a body moving under a force inversely proportional to the square of the distance moves on an elliptical path.

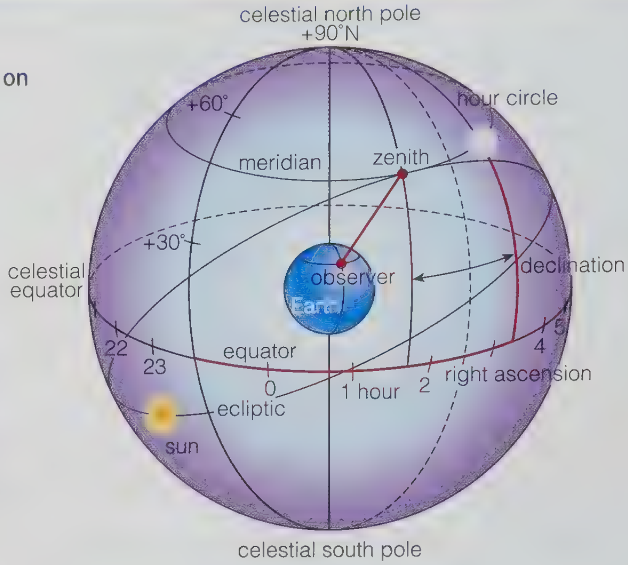
The *Principia* is considered one of the greatest single contributions in the history of science.

Among the Stars

FINDING YOUR WAY IN THE STARS

Astronomers use two main systems to pinpoint objects on the celestial sphere.





Horizon System

It is important to realize that angular distances between objects on the celestial sphere do not represent real distances between them. Two stars that appear close on the celestial sphere can actually be very far apart. The simplest coordinate system is called the *horizon system*.

- The observer projects the horizon onto the celestial sphere to form a great circle called the *astronomical horizon*.
- Starting from the zenith, one can draw an arc, called a *vertical circle*, that intersects the astronomical horizon.
- The height of the star in degrees on the vertical circle above the astronomical horizon is called the *altitude* or *elevation* of the star. The altitude of the zenith is precisely 90 degrees.
- The angle along the astronomical horizon from the direction of the North Pole to the vertical circle of the star is called the *azimuth* of the star.
- The azimuth is counted turning from north to east to south to west. For example, the azimuth of a star directly west is 270 degrees.
- The altitude and the azimuth together determine the position of the star on the celestial sphere.

Disadvantages. The horizon system has two disadvantages.

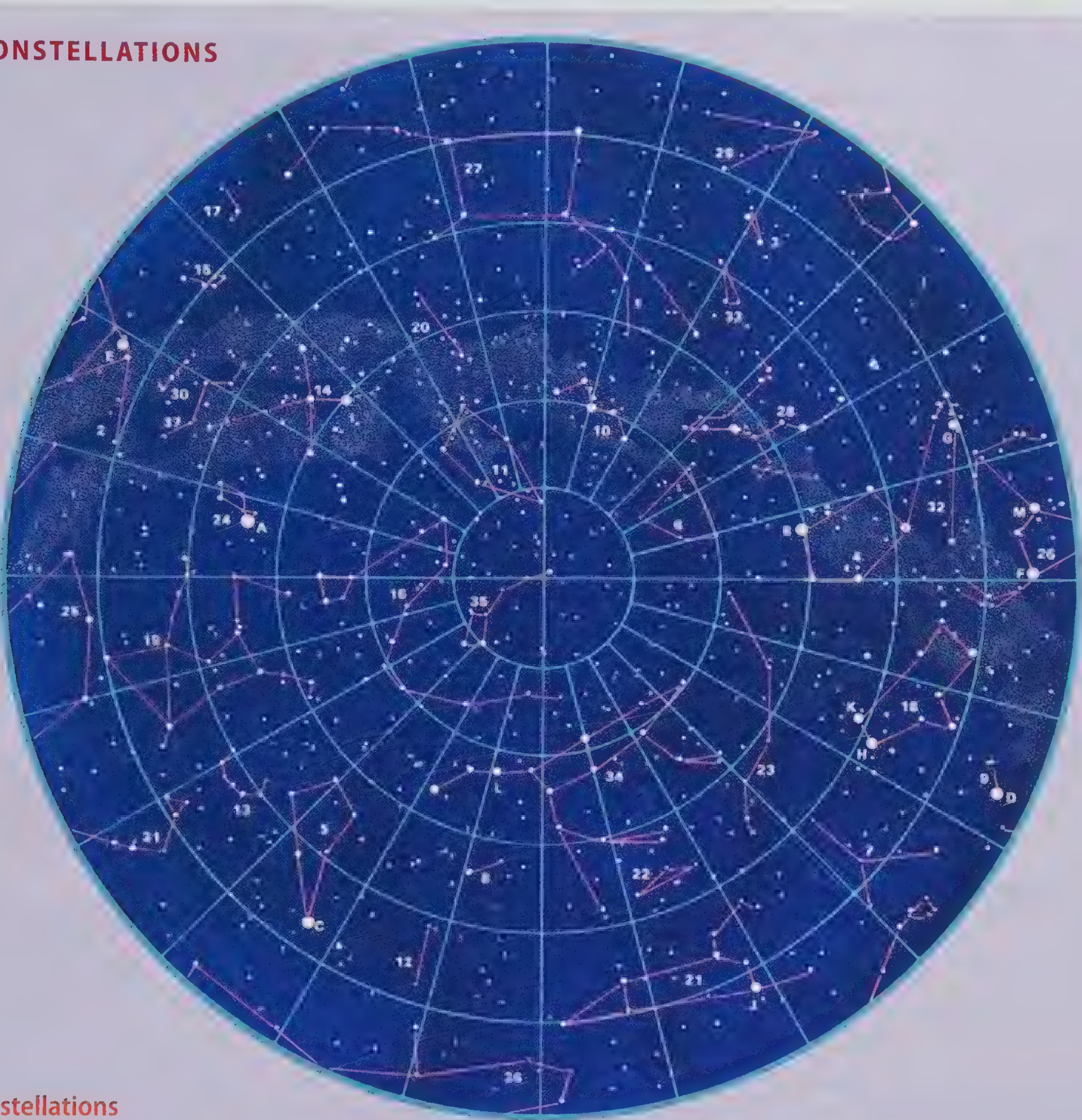
1. The position of the star depends on the location of the observer on Earth.
2. As Earth rotates, the coordinates of the star change continuously with time.

Equatorial System

Astronomers prefer to use equatorial coordinates, a system that is independent of the rotation of Earth and of the location of the observer on Earth.

- The celestial sphere is assumed to rotate around the same axis as Earth's axis.
- The *celestial poles* are the points around which the celestial sphere seems to rotate.
- The *celestial equator* is the projection of Earth's equator on the celestial sphere.
- The *ecliptic* is the apparent path of the sun around the celestial sphere.
- The ecliptic intersects the celestial equator in two points, called *equinoxes*.
- Starting from the celestial pole, an imaginary circle called an *hour circle* intersects the celestial equator.
- The *declination* of a star is its angular distance north or south of the celestial equator, measured along the hour circle passing through the star.
- The angular distance between the hour line of a star and the vernal equinox (the point the sun crosses the equator on the first day of spring) is called the *right ascension* of the star.
- The right ascension is measured, turning west-south, in hours, minutes, and seconds rather than arc degrees.
- The choice of time units is easily understandable if one remembers that it takes a star 24 hours to describe a full circle in the sky.

THE CONSTELLATIONS



The Constellations

Latin Name	English Name
1. Andromeda	The Chained Lady
2. Aquila	The Eagle
3. Aries*	The Ram
4. Auriga	The Charioteer
5. Boötes	The Herdsman
6. Camelopardalis	The Giraffe
7. Cancer*	The Crab
8. Canes Venatici	The Hunting Dogs
9. Canis Minor	The Lesser Dog
10. Cassiopeia	The Lady in the Chair
11. Cephus	Cassiopeia's Consort
12. Coma Berenices	Bernice's Hair

Latin Name	English Name
13. Corona Borealis	The Northern Cross
14. Cygnus	The Swan
15. Delphinus	The Dolphin
16. Draco	The Dragon
17. Equuleus	The Lesser Horse
18. Gemini*	The Twins
19. Hercules	Hercules
20. Lacerta	The Lizard
21. Leo*	The Lion
22. Leo Minor	The Lesser Lion
23. Lynx	The Lynx
24. Lyra	The Lyre
25. Ophiuchus	The Serpent Holder

Latin Name	English Name
26. Orion	The Hunter
27. Pegasus	The Winged Horse
28. Perseus	Persons
29. Pisces*	The Fishes
30. Sagitta	The Arrow
31. Serpens	The Serpent
32. Taurus*	The Bull
33. Triangulum	The Triangle
34. Ursa Major	The Great Bear (The Big Dipper)
35. Ursa Minor	The Little Bear (The Little Dipper)
36. Virgo*	The Virgin
37. Vulpecula	The Fox

Brightest Stars

A. Vega
B. Capella
C. Arcturus
D. Procyon
E. Altair
F. Betelgeuse
G. Aldebran
H. Pollux
I. Deneb
J. Regulus
K. Castor
L. Alioth
M. Bellatrix

*Located in the Zodiac

Beyond the Telescope

Vast Distances

Many distances involved in astronomy are so huge that they are measured in special units. To get a sense of the vast sizes and distances involved in the study of space objects, consider the tables below.

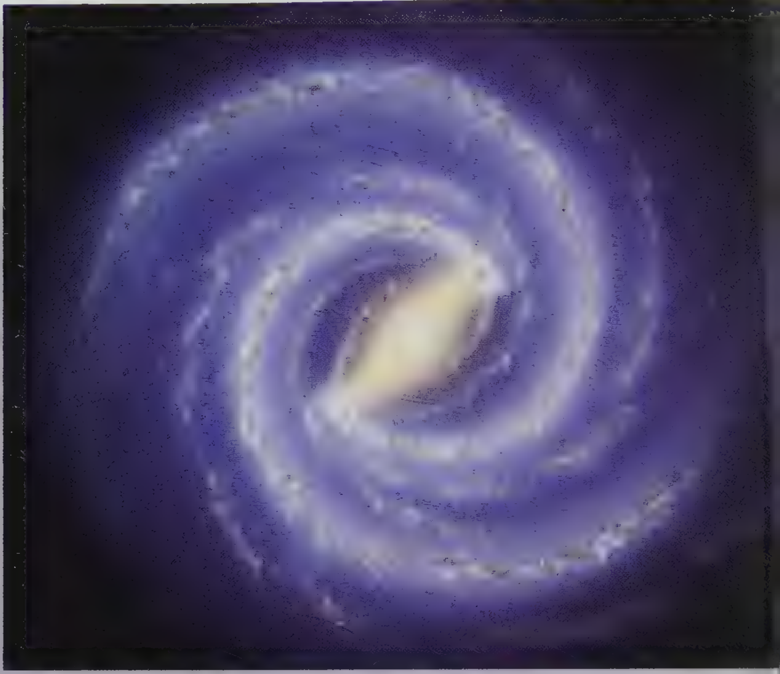
Light-Years. One such unit is the *light-year,* the distance that light travels in a vacuum in a year.

- This distance equals about 9.46 trillion kilometers (5.88 trillion miles).
- The star nearest the sun, Proxima Centauri, is about 4 light-years from Earth.
- The Milky Way is about 100,000 light-years across.
- The sun is roughly 25,000 light-years from the center.
- The nearest large galaxy is the Andromeda Galaxy. It is about 2.5 million light-years away.
- Some galaxies are more than 13 billion light-years distant.

Astronomical Units. Astronomers measure distances in the solar system in *astronomical units (AU).*

- One AU is the average distance from Earth to the sun—about 93,000,000 miles (150,000,000 kilometers).
- This distance equals about 8 light-minutes.
- The average distance from the sun to Neptune, the farthest planet, is about 30 AU.

Parsecs. In their technical work with extremely long distances, astronomers use a unit called a *parsec,* rather than the light-year. The parsec is based on *parallax,* an angular measurement. One parsec equals about 3.26 light-years.



▲ **The Milky Way** measures about 100,000 light-years across.

Sizes of Astronomical Objects

Object	Diameter	Equivalent Units
Earth	12,700 km (7,900 mi.)	Jupiter, the biggest planet in the solar system, is more than 11 times as large.
The sun	1,390,000 km (864,000 mi.)	The sun is about 109 times as large as Earth. The largest stars are about 1,500 times as large as the sun.
The Milky Way	100,000 light-years	About 700 billion times as large as the sun
Markarian's Chain of galaxies	2.5 million light-years	About 25 times the diameter of the Milky Way, and located about 70 million light-years away

Units of Distance in Astronomy

Unit	Metric	English	Equivalent Units	
Astronomical unit	150.0 million kilometers	93.0 million miles	0.0000158 light-years	0.00000485 parsecs
Light-year	9.46 trillion kilometers	5.88 trillion miles	63,200 astronomical units	0.307 parsecs
Parsec	30.9 trillion kilometers	19.2 trillion miles	206,000 astronomical units	3.26 light-years

Space Rocks

Direct sampling is the examination of pieces of material from celestial objects. In their examinations, scientists often use techniques of geology, including chemical analysis.

Meteorites. The most common samples are *meteorites,* rocks that fell through the atmosphere from farther out in our solar system.

Thousands of meteorites have been found. Most come from asteroids. A handful come from the moon or Mars. The best place to find meteorites is Antarctica. They show up on the polar ice much more clearly than they do among ordinary rocks elsewhere.

Moon Rocks. Scientists have also studied hundreds of kilograms of moon rocks brought to Earth by astronauts from 1969 to 1972.

Researchers have also analyzed bits of space dust collected by devices on high-altitude aircraft. They have determined that some of the dust came from beyond our solar system.

Computer Modeling

Astronomers use computers to build scientific models (sets of mathematical equations) that represent certain processes, such as the formation of a star. After entering the equations into a computer, an astronomer inserts numbers into the equations. The computer then simulates how the process would develop.

Speeding the Process. In some cases, the computer produces a moving picture on a computer screen. The picture can run much faster than the actual process. This kind of model can help astronomers because many important processes occur much too slowly for astronomers to observe.

Other important processes that can be simulated occur in inaccessible places, such as the interiors of stars.

CAPTURING PARTICLES

From 2001 to 2004, NASA's Genesis spacecraft gathered samples of the solar wind, a continuous flow of particles from the sun. Genesis returned to Earth in September 2004. (See page 938.)

Also in 2004, NASA's Stardust spacecraft passed near Comet Wild 2 and captured particles from its coma, a cloud of gas and dust surrounding the comet's core. Stardust dropped a capsule containing the samples to Earth in 2006.

In 2009, NASA scientists revealed the samples contained an amino acid, a building block of life.

◀ **The Eagle Nebula.** These gas pillars in the Eagle Nebula are about 7,000 light-years away from Earth. The pillars—made up of hydrogen gas and dust—are about 4 light-years (23.5 trillion miles) in length.

Around the Sun

Planets move in two main ways. They travel around the sun in paths called *orbits*. Each planet also *rotates* on its *axis*, an imaginary line through its center.

Orbits

The planets of the solar system all move approximately in the same plane of orbit as Earth. Because of this, when seen from Earth, all the planets occupy positions near the ecliptic. The *ecliptic* is the apparent path of the sun around the celestial sphere.

Prograde Motion. Most of the time, the planets move westward across the sky slightly more slowly than the stars do. As a result, the planets seem to drift eastward relative to the background stars. This motion is called *prograde*.

Retrograde Motion. For a while each year, however, the planets seem to reverse their direction. This backward motion is called *retrograde*.

Retrograde motion occurs because Earth moves faster in its orbit than the planets that are farther from the sun. The planets that are closer to the sun move faster in their orbits than Earth travels in its orbit.

Retrograde motion occurs whenever Earth passes an outer planet traveling around the sun or an inner planet passes Earth.

OTHER ORBITERS

Besides the sun and the planets, several other types of celestial bodies make up the solar system: satellites (or moons) of planets, asteroids, comets, meteoroids, and interplanetary dust.

The Solar System

	Sun	Mercury	Venus	Earth	Mars	Jupiter
Distance from sun		57,910,000 km (35,980,000 mi.)	108,210,000 km (67,240,000 mi.)	149,600,000 km (92,960,000 mi.)	227,920,000 km (141,620,000 mi.)	778,570,000 km (483,780,000 mi.)
Length of year (in Earth days)	225 million years	87.969	224.701	365.256	686.980	4,332.589 (11.86 yr.)
Diameter	865,000 mi.	4,879 km (3,032 mi.)	12,104 km (7,521 mi.)	12,756 km (7,926 mi.)	6,792 km (4,221 mi.)	142,984 km (88,846 mi.)
Length of day	25 Earth days	59 Earth days	About 243 Earth days	23 hr., 56 min.	24 hr., 37 min.	9 hr., 55 min.
Mass relative to Earth	333,000	0.0553	0.815	1.00	0.107	317.83
Average surface temperature*	10,000°F	332°F	867°F	59°F	−85°F	−166°F*
Atmosphere	Hydrogen, helium	Minimal, but includes oxygen, sodium, and carbon dioxide	Carbon dioxide, nitrogen	Nitrogen, oxygen, argon, carbon dioxide	Carbon dioxide, argon, nitrogen	Hydrogen, helium, methane, ammonia, water
Gravity relative to Earth		0.378	0.905	1.00	0.377	2.364
Number of moons		none	none	1	2	67 known

*Effective temperature, a measure of the amount of energy given off by the body

Rotation

Planets rotate at different rates. One day is defined as how long it takes Earth to rotate once. Jupiter and Saturn spin much faster, in only about 10 hours. Venus rotates much more slowly, in about 243 Earth days.

Prograde Rotation. Most planets rotate in the same direction in which they revolve around the sun, with their axis of rotation standing upright from their orbital path.

A law of physics holds that such rotation does not change by itself. So astronomers think that the solar system formed out of a cloud of gas and dust that was already spinning.

Uncommon Motion. Uranus and Pluto (a dwarf planet) are tipped on their sides, however, so that their axes lie nearly level with their paths around the sun.

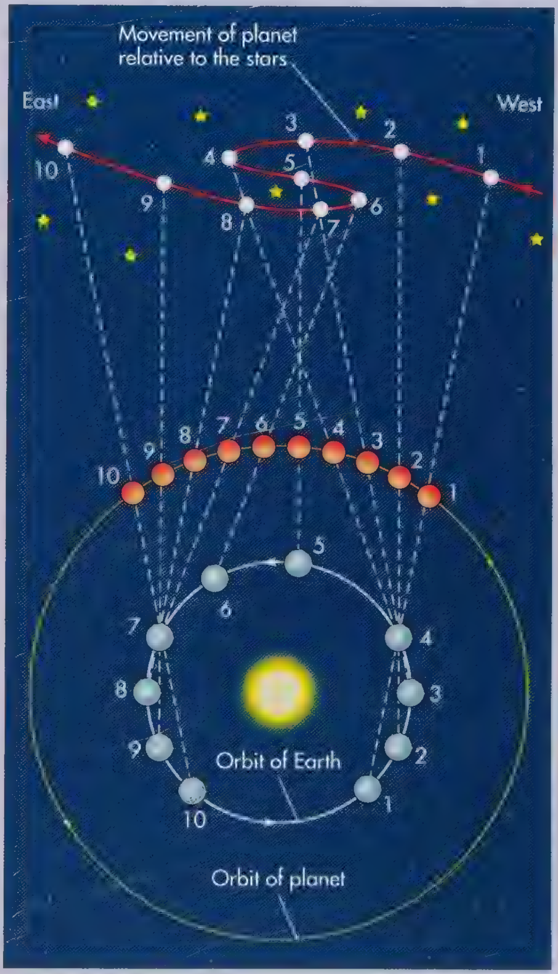
Venus is tipped all the way over. Its axis is almost completely upright, but the planet rotates in the direction opposite from the direction of its revolution around the sun. (See page 921.)

Most astronomers think that some other objects in the solar system must have collided with Uranus, Pluto, and Venus and tipped them.

IN RETROGRADE

The planets sometimes appear to change direction because of differences in orbital speed. In this diagram, each set of three numbers—for example, the three 5's—represents the same moment of time.

Earth orbits the sun more rapidly than does a more distant planet. As Earth passes a more distant planet, that planet's movement appears to briefly change direction as seen from Earth.



Saturn	Uranus	Neptune
1,433,530,000 km (890,750,000 mi.)	2,872,460,000 km (1,784,860,000 mi.)	4,495,060,000 km (2,793,100,000 mi.)
10,759.22 (29.46 yr.)	30,685.4 (84.01 yr.)	60,189 (164.79 yr.)
120,536 km (74,898 mi.)	51,118 km (31,763 mi.)	49,528 km (30,775 mi.)
10 hr., 33 min.	17 hr., 14 min.	16 hr., 7 min.
95.161	14.536	17.148
–148°F*	–344°F	–365°F
Hydrogen, helium, methane, ammonia	Hydrogen, helium, methane	Hydrogen, helium, methane
1.07	0.905	1.14
62 known	27 known	14 known

Saturn and Uranus

Saturn

- Saturn is second only to Jupiter in size, mass, and speed of rotation.
- Its diameter is 120,500 kilometers (74,875 miles).
- Saturn's mass is 95 times that of Earth.
- The mean distance from the sun is 1,427 million kilometers (886.7 million miles).
- Its rotational period is 10 hours, 39.4 minutes.
- It orbits the sun in 29.5 years.
- Saturn is the least dense of all planets.
- Many features of Saturn's atmosphere and inner structure are similar to those of Jupiter.

Exploration. Much has been learned about Saturn from space probes. Pioneer 11 reached Saturn in September 1979 and approached within 20,880 kilometers (12,974 miles).

Voyager 1 reached the planet in November 1980, and Voyager 2 in August 1981. Voyager 1 passed close by Titan, Saturn's largest satellite. Voyager 2 continued its voyage to Uranus and Neptune.

New Rings. The pictures of Saturn's ring system taken by the Pioneer and Voyager spacecraft count among the most spectacular ever obtained in astronomy. The F ring, never observed from Earth before, became visible in the photographs.

Spokes. In the B ring, are a number of *spokes*—radial darker zones—became visible. Voyager 2 performed measurements that showed that the ring system is very thin, at most 150 meters (490 feet) thick.

Orbiting Saturn. After traveling for 7 years, the Cassini-Huygens spacecraft became the first to orbit Saturn when it arrived in June 2004.

In January 2005, the Huygens probe, which had detached from the Cassini spacecraft, landed on Saturn's largest moon, Titan. Scientists believe conditions on Titan to be comparable to that of early Earth and hope that study of Titan will shed light on Earth's beginnings. Early photos from Titan indicated rain and rivers of methane.

Saturn's Rings

Distance from center of Saturn to inner edge of ring.

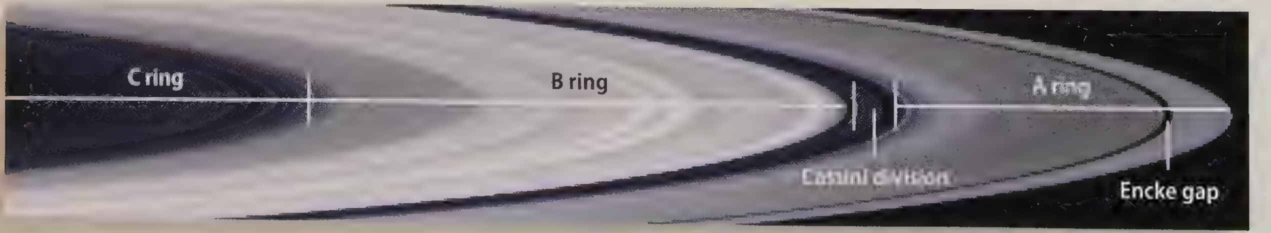
Name	Distance in km (mi.)
D	66,900 (41,600)
C	74,700 (44,700)
B	92,000 (57,300)
Cassini division	117,500 (73,000)
A	122,300 (76,000)
Encke gap	133,600 (82,900)
F	139,800 (86,900)
G	166,000 (103,000)
E	180,000 (111,200)

SATURN'S RINGS

By far Saturn's most outstanding feature is its ring system. The seven main rings surround the planet at its equator but do not touch it. They consist mainly of pieces of ice, ranging from dust-sized grains to chunks more than 3 meters (10 feet) in diameter.

Many of the rings feature thin bands of varying brightness, called *ringlets*. Gaps of several thousand miles separate most of the major rings.

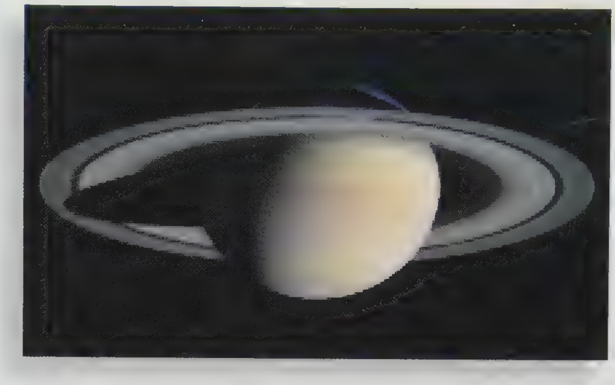
In 2009, scientists using the Spitzer Space Telescope discovered a diffuse doughnut-shaped ring far beyond the orbit of the previously known rings. The ring is centered around the orbit of the moon Phoebe. The ring is roughly 13 million kilometers (8 million miles) thick.



Saturn's Moons. With the discovery of new satellites by spacecraft, the total number of known moons of Saturn has grown to 62.

Saturn's largest moon, Titan, was discovered by Dutch astronomer Christiaan Huygens in 1655. Voyager photographs of Titan show that it is covered by a thick orange atmosphere mainly containing nitrogen. The atmospheric pressure is 1.6 times that of Earth.

Discoveries from the Cassini-Huygens mission include previously unknown moons, towering sand dunes and liquid seas on Titan, and an atmosphere on Enceladus. Additionally, partial rings have been noted orbiting two of Saturn's small inner moons.



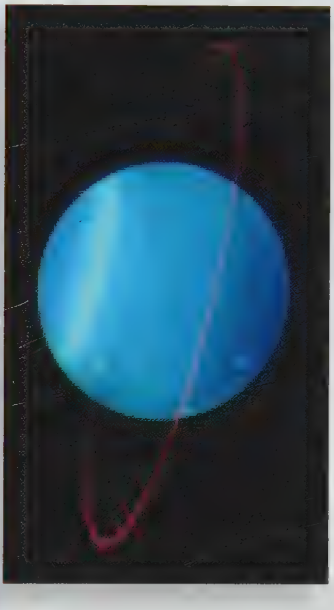
▲ **Saturn,** in a photograph taken by the Cassini-Huygens spacecraft

Uranus

- With a magnitude of 5.5, Uranus is the farthest planet that can be seen without a telescope.
- Uranus is half the size of Saturn, with a diameter of 51,100 kilometers (31,690 miles).
- The mean distance from the sun is 2,871 million kilometers (1,784 million miles).
- Uranus orbits the sun in 84 years and rotates around its axis in 17.2 hours.
- Like Venus, Uranus rotates in retrograde, but its axis is tilted over 98° to its orbital plane. (See page 919.)
- Consequently, unlike the other planets, Uranus does not have day and night.
- Its northern and southern halves are pointed toward the sun alternately every 42 years.
- At the present, Uranus has been found to have 13 rings that are similar to Saturn's rings.

Exploration. Voyager 2 visited the planet in December 1985. Pictures taken then show that Uranus is covered by a thick atmosphere colored greenish blue by trace amounts of methane.

The planet below the atmosphere may include water, ammonia, and methane ices up to 8,000 kilometers (5,000 miles) deep. The central part of the planet may consist of a rocky core the size of Earth.



◀ **Uranus's atmosphere** features complex cloud patterns colored blue-green by methane gas.

Moons. Uranus has 27 known moons. The five largest moons are Ariel, Miranda, Oberon, Titania, and Umbriel. They consist of about 60 percent water ice and 40 percent rock.

Pictures of Miranda revealed structures never seen on other celestial bodies in the solar system. Rectangular and circular features cover large parts of its surface, hinting that Miranda never completed the process in which heavier elements sank to form the core and lighter elements formed the crust.

Neptune and Dwarf Planets

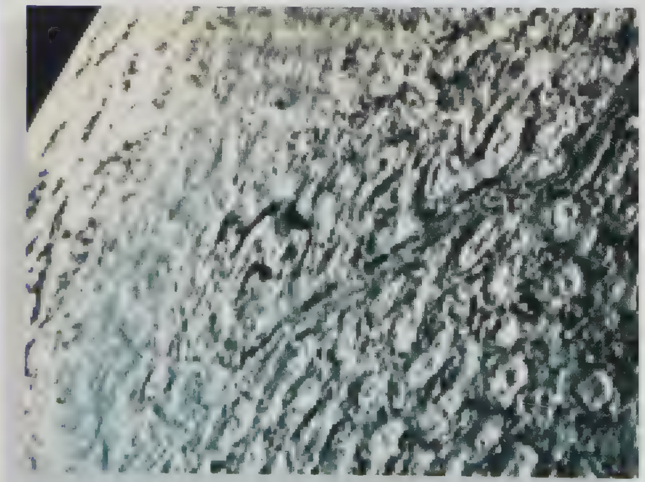
Neptune

- With a magnitude of 8, Neptune is invisible to the naked eye, but can be observed through binoculars.
- The average distance from the sun is 4,504 million kilometers (2,799 million miles).
- The planet completes an orbit around the sun in 164.8 years.
- Neptune's rotation period is 16 hours, 7 minutes.
- Its average temperature is –201°C (–330°F).
- The planet's atmosphere is made up of hydrogen, helium, methane, and acetylene.
- Neptune has at least 14 satellites.

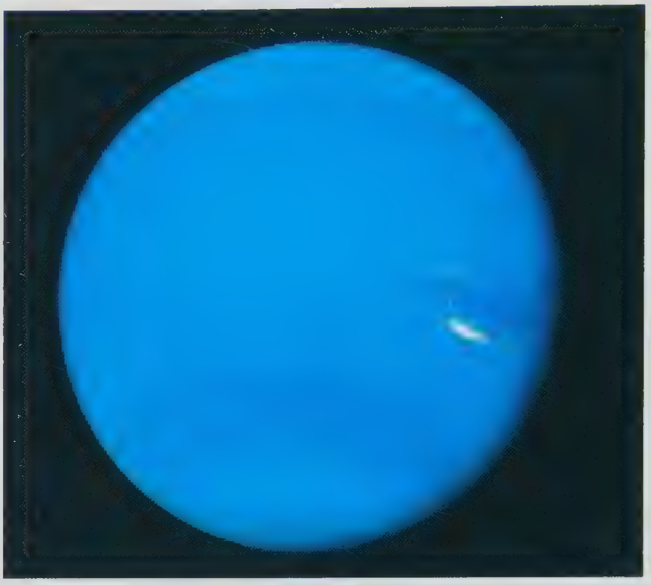
Exploration. Voyager 2 reached Neptune in 1989. It detected Neptune's magnetic field and measured its rotational period, which is 16 hours, 7 minutes, and corresponds to the rotational period of the planetary core. The cloud formations rotate more slowly, with a period of more than 18 hours near the equator.

Voyager 2 discovered six new moons in addition to the previously known moons Triton and Nereid. Six more have since been discovered.

Voyager 2 also produced the first images of Neptune's system of six dim rings. All of these rings are much fainter and darker than the rings of Saturn. They appear to consist of particles of dust.



▲ **Trapped Comet.** Triton may once have been a large comet before being captured by Neptune's gravity.



▲ **Neptune and Triton.** The blue clouds of Neptune are mostly frozen methane, the main chemical in natural gas.

HOW DO WE KNOW THAT

THE DISCOVERY OF NEPTUNE

Neptune was discovered mathematically before it was seen through a telescope. Astronomers thought Uranus was the most distant planet, but they noticed that it was not always in the position they predicted. The force of gravity of an unknown planet seemed to be influencing Uranus.

In 1843, English astronomer and mathematician John C. Adams began working to find the location of the unknown planet. Adams predicted the planet would be about 1.6 billion kilometers (1 billion miles) farther from the sun than Uranus. In 1845, Adams sent his remarkably accurate work to Sir George B. Airy, the Astronomer Royal of England. However, Airy did not look for the planet with a telescope. Apparently, he lacked confidence in Adams.

By mid-1846, French mathematician Urbain J. J. Leverrier also had predicted Neptune's position. He sent his predictions, which were similar to those of Adams, to the Urania Observatory in Berlin, Germany. On September 23, 1846, astronomers at the observatory found Neptune near the position predicted by Leverrier. Today, both Adams and Leverrier are credited with the discovery.

Dwarf Planets

A *dwarf planet,* also called a *plutoid,* is a celestial body that orbits the sun, is near-spherical in shape, is not a satellite, and, because of its small size, has not cleared the orbit in its neighborhood.

Pluto is the best known dwarf planet, having been considered the ninth planet for 76 years.

- Pluto's average distance from the sun is around 5,906,380,000 kilometers (3,670,050,000 miles).
- Pluto completes one orbit every 248 Earth years.
- For 20 years of each orbit, it comes closer to the sun than the orbit of Neptune. Pluto last entered Neptune's orbit in 1979 and remained there until 1999.
- Pluto rotates about once every 6 Earth days.
- Pluto has an estimated diameter of 1,473 miles (2,370 kilometers), only about two-thirds the diameter of Earth's moon.
- Pluto's surface is one of the coldest places in our solar system. Astronomers think the temperature on parts of Pluto is about –232°C (–385°F).
- Pluto has five known moons: *Charon, Nix, Hydra, Kerberos* and *Styx.*

Crowded Orbit. Unlike a planet, a dwarf planet lacks the gravitational pull to sweep other objects from the region of its orbit. As a result, dwarf planets are found among populations of smaller bodies.

The dwarf planet Ceres, for example, orbits in the Main Belt of asteroids, between the orbits of Mars and Jupiter.

Other Plutoids. Other dwarf planets orbit primarily beyond Neptune in the Kuiper belt. Compared with the planets, KBOs tend to follow irregular, elongated orbits.

The first KBOs to be classified as dwarf planets include Eris, Haumea, Makemake, and Pluto.

THE PLUTO CONTROVERSY

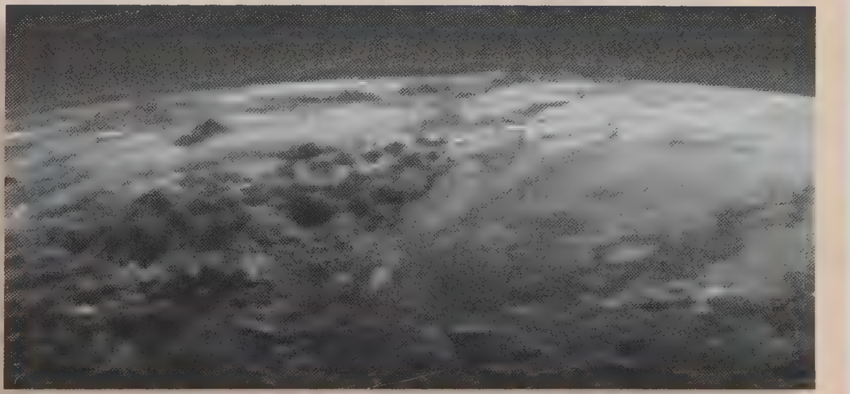
Pluto was discovered in 1930 and for 76 years was considered a planet. But its small size and irregular orbit led many astronomers to question whether Pluto should be grouped with such worlds as Earth and Jupiter.

The debate intensified in the 1990s with the discovery of the Kuiper belt objects (KBOs), a band of icy, rocky objects in the outer regions of our solar system. KBOs have many similarities with Pluto.

In 2006, the International Astronomical Union (IAU) created a new category called dwarf planets. Pluto was reclassified and is now considered a dwarf planet.

PLUTO'S SURFACE

In July 2015, the New Horizons probe returned images of Pluto with blue skies over mountains and plains of ice. Areas observed without craters suggests Pluto was recently or is currently geologically active.



Classifying the Stars

Astronomers classify stars based on their characteristics. The main characteristics by which stars are classified are

- *luminosity,* the rate at which a star emits energy.
- surface temperature.
- *absolute magnitude,* the apparent magnitude a star would have if it were at a distance of 10 parsecs, or 32.6 light-years.

GRAPHING THE STARS

At the beginning of the 20th century, two astronomers—Ejnar Hertzsprung of Denmark and American Henry Russell—independently developed graphs to record the relationship between the color and absolute magnitude of a star.

Today, astronomers use one diagram—the Hertzsprung-Russell diagram—to display the main characteristics of stars.

Luminosity Classes

In the 1930s, American astronomers William W. Morgan and Philip C. Keenan invented what came to be known as the *MK luminosity classification system* to group different kinds of stars. Astronomers revised and extended this system in 1978.

In the MK system, the largest and brightest classes have the lowest classification numbers.

The MK classes are: Ia, bright supergiant; Ib, supergiant; II, bright giant; III, giant; IV, subgiant; and V, main sequence or dwarf.

Overlapping Classes. Because temperature also affects the luminosity of a star, stars from different luminosity classes can overlap. For example, Spica, a class V star, has an absolute magnitude of –3.2; but Pollux, a class III star, is dimmer, with an absolute magnitude of 0.7.

Spectral Classes

In the MK system, there are eight spectral classes, each corresponding to a certain range of surface temperature.

- From the hottest stars to the coolest, these classes are: O, B, A, F, G, K, and M.
- Each spectral class, in turn, is made up of 10 spectral types, which are designated by the letter for the spectral class and a numeral.
- The hottest stars in a spectral class are assigned the numeral 0; the coolest stars, the numeral 9.

A complete MK designation thus includes symbols for luminosity class and spectral type.

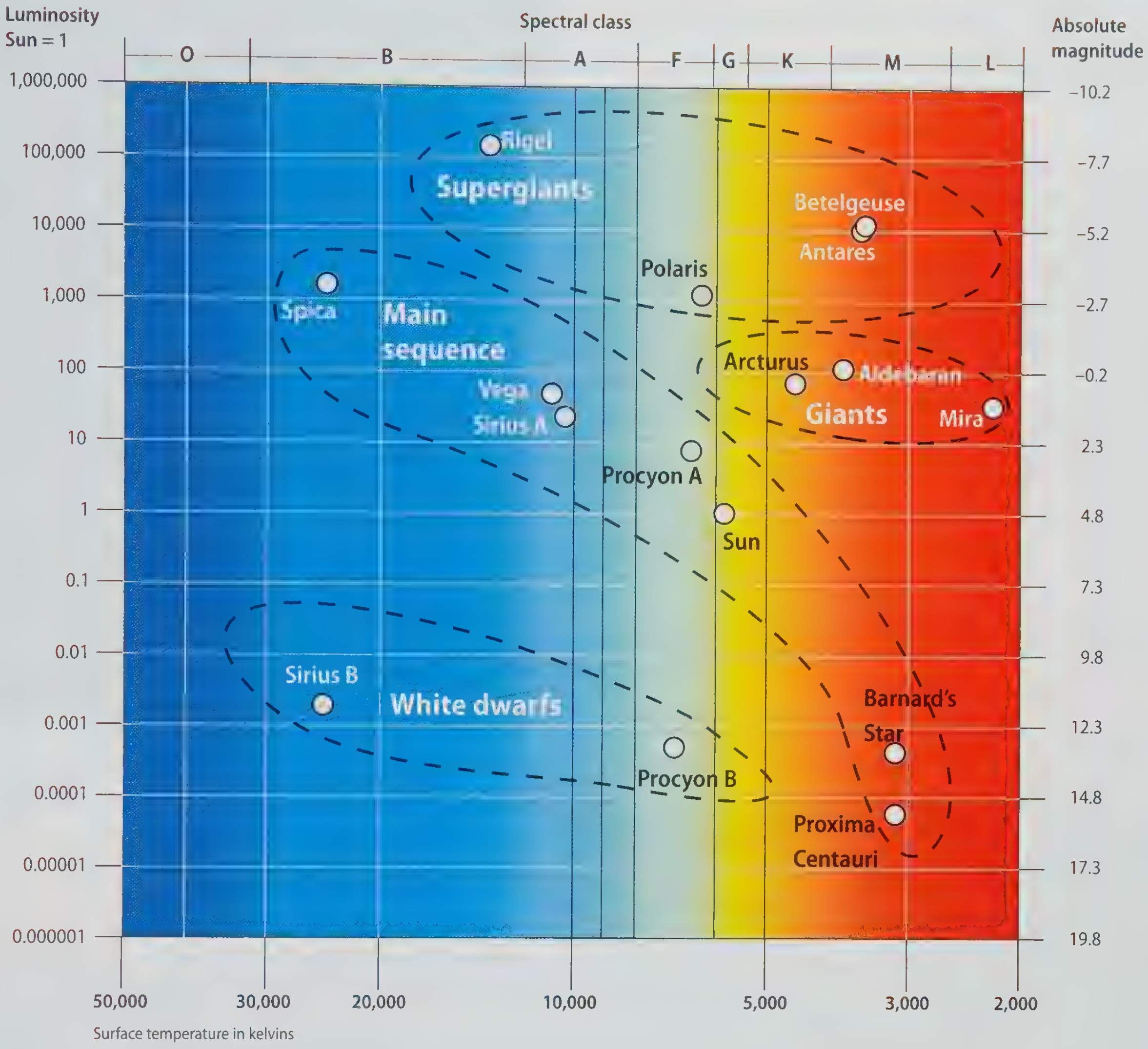
- The complete designation for the sun is G2V.
- Alpha Centauri A is also a G2V star.
- The complete designation for Rigel is B8Ia.

STELLAR CLASSIFICATION

Luminosity Classes	
Ia	bright supergiant
Iab	less bright supergiant
Ib	supergiant
II	bright giant
III	giant
IV	subgiant
V	main sequence
VI	subdwarf
VII	white dwarf

Spectral Classes		
Type	Temperature Range (Kelvin)	Color
O	100,000–30,000	blue
B	25,000–12,000	blue-white
A	11,000–8,000	white
F	7,800–6,200	white-yellow
G	6,000–4,600	yellow
K	4,900–3,350	orange
M	3,400–2,600	red

THE HERTZSPRUNG-RUSSELL DIAGRAM



The Hertzsprung-Russell (H-R) diagram is one of the most important graphs of astrophysics. It shows that the great majority of stars fall on the *main sequence,* a nearly straight line running from highly luminous, hot, bluish stars down to faint, relatively cool, reddish stars. A given temperature and size go together. Thus, along the main sequence, any stars having the same temperature will also be of the same magnitude.

Mass-Luminosity Relation. Measurements of a small number of stars have shown that the luminosity L is proportional to the third power of the mass of a star. This relationship—the *mass-luminosity relation*—implies that for main sequence stars, the bright stars are much more massive than the faint stars.

Exceptions to the Rule. Spectral type does not always correspond to only one absolute magnitude. For example, Alpha Aurigae and Alpha Centauri are both G-type stars, but they differ strongly in absolute magnitude, 0.3 and 4.4, respectively.

Since the surface temperature has to be approximately the same, the much higher luminosity can only be explained by assuming that the star is much larger. Hertzsprung introduced the term *giant* for these large stars. The stars that lie above the main sequence in the H-R diagram include the red and yellow giants and the very large *supergiants.*

Similarly, there are groups of stars that are much fainter than their counterparts in the main sequence. These stars are much smaller and are called white dwarfs.

Black Holes and Binary Stars

Black Holes

A *black hole* is a region of space whose gravitational force is so strong that nothing can escape from it. A black hole is invisible because it even traps light.

Neutron Star Limits. Just as there is a limit to the mass of a dwarf star consisting of electron-degenerate matter, there is an upper mass limit to the existence of a neutron star. (See pages 946–947.)

This limit is estimated to be about 3 solar masses. Above that value, the stellar core of neutron-degenerate matter cannot support itself and begins collapsing even further into a black hole.

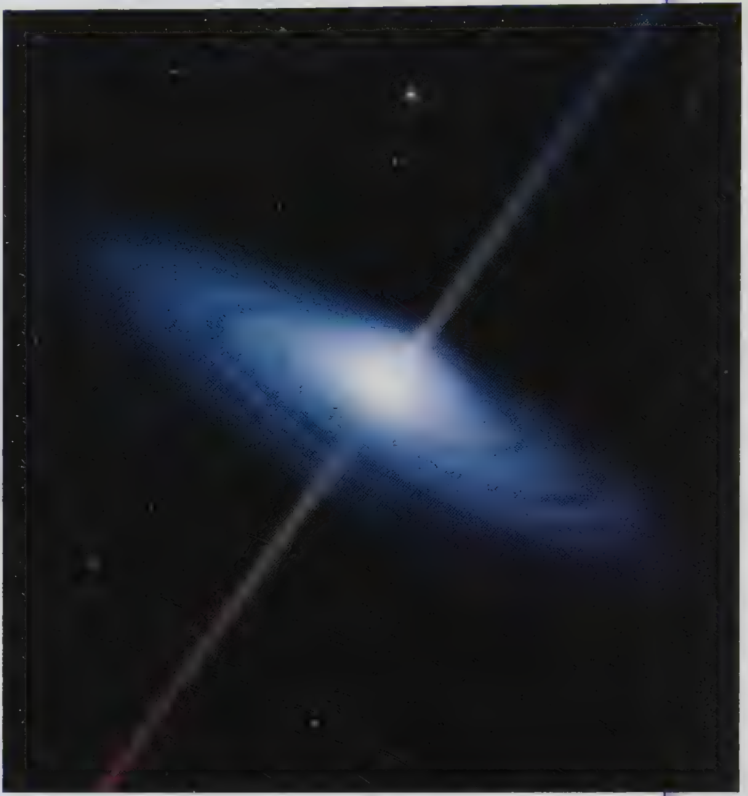
Relativity and Black Holes. The fundamental descriptions of black holes are based on equations in the theory of general relativity developed by the physicist Albert Einstein. Einstein's theory describes gravity as the degree of curvature of space-time.

Characteristics of Black Holes

- The gravitational force is strong near a black hole because all the black hole's matter is concentrated at a single point in its center—the *singularity*.
- The singularity of a black hole is believed to be much smaller than an atom's nucleus.
- A black hole is surrounded by a region around its singularity called the *event horizon*.
- At the event horizon, the pull of gravity becomes infinitely strong. An object can exist there for only an instant as it plunges inward at the speed of light.
- The radius of the event horizon depends on the amount of mass that has collapsed.
- Astronomers use the event horizon's radius to specify the size of a black hole.

Proving Their Existence. Astronomers have found compact objects that are likely black holes, but have not yet discovered a black hole for certain.

To prove that a compact object is a black hole, scientists would have to measure such effects as severe bending of a light beam and an extreme slowing of time.



▲ **A supermassive black hole** sucks in a swirling disk of matter, shooting out beams of particles in this artist's conception. A supermassive black hole has a mass from millions to billions of times that of the sun.

PROBABLE BLACK HOLE

There are two X-ray sources that may be black holes. The most promising candidate, discovered by the *Uhuru* satellite, is called Cygnus X-1. Other satellites have also discovered gamma rays coming from this object.

Cygnus X-1 is at a distance of 6,000 light-years from Earth and consists of a blue supergiant star of about 30 solar masses around which rotates a highly condensed object of about 14 solar masses.

Cygnus X-1 emits X-rays that astronomers think are produced by matter flowing from the supergiant to the black hole. The matter spirals in a disklike region around the black hole before being swallowed by it. As the matter swirls toward the black hole, it becomes more and more concentrated and heated, and it begins emitting X-rays.

Binary Stars

A *binary star,* or a *double star,* is a pair of stars that revolve around each other and are held together by gravity.

Bigger, Closer, Faster. The orbital speed of binary stars depends mostly on two factors.

- **Mass:** The greater a star's mass, the stronger its gravity and the faster it pulls its companion around in orbit.
- **Proximity:** The closer the two stars are to each other, the faster they revolve.

Some are so close as to almost touch and can go around each other in just a few hours or minutes. Other pairs may be separated by many times the diameter of the solar system and take a million years to revolve around each other.

The Numbers. Perhaps as many as 25 percent of the stars in our galaxy have a partner. Some stars have more than one partner. The "star" Castor in the Gemini constellation is at least six stars.

Most stars belong to double or multiple star systems.

Contact Binaries. The stars in some binaries are so close that they touch. In these *contact binaries,* the gravity of each star distorts its companion, causing enormous tides to form on the surfaces of both stars. Violent events can occur in such close pairs.

- One star may become an *X-ray pulsar,* a star that radiates X-rays in precisely timed bursts.
- Other close pairs radiate powerful radio waves.
- In others, one of the stars pulls matter to itself from the other star. This matter may explode in nuclear reactions, causing the star to flare brightly.

fyi! *The fastest binaries can complete an orbit in less than 6 minutes traveling at over 480 kilometers (300 miles) per second.*

Ghostly Binaries. The star Algol, also called Beta Persei, derives its name from an Arabic word meaning *ghoul,* likely due to the star's unusual periodic brightening and dimming.

Algol appears to be a single star, but it is actually a type of double star called an *eclipsing binary.* In such a binary, a pair of stars revolve around each other so that one periodically blocks the light of the other. This action reduces the brightness of the double star as viewed from the earth.

Finding Binaries. The existence of close binaries can be revealed by a spectrograph.

- When, in its motion, a component approaches Earth, its spectral lines are shifted by the Doppler effect toward the blue end of the spectrum and reduced in wavelength.
- Half a cycle later, when this component recedes from Earth, the lines shift toward the red and the wavelengths increase.
- This Doppler shift can easily be measured. It has led to the discovery of many spectroscopic binaries.

▲ **The binary star system Albireo** consists of two bright, orbiting stars 400 light-years from Earth. They may take about 75,000 years to orbit each other once.

THE UNIVERSE is everything. It consists of all matter and all light and other forms of radiation and energy. It includes everything that exists anywhere in space and time.

All stars, including the sun, are part of the universe. Some other stars also have planetary systems. In addition to planets, stars, and other bodies, the universe contains gas, dust, magnetic fields, and cosmic rays.

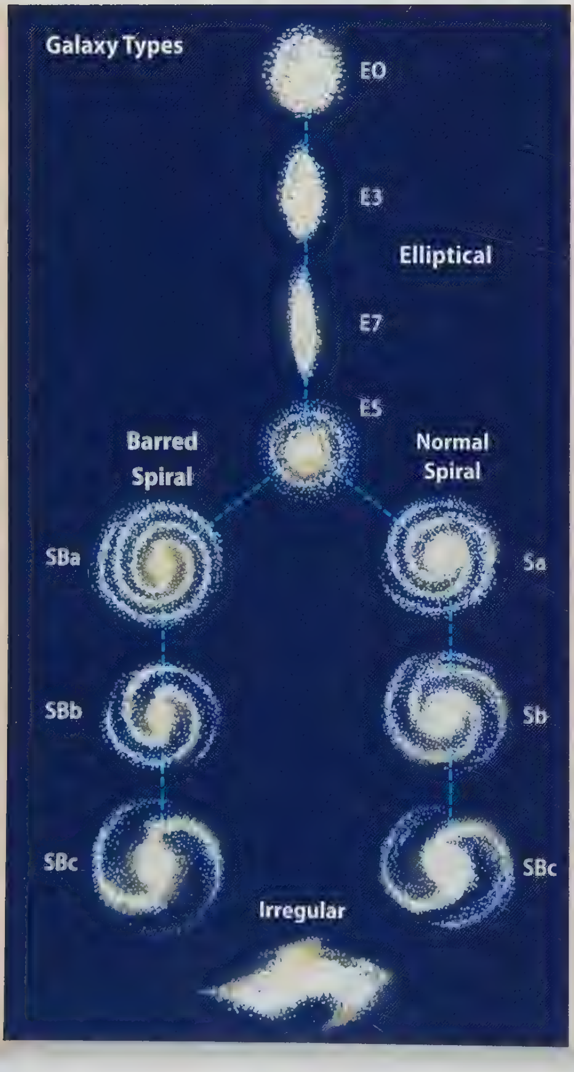
Galaxies

In the 1920s, the American astronomer Edwin P. Hubble discovered that the universe contains many other galaxies beyond our own. The observable universe includes billions, possibly even trillions, of galaxies. Each galaxy consists of from hundreds of thousands to trillions of stars.

Classification

Galaxies come in a variety of sizes and shapes.

- The smallest galaxies, called *dwarf galaxies,* are 2,000 parsecs in diameter and have a mass 10 million times that of the sun.
- The largest galaxies, the *giant ellipticals,* are some 100,000 parsecs in diameter and have a mass 1,000 billion times that of the sun.



HUBBLE CLASSIFICATION

Edwin Hubble in 1925 introduced a classification system for galaxies based on their forms and structure.

Elliptical Galaxies

- These galaxies are designated with the letter E followed by an index suggesting the ellipticity of the galaxy.
- E0 is used for spherical galaxies.
- E7 is used for very elongated ellipses.

Spiral Galaxies

- Normal spiral galaxies are denoted with the letter S.
- Barred spirals are designated with SB.

Both types are subdivided as a, b, and c types.

- The a types have large and very bright nuclei or central bars.
- The b types have a less pronounced nucleus.
- The c types have most of their light coming from the spiral arms.

S0-type galaxies are an intermediate form between elliptical and spiral galaxies. They have a bright nucleus surrounded by an elliptical zone with little evidence of spiral arms. Spiral galaxies range in size from 50,000 to 200,000 light-years.

Spiral Galaxies. About 17 percent of galaxies that astronomers can observe through a telescope have a structure with spiral arms comparable to that of our own galaxy or to that of the Andromeda galaxy. However, among the 1,000 brightest galaxies, almost 70 percent have a spiral structure.

Elliptical Galaxies. Eighty percent of all galaxies are elliptical, with no spiral arms and little or no visible structure.

Irregular Galaxies. A small percentage of galaxies have irregular shapes. The Large and Small Magellanic Clouds are irregular galaxies.

Clean or Dusty. Spiral galaxies and irregular galaxies contain large amounts of interstellar gas and dust and large numbers of newly formed stars.

It is believed that supernovas, which occur much more frequently in elliptical galaxies, keep these galaxies clean of interstellar gas and dust and thus prevent the formation of new stars.

▲ **The Andromeda Galaxy** and our own galaxy—the Milky Way—are the largest galaxies in the cluster known as the Local Group.

Distribution

Few galaxies are truly isolated. Most galaxies, including our own, inhabit *groups,* containing dozens of galaxies, or *clusters,* containing from hundreds to thousands of members. Groups and clusters, in turn, reside in large *superclusters.*

The Local Group. Our galaxy has several neighbors, of which the Small and Large Magellanic Clouds are the closest. These galaxies are members of the so-called *Local Group,* which also includes the Andromeda galaxy and dozens of smaller galaxies in a diameter of about 6.5 million light-years.

These galaxies are so close that they interact gravitationally. For example, the plane of the Andromeda galaxy is somewhat warped by the presence of two satellite galaxies and, under mutual gravitational attraction, it approaches our galaxy with a speed of 110 kilometers (70 miles) per second.

Galaxy Clusters

- Over 2,000 clusters of galaxies are known, and it is believed that billions exist.
- Most of the clusters contain at least 50 members.
- Regular clusters are generally spherical, with more galaxies toward the center.
- They contain mainly E and S0 galaxies and have a total mass 10,000 times the mass of our galaxy.

Irregular clusters are less symmetrical and contain fewer galaxies. They usually contain about 100 times the mass of our galaxy. Irregular clusters with a small number of galaxies generally contain spiral galaxies. Those with a large number of galaxies generally contain elliptical galaxies.

Supercluster. The Local Group, the Virgo Cluster, and the Coma Cluster are relatively close and form a supercluster. About 50 superclusters, which are clusters of clusters, have been identified in the universe.

Time Line: 1900–1959

The challenges involved in studying space are great—massive distances and environments inhospitable to humans. But these challenges have been met, to an extent, by the human curiosity and ingenuity that fuels space exploration.

Use the legend, found on each two-page section, to identify the country that accomplished each achievement in space exploration. For example, milestones reached by the United States are identified by a blue circle.

Legend

- ● United States
- ● Soviet Union/Russia
- ● Germany
- ● Japan
- ● China
- ● European Space Agency
- ● India

1942–1943
● Germany launches thousands of V-2 rockets, each carrying a ton of high explosive, on London. Breakthrough achievements include liquid oxygen and alcohol producing 55,000 pounds of thrust and a ballistic trajectory of over 100 miles at supersonic speed.

1900 1910 1920 1930 1940

1903
● The Wright brothers launch man's dream of controlled flight with a 120-foot flight.

1912
An unnamed European balloon flight results in the discovery of cosmic rays.

1923
● Hermann Oberth becomes the leader of interest in rocket development in Germany by publishing *The Rocket to Interplanetary Space.*

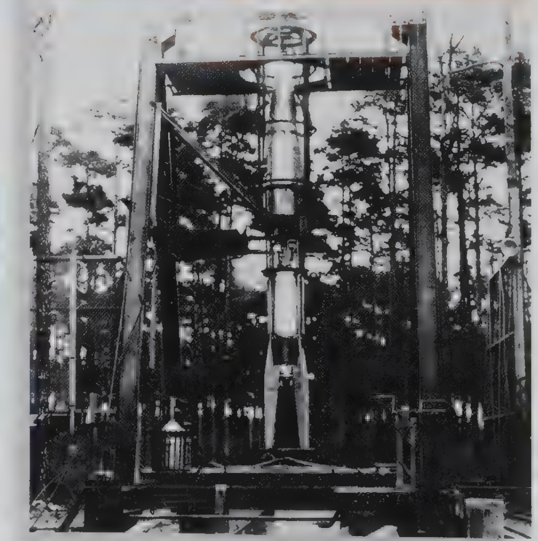
1931
● The German Rocket Society develops a liquid-propellant rocket engine producing 110 pounds of thrust.

1946–1947
● The first captured V-2 rockets are fired at White Sands Proving Ground, New Mexico, under the direction of Werner von Braun and about 100 of his top engineers, who had surrendered to the U.S. Army in 1945.

1934
● German Rocket Society disbands as German army takes over rocket development. Werner von Braun develops the A-3 rocket with 660 pounds of thrust.

1919
● The Smithsonian Institution publishes Robert Goddard's *A Method of Reaching Extreme Altitudes.*

1926
● Goddard launches the first liquid propellant rocket at Auburn, Massachusetts. It travels 120 feet.



Oct. 4, 1957

- Sputnik 1, the first man-made Earth satellite, is launched by the Soviet Union. It circles Earth once every 96 minutes at a speed of 29,000 kph (18,000 mph) until it falls to Earth on January 4, 1958.

1958

- The National Aeronautics and Space Administration (NASA) is established to oversee and coordinate all U.S. space missions and activities.

1950 1955 1959

Nov. 3, 1957

- The Soviet Union launches Sputnik 2, which carries a dog named Laika, the first animal to soar into space. Laika survives the launch, but the technology does not yet exist to bring Sputnik 2 back safely to Earth. Laika does not survive the mission, but she proves that animals can survive a launch into space and the effects of microgravity.

Jan. 31, 1958

- The United States launches its first satellite—Explorer 1. Data from Explorer 1 leads to the discovery of the Van Allen belts, two bands of radiation that surround Earth high above its surface.

Nov. 17, 1958

- The U.S. launches its second satellite, Vanguard 1, which is a tiny sphere just 6.4 inches in diameter. Vanguard 1 relays information showing Earth is pear-shaped.

Jan. 2, 1959

- The Soviet Union launches Lunik 1. This satellite makes the first lunar flyby and sends back data that leads to the discovery of the solar wind.

Feb. 17, 1959

- The U.S. launches Vanguard 2, the first satellite to send back weather data.

May 28, 1959

- The U.S. launches two monkeys—a rhesus monkey named Able and a squirrel monkey named Baker—into space in the nose cone of a Jupiter missile. The monkeys return unhurt after a 300-mile-high suborbital flight.

Sept. 12, 1959

- The Soviet Union launches Lunik 2. Two days later, Lunik 2 becomes the first spacecraft to impact the moon's surface.

Oct. 4, 1959

- The Soviet Union launches Lunik 3 into a huge orbit around both Earth and the moon. Lunik 3 returns the first images of the moon's perpetually far side on October 7.

Space Science Glossary

absolute magnitude the apparent magnitude that a star would have if it were observed from a distance of 10 parsecs (32.6 light-years).

active galaxy any galaxy that is a very powerful transmitter of radiation in certain ranges of the electromagnetic spectrum.

annular eclipse an eclipse in which the moon darkens only the middle of the sun, leaving a bright ring around the edges.

asteroid a rocky or metallic object smaller than a planet that orbits the sun.

astronomical unit the average distance from Earth to the sun—about 150,000,000 kilometers (93,000,000 miles); used as a unit in measuring the distances between stars and planets.

astronomy the study of the universe and the objects in it.

azimuth the angle along the astronomical horizon from the direction of the North Pole to the vertical circle that passes through a star.

big bang the initial explosion of a very high concentration of matter and energy that scientists think started the expansion of the universe. The big bang theory ranks as the most widely held scientific theory of the universe's origin. According to the theory, the big bang occurred about 14 billion years ago.

binary star a pair of stars that revolve around each other and are held together by gravity.

black hole a region of space whose gravitational force is so strong that nothing can escape from it; it is invisible because it traps light as well.

celestial poles the points around which the celestial sphere appears to rotate.

celestial sphere the imaginary sphere that rotates around Earth. The stars, planets, and other celestial bodies appear to be fixed on the celestial sphere.

Cepheid variable variable star whose changes in brightness recur in a characteristic manner with a relatively short periodicity, and may be due to expansion and contraction.

chromosphere the layer of the sun's atmosphere above the photosphere; extends to an altitude of several thousand kilometers.

closed universe one of three possibilities that cosmologists use to describe the nature of the universe (the others being *open* and *flat*). In a closed universe, the average density of the universe is high enough that gravitational forces will slow and eventually reverse expansion, and the universe will collapse on itself.

cluster an association of stars. *Globular clusters* are spherical associations of stars found in galaxies that contain up to several hundred thousand stars. Large associations of galaxies are also called clusters.

corona a thin envelope of gas surrounding the sun and extending far from the sun to a distance as great as several solar radii.

cosmic microwave background (CMB) radiation energy left over from the early universe coming from all directions in the universe; believed to be remnants from the big bang.

cosmic rays electrically charged, high-energy subatomic particles moving at very high speeds that reach Earth from all directions, coming from the sun and from interstellar space.

cosmogony the study of the origin of the solar system.

cosmological model a mathematical or physical description of the structure of the universe.

cosmology the scientific study of the general structure, origin, and evolution of the universe.

dark energy a theoretical form of energy that apparently makes the universe expand more and more rapidly.

dark matter invisible matter that does not give, reflect, or absorb light rays; believed by scientists to make up about 85 percent of the matter in the universe.

declination a star's angular distance north or south of the celestial equator, measured along the hour circle passing through the star.

Doppler effect the apparent change in wave frequency when either the source of waves or the observer moves toward or away from the other.

dwarf planet a celestial body that orbits the sun, is near-spherical in shape, is not a satellite, and, because of its small size, has not cleared the orbit in its neighborhood; also called a plutoid.

eclipse the obscuring of light from one celestial body by another. In a *solar eclipse,* the moon moves between the sun and Earth so as to obscure the sun as seen from Earth. In a *lunar eclipse,* Earth moves between the sun and the moon so that the moon is in Earth's shadow.

ecliptic the apparent path of the sun around the celestial sphere.

equinox one of two points where the ecliptic intersects the celestial sphere.

event horizon the boundary of the region around a black hole from which no light or matter can escape. At the event horizon, the pull of gravity becomes infinitely strong.

extinction In astronomy, the absorption and scattering of starlight by dust.

flat universe one of three possibilities that cosmologists use to describe the nature of the universe (the others being *open* and *closed*). In a flat universe, the universe will expand forever, but at a lower rate than that expected in an open universe.

gas giant any one of the four planets with orbits beyond that of Mars. They are Jupiter, Saturn, Uranus, and Neptune. The gas giants have no solid surface, and they consist mostly of hydrogen and helium. Also called the Jovian planets.

geocentric Earth-centered; early astronomers believed the universe to be geocentric.

globular cluster a tightly packed spherical collection of stars found in galaxies that contain up to several hundred thousand stars.

heliocentric sun-centered; early astronomers believed the universe to be heliocentric.

inflation theory theory that holds that the early universe went through an extremely brief period of particularly rapid expansion.

Jovian planet any one of the four planets with orbits beyond that of Mars. They are Jupiter, Saturn, Uranus, and Neptune. Jovian planets have no solid surface, and they consist mostly of hydrogen and helium. Also called gas giants.

Kuiper belt objects a band of icy, rocky objects in the outer regions of the solar system.

light-year the distance that light travels in a vacuum in a year; used as a unit to measure astronomical distances.

Local Group a concentration of galaxies that includes our own galaxy—the Milky Way—as well as many nearby galaxies.

luminosity the rate at which a star emits energy; a star's luminosity depends directly on its size and on how brightly each unit of area shines.

lunar eclipse an eclipse in which Earth moves between the sun and the moon so that the moon is in Earth's shadow.

magnitude the measure of the brightness of a star. *Apparent magnitude* is the brightness of a star as it appears to an observer on Earth. *Absolute magnitude* is the apparent magnitude that a star would have if it were observed from a distance of 10 parsecs.

Main Belt a belt of asteroids that orbits the sun between Mars and Jupiter.

main sequence star a star that produces nearly all of its energy by combining hydrogen nuclei to form helium nuclei in its core.

meteor a mass of stone or metal that enters Earth's atmosphere from outer space with enormous speed; meteors are visible in the sky as falling stars.

meteorite the portion of a meteoroid that survives as it passes through Earth's atmosphere and reaches the surface.

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Preface

Welcome to *Southwestern Advantage Social Studies & Language Arts*. We are pleased to bring you this unique, user-friendly reference book. It has been designed in such a way that students can spend "more time learning, less time looking." The pages are open and inviting and organized into information boxes, bulleted lists, and other easily usable and understandable pieces.

Now You Know boxes are used frequently to briefly summarize the information on the preceding few spreads. **The Basics** boxes outline information essential to the topic at hand. **Watch Out** boxes alert students to things that might be easily confused or that might give students difficulty. **FYI** boxes give additional small nuggets or bits of information.

Timelines are used widely to provide a visual overview of events that occurred at the same time in various parts of the world. They can help students correlate what they may be studying in several different areas of their schoolwork.

Students (or their parents) can also go to SWadvantage.com, where they will find additional, more in-depth information on a wide range of subject matter.

We hope you will find this book both useful and enjoyable. Every effort has been made to ensure that the information in this book is as accurate as possible. If errors should be found, however, we would appreciate hearing from you. Please share your comments or suggestions with us at 1-888-551-5901 or send them to Editor c/o Southwestern Advantage, P.O. Box 305142, Nashville, TN 37230.

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These books have been designed so that information can be accessed easily. Social Studies has been divided into six "strands": United States History; Canadian History; Government; Economics; World History; and Geography. Language Arts has likewise been divided into six strands: Grammar; Writing and Research; Vocabulary; Reading; Speaking; and Literature. Each strand is then divided into smaller units.

The first navigational tool is the detailed, color-coded Table of Contents. The contents pages also indicate separately where the special features of the book can be found, such as the Table of Shakespeare's Plays and the history time lines.

UNITED STATES HISTORY

CANADIAN HISTORY

GOVERNMENT

ECONOMICS

WORLD HISTORY

GEOGRAPHY

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Strand color bar

fyi!

Though the Stone Age in the Near East ended in about 3000 BC, not all societies left it behind at once.

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READING

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LITERATURE

SPECIAL ADVANTAGES

Famous Speeches (annotated) 908
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Table of Shakespeare's Plays 960

THE ROSETTA STONE

For over 1,000 years, scholars tried but failed to decipher the writing system of ancient Egypt. Then, in 1799, a rock slab with ancient Greek and Egyptian writing was found outside Rosetta, a city near Alexandria.

Features

THE NILE VALLEY **WORLD HISTORY**

Unit within World History

Name of strand World History strand

Unit color bar

Spotlight on...

ALEXANDER THE GREAT

356–323 BC

As one of the greatest generals in history, Alexander the Great was responsible for the spread of Greek culture throughout western Asia and Egypt.

- He was educated by the famous Greek philosopher Aristotle.

THE basics

WORLD WAR II: CAUSES AND BEGINNINGS

✓ Worldwide economic depression during the 1930s helped create military dictatorships in Europe and Asia.

WATCH OUT!

Set a goal for each writing session. Whether it's one paragraph or one page, having a goal will help you focus on that particular section.

✓ The end of the Ice Age, about 11,500 years ago, marked the beginning of the Neolithic era.

Know now you Know!

Strand color tab

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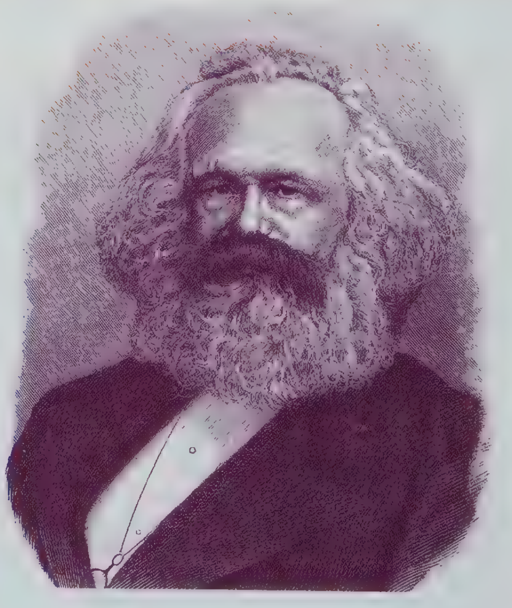

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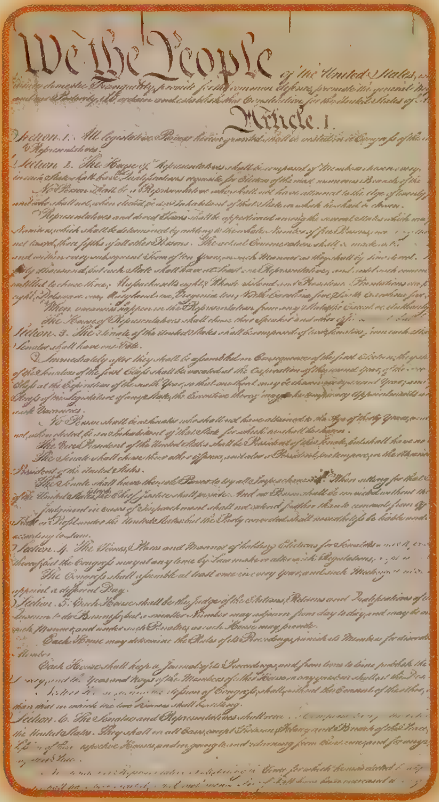
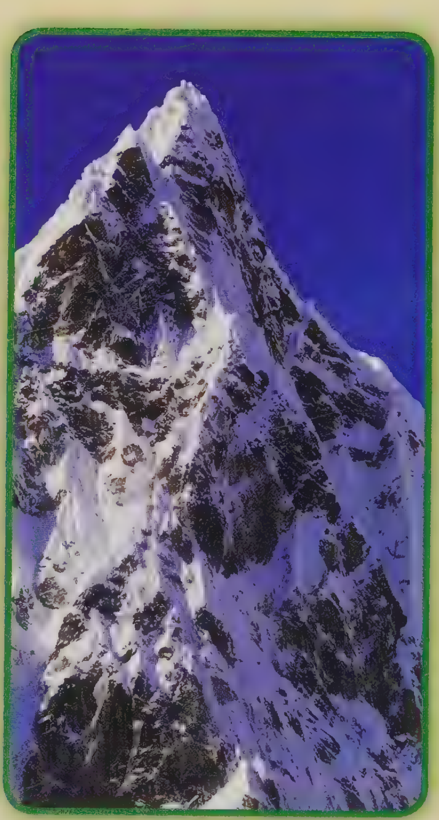


We the People of the United States,

Article. 1

United States History
Canadian History
Government
Economics
World History
Geography



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European Explorers

Where They Came From

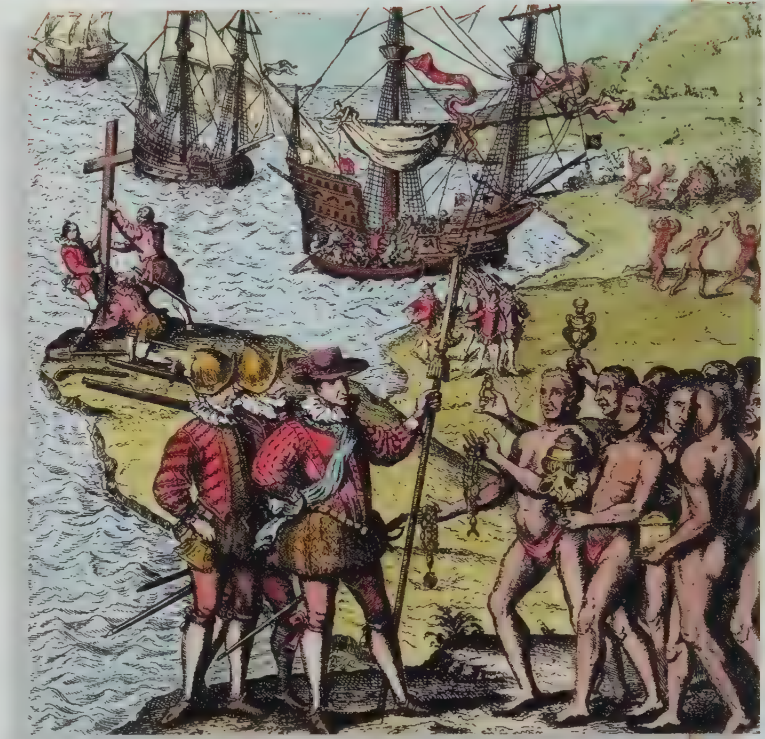
Norway, Sweden, Denmark. The Vikings sailed westward into the Atlantic Ocean from the 800s on. After settling Iceland and Greenland, their travels took them to present-day Newfoundland and Labrador. Repeated attacks by the native populations prevented them from establishing settlements there.

Portugal. During the early 1400s, Portuguese explorers concentrated their attention on the west coast of Africa. Their expeditions south along the coast of Africa resulted in the discovery of a sea route to India, which other nations soon followed.

Spain. Christopher Columbus believed he could find a sea route to the East by sailing west. When his voyages took him to several islands in the Caribbean Sea, he was convinced he'd reached Asia. Later Spanish expeditions led by explorers such as Hernando de Soto and Francisco Coronado also went west, landing in Mexico, Central America, South America, and North America.

France. In 1524, the king of France sent Giovanni da Verrazano to explore the North American coast and find a "Northwest Passage" through North America to Asia. Verrazano did not find this passage, but on the basis of his explorations, France laid claim to North America. Later French expeditions traveled deeper into North America.

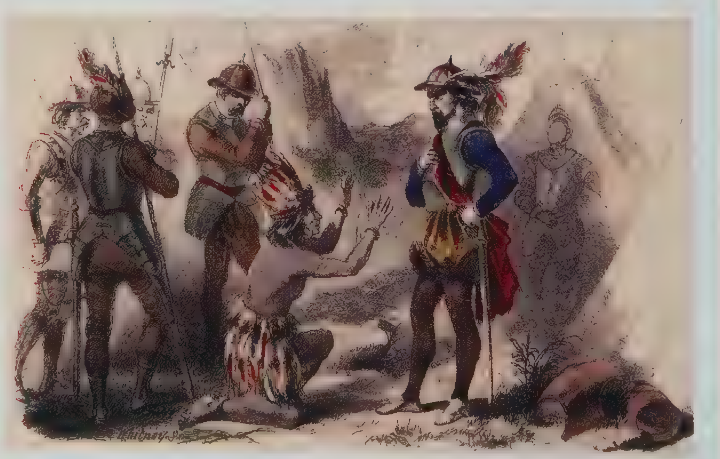
England. England had begun sending its explorers to North America as early as 1497. Many were traveling in search of a Northwest Passage. They were unsuccessful, but explorers such as John Cabot claimed the "new-found-land" for England.



▲ **Christopher Columbus** arrived at the island now called Hispaniola, in the Caribbean.

IMPACTS

The age of exploration brought great wealth to Europe and catastrophe to the native tribes living in the New World. Explorers and colonists brought foreign diseases that devastated whole Indian tribes. Friendly relations gave way to violent conflicts over land. In South America, conquered tribes were enslaved and stripped of their wealth.



Francisco Pizarro of Spain conquered the Incas of Peru. In this image, the Inca ruler Atahualpa pleads for mercy; Pizarro later had Atahualpa killed anyway. ▶

Key Explorers and Expeditions

Name	Sailed For	Dates	Expedition
Bartholomeu Dias	Portugal	1487–1488	• Sailed down the western coast of Africa, near the southern tip of the continent
Vasco da Gama	Portugal	1497–1498	• Followed Dias's route • Continued through the Indian Ocean, and on to India
Christopher Columbus	Spain	1492–1504	• Sought a westward path to Asia • Traveled with three ships—the *Niña,* the *Pinta*, and the *Santa Maria* • Landed on the islands of San Salvador, Hispaniola, and Cuba, believing they were part of Asia
Juan Ponce de León	Spain	1513	• Found gold in Puerto Rico • Explored the coasts of Florida seeking a legendary "Fountain of Youth"
Ferdinand Magellan	Spain	1519–1522	• Searched for a westward route to the Spice Islands • Killed in 1521 in the Philippines • His expedition circumnavigated, or traveled completely around, the globe in 1522
Hernando de Soto	Spain	1539	• Sought gold • Expeditions began in Florida, went through the Carolinas • Traveled as far west as present-day Oklahoma
Francisco Coronado	Spain	1539–1542	• Inspired by tales of wealth from explorer Alvar Nuñez Cabeza de Vaca • Explored the U.S. Southwest, including the Grand Canyon
Jacques Cartier	France	1535–1536	• Failed to find a Northwest Passage • First European to see the St. Lawrence River
René-Robert Cavelier, Sieur de la Salle	France	1643–1687	• Traveled down the Mississippi River to its mouth at the Gulf of Mexico • Claimed the entire region drained by the Mississippi for France and named it Louisiana in honor of King Louis XIV
John Cabot	England	1497–1498	• Explored the coast of North America in search of a water route across North America to Asia • Claimed the land for England
Henry Hudson	Netherlands	1607–1611	• Traveled deep into Canada searching for a Northwest Passage • Claimed the region along the Hudson River (into present-day New York) for the Dutch

Pilgrims and Puritans

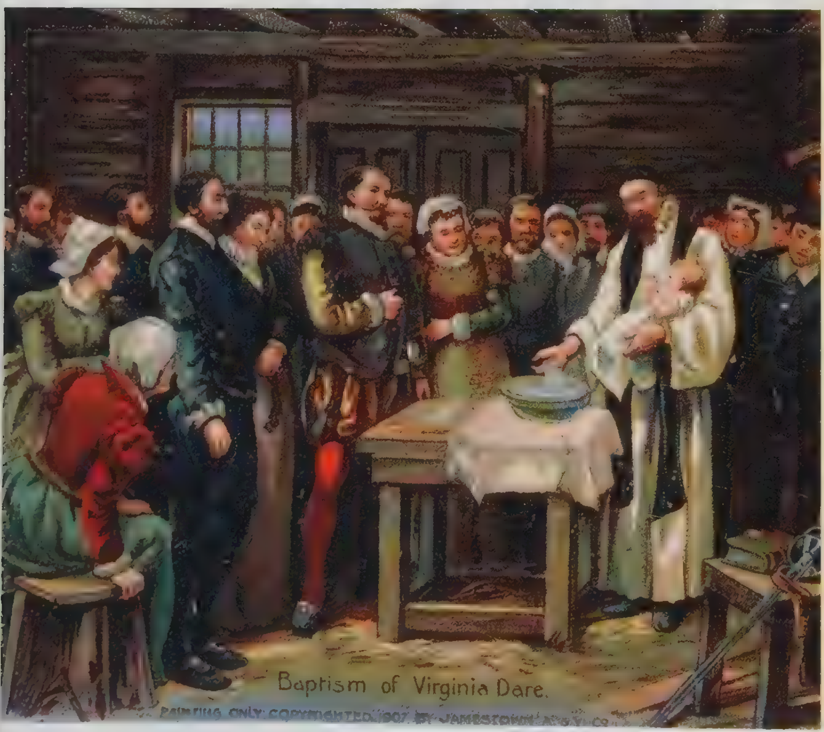
New England

The founding of much of New England came, in part, from trouble in Massachusetts. The Puritan leaders there demanded that people living in Massachusetts follow Puritan beliefs and practices. As a result, some groups left Massachusetts Colony in search of greater religious or political freedom and began their own colonies. The New England area was characterized by rocky soil, plentiful fishing, and woodlands.

Colony	Early Events
Massachusetts	Begins with settlements founded by the Pilgrims (1620) and Puritans (1628). Strict Puritan laws and codes.
New Hampshire	Charter granted to John Mason and Sir Ferdinando Gorges in 1622. Originally consisted of New Hampshire and Maine, but the land is divided in 1629. Made part of Massachusetts in 1641. But in 1680, King Charles II of England made New Hampshire a separate province.
Connecticut	Dissenters from Massachusetts, led by Thomas Hooker, establish a town called Hartford. Hartford joins with Windsor and Wethersfield in 1636 to form Connecticut Colony.
Rhode Island	Begins with settlement in Providence, founded after Roger Williams is banished from Massachusetts.

THE LOST COLONY

One of the earliest English settlement attempts ended with a mystery that is still unsolved. In 1585 and 1587, English adventurer Sir Walter Raleigh sent expeditions to Virginia. The settlers landed on Roanoke Island, off the coast of present-day North Carolina. Their ship sailed back to England for supplies, but was delayed there. When the ship returned in 1591, nothing remained of the colony except the word Croatoan—the name of a local friendly Indian tribe—carved into a tree. All of the colonists had disappeared.


Baptism of Virginia Dare.
PAINTING ONLY COPYRIGHTED 1907 BY JAMESTOWN A. & V. CO.

Virginia Dare. This image shows the baptism of Virginia Dare, the first English child born in North America. Dare disappeared along with the rest of Roanoke Colony.

The Middle Colonies

The first settlers in this area had been Dutch, and much of the region was claimed by the Netherlands. The English seized control of New Netherland in 1664. This area had richer soil than the New England region and therefore was home to more farming.

Colony	Early Events
New York	Initially the Dutch settlement of New Amsterdam. Dutch settlers did not resist the English takeover in the late 1600s, due to their dislike of the current Dutch governor, Peter Stuyvesant. Renamed New York, after the English king appointed the Duke of York as its proprietor.
Delaware	Originally settled by the New Sweden Company in 1638. Taken over by the Dutch in 1655. The English take control of the area in 1664.
Pennsylvania	Created by William Penn, a Quaker, in response to religious persecution. Charter granted in 1681; government established the following year. Advertised by pamphlets throughout the countries of Western Europe. The colony granted religious freedom to Quakers, Protestants, Catholics, and Jews.
New Jersey	Given by the Duke of York to two of his friends, Lord John Berkeley and Sir George Carteret. Initially divided into East Jersey and West Jersey, but eventually combined into a single colony.

The Southern Colonies

In this region, rich soil, flat land, and a warm climate combined to promote large-scale agriculture.

Colony	Early Events
Virginia	The first permanent English settlement, dating back to Jamestown in 1607.
Maryland	Founded to offer religious freedom to Roman Catholics, who were persecuted in England. In 1649, the Maryland Toleration Act granted religious freedom to all Christians who settled there.
North and South Carolina	Began as a single large colony called Carolina, under the control of eight proprietors. Conflicts with the Spanish and the Indians, and within the colony itself, led to it being controlled by the English Crown.
Georgia	Last of the 13 colonies to be established. Founded as a place where people in debt could start over, instead of going to debtors' prison in England.

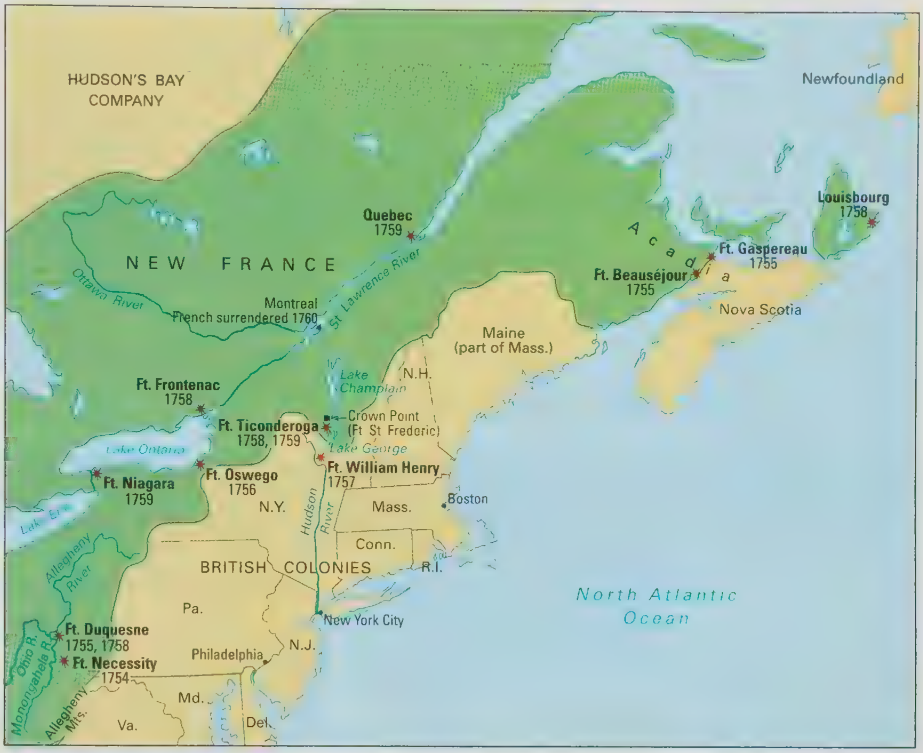
England vs. France

As the American colonies expanded into the West, their claims began to conflict with the lands claimed by the French. This rivalry led to a series of wars, fought from 1689 to 1763, which came to be called the French and Indian Wars. (See also pages 252–257.)

The French and Indian Wars, 1689–1763

War	Causes and Events	Results
King William's War (1689–1697)	• Named after King William III of England. • In New York, Indian allies of the English challenged French control of the fur trade. • In New England, Indian allies of the French resisted English expansion. • War broke out between England and France.	• Fighting in North America was inconclusive. • Treaty of Ryswick (1697) restored all conquered lands to their original owners. • France stepped up settlement in the Ohio and Mississippi river valleys.
Queen Anne's War (1702–1713)	• Named after Queen Anne of England. • Britain went to war against France to prevent France from gaining control of the Spanish throne. • English colonists attacked and burned Spanish-held St. Augustine (1702), captured all but one of the 14 missions in northwestern Florida (1704), and captured Port Royal, Nova Scotia (1710). • The colonists failed to take Quebec (1711).	• The American phase of the war went in favor of the English colonists. • Treaty of Utrecht (1713) gave Britain Hudson Bay, Newfoundland, and Nova Scotia. • France became determined to develop its western holdings.
King George's War (1744–1748)	• Named for King George II of Britain. • Fighting in North America began when the French tried to regain Nova Scotia. • The greatest battle of the war occurred in 1745, when New England colonial troops captured the French fortress of Louisbourg on Cape Breton Island.	• The Treaty of Aix-la-Chapelle, which ended the war, gave back to Britain and France the territory each side had lost in the war.
The French and Indian War (1754–1763)	• The final conflict between England and France in North America. • The French moved to gain control of the Ohio Valley. • In 1754, a Virginia force was sent to order French withdrawal and establish a fort at the site of present-day Pittsburgh, but the French had already established Fort Duquesne. • Virginia forces established Fort Necessity nearby at Great Meadows but were forced to withdraw.	• After initial French successes, the tide of war turned against them. • The British took forts Duquesne (1758) and Ticonderoga (1759) and captured Quebec (1759) and Montreal (1760). • The Treaty of Paris (1763) ended the war. France surrendered all its territories east of the Mississippi River, except for New Orleans.

The French and Indian Wars eventually led to the end of France's colonial power in North America. This map shows the location of several key battles.


British possession
French possession
Colonial boundary
Major battle
Fort
City
0 200 Miles
0 200 Kilometers
HUDSON'S BAY COMPANY
NEW FRANCE
Newfoundland
Quebec 1759
Louisbourg 1758
Acadia
Ft. Gaspereau 1755
Ft. Beauséjour 1755
Nova Scotia
St. Lawrence River
Ottawa River
Montreal
French surrendered 1760
Maine (part of Mass.)
N.H.
Lake Champlain
Ft. Frontenac 1758
Ft. Ticonderoga 1758, 1759
Crown Point (Ft St Frederic)
Lake George
Ft. William Henry 1757
Lake Ontario
Ft. Niagara 1759
Ft. Oswego 1756
N.Y.
Hudson River
Mass.
Boston
Lake Erie
Allegheny River
Conn.
R.I.
BRITISH COLONIES
North Atlantic Ocean
Pa.
New York City
Ft. Duquesne 1755, 1758
Ft. Necessity 1754
Philadelphia
N.J.
Ohio R.
Monongahela R.
Allegheny Mts.
Md.
Va.
Del.

Now you Know!

- ✔ Some colonizers of North America were interested mostly in the opportunities the colonies offered to build wealth. Other colonizers, such as the English Puritans and Pilgrims, were primarily interested in the opportunities the colonies offered to create societies based on their religious beliefs.
- ✔ The colonies of the Atlantic coast fell into groups according to geography: New England, in the north, Middle colonies, and Southern colonies. From 1607 to 1730, 13 colonies became established in the region.
- ✔ The colonial economy was largely agricultural. Small farms were characteristic of New England, whereas conditions in the Middle and Southern colonies supported large-scale farming of such crops as cotton, tobacco, and wheat.
- ✔ Government of the colonies varied with their types. The monarch ruled royal colonies; one or more proprietors (owners) governed proprietary colonies; and some colonies were self-governing.
- ✔ Such parts of colonial government as the House of Burgesses, a form of legislature, in Virginia, and the Fundamental Orders of Connecticut, a type of constitution, would serve as models for similar parts of the government of the United States.
- ✔ Native Americans and colonists largely coexisted at first. However, as the colonies began to prosper and take over more and more Native American land, the coexistence turned into conflict. Wars between colonists and Native Americans took place from the early 1600s through the early 1700s.
- ✔ The English and French fought for control of territory in what are now Canada and the United States in the French and Indian Wars. The wars erupted periodically from the late 1600s through the mid-1700s. At the end of the wars, the English controlled much more territory than the French.

INDEPENDENCE

The Revolutionary War

The Revolutionary War began on April 19, 1775, when British soldiers and American patriots clashed at Lexington, Massachusetts, and at nearby Concord. The war would last 8 years. Although the British won many battles, they gained little from their victories. The American patriots were able to form new forces and fight on.

The American Revolution: 1766–1783

1766–1774

March 8, 1766
Repeal of the Stamp Act

March 5, 1770
Boston Massacre

June 9, 1772
Burning of the customs schooner *Gaspé*

Dec. 16, 1773
Boston Tea Party

Sept. 5, 1774
First Continental Congress (to Oct. 6, 1774)

1775

April 18–19, 1775
Paul Revere's ride and battle of Lexington and Concord

May 10
American forces under Ethan Allen and Benedict Arnold capture Ticonderoga

June 17
Battle of Bunker Hill: The British, under Gov. Thomas Gage, drive the Americans back

July 3
Washington assumes command of the Continental Army at Boston

July
Siege of Boston (to Mar. 1776)

Sept. to Dec.
American invasion of Canada

Nov. 12
American forces capture Montreal

1776

March 1776
The British evacuate Boston

July 4
Declaration of Independence adopted by Continental Congress in Philadelphia

Sept. 15
Occupation of New York City by the British

Sept. 22
Nathan Hale executed as a spy

Oct. 11–13
Naval battle on Lake Champlain: The British defeat American forces led by Benedict Arnold

Nov. to Dec.
Washington's forces retreat through New Jersey

Dec. 25
Washington's forces cross the Delaware River

Dec. 26
Battle of Trenton, NJ: Washington's troop defeats Hessian (German) mercenaries in a surprise attack

1777

Jan. 3, 1777
Washington gains a victory at the Battle of Princeton, NJ

April 25–26
British raid and burn Danbury, CT

Sept. 11
British victory at the Battle of Brandywine, PA

Sept. 19
Continental Congress flees from Philadelphia to York, PA

Sept. 26
British occupy Philadelphia

Oct. 4
Washington's forces defeated at Battle of Germantown, PA

Oct. 17
Surrender of the British Army at Saratoga, NY, by Sir John Burgoyne. The victory, a turning point in the war, inspires the French to commit their support to the Americans.

Nov. 15
Articles of Confederation submitted to the states for ratification

Dec. 19
Washington's army encamps at Valley Forge, PA (to June 17, 1778)

1778

Feb. 6, 1778
Franco-American treaty of alliance signed in Paris

June 18
British evacuate Philadelphia

July 4
British forces defeat American troops in what comes to be called the Wyoming Massacre (in PA)

July 11
Arrival of French fleet under Comte d'Estaing off Sandy Hook, NY

Nov. 11
Cherry Valley Massacre (in PA) by Indians led by Chief Joseph Brant, allied with the British

Dec. 29
Capture of Savannah, GA, by British general Robert Howe

1779

Jan. 29 , 1779
British conquest of Georgia succeeds

Feb. 25
Capture of Vincennes, IN, by American forces

June 21
Spain declares war on Great Britain

July 5
Plundering and burning of New Haven, CT, by British forces

Sept. 23
The *Bonhomme Richard*, an American ship, captures the British *Serapis* in a naval battle

Oct. 9
American forces launch an assault on Savannah. Washington's aide-de-camp, Casimir Pulaski, is killed.

1780

May 12, 1780
British siege of Charleston, SC, ends in American surrender

July 10
Arrival of French troops under Rochambeau at Newport, RI

July 25
Gen. Horatio Gates assumes command of American army in the South

Aug. 16
Battle of Camden, SC, ends in a British victory under Lord Cornwallis

Sept. 21
Benedict Arnold's treason uncovered

Dec. 4
Gen. Nathanael Greene assumes command of the American army in the South

1781–1783

March 1, 1781
Articles of Confederation become effective

Sept. 28 to Oct. 19, 1781
Siege of Yorktown, VA

Oct. 19, 1781
The British surrender at Yorktown, VA, to forces led by George Washington, ending the war

July 11, 1782
British evacuate Savannah, GA

Nov. 30, 1782
Preliminary Treaty of Peace between the United States and Great Britain signed in Paris

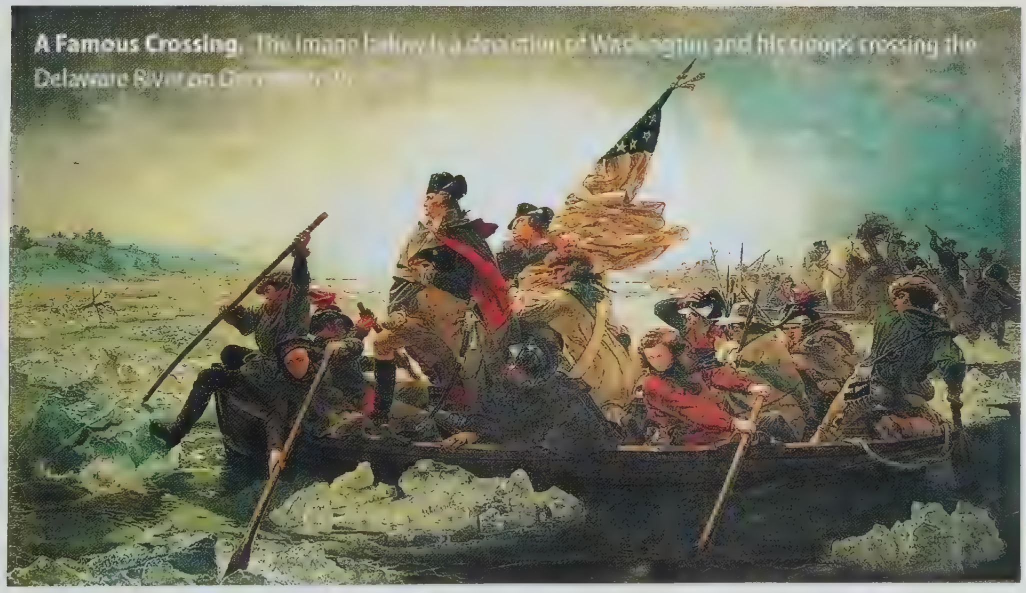
Dec. 14, 1782
British evacuate Charleston, SC

Sept. 3, 1783
The United States and Great Britain sign the definitive Treaty of Paris

Nov. 25, 1783
British evacuate New York City

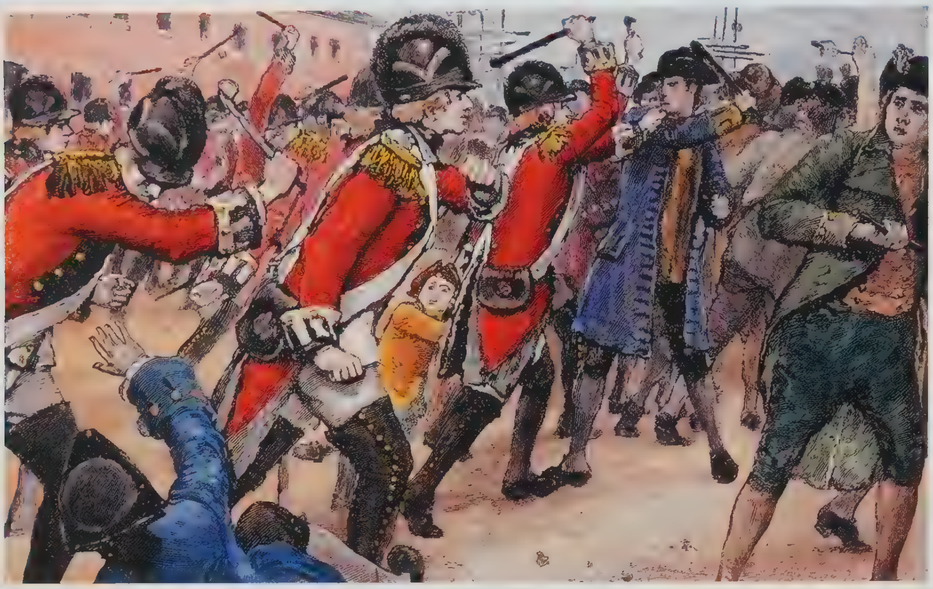
Dec. 4, 1783
Washington delivers his Farewell Address to his officers in Fraunces Tavern, New York City

Dec. 23, 1783
Washington resigns as commander in chief of the American army



A Famous Crossing. The image below is a depiction of Washington and his troops crossing the Delaware River on [illegible]

Growing Resistance



The Boston Massacre had both immediate and long-term effects. It resulted in the repeal of several unpopular laws. Additionally, two of the British soldiers involved in the incident were convicted of manslaughter. In the long term, it was also one step closer to full revolution.

The Boston Massacre

The Boston Massacre was the first conflict between Britain and the colonies to become violent. Still, it was not truly a massacre; the term was used to create anti-British sentiment in the colonies.

Action

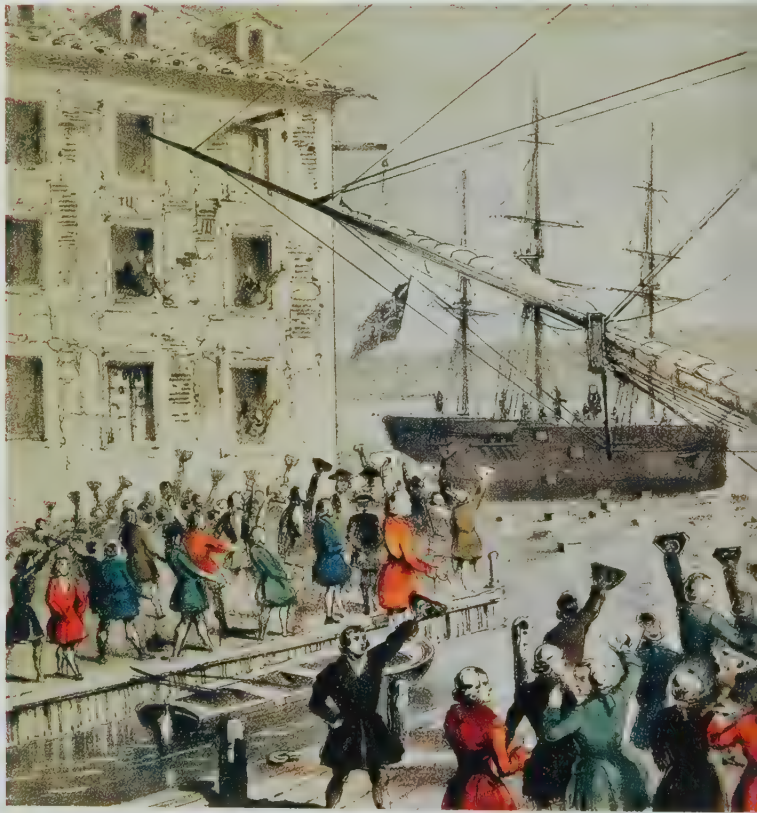
- British troops were stationed in Boston after protests against the Townshend Acts (see preceding page).
- Under the Quartering Act, the citizens of Boston were forced to provide housing for the troops.

Colonial Response

- On March 5, 1770, the Bostonians' anger exploded into violence. As a crowd of Bostonians taunted red-coated soldiers standing guard near the Customs House, a shot rang out, and the soldiers opened fire on the crowd. When the smoke cleared, five of the colonists lay dead or dying in the snow.

Results

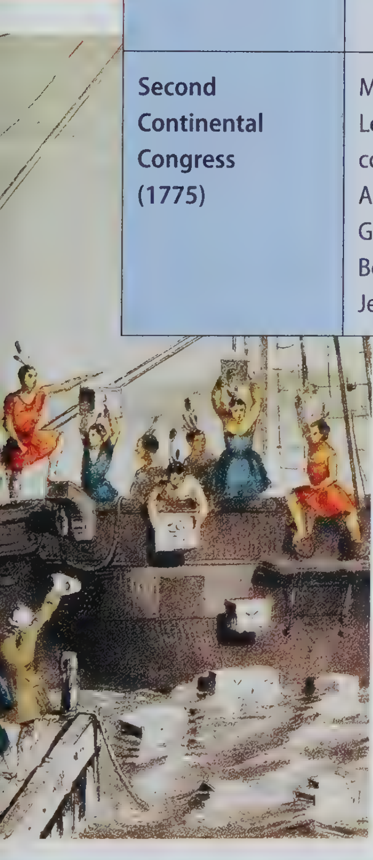
- Word of this "Boston Massacre" spread quickly through the colonies. In the next month, the British government repealed all the Townshend duties except the one on tea. It left that as an indication of its right to tax the colonists.



Organized Resistance

Organized resistance took a few different forms, with the colonies gradually forming a large united front against British actions.

Movement	Characteristics	Actions
Committees of Correspondence	Started by Samuel Adams after the Boston Massacre. Soon there were Committees in nearly every colony. Members included John Hancock and James Otis of Massachusetts, and Patrick Henry and Thomas Jefferson of Virginia.	• Kept tabs on British actions in the colonies • Passed information along to other colonists • Encouraged opposition to British actions
First Continental Congress (1774)	Included representatives of all the colonies (except Georgia). The Congress met to draw up a united protest against British acts.	• Declared the Coercive Acts null and void • Pledged that their colonies would not buy goods from Britain • Urged the colonists of Massachusetts not to pay any taxes to their military government • Encouraged Massachusetts to organize a militia • Sent a formal protest to King George III • Planned to meet again the following May
Second Continental Congress (1775)	Met after military clashes at Lexington and Concord. Included colonists from every colony. Among the representatives: George Washington, John Adams, Benjamin Franklin, Thomas Jefferson, and John Hancock.	• Petitioned the king to repeal the Intolerable Acts • Authorized the creation of the Continental Army • Named George Washington commander of the Continental Army's forces



THE BOSTON TEA PARTY

In 1773, the British passed the Tea Act, which favored the East India Company, a British tea-selling company, and threatened to put colonial tea merchants out of business. In the dark of night on December 16, 1773, some 50 Sons of Liberty disguised as Indians stole aboard three ships anchored in Boston Harbor that were loaded with tea from the hated British tea company. They threw all of the tea overboard. This "Boston Tea Party" enraged the British government.

The Boston Tea Party brought immediate backlash from Britain: the Intolerable Acts. (See page 49.)

We the People

The writers of the Constitution had thrown off a government they found oppressive. They were very concerned about the same oppression developing in the new nation. At the same time, they needed a government with enough power to address the needs of a large—and still growing—nation.

Principles

The Constitution aims to spell out both the responsibilities of the United States government and the structure of that government.

The purposes of the government are laid out in the Constitution's first section, or Preamble:

- "establish justice"—mediate between citizens
- "ensure domestic tranquility"—protect citizens
- "provide for the common defense"—conduct foreign affairs
- "promote the general welfare"—provide services for citizens
- "secure the blessings of liberty"—promote prosperity.

Structure

The Preamble is followed by seven sections, called Articles:

- Article I – The Legislative Branch
- Article II – The Executive Branch
- Article III – The Judicial Branch
- Article IV – Relation of the states to each other, Federal-state relations
- Article V – Amending the Constitution
- Article VI – National Debts
- Article VII – Ratifying the Constitution

Following the Articles is a section of Amendments—additions or changes to the Constitution—made since 1787.

The Constitution Since 1787

The Constitution remains effective to this day because it can be amended, or changed, to meet the nation's changing needs.

Amendments. Amendments address issues missing from the original—for example, how to fill a vacant vice president's seat (25th amendment)—or extend rights to formerly excluded groups. Among other things, constitutional amendments have:

- Granted citizenship to former slaves
- Granted the right to vote to women
- Lowered the voting age for all citizens

The Bill of Rights. The first 10 amendments to the Constitution are called the Bill of Rights. These amendments, added in 1791, protect individual liberties such as freedom of speech, religion, and the press.

ESSENTIALS OF THE CONSTITUTION

Separation of Powers. The responsibilities of the government are carefully divided among the three branches: the legislative, the executive, and the judicial.

Checks and Balances. The Constitution provides for checks and balances that offer each branch protection against the others.

- The president (executive branch) has the power of veto over acts of Congress; he or she may simply refuse to sign into law a bill Congress has passed.
- The Congress (legislative branch) may override a presidential veto, if both the House and the Senate vote to override by a two-thirds majority.
- The judiciary (judicial branch) determines the constitutionality of laws passed by Congress and signed by the president.

The ultimate check in this system is that the president, the vice president, and federal judges are subject to impeachment and removal for violation of their oaths of office. (See also page 340.)

Constitution of the United States

Preamble

We the people of the United States, in order to form a more perfect union, establish justice, insure domestic tranquility, provide for the common defense, promote the general welfare, and secure the blessings of liberty to ourselves and our posterity, do ordain and establish this Constitution for the United States of America.

Note: The titles and annotations in this column are aids to understanding. They are not part of the Constitution itself.

Article I

The Legislative Branch

Section 1. All legislative powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

House of Representatives

Section 2. The House of Representatives shall be composed of members chosen every second year by the people of the several States, and the electors in each State shall have the qualifications requisite for electors of the most numerous branch of the State legislature.

No person shall be a Representative who shall not have attained to the age of twenty-five years, and been seven years a citizen of the United States, and who shall not, when elected, be an inhabitant of that State in which he shall be chosen.

Representatives and direct taxes shall be apportioned among the several States which may be included within this Union, according to their respective numbers, which shall be determined by adding to the whole number of free persons, including those bound to service for a term of years, and excluding Indians not taxed, three-fifths of all other persons. The actual enumeration shall be made within three years after the first meeting of the Congress of the United States, and within every subsequent term of ten years, in such manner as they shall by law direct. The number of Representatives shall not exceed one for every thirty thousand, but each State shall have at least one Representative; and until such enumeration shall be made, the State of New Hampshire shall be entitled to choose three; Massachusetts, eight; Rhode Island and Providence Plantations, one; Connecticut, five; New York, six; New Jersey, four; Pennsylvania, eight; Delaware, one; Maryland, six; Virginia, ten; North Carolina, five; South Carolina, five; and Georgia, three.

This section set up the national census to determine how many representatives in Congress each state would have. Originally, slaves and Indians were not counted "whole persons" in the census, but the 14th Amendment gave former slaves the full rights of citizenship. The number of congressmen from each state is still determined by the number of people in the state, but the total membership of the House is limited to 435.

When vacancies happen in the representation from any State, the executive authority thereof shall issue writs of election to fill such vacancies.

The House of Representatives shall choose their Speaker and other officers, and shall have the sole power of impeachment.

"Impeachment" means accusing an official of wrong conduct in office. The House of Representatives makes these charges, and the Senate acts as the court where they are tried (see Section 3).

Senate

Since the 17th Amendment was passed in 1913, senators have been chosen by direct popular vote, not by the legislatures.

This clause set up a system of staggered elections to the Senate. All senators now have 6-year terms, but the terms expire at different times.In one election year, only one-third of the senators are up for election; the others still have 2 or 4 more years to serve. This gives the Senate more continuity than the House, where all members are up for election every 2 years.

Since there is always an even number of senators, tie votes are possible, and so the vice president was given the power to break ties.

Section 3. The Senate of the United States shall be composed of two Senators from each State, chosen by the legislature thereof for six years; and each Senator shall have one vote.

Immediately after they shall be assembled in consequence of the first election, they shall be divided as equally as may be into three classes. The seats of the Senators of the first class shall be vacated at the expiration of the second year, of the second class at the expiration of the fourth year, and of the third class at the expiration of the sixth year, so that one-third may be chosen every second year; and if vacancies happen by resignation, or otherwise, during the recess of the legislature of any State, the executive thereof may make temporary appointments until the next meeting of the legislature, which shall then fill such vacancies.

No person shall be a Senator who shall not have attained the age of thirty years, and been nine years a citizen of the United States, and who shall not, when elected, be an inhabitant of that State for which he shall be chosen.

The Vice President of the United States shall be President of the Senate, but shall have no vote, unless they be equally divided.

The Senate shall choose their other officers and also a President pro tempore, in the absence of the Vice President, or when he shall exercise the office of President of the United States.

The Senate shall have the sole power to try all impeachments. When sitting for that purpose, they shall be on oath or affirmation. When the President of the United States is tried, the Chief Justice shall preside; and no person shall be convicted without the concurrence of two-thirds of the members present.

Judgment in cases of impeachment shall not extend further than to removal from office, and disqualification to hold and enjoy any office of honor, trust, or profit under the United States; but the party convicted shall nevertheless be liable and subject to indictment, trial, judgment and punishment, according to law.

Congressional Elections

Section 4. The times, places, and manner of holding elections for Senators and Representatives shall be prescribed in each State by the legislature thereof; but the Congress may at any time by law make or alter such regulations, except as to the places of choosing Senators.

The 20th Amendment changed this meeting time to noon, January 3.

The Congress shall assemble at least once in every year, and such meeting shall be on the first Monday in December, unless they shall by law appoint a different day.

Procedures

Section 5. Each House shall be the judge of the elections, returns, and qualifications of its own members, and a majority of each shall constitute a quorum to do business; but a smaller number may adjourn from day to day, and may be authorized to compel the attendance of absent members, in such manner, and under such penalties, as each House may provide.

Each House may determine the rules of its proceedings, punish its members for disorderly behavior, and with the concurrence of two-thirds, expel a member.

Each House shall keep a journal of its proceedings, and from time to time publish the same, excepting such parts as may in their judgment require secrecy, and the yeas and nays of the members of either House on any question shall, at the desire of one-fifth of those present, be entered on the journal.

In addition to the *Congressional Record*, which is published every day, both houses of Congress keep a record of their proceedings.

Neither House, during the session of Congress, shall, without the consent of the other adjourn for more than three days, nor to any other place than that in which the two Houses shall be sitting.

Section 6. The Senators and Representatives shall receive a compensation for their services, to be ascertained by law and paid out of the Treasury of the United States. They shall, in all cases except treason, felony, and breach of the peace, be privileged from arrest during their attendance at the session of their respective Houses, and in going to and returning from the same; and for any speech or debate in either House they shall not be questioned in any other place.

Payment and Privileges

These privileges are called "congressional immunity."

No Senator or Representative shall, during the time for which he was elected, be appointed to any civil office under the authority of the United States, which shall have been created, or the emoluments whereof shall have been increased during such time; and no person holding any office under the United States shall be a member of either House during his continuance in office.

"Emoluments" are salaries. This section prevents federal officials from being members of Congress at the same time.

Section 7. All bills for raising revenue shall originate in the House of Representatives; but the Senate may propose or concur with amendments as on other bills.

Relation to Executive

Every bill which shall have passed the House of Representatives and the Senate shall, before it become a law, be presented to the President of the United States; if he approve he shall sign it, but if not he shall return it, with his objections, to that House in which it shall have originated, who shall enter the objections at large on their journal and proceed to reconsider it. If after such reconsideration two-thirds of that House shall agree to pass the bill, it shall be sent, together with the objections, to the other House, by which it shall likewise be reconsidered, and if approved by two-thirds of that House, it shall become a law. But in all such cases the votes of both Houses shall be determined by yeas and nays, and the names of the persons voting for and against the bill shall be entered on the journal of each House respectively. If any bill shall not be returned by the President within ten days (Sundays excepted) after it shall have been presented to him, the same shall be a law, in like manner as if he had signed it, unless the Congress by their adjournment prevent its return, in which case it shall not be a law.

This section describes the president's veto power. Even if a bill has been passed by both the Senate and the House, the president can veto it, or turn it down, instead of signing it and making it a law. However, a two-thirds vote by both houses can pass the bill over his veto. Simply holding the bill when Congress is about to adjourn is a "pocket veto."

Every order, resolution or vote to which the concurrence of the Senate and House of Representatives may be necessary (except on a question of adjournment) shall be presented to the President of the United States; and before the same shall take effect shall be approved by him, or being disapproved by him, shall be repassed by two-thirds of the Senate and House of Representatives, according to the rules and limitations prescribed in the case of a bill.

Specific Powers

Section 8. The Congress shall have power to lay and collect taxes, duties, imposts and excises, to pay the debts and provide for the common defense and general welfare of the United States; but all duties, imposts and excises shall be uniform throughout the United States;

To borrow money on the credit of the United States;

To regulate commerce with foreign nations, and among the several States, and with the Indian tribes;

To establish an uniform rule of naturalization, and uniform laws on the subject of bankruptcies throughout the United States;

To coin money, regulate the value thereof, and of foreign coin, and fix the standard of weights and measures;

To provide for the punishment of counterfeiting the securities and current coin of the United States;

To establish post offices and post roads;

This section allows Congress to pass laws about patents and copyrights.

To promote the progress of science and useful arts by securing for limited times to authors and inventors the exclusive right to their respective writings and discoveries;

To constitute tribunals inferior to the Supreme Court;

To define and punish piracies and felonies committed on the high seas, and offenses against the law of nations;

Only Congress can declare war, but the president, as commander in chief, can order the armed forces to act.

To declare war, grant letters of marque and reprisal, and make rules concerning captures on land and water;

To raise and support armies, but no appropriation of money to that use shall be for a longer term than two years;

To provide and maintain a navy;

To make rules for the government and regulation of the land and naval forces;

To provide for calling forth the militia to execute the laws of the Union, suppress insurrections, and repel invasions;

To provide for organizing, arming and disciplining the militia, and for governing such part of them as may be employed in the service of the United States, reserving to the States respectively the appointment of the officers, and the authority of training the militia according to the discipline prescribed by Congress;

This gave Congress the authority to establish and govern the District of Columbia.

To exercise exclusive legislation in all cases whatsoever over such district (not exceeding ten miles square) as may, by cession of particular States and the acceptance of Congress, become the seat of the government of the United States, and to exercise like authority over all places purchased by the consent of the legislature of the State in which the same shall be, for the erection of forts, magazines, arsenals, dockyards, and other needful buildings;—And

To make all laws which shall be necessary and proper for carrying into execution the foregoing powers, and all other powers vested by this Constitution in the Government of the United States, or in any department or officer thereof.

This is sometimes called the "elastic clause," as it can be interpreted to give many powers not actually mentioned in the Constitution.

Section 9. The migration or importation of such persons as any of the States now existing shall think proper to admit shall not be prohibited by the Congress prior to the year one thousand eight hundred and eight, but a tax or duty may be imposed on such importation, not exceeding ten dollars for each person.

Limitations on Congress

This paragraph set up a waiting period for action on the slave trade; Congress did abolish it in 1808.

The privilege of the writ of habeas corpus shall not be suspended, unless when in cases of rebellion or invasion the public safety may require it.

Habeas corpus guards against unjust imprisonment by requiring a judge or court to decide if a person may be held.

No bill of attainder or ex post facto law shall be passed.

No capitation or other direct tax shall be laid, unless in proportion to the census or enumeration herein before directed to be taken.

An *ex post facto* law applies to acts committed before the law was passed. The 16th Amendment allowed the income tax, which is not related to the census.

No tax or duty shall be laid on articles exported from any State.

No preference shall be given by any regulation of commerce or revenue to the ports of one State over those of another; nor shall vessels bound to or from one State be obliged to enter, clear or pay duties in another.

No money shall be drawn from the Treasury but in consequence of appropriations made by law; and a regular statement and account of the receipts and expenditures of all public money shall be published from time to time.

No title of nobility shall be granted by the United States; and no person holding any office of profit or trust under them shall, without the consent of the Congress, accept of any present, emolument, office, or title of any kind whatever from any king, prince, or foreign state.

In fact, presidents often exchange gifts with important foreign visitors, but the gifts are considered as gifts to the country.

Section 10. No State shall enter into any treaty, alliance, or confederation; grant letters of marque and reprisal; coin money; emit bills of credit; make anything but gold and silver coin a tender in payment of debts; pass any bill of attainder, ex post facto law or law impairing the obligation of contracts, or grant any title of nobility.

Limitations on States

No State shall, without the consent of the Congress, lay any imposts or duties on imports or exports, except what may be absolutely necessary for executing its inspection laws; and the net produce of all duties and imposts, laid by any State on imports or exports, shall be for the use of the Treasury of the United States; and all such laws shall be subject to the revision and control of the Congress.

No State shall, without the consent of Congress, lay any duty of tonnage, keep troops or ships of war in time of peace, enter into any agreement or compact with another State or with a foreign power, or engage in war, unless actually invaded or in such imminent danger as will not admit of delay.

Jefferson and Monroe

Thomas Jefferson

In the presidential election of 1800, Federalist John Adams was defeated by Thomas Jefferson, and the Democratic-Republicans took over. Jefferson was a firm believer in limited government and was in constant conflict with Federalist leader Alexander Hamilton over the government's power. (See page 67.)

Presidency (1801–1809). Jefferson's first term was a time of growth and prosperity. The United States increased greatly in size, and its economy expanded. Before his second term was over, however, conflict overseas nearly pushed the nation into war.

THE WORLD OF JEFFERSON

1803

- Dalton's Theory of the Atom, one of the foundations of chemistry, was developed.

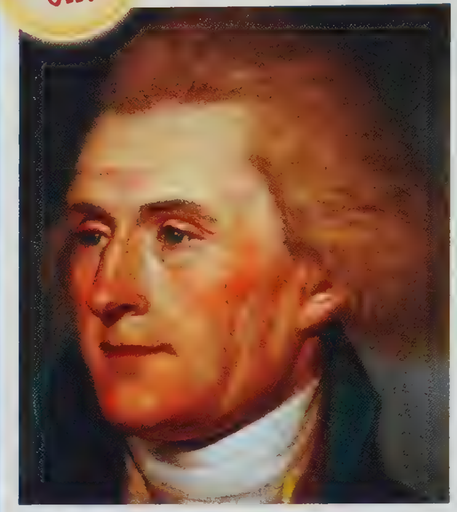
1804

- Napoleon crowned himself emperor of France.
- Richard Trevithick, an English engineer, invented the steam-powered locomotive.

1807

- American inventor Robert Fulton began the first successful steamboat service.

Spotlight on... THOMAS JEFFERSON



1743–1826

- Designed the Virginia Capitol, the University of Virginia, and his home at Monticello
- His library was the basis of the Library of Congress
- Wrote the first draft of the Declaration of Independence
- Served as vice president under John Adams

Major Events	Causes and Details	Effects
Tripolitan War (1801–1805)	Tripoli (a Barbary State on the coast of north Africa) demanded increased tributes to protect American ships in the Mediterranean. Jefferson refused to pay and blockaded the Tripoli port; war followed.	A peace settlement ended the tributes to Tripoli. The U.S. paid tribute to other Barbary States until 1815.
The Louisiana Purchase (1803)	Napoleon offered to sell the vast area between the Mississippi River and the Rocky Mountains for only $15 million. In 1810, West Florida was also annexed.	The U.S. doubled in size. Congress extended the Mississippi Territory and created the Michigan and Illinois territories.
Embargo Act (1807)	Britain and France had interfered with American ships for years. The Embargo Act forbade American ships to call on foreign ports.	The Act hurt American traders and did not stop British and French interference. Trade resumed with all nations except Britain and France in 1809.

Major Events	Causes and Details	Effects
"The American System"	• In 1817, Congress passed a bill to help finance the construction of "a perfect system of roads and canals." • The Erie Canal was approved by the New York legislature, but not by Congress.	• Completed in 1825, the Canal ran from Albany to Buffalo on Lake Erie. • Opened a shipping passage from the Atlantic Ocean to the Great Lakes. • Aided the growth of industry and the westward movement of settlers.
Seminole War	• In 1817, fighting broke out between Georgia settlers and Seminole Indians in Spanish Florida, partly because the Americans were not honoring treaties. • Monroe ordered Major General Andrew Jackson to attack the Indians.	• Jackson captured Pensacola, Spanish Florida's capital. • In 1819, the Adams-Onis Treaty gave Florida to the United States in return for the cancellation of $5 million in Spanish debt to the U.S.

James Monroe

In 1816, the Democratic-Republican candidate, James Monroe of Virginia, won the presidency. Monroe became president after more than 40 years of public service.

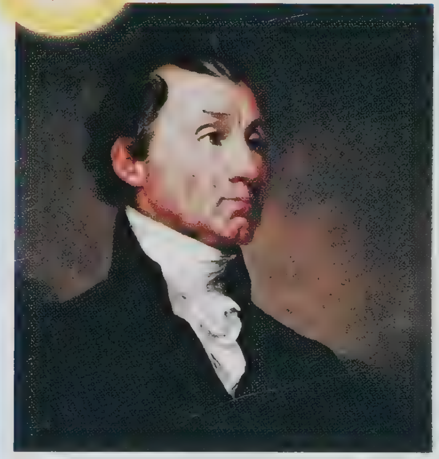
Presidency (1817–1825). Following the War of 1812, America took its place as a full-fledged nation in the world. At home, there was a growing sense of unity and nationalism. As a result of this, the two terms of Monroe's presidency are often referred to as "the Era of Good Feelings."

THE MONROE DOCTRINE

This foreign policy declaration became firmly established as part of the nation's political practice.

- In the early 1800s, Spanish colonies in Latin America began declaring their independence.
- Monroe and (Secretary of State) John Quincy Adams worried that Spain and its European allies might attempt to reclaim the colonies.
- In 1823, Monroe recognized the colonies' independence and warned Europe against any attempt at conquest or colonization in the Western Hemisphere.

Spotlight on... JAMES MONROE



1758–1831

- Studied law under Thomas Jefferson
- With James Madison and Thomas Jefferson, helped form the Democratic-Republican party
- As U.S. minister to France, helped arrange the Louisiana Purchase

Abolitionists Speak

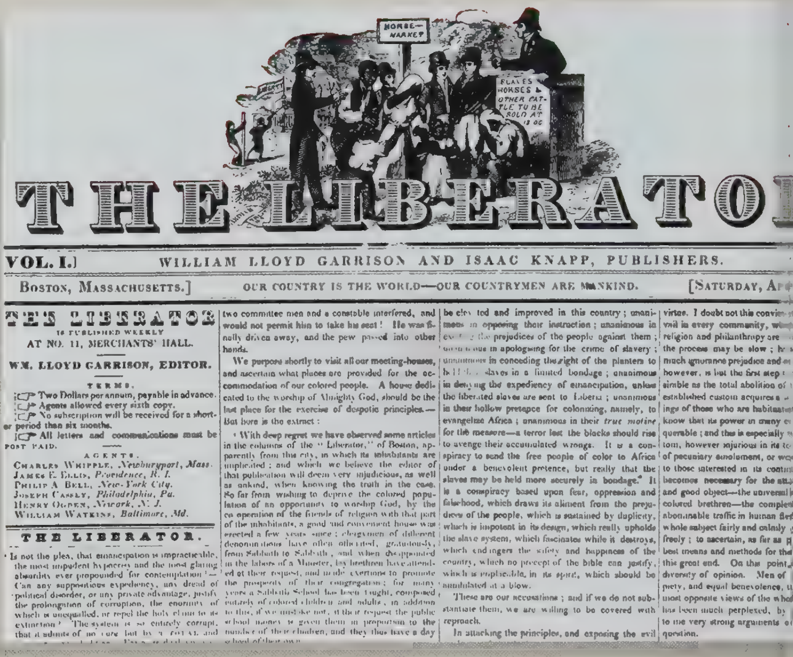
During the 1700s, noted philosophers and religious leaders in Europe and North America began to condemn slavery. They declared that slavery violated human rights and God-given law. From this sentiment grew a movement to abolish, or ban, slavery.

The Abolitionist Movement

There had always been some moral qualms about slavery in the nation. In 1696, Quakers in Pennsylvania took the first action against slavery by banning the importation of slaves into the colony. But opposition to slavery did not attract real support until the American Revolution brought questions about liberty and justice squarely before the colonists.

Effects of the Revolution. Some 5,000 African Americans had fought with the colonists against the British during the Revolutionary War. Prominent leaders of the American revolutionary movement, including Thomas Jefferson (himself a Southern slaveholder), Patrick Henry, and Thomas Paine, spoke out against slavery. The drafting of the Constitution again raised the issue of abolishing slavery. However, Southern states, whose economy depended on slavery, refused to abolish it.

Organizations and Leaders. In 1833, William Lloyd Garrison helped organize the American Anti-Slavery Society. He served as its president for two decades. One of the most effective speakers of the 1,000-chapter Anti-Slavery Society was Frederick Douglass. Douglass and Sojourner Truth, another freed slave, sought to influence public opinion by telling the stories of their own lives.


HORSE-MARKET
SLAVES HORSES & OTHER CATTLE TO BE SOLD AT 12 OC

THE LIBERATO

VOL. I.] WILLIAM LLOYD GARRISON AND ISAAC KNAPP, PUBLISHERS.

BOSTON, MASSACHUSETTS.] OUR COUNTRY IS THE WORLD—OUR COUNTRYMEN ARE MANKIND. [SATURDAY, A

THE LIBERATOR
IS PUBLISHED WEEKLY
AT NO. 11, MERCHANTS' HALL.

WM. LLOYD GARRISON, EDITOR.

TERMS.

☞ Two Dollars per annum, payable in advance.
☞ Agents allowed every sixth copy.
☞ No subscription will be received for a shorter period than six months.
☞ All letters and communications must be POST PAID.

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James E. Ellis, *Providence, R. I.*
Philip A. Bell, *New-York City.*
Joseph Cassey, *Philadelphia, Pa.*
Henry Ogden, *Newark, N. J.*
William Watkins, *Baltimore, Md.*

THE LIBERATOR.

Is not the plea, that emancipation is impracticable, the most impudent hypocrisy and the most glaring absurdity ever propounded for contemplation?—Can any supposititious expediency, any dread of political disorder, or any private advantage, justify the prolongation of corruption, the enormity of which is unequalled, or repel the holy claim to its extinction? The system is so entirely corrupt, that it admits of no cure but by a TOTAL and

two committee men and a constable interfered, and would not permit him to take his seat! He was finally driven away, and the pew passed into other hands.

We purpose shortly to visit all our meeting-houses, and ascertain what places are provided for the accommodation of our colored people. A house dedicated to the worship of Almighty God, should be the last place for the exercise of despotic principles.—But here is the extract:

'With deep regret we have observed some articles in the columns of the "Liberator," of Boston, apparently from this city, in which its inhabitants are implicated: and which we believe the editor of that publication will deem very injudicious, as well as unkind, when knowing the truth in the case. So far from wishing to deprive the colored population of an opportunity to worship God, by the co-operation of the friends of religion with that part of the inhabitants, a good and convenient house was erected a few years since; clergymen of different denominations have often officiated, gratuitously, from Sabbath to Sabbath, and when disappointed in the labors of a Minister, lay brethren have attended at their request, and made exertions to promote the prospects of their congregation; for many years a Sabbath School has been taught, composed entirely of colored children and adults, in addition to this, if we mistake not, at their request the public school money is given them in proportion to the number of their children, and they thus have a day school of their own.

be elevated and improved in this country; unanimous in opposing their instruction; unanimous in exciting the prejudices of the people against them; unanimous in apologising for the crime of slavery; unanimous in conceding the right of the planters to hold their slaves in a limited bondage; unanimous in denying the expediency of emancipation, unless the liberated slaves are sent to Liberia; unanimous in their hollow pretence for colonizing, namely, to evangelize Africa; unanimous in their *true motive* for the measure—a terror lest the blacks should rise to avenge their accumulated wrongs. It is a conspiracy to send the free people of color to Africa under a benevolent pretence, but really that the slaves may be held more securely in bondage.* It is a conspiracy based upon fear, oppression and falsehood, which draws its aliment from the prejudices of the people, which is sustained by duplicity, which is impotent in its design, which really upholds the slave system, which fascinates while it destroys, which endangers the safety and happiness of the country, which no precept of the bible can justify, which is implacable in its spirit, which should be annihilated at a blow.

These are our accusations; and if we do not substantiate them, we are willing to be covered with reproach.

In attacking the principles, and exposing the evil

virtue. I doubt not this convi
vail in every community, w
religion and philanthropy are
the process may be slow; h
much ignorance prejudice and
however, is but the first step
sirable as the total abolition of
established custom acquires a
ings of those who are habituat
know that its power in many c
querable; and this is especially
tom, however injurious in its t
of pecuniary emolument, or w
to those interested in its conti
becomes necessary for the at
and good object—the universal
colored brethren—the complet
abominable traffic in human fle
whole subject fairly and calmly
freely; to ascertain, as far as
best means and methods for the
this great end. On this point
diversity of opinion. Men of
piety, and equal benevolence,
most opposite views of the who
has been much perplexed, by
to the very strong arguments o
question.

Voice of Liberation. *The Liberator* was perhaps the best known of the abolitionist newspapers.

ABOLITIONISTS IN PRINT

Throughout the early 1800s, abolitionist books and newspapers helped to stir antislavery feelings throughout the country.

1827 John Russworm, a former slave, founds *Freedom's Journal*, the first African-American newspaper.

1829 David Walker, a free black, publishes *The Appeal*, urging black Americans to strike for their freedom.

1831 William Lloyd Garrison begins publishing a newspaper called *The Liberator*, in which he demands freedom for slaves, claiming that the Declaration of Independence forbids slavery.

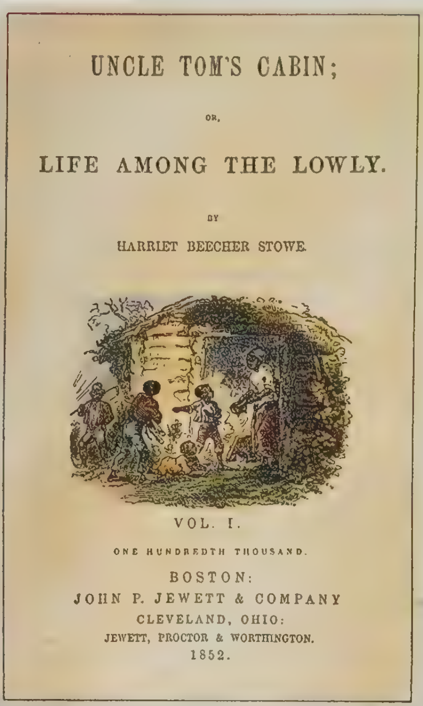
1847 Former slave and famous antislavery orator Frederick Douglass founds the newspaper *North Star*.

▲ **William Lloyd Garrison's** printed attacks on slave dealers caused him to be jailed for 7 months.

Sojourner Truth (born Isabella Baumfree) was the first black female orator to speak out against slavery. Born a slave, she became free when New York State outlawed slavery in 1828.

UNCLE TOM'S CABIN

In 1852, author Harriet Beecher Stowe published *Uncle Tom's Cabin*, a novel. The work described the cruelties of slave life and the desperation of the slaves who fled from it. It quickly became a best-seller in the United States and Great Britain. In the North, it stirred antislavery feelings to a fever pitch; in the South, Stowe became a hated figure.



UNCLE TOM'S CABIN;

OR,

LIFE AMONG THE LOWLY.

BY

HARRIET BEECHER STOWE.

VOL. I.

ONE HUNDREDTH THOUSAND.

BOSTON:
JOHN P. JEWETT & COMPANY
CLEVELAND, OHIO:
JEWETT, PROCTOR & WORTHINGTON.
1852.

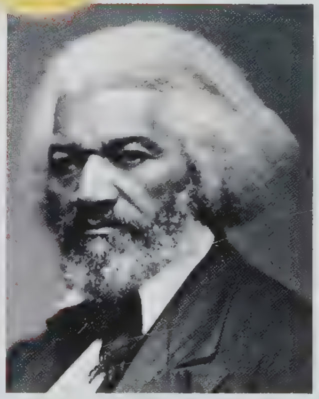
MORE FAMOUS ABOLITIONISTS

- Poets John Greenleaf Whittier and James Russell Lowell
- Reformers Wendell Phillips and Theodore Weld
- Women's rights activist Lucretia Mott
- Angelina and Sarah Grimke, wealthy sisters from the South who had left South Carolina in protest against slavery

fyi!

In 1840, abolitionists formed the Liberty Party and ran a presidential candidate. In 1854, the antislavery factions within the nation's major political parties—the Whigs and the Democrats—split from their parties and formed the Republican Party.

Spotlight on... FREDERICK DOUGLASS



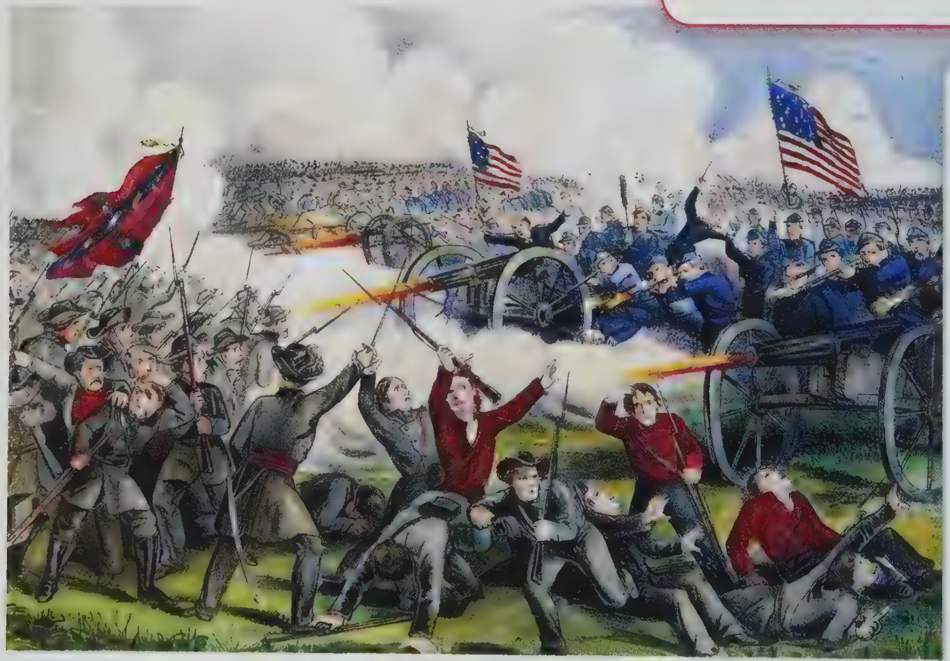
1818?–1895

- U.S. writer and speaker, a leading voice of the abolitionist movement.
- Born a slave, Douglass learned to read and write from his owner's wife. His eloquent *Narrative of the Life of Frederick Douglass, an American Slave* (1845), is a classic.
- In 1838, he escaped to New York and became a speaker for the Massachusetts Anti-Slavery Society. To avoid reenslavement, he went to England to lecture on slavery and women's rights.
- On his return, he founded the antislavery weekly *North Star*, published until 1860.
- After the Civil War, Douglass involved himself in the issues of Reconstruction and held federal office.

THE CIVIL WAR

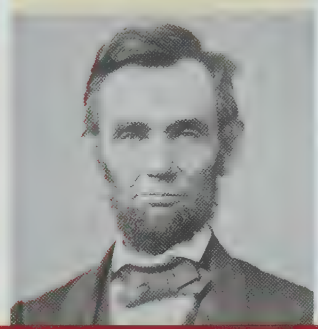
Between December of 1860 and February of 1861, six states seceded (withdrew) from the United States. They banded together to form the Confederate States of America. (Five more would leave the Union and join the Confederacy in March and April 1861.) In April 1861, Confederate forces bombarded Fort Sumter. After decades of conflict, war had finally broken out between the North and the South. At the beginning of the war, the North and South each hoped to gain victory in a short war—90 days perhaps. But the war would last 4 years.

Union artillery fires upon advancing Confederate troops during the Battle of Gettysburg (July 1–3, 1863). This Pennsylvania battle is considered a turning point in the war. ▼



The Civil War: Time Line

Nov. 6
Abraham Lincoln, running as the candidate of the Republican Party, is elected president. ▼



1860

Dec. 20
South Carolina secedes from the Union. Five other states of the lower South—Mississippi, Florida, Alabama, Georgia, and Louisiana—follow suit.

Feb. 9
Representatives of the six states that seceded meet in Montgomery, AL, to form a new nation: the Confederate States of America. They elect as president Jefferson Davis of Mississippi.

1861

Feb. 22
Davis inaugurated at Montgomery, AL.

March 2
Texas joins the Confederacy.

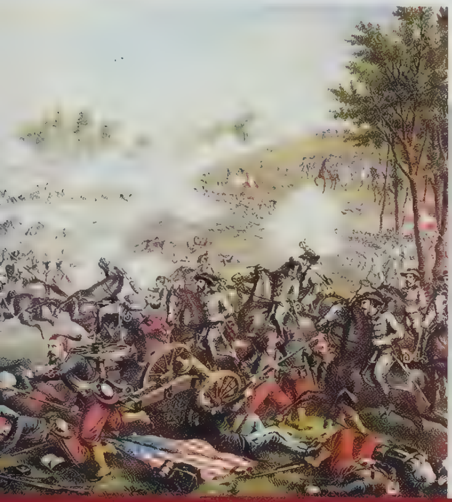
Legend

✶ Union victory

✶ Confederate victory

✶✶ Draw

(U) Union general
(C) Confederate general
(k) killed
ital victorious general



March 4
Abraham Lincoln is inaugurated.

April 12–13
The Confederates attack federally held Fort Sumter in the harbor of Charleston, SC, triggering the Civil War.

April 19
Blockade of Southern ports declared by President Lincoln.

May
Virginia, Tennessee, Arkansas, and North Carolina join the Confederacy. The Confederate capital moves from Montgomery, AL, to Richmond, VA, on May 21.

◄ July 21
The first battle of Bull Run (First Manassas): Union forces marching from Washington, D.C., clash with Confederate troops at Manassas Junction in Virginia. The Union army is forced to retreat. Irvin McDowell (U); *J. E. Johnston* and *P. G. T. Beauregard (C).*

Aug. 10
Battle of Wilson's Creek, MO: Nathaniel Lyon (U)(k); *Ben McCulloch (C).*

Oct. 21
Battle of Ball's Bluff, VA: C. P. Stone (U); *N. G. Evans (C).*

Nov. 7
Battle of Belmont, MO: Ulysses S. Grant (U): *L. Polk (C).*

Nov. 8
Union seizure of Confederate commissioners James Mason and John Slidell from the British steamer *Trent.*

Dec. 9
Creation of Committee on the Conduct of the War by Union Congress.

1862

Jan. 19
Battle of Mill Springs (or Logan's Cross Roads), KY: *G. H. Thomas (U);* G. B. Crittenden (C).

Feb. 6
Fall of Fort Henry, TN: *Ulysses S. Grant (U);* L. Tilghman (C).

Feb. 14–16
Siege and surrender of Fort Donelson, TN: *Ulysses S. Grant (U);* S. B. Buckner (C).

March–June 9
Jackson's Valley campaign (VA): N. P. Banks (U); *T. J. Jackson (C).*

March 9
The Union warship *Monitor* forces the Confederate ship *Merrimack* to a draw. The *Merrimack* retires. This is the first battle in history between ironclad warships.

April 5–May 4
Siege of Yorktown, VA: *G. B. McClellan (U);* J. B. Magruder and J. E. Johnston (C).

April 6–7
At Shiloh, TN, a Confederate attack on General Grant's Union army fails. The Confederate troops withdraw after 2 days of bloody fighting. *Ulysses S. Grant (U);* A. S. Johnston (k) and P. G. T. Beauregard (C).

April 16
First Confederate Conscription Act.

May 1
Surrender of New Orleans, LA: *B. F. Butler (U);* Mansfield Lovell (C).

May 31–June 1
Battle of Seven Pines (or Fair Oaks), VA: *G. B. McClellan (U);* J. E. Johnston and G. W. Smith (C).

June 1
Robert E. Lee appointed to command the Confederate Army of Northern Virginia.

June 25–July 1
Seven Days battles, VA: G. B. McClellan (U); *Robert E. Lee (C).*

The Civil War: Time Line

July 17
Second Union Confiscation Act passed.

Aug. 29–30
The Second Battle of Bull Run (Second Manassas) takes place as Union troops, trying to capture the Confederate capital at Richmond, VA, are defeated by Confederate troops. John Pope (U); *Robert E. Lee (C).*

Sept. 17
At Antietam Creek in Maryland, Confederate troops marching to attack Pennsylvania are stopped by Union troops at the Battle of Antietam (Sharpsburg). They retreat to Virginia. *G. B. McClellan (U);* Robert E. Lee (C).

Sept. 19
Battle of Iuka, MS: *W. S. Rosecrans (U);* Sterling Price (C).

Sept. 24
President Lincoln suspends the writ of *habeas corpus* rights in the Union.

Oct. 3–4
Battle of Corinth, MS: *W. S. Rosecrans (U);* E. Van Dorn (C).

Oct. 8
Battle of Perryville, KY: D. C. Buell (U); Braxton Bragg (C). Draw.

Dec. 13
At Fredericksburg, VA, Union forces are defeated again as they try for Richmond. A. E. Burnside (U); *Robert E. Lee (C).*

Dec. 20
Holly Springs, MS, depot captured and burned; R. C. Murphy (U); *E. Van Dorn (C).*

Dec. 31–
Jan. 2, 1863
Battle of Stones River (or Murfreesboro), TN: W. S. Rosecrans (U); Braxton Bragg (C). Draw. ▶

1863

Jan. 1
Lincoln signs the Emancipation Proclamation, declaring all slaves in Confederate states free.

Jan. 1
Galveston, TX, recaptured: I. S. Burrell (U); *J. B. Magruder (C).*

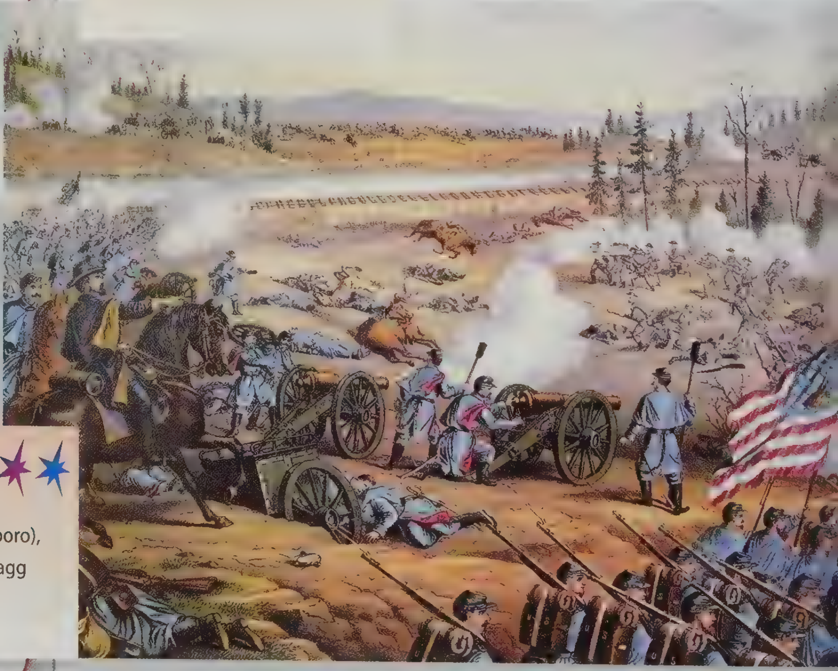
Feb. 25
Union National Bank Act effective.

March 3
Union Federal Draft Act.

April 11–May 4
Siege of Suffolk, VA: *John J. Peck (U);* James Longstreet (C).

May 1–May 4
At the Battle of Chancellorsville, in Virginia, Lee's troops defeat another Union attempt on Richmond. "Stonewall" Jackson is wounded and dies on May 10. Joseph Hooker (U); *Robert E. Lee (C).*

June 9
Cavalry battle of Brandy Station, VA: Alfred Pleasanton (U); *Jeb Stuart (C).*



A greater proportion of men were killed, wounded, or captured during the Battle of Stones River than in any other Civil War battle.

June 20
West Virginia admitted to Union.

June 23–30
Tullahoma, TN, campaign: *W. S. Rosecrans (U);* Braxton Bragg (C).

July 1–3
At Gettysburg, Lee's march into Pennsylvania is repulsed, and he retreats to Virginia. This loss marks the beginning of the end for the South. *George G. Meade (U);* Robert E. Lee (C).

May 19–July 4
Siege and surrender of Vicksburg, MS. The campaign started in November 1862. *Ulysses S. Grant (U);* John C. Pemberton (C).

May 27–July 9
Siege and surrender of Port Hudson, LA. Together with the fall of Vicksburg, this victory placed the Mississippi River under Union control. *N. P. Banks (U);* Franklin Gardner (C).

July 13–16
In March, Congress had instituted the first national military draft. Because only the wealthy could escape conscription by paying $300 or hiring a substitute, the act became known as the poor man's draft. It triggers 4 days of bloody July riots in New York City.

Sept. 19–20
In the Battle of Chickamauga, Confederate forces overwhelm the Union forces. W. S. Rosecrans (U); *Braxton Bragg (C).*

Sept. 21–Nov. 25
Siege of Chattanooga, TN, begins on Sept. 21. The Battle of Chattanooga takes place from Nov. 23 to Nov. 25: *Ulysses S. Grant (U)*; Braxton Bragg (C).

Nov. 17–Dec. 4
Siege of Knoxville, TN: *A. E. Burnside (U)*; J. Longstreet (C).

Nov. 19
Lincoln delivers the Gettysburg Address at the battlefield in Gettysburg, PA.

1864

Feb. 3–March 5
Meridian, MS, campaign: W. T. Sherman (U); L. Polk (C). No decision.

Feb. 20
Battle of Olustee (or Ocean Pond), FL: Truman Seymour (U); *Jos. Finegan (C).*

March 9
Ulysses S. Grant is appointed lieutenant general of Union armies.

March 10–May 22
Red River, LA, campaign: N. P. Banks (U); *Richard Taylor (C).*

April 12
Capture of Fort Pillow, TN: L. F. Booth (U)(k); *N. B. Forrest (C).*

May 5–6
Battle of the Wilderness, VA: *Ulysses S. Grant (U);* Robert E. Lee (C).

May 4
The campaign to take Atlanta begins in Chattanooga. In a series of chesslike maneuvers, W. T. Sherman (U) pushes J. E. Johnston and J. B. Hood (C) southeast toward Atlanta.

May 8–19
Operations about Spotsylvania Court House, VA: Ulysses S. Grant (U); Robert E. Lee (C). No decision.

May 11
Cavalry battle at Yellow Tavern, VA: *P. H. Sheridan (U);* Jeb Stuart (C)(k).

May 15
Battle of New Market, VA: Franz Sigel (U); *J. C. Breckinridge (C).*

May 25–26
Battle of New Hope Church, GA: J. Hooker (U); *J. E. Johnston (C).*

The Civil War: Time Line

June 1–3
Battle of Cold Harbor, VA: Ulysses S. Grant (U); *Robert E. Lee (C).*

June 10
Battle of Brice's Cross Roads, MS: S. D. Sturgis (U); *N. B. Forrest (C).*

June 20
Siege of Petersburg, VA, begins as part of the Union effort to cut off Richmond's supply lines. Trench warfare, a precursor to WWI tactics, lasts until April 1865.

June 19
Naval battle between the *Alabama* and *Kearsarge,* off the French coast; *John A. Winslow (U);* Raphael Semmes (C). ▶

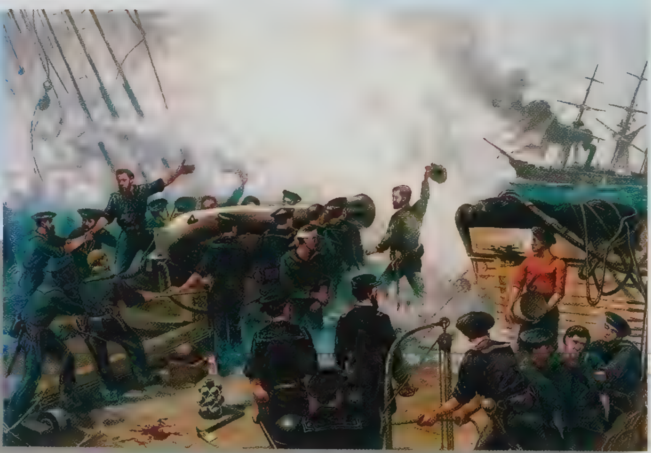
July 5–12
Early's raid on Washington, D.C.: *Ulysses S. Grant (U);* J. E. Early (C).

July 9
Battle of Monocacy, MD: Lew Wallace (U); *J. E. Early (C).*

Aug. 5
Naval battle in Mobile Bay, AL: *D. G. Farragut (U);* Percival Drayton (C).

Aug. 7–Oct. 25
Union forces under Philip H. Sheridan invade the Shenandoah Valley. Grant charges Sheridan with flushing Jubal Early out of the valley and removing the threat to Washington, D.C.

Sept. 1
Atlanta falls to Union forces and the city is evacuated. *W. T. Sherman (U);* J. B. Hood (C).



Oct. 19
Toward the end of the Shendandoah Valley campaign, Sheridan inflicts a crushing defeat upon Early at the Battle of Cedar Creek, VA. *P. H. Sheridan (U);* J. E. Early (C).

Oct. 23
Battle of Westport, MO: *S. R. Curtis (U);* Sterling Price (C).

Nov. 8
Lincoln reelected president.

Nov. 15–Dec. 21
"Sherman's March to the Sea" begins as Union troops under the command of General Sherman cut a path of destruction from Atlanta to Savannah, GA. Savannah falls on December 21.

Nov. 30
Battle of Franklin, TN: *J. Scofield (U);* J. B. Hood (C).

Dec. 15–16
Battle of Nashville, TN: *G. H. Thomas (U);* J. B. Hood (C).

1865

Jan. 13–15
Battle of Fort Fisher, NC: *A. H. Terry (U);* W. H. Whiting (C)(k).

Feb. 3
Hampton Roads, VA, peace conference.

Feb. 6
Robert E. Lee is appointed general in chief of all Confederate forces.

Feb. 17
Columbia, SC, captured and destroyed by fire: *W. T. Sherman (U).*

Feb. 18
Charleston, SC, evacuated by Confederate troops.

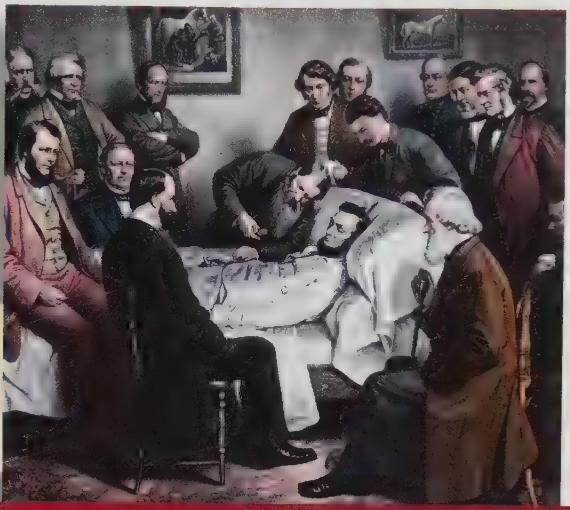
March 4
Abraham Lincoln inaugurated for second term.

April 1
Battle of Five Forks, VA: *P. H. Sheridan (U)*; G. E. Pickett (C).

April 2
Capture of Selma, AL: *J. H. Wilson (U)*; N. B. Forrest (C).

April 12
Capture of Mobile, AL: *E. R. S. Canby (U)*; D. H. Maury (C).

April 15
Death of President Lincoln. Andrew Johnson inaugurated as president.



April 2
The Siege of Petersburg ends as Union forces pierce the city's defenses and Confederate forces evacuate after nightfall. The Confederate government evacuates Richmond, the capital, the same day.

April 3
Union forces occupy Richmond.

April 26
Confederate surrender at Durham, NC: *W. T. Sherman (U)*; J. E. Johnston (C).

May 4
Surrender of Confederate forces in Louisiana: *E. R. S. Canby (U)*; R. Taylor (C).

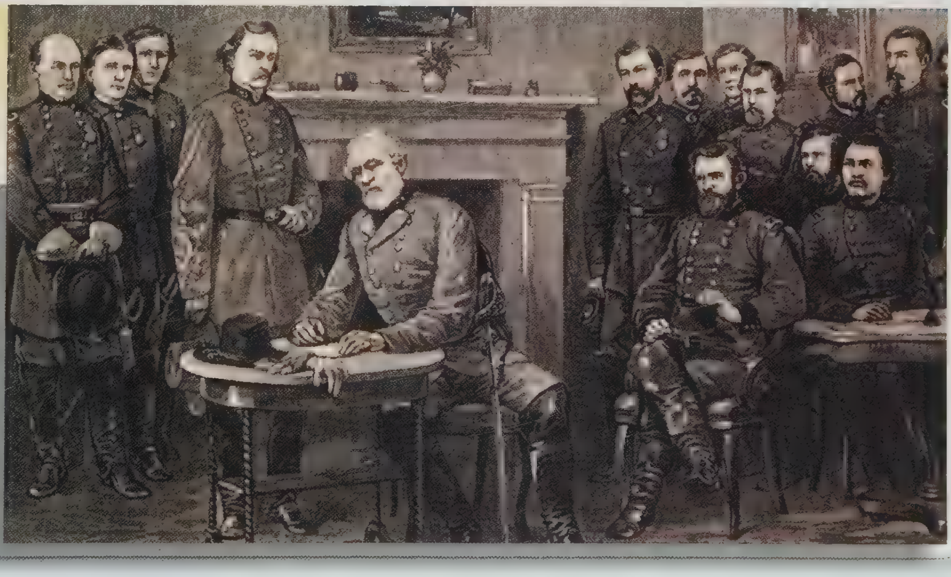
May 10
Jefferson Davis captured at Irwinsville, GA.

May 26
Surrender of Trans-Mississippi Dept.: *E. R. S. Canby (U)*; Kirby Smith (C).

June 2
Surrender of Galveston, TX: H. K. *Thatcher (U)*; Kirby Smith (C).

Nov. 6
Final Confederate surrender—*Shenandoah*, Capt. James Waddell, to British authorities at Liverpool, England.

April 9
Lee surrenders to Grant in the house of Wilmer McClean, in Appomattox Court House, VA.



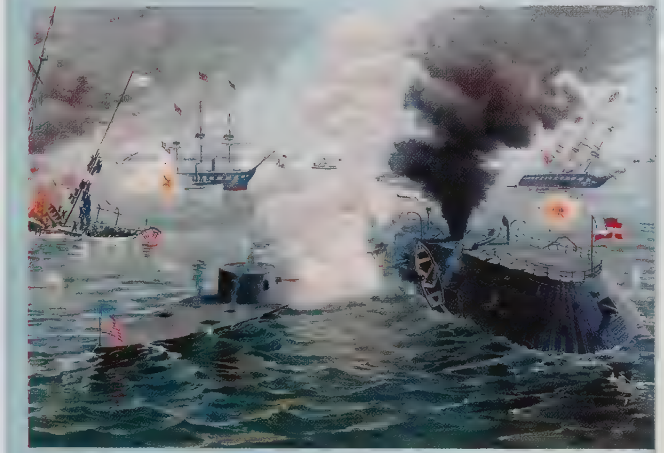
Appomattox Court House. Lee, always considerate of his men, knew only senseless slaughter awaited them after Grant pursued him deep into Virginia. In this depiction, the two great generals sign the surrender terms in the McLean home.

Battles and Campaigns

The Anaconda Plan

The South planned to wear the North down with a defensive war. The North, on the other hand, planned to invade and isolate the South with General Winfield Scott's Anaconda Plan (named after the python that crushes its victims). The plan would split the South into three parts.

1. Capture the Mississippi River valley and isolate states west of it.
2. Split the Confederacy east and west of the Appalachians.
3. Strangle the South with a naval blockade extending from Virginia to Texas.



BATTLE OF THE IRONCLADS

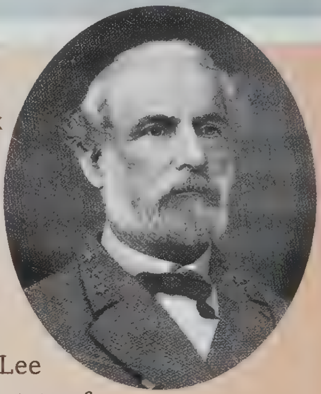
In 1861, the Confederates had raised a sunken federal ship, the *Merrimack*, off Norfolk, Virginia, covered the wooden vessel with iron plates, and renamed it the *Virginia*. On March 8, 1862, the *Virginia* attacked the Northern ships at Hampton Roads, a channel in Virginia. It destroyed two Northern vessels and grounded three others. When the ship returned the next day to finish the job, it faced the *Monitor*, an ironclad ship designed especially for the Northern Navy. History's first battle between ironclad warships followed. Although neither ship won, the *Monitor* proved to be the superior vessel. Later, the U.S. Navy built a large ironclad fleet modeled after it.

Early Battles

Several early battles were fought in an attempt by the North to capture the Confederate capital, Richmond. Union forces hoped the capture would demoralize the South.

First Bull Run. In July 1861, the Union advance was stopped by General Thomas Jonathan Jackson, who earned his nickname "Stonewall" in this battle by standing (it was later claimed) calmly in the midst of the fighting, like a stone wall.

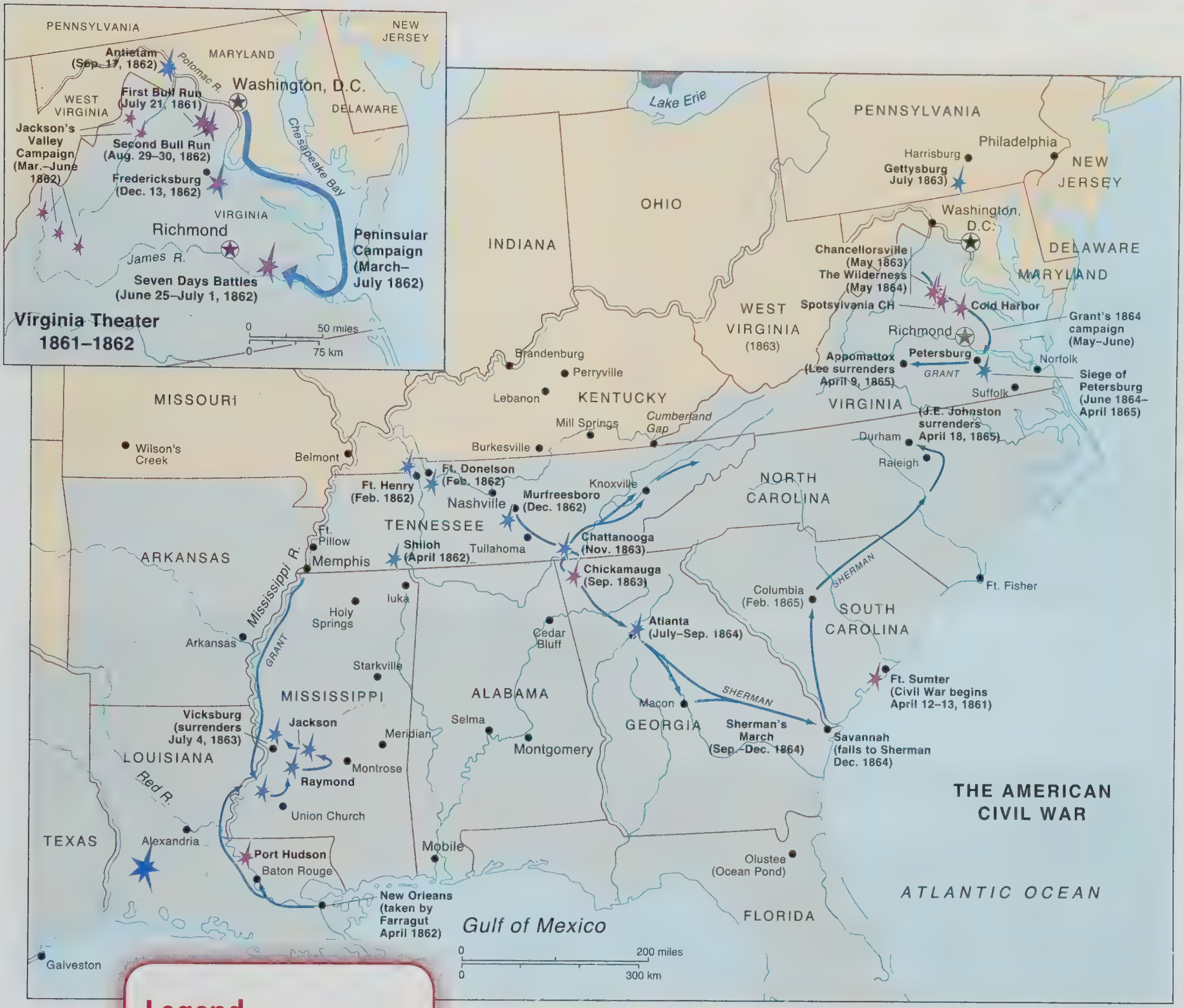
Second Bull Run. Union forces under General John Pope joined with General George B. McClellan's forces and marched on Richmond in the summer of 1862. But like the Union army of 1861, they too were stopped at Bull Run in a battle on August 29 and 30.



General Robert E. Lee hoped to invade Maryland and wreak havoc—cut off roads and rail lines, take prisoners, capture supplies, and demoralize the North. ▶

Antietam. While the Union tried to capture Richmond, Lee set out to invade the border state of Maryland. On September 17, 1862, Union and Confederate forces met in the bloodiest single-day battle of the war, at Antietam Creek in Maryland. Almost 5,000 died and more than 18,000 were wounded. McClellan fought the Confederate army to a draw, but allowed Lee's army to retreat.

Fredericksburg and Chancellorsville. Union troops under General Ambrose E. Burnside made another attempt on Richmond in mid-December 1862. Lee's smaller army stopped the Union Army at Fredericksburg. General Joseph "Fighting Joe" Hooker marched on Richmond in late April 1863, but only got as far as Chancellorsville. The Union defeat came with 17,000 dead, wounded, or missing.



Virginia Theater
1861–1862
PENNSYLVANIA
NEW JERSEY
MARYLAND
DELAWARE
WEST VIRGINIA
VIRGINIA
Antietam (Sep. 17, 1862)
Potomac R.
Washington, D.C.
First Bull Run (July 21, 1861)
Jackson's Valley Campaign (Mar.–June 1862)
Second Bull Run (Aug. 29–30, 1862)
Fredericksburg (Dec. 13, 1862)
Chesapeake Bay
Richmond
James R.
Peninsular Campaign (March–July 1862)
Seven Days Battles (June 25–July 1, 1862)
0 50 miles
0 75 km
THE AMERICAN CIVIL WAR
Lake Erie
PENNSYLVANIA
Philadelphia
Harrisburg
Gettysburg July 1863)
NEW JERSEY
OHIO
INDIANA
Washington, D.C.
DELAWARE
MARYLAND
Chancellorsville (May 1863)
The Wilderness (May 1864)
Spotsylvania CH
Cold Harbor
Grant's 1864 campaign (May–June)
WEST VIRGINIA (1863)
Richmond
Appomattox (Lee surrenders April 9, 1865)
Petersburg
GRANT
Norfolk
Siege of Petersburg (June 1864–April 1865)
Suffolk
VIRGINIA
(J.E. Johnston surrenders April 18, 1865)
Durham
Raleigh
Brandenburg
Perryville
Lebanon
KENTUCKY
Mill Springs
Cumberland Gap
Burkesville
MISSOURI
Wilson's Creek
Belmont
Ft. Henry (Feb. 1862)
Ft. Donelson (Feb. 1862)
Nashville
Murfreesboro (Dec. 1862)
Knoxville
NORTH CAROLINA
TENNESSEE
Ft. Pillow
Shiloh (April 1862)
Tullahoma
Chattanooga (Nov. 1863)
Chickamauga (Sep. 1863)
ARKANSAS
Mississippi R.
Memphis
Iuka
Holy Springs
Arkansas
GRANT
Columbia (Feb. 1865)
SHERMAN
Ft. Fisher
SOUTH CAROLINA
Cedar Bluff
Atlanta (July–Sep. 1864)
Starkville
MISSISSIPPI
ALABAMA
Ft. Sumter (Civil War begins April 12–13, 1861)
Vicksburg (surrenders July 4, 1863)
Jackson
Meridian
Selma
Macon
SHERMAN
GEORGIA
Sherman's March (Sep.–Dec. 1864)
Savannah (falls to Sherman Dec. 1864)
Montgomery
LOUISIANA
Montrose
Raymond
Red R.
Union Church
TEXAS
Alexandria
Port Hudson
Baton Rouge
Mobile
Olustee (Ocean Pond)
New Orleans (taken by Farragut April 1862)
Gulf of Mexico
FLORIDA
ATLANTIC OCEAN
Galveston
0 200 miles
0 300 km

Legend

✶ Confederate victory

✶ Union victory

↑ Major Union campaign

THE EMANCIPATION PROCLAMATION

Toward the end of 1862, with Union morale low after many losses, Lincoln decided to take a moral stand on slavery. On January 1, 1863, the Emancipation Proclamation was issued.

- It proclaimed that all slaves in areas still in a state of rebellion were free.
- Since it did not free slaves in Union-held territories or in border states loyal to the Union, it did not actually free any slaves.
- It also announced Lincoln's decision to use black troops.
- Now the war had become a crusade for the oppressed. This also made it more difficult for Great Britain, which opposed slavery, to recognize the Confederacy.

A CHANGING SOCIETY

In the late 1800s and early 1900s, changes in industry and technology went hand in hand with a shifting culture. Floods of new immigrants brought their own cultures to the American "melting pot." Many Americans called for changes in the country's economic, political, and social systems. They wanted to reduce poverty, improve the living conditions of the poor, and regulate big business. Meanwhile, segregationists in the South fought to maintain the economic and racial divisions of the era before the Civil War.

Immigration

Between 1815 and the 1920s, a flood of some 35 million immigrants—most of them Europeans—arrived on American shores. From 1815 to 1890, 15 million people came. The peak came in 1882, when 800,000 immigrants arrived in a single year. Yet the millions of immigrants who came during the early 1900s dwarfed even that figure.

Immigration Patterns

The United States has had four major waves of immigration. The migration before 1885 is often referred to as the "old migration." This migration began with the first colonists, and consisted mostly of people from northern and western Europe. The migration after 1885 is known as the "new migration." It consisted of many more immigrants from southern and eastern Europe.

Immigration to the United States

Who	When	How Many?	Why?
Irish	1840 to 1850s	About 1.5 million	Famine resulting from potato crop failure
Germans	1840s to 1880s	About 4 million	Severe economic depression and unemployment; political unrest
Danes, Norwegians, and Swedes	1870s to 1900s	About 1.5 million	Poverty; shortage of farmland
Poles	1880s to 1920s	About 1 million	Poverty; political repression; cholera epidemics
Jews from eastern Europe	1880s to 1920s	About 2.5 million	Religious persecution
Austrians, Czechs, Hungarians, and Slovaks	1880s to 1920s	About 4 million	Poverty; overpopulation
Italians	1880s to 1920s	About 4.5 million	Poverty; overpopulation

Ellis Island. Newcomers from Europe typically landed at Ellis Island in New York Harbor. There they were questioned by government officials and examined by doctors before being allowed to enter the country.

Northern and Western Europe

In the 1840s, the surge of immigrants from northern and western Europe nearly tripled.

- In the 1840s, 1,700,000 immigrants—many from Ireland and Germany—arrived.
- In the 1850s, 2,600,000 more arrived.
- From 1860–1890, another 10 million arrived, mainly from Great Britain, Ireland, Germany, and Scandinavia.

Many of these groups migrated to the Midwest and West, where they built new farms.

Southern and Eastern Europe

The late 1800s brought a shift in the origins of Europeans emigrating to America.

- New arrivals continued to come from England, Scotland, Ireland, Germany, France, and other parts of northern Europe.
- A much larger number came from southern and eastern Europe—Italians, Poles, Russians, and others.
- Jews came to America from Russia and elsewhere in eastern Europe after 1880, largely to escape pogroms (massacres) and other anti-Semitic persecutions.

These immigrants tended to settle in ethnic neighborhoods in cities, rather than on farms, because much of America's farmland had already been settled.

China

San Francisco was the main port of entry for Chinese immigrants, many of whom came looking for work during the California gold rushes.

- They called the city "the Golden Mountain" (Jinshan or Gam Saan), and established the first Chinatown in America.
- Chinese immigrants generally planned on staying for only a few years, hoping to make enough money to return to China and live comfortably.
- Between 1850 and 1882, for example, some 322,000 Chinese arrived in the United States, but only about 100,000 actually remained. (See also page 141.)

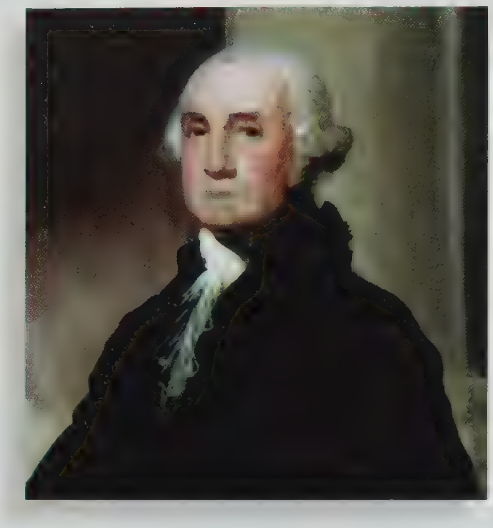
European immigrants arrive in New York. Due to poor nutrition and disease on the long transatlantic voyage, a significant number never lived to see America.

Restrictions

The wave of immigrants from southern/eastern Europe and Asia led to a backlash of nativism—opposition to immigrants. Some feared the immigrants would threaten American unity. Others objected that immigrant workers kept wages low. For the first time, the United States began to restrict immigration.

- In 1882, Congress passed the Chinese Exclusion Act, which prohibited Chinese laborers from coming to the United States. Only certain "exempt" groups—such as clergy, scholars, and diplomats—were permitted.
- In 1907, the United States concluded a "gentlemen's agreement" with Japan to exclude any further immigration from that country.
- Legislation during the 1920s set up quotas that discriminated against immigrants not from northern and western Europe.

Biographies of Selected Presidents



George Washington, the first U.S. president and the foremost of the country's founding fathers, served from 1789 to 1797. More than any other individual, he was the architect of American government after the colonies broke away from British rule.

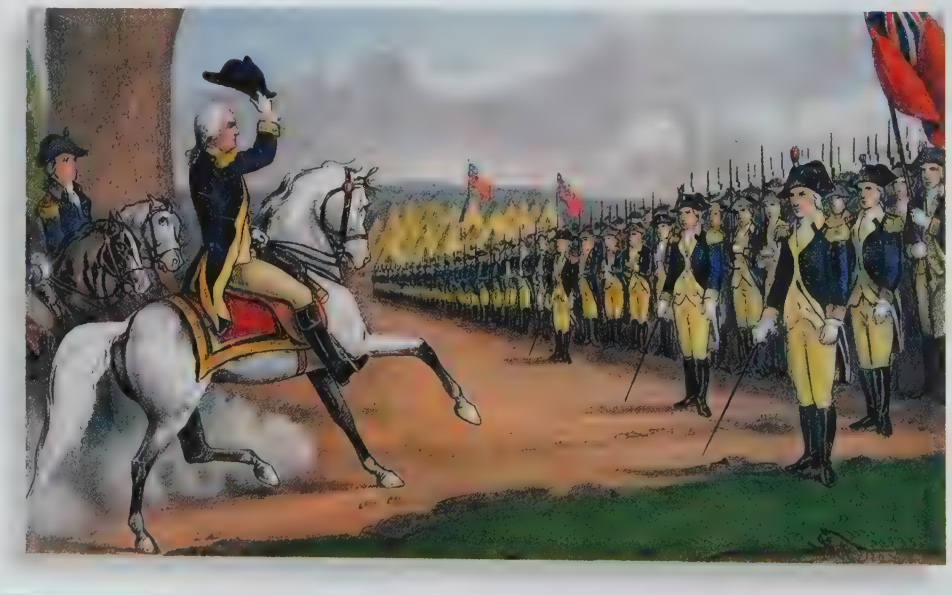
Born February 22, 1732, in Westmoreland County, Virginia, Washington was a key figure in the American Revolution. As commander in chief of the Continental Army, he was tasked with leading inexperienced troops through the bloody Revolutionary War.

After his victory in 1787, Washington presided over the Philadelphia Convention, where the Constitution was drafted. When the document was ratified, Washington was unanimously elected president. He took office April 30, 1789.

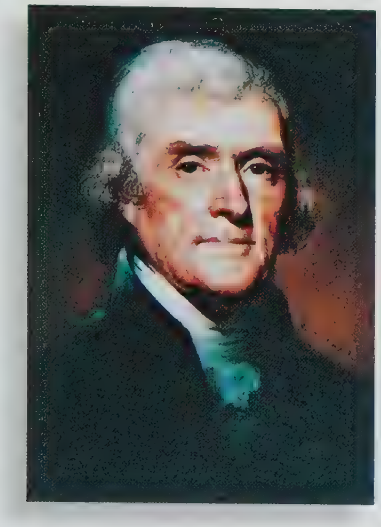
As the leader of a brand new country, Washington attended to a host of domestic issues. Foremost among them was passing tax legislation to pay the nation's mounting debts. But more than any single action he took as president, Washington's overarching contribution was setting a precedent for the executive branch. The way he handled certain aspects of government—for instance, cabinet appointments—still informs how the government operates today. The president carefully weighed all of his actions in light of this responsibility.

Washington's foreign policy was to remain neutral in international affairs whenever possible. He believed that the new country needed to focus its attention inward to grow stronger. It was a controversial position, particularly during the French Revolution, which some Americans felt obligated to support.

His long service took a toll on Washington, who retired after his second term. Three years later, in 1799, he died of a throat infection.



Washington taking command of the Continental Army



Thomas Jefferson, the nation's third president, held the office from 1801 to 1809. He is celebrated as the main author of the Declaration of Independence and for doubling the size of the country with the Louisiana Purchase.

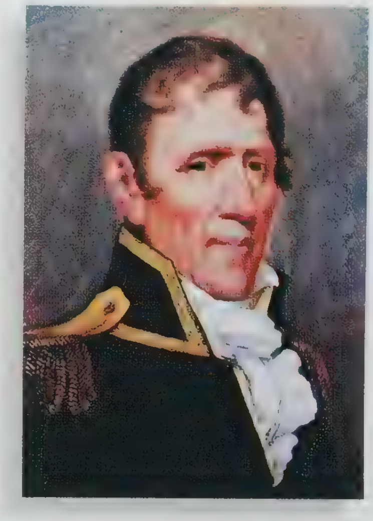
Born April 13, 1743, in Shadwell, Virginia, Jefferson was a key figure on the political scene well before he became president. In 1776, he wrote the Declaration of Independence, which announced the American colonies' break from Great Britain. The principles he articulated in that document—including the belief that all men are created equal—informed Jefferson's political career. But many people question his personal commitment to those principles since he was a slaveholder.

In the years leading up to his presidency, Jefferson held a number of important political offices, including governor of Virginia, U.S. secretary of state, and vice president under John Adams. As president, two of his first acts were to slash spending and lower taxes.

Over time, Jefferson found himself drawn into international affairs despite his wishes. Sometimes the results were positive, like his triumphant Louisiana Purchase. Short on funds, the seller, Napoleon, was willing to part with much more land than Jefferson had anticipated going into negotiations.

But Jefferson made a major misstep in foreign affairs. In response to the French and English abuse of U.S. ships during the Napoleonic wars, he called for an embargo that blocked trade with Europe. The embargo backfired, seriously damaging the American economy.

After he retired, Jefferson lived at Monticello, his home, until he died in 1826.



Andrew Jackson, the founder of the Democratic Party, served from 1829 to 1837. His win in the election of 1828 represented a shift in the political landscape in many respects.

Born poor, Jackson was a self-taught lawyer with relatively little experience as a politician. Instead of working his way up to the presidency through the usual channels, he was propelled into the position by the American public.

Considering himself a mouthpiece of the people, Jackson took a heavy hand in domestic policy. His forceful agenda and penchant for veto earned him the nickname of King Andrew I among his critics.

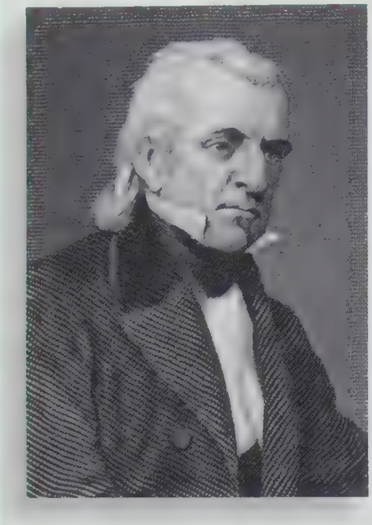
Jackson shaped his policy around the national agenda, which he usually valued over the rights of individual states. One of the events that shaped Jackson's presidency was the nullification crisis, a rift with South Carolina that occurred when the state resisted a federally imposed tariff. The crisis was resolved before Jackson resorted to military force.

Another controversy during Jackson's presidency was his insistence upon moving federal funds from the centralized Second Bank of the United States to a variety of state-run banks. Many economists believe this incident, known as the Bank War, had a negative impact on the economy.

Jackson's largest failure as president was the Indian Removal Act, which he signed into law. It directly resulted in the Trail of Tears and other atrocities against Native Americans.

At the end of his second term, Jackson retired to his home outside of Nashville, Tennessee, where his health declined steadily until his death in 1845.

Biographies of Selected Presidents



James K. Polk, a Democrat who served from 1845 to 1849, is remembered for his focus on westward expansion.

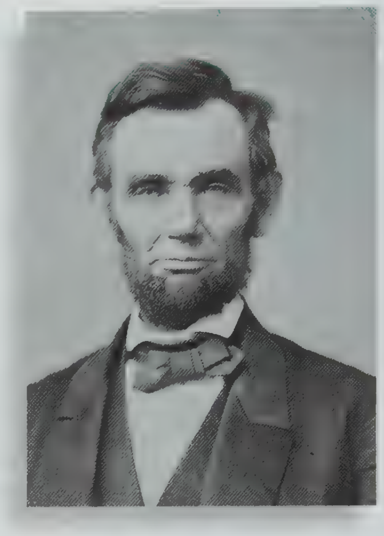
Polk, the eldest of 10 children, was born in Mecklenburg County, North Carolina, on November 2, 1795. He began moving in political circles as a young lawyer in Tennessee. After a stint in the state legislature, he served as Speaker of the House of Representatives from 1835 until he became governor of Tennessee in 1839.

Polk's close relationship with Andrew Jackson led to his nomination for the presidency in 1844. Polk is often referred to as a "dark horse" candidate because he wasn't well known to the American public at the time.

By the end of Polk's term, the country's footprint had grown by one-third. He acquired huge chunks of land in the west, including (but not limited to) Oregon, Texas, California, and New Mexico. He settled the boundaries of Oregon through peaceful negotiations with the British, but the other acquisitions were more difficult, resulting in the Mexican-American War.

While Polk's ambitious expansion was successful in terms of territorial gains, it ignited a national debate about whether or not slavery should be legal in the newly annexed areas. This issue deepened the divide between the North and the South, which eventually led to the Civil War.

Polk's reputation as a workaholic took a toll on his health during his one-term presidency. (He never intended to run for reelection.) He died just a few months after he left the White House in 1849.



Abraham Lincoln, known as the Great Emancipator for ending slavery in the United States, served from 1861 to 1865.

Born February 12, 1809, in Hardin County, Kentucky, Lincoln rose from humble beginnings to become a self-taught lawyer. One of the most widely admired figures in American history, his presidency was consumed by the Civil War. While he ultimately preserved the Union, the war lasted more than 4 years and left 1 million soldiers dead or wounded.

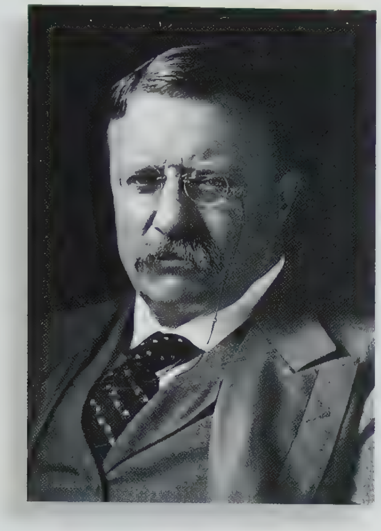
Lincoln's election precipitated Southern secession; seven states left the Union before he even took office. Worried that Lincoln would abolish slavery despite his pledge to the contrary, these states (and, eventually, four others) formed the Confederate States of America—an act that Lincoln considered illegal.

The Civil War began on April 12, 1861, when Confederates attacked Fort Sumter. Citing a national emergency, Lincoln expanded his powers as president. His goal was to reunite the country at all costs.

The war took a heavy toll on the country and on Lincoln, who made a point of being personally involved. He spent time with soldiers and considered their appeals for clemency. While his character is universally praised today, he was heavily criticized at the time.

On January 1, 1863, Lincoln signed the Emancipation Proclamation. It freed Confederate slaves and shifted the focus of the Civil War to freedom. Many black soldiers joined the Union Army.

Lincoln was shot and killed by an assassin, John Wilkes Booth, in April 1865, less than a week after Confederate forces surrendered.



Theodore Roosevelt, who became president after William McKinley was assassinated in 1901, spent his presidency expanding the government's reach to help more people at home and abroad.

Born in New York City on October 27, 1858, Roosevelt studied law, served in the Spanish-American War, and governed the state of New York before he became vice president under McKinley. At age 42, he became the country's youngest president.

Roosevelt developed a domestic agenda he nicknamed the Square Deal. His primary goals were to resolve disputes between laborers and employers, to oversee and regulate industry by enforcing antitrust laws, and to promote conservation by creating national forests and wildlife preserves. As he implemented the Square Deal, Roosevelt expanded the powers of the office of president.

At the same time, Roosevelt wanted the United States to become more involved in international affairs. To that end, he expanded the nation's role in Latin American countries and mediated a dispute between France and Germany. He also won the Nobel Peace Prize for mediating the Russo-Japanese War.

His biggest achievement in foreign affairs was developing the Panama Canal, a waterway that cut travel time between the Atlantic and Pacific oceans. Getting permission to build involved a complex series of negotiations with Great Britain, Colombia, and Panama.

After he left office in 1909, Roosevelt took a brief break from politics before running in the 1912 presidential election. (He lost to Woodrow Wilson.) He died in 1919.

Woodrow Wilson, who served from 1913 to 1921, is remembered for his ambitious domestic policy and for leading the country through World War I.

Wilson was born December 28, 1856, in Staunton, Virginia. He was a lawyer and an academic before he became a politician. As president, he aggressively pursued a progressive domestic agenda called the New Freedom program. A string of legislative victories—including labor regulations and bank reform—increased the government's influence over the American economy.

Wilson's early forays into foreign policy focused on promoting democracy in Latin America, but soon all of his attention was directed toward World War I. In the first years of the war, which began in 1914, Wilson tried not to take sides. He had little choice but to join the fray in 1917 when Germany continually attacked neutral ships. After the United States joined the war effort, the Allied forces (which included the United States, Great Britain, and France, among others) declared victory in 1918.

After the war, Wilson made many vital contributions to the Treaty of Versailles, an important peace agreement that created the League of Nations. A precursor to the United Nations, the League was an international peacekeeping organization. In 1919, Wilson tried (and failed) to rally support for the League in the United States. His exertions led to a debilitating stroke. While he carried out the remainder of his term, Wilson never fully recovered before he died in 1924.

Biographies of Selected Presidents

Franklin Delano Roosevelt served from 1933 to 1945—the longest tenure of any American president. After he guided the nation through the worst of the Great Depression, he helped lead the Allies to victory in World War II.

Roosevelt was born January 30, 1882, in Hyde Park, New York. During his first years as president, he quickly developed and implemented a broad series of public programs called the New Deal. Designed to boost the economy and to broaden social benefits, the programs met with mixed success.

Initially, Roosevelt focused on emergency relief and preliminary steps toward recovery by stabilizing banks, giving cash to the poor, helping farmers, and creating new jobs. Later, he worked toward the broader goal of laying the foundation for a stable financial future for all Americans.

Ultimately, it was World War II that healed the economy as the country raced to manufacture materials for the war effort. Still, the soothing influence of Roosevelt's optimism over the American people through the darkest days of the Depression would be difficult to overstate.

Before the surprise attack on Pearl Harbor in 1941, Roosevelt had provided extensive aid to Great Britain while maintaining an official policy of neutrality toward the conflict. After the Japanese assault drew the United States into the fray, Roosevelt proved himself an effective wartime leader.

Roosevelt died of a cerebral hemorrhage on April 12, 1945, less than a month before the Germans surrendered and the Allied victory transformed the United States into a global power.

Harry S. Truman (born May 8, 1884) became president following the sudden death of Franklin Delano Roosevelt on April 12, 1945.

With the United States mired in World War II, the new leader faced many challenges. While the Germans surrendered mere weeks after Truman took office, Japan showed no sign of relenting. Appalled by the prospect of losing more American lives, he decided to hasten the end of the war by dropping atomic bombs on the Japanese cities of Hiroshima and Nagasaki. The bombs killed more than 100,000 people instantly, and the war was over within the month.

Truman continued to face tough foreign policy decisions throughout his presidency. He worked hard to contain the growing influence of the Soviet Union, which had become increasingly antagonistic toward the United States after the war. He supported countries in Europe that were vulnerable to Communist influence. He intervened in the Korean War with the support of the United Nations, which he had helped establish.

On the domestic front, Truman dealt with public unrest as the economy switched gears after the war. While the transition was shaky at first, it was ultimately successful. Truman started pursuing his domestic program, the Fair Deal, in 1949. It called for a wide variety of public programs and reforms, including civil rights and national health insurance. While most of his wish list never made it past Congress, Truman was successful in raising the minimum wage and expanding Social Security.

After Truman left office in 1953, he lived in Missouri until his death in 1972.

John F. Kennedy, who served from 1961 to 1963, is perhaps best remembered for his untimely death. The wildly popular president's assassination in Dallas, Texas, rocked and traumatized the nation.

Kennedy was born on May 29, 1917, in Brookline, Massachusetts. He was only 43 years old when he became president—the youngest person to be elected president in American history.

The threat of Communism was Kennedy's greatest challenge when he entered office. The Cold War was well under way, and Americans lived under the constant threat of nuclear attack by the Soviet Union. Kennedy tried to ease that strain by promising to eradicate Communism around the world.

In 1961, Kennedy secretly backed a group of Cuban rebels that tried to overthrow Fidel Castro. That failed attempt, known as the Bay of Pigs Invasion, was both an embarrassment and a setback for the American government.

In 1962, the Cuban Missile Crisis erupted when it became known that the Soviets were building stockpiles of missiles in Cuba. A tense standoff eventually gave way to an important nuclear weapons treaty between the United States and the Soviet Union.

At home, Kennedy's domestic agenda was thwarted somewhat by a lack of Congressional support. He enjoyed limited success in passing legislation that promoted economic and social programs.

While Kennedy supported civil rights for African Americans, he felt he had limited power to introduce legislation during his first term. He enforced desegregation in Southern schools with military troops and upheld other antidiscrimination laws, but leaders in the civil rights movement, including Martin Luther King, Jr., pressed him for further action. The president started making more progress on this front shortly before he was killed.

Kennedy giving his inaugural address. At his left is Vice President Lyndon Johnson.

Biographies of Selected Presidents

Ronald Reagan, a popular Republican, focused on the economy and the strength of the U.S. military during his two-term presidency (1981 to 1989).

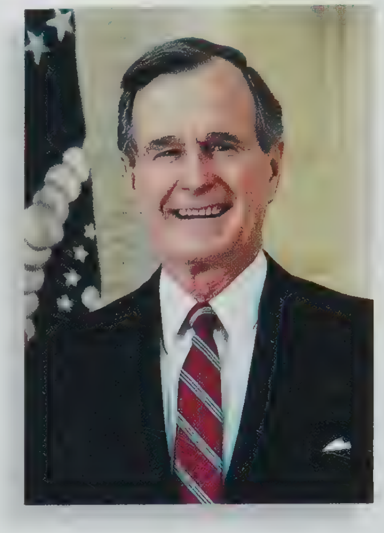
Born February 6, 1911, in Tampico, Illinois, Reagan spent his formative years in the Midwest before he moved to Hollywood to pursue a career in radio and film. As an actor and president of the Screen Actors Guild, he started moving in Republican circles. His anti-Communist stance helped launch his career as a politician. He served as governor of California from 1967 to 1975.

In 1981, just 2 months into his presidency, Reagan was shot in Washington, D.C. While he was gravely injured, his positive attitude and quick recovery cemented his popularity with the American public.

Throughout his presidency, Reagan's domestic policy was centered on economic reform. He decreased government spending on public programs and lowered taxes. While the U.S. economy rebounded from a recession during his watch, the national debt increased and homelessness became a prominent issue.

Reagan's foreign policy was based on a strong defensive stance. During his 8 years in office, the U.S. defense budget was increased by 35 percent. Where his predecessors carefully handled the country's strained relationship with the Soviet Union, Reagan was more bold. Still, by the end of his term, he had negotiated important arms reductions agreements with Mikhail Gorbachev, which helped end the Cold War.

In 1994, five years after he left the White House, Reagan announced that he had been diagnosed with Alzheimer's disease. He died in 2004.



George Herbert Walker Bush's presidency (1989 to 1993) coincided with a tumultuous time in history. His foreign-policy decisions helped the U.S. maintain its steady footing as the rest of the world changed.

Bush was born June 12, 1924, in Milton, Massachusetts. Before college, he served as a distinguished Navy pilot during World War II. After establishing himself in the oil industry in Texas, he held a wide variety of prestigious political offices, including Ambassador to the United Nations and Director of the Central Intelligence Agency. These appointments culminated in the two terms he served as vice president under Ronald Reagan.

His extensive experience in Washington proved valuable as Bush embarked upon his own presidency, when the world's political landscape was rapidly shifting. He skillfully handled the country's relationship with the East through the collapse of the Soviet Union in 1991. He also removed Communist dictator General Manuel Noriega from power in Panama.

Bush's biggest accomplishment was his victory in the Gulf War, which curtailed the aggression of Iraq against Kuwait. Bush organized an international coalition of countries in a war effort that lasted less than 2 months, temporarily restoring stability to the Middle East.

Meanwhile, on the domestic front, Bush faced challenges such as a large national deficit, a lack of funding for federal programs, and a failing economy. The public perceived him as placing too much emphasis on foreign affairs—a mistake that cost Bush the presidential election in 1992 to Bill Clinton.

THE PRESIDENTS: FREQUENTLY ASKED QUESTIONS

Forty-four presidents have served in the United States since 1789, for an average of about 5 years each. In recent times, as the United States has become a world power, the office of the president has become one of the most influential positions—as well as the most difficult to manage—in the world.

Looked at all together, the presidents of the United States are an interesting group. The following are some frequently asked questions about U.S. presidents, and the answers to those questions.

Which presidents served the longest and shortest times in office?

Longest: Franklin D. Roosevelt served from March 1933 to April 1945, a total of 12 years, 1 month. He had been elected for a fourth term and would have served 16 years had he not died in office. The 22nd Amendment (1951) limits presidents to two terms.

Shortest: William Henry Harrison served from March 4 to April 4, 1841. He caught a cold at his inauguration and died of pneumonia just 1 month after taking office. James A. Garfield, another president, died of gunshot wounds in September 1881. He had served only 6 months.

How many presidents died in office?

Eight. They are: William Henry Harrison, 1841, illness; Zachary Taylor, 1850, illness; Abraham Lincoln, 1865, assassination; James Garfield, 1881, assassination; William McKinley, 1901, assassination; Warren Harding, 1923, illness; Franklin D. Roosevelt, 1945, illness; and John F. Kennedy, 1963, assassination.

Were any presidents related to each other?

Yes. John Adams and John Quincy Adams were father and son. William Henry Harrison and Benjamin Harrison were grandfather and grandson. Theodore and Franklin Roosevelt were cousins. George H. W. Bush and George W. Bush are father and son.

Were all the presidents married?

All except one were married at some time in their lives. The exception is James Buchanan. Several presidents have been married more than once, usually after the death of their first wives. The only president who had been divorced was Ronald Reagan.

Did all presidents retire after serving as president?

Most presidents did retire after their terms ended, but not all.

- John Quincy Adams returned to the House of Representatives and served there for 17 years after his presidency.
- William Howard Taft became a law professor and later served 9 years as chief justice of the Supreme Court.

PRESIDENTIAL ELECTIONS: A SCHEDULE

Popular election	First Tuesday after first Monday in November	Candidate with plurality in each state wins that state's electoral votes.
Electoral College meetings	First Monday after second Wednesday in December	Electors meet in state capitals to cast their votes. States certify the votes and send them to Congress.
Counting the electoral vote		Results of the Electoral College vote are tallied by the president of the Senate (usually the vice president) at a joint session of Congress. If there is a tie in the electoral vote, the House of Representatives must elect the president, each state receiving one vote.
Inauguration	January 20	The president-elect is sworn in at noon and takes office.

Canadian History

ARCTIC OCEAN
GRANT
BANKS
VICTORIA
DOMINION OF CANADA
MACKENZIE
DAWSON
GREAT BEAR LAKE
GREAT SLAVE LAKE
HUDSON BAY
BELCHER
NELSON
ROCKY
ATHABASKA
FORT VERMILION
HAZELTON
FT. FRASER
EDMONTON
PRINCE ALBERT
SASKATCHEWAN R.
SASKATOON
CALGARY
WINNIPEG
QUEEN CHARLOTTE ISLANDS
VANCOUVER
MISSOURI R.
SIOUX CITY

Settlement to Confederation 240

Confederation to the Present 270

Canadian National Elections 308

FIRST NATIONS

The first human beings to inhabit what is now Canada were the ancestors of the present-day Indians and Inuit. They began arriving nearly 30,000 years ago from what today are known as Asia and Alaska. Over the centuries, some of these aborigines, moving south and east, spread themselves sparsely over the huge expanse of northern North America. These people came to constitute the tribes that the Europeans called Indians. The tribes developed distinctive languages, cultures, and ways of making a living. Today, Canada's native people are often known as *First Nations peoples* or *Aboriginals.*

Lands of the Inuit

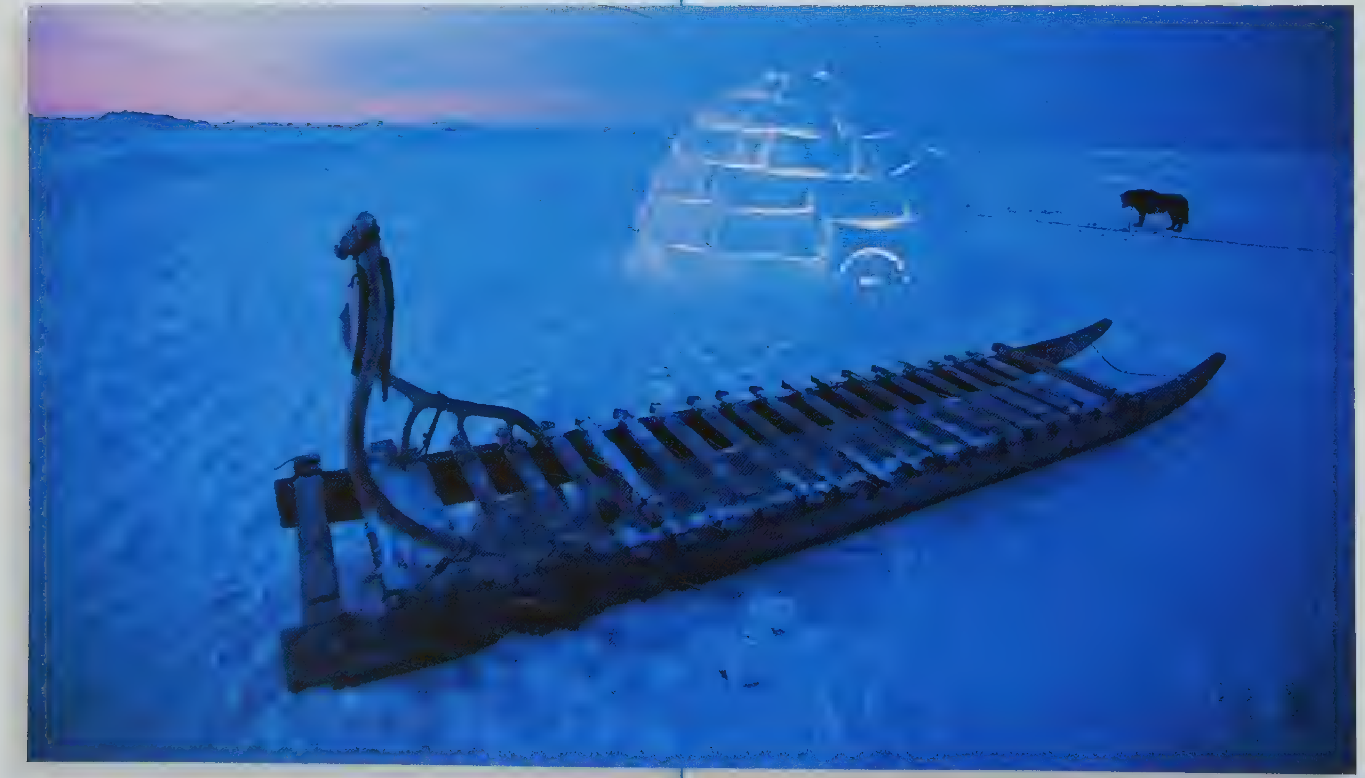
Inuit culture developed over 1,000 years ago in the far northern areas of Canada, which consist mostly of treeless tundra. *Inuit* means *people* in the Inuit-Inupiaq language. The term is used today instead of the old term *Eskimos*, a word of Native American origin that could mean *eaters of raw meat, netter of snowshoes*, or *speaker of a foreign language*. Many Inuit consider the term Eskimo insulting.

Ways of Life

The Inuit have always lived in small groups scattered over a huge region.

- They lived in groups that varied in size from a single family to several hundred people.
- The size of the groups depended on the amount of food available in different seasons.

Igloos are built by stacking blocks of hard snow in coils to form a dome. Few Inuit would have lived in an igloo year-round, however. Igloos were mainly used for temporary shelter. The sled shown would have been the most common form of transportation in winter. ▼



◀ **Today's Inuit** are partly integrated into the rest of the Canadian economy, as fishermen, fur traders, tour guides, and producers of arts and crafts.

Food and Shelter. The Inuit depended on fish, waterfowl, sea mammals, and the few land animals of the far north, such as bears and caribou, for food and for skins to be used for clothing and shelter.

Inuit families would have had both summer and winter housing:

- in summer, wood-frame tents covered with animal skins (probably seal or caribou)
- in winter, sod houses or snowhouses (igloos)

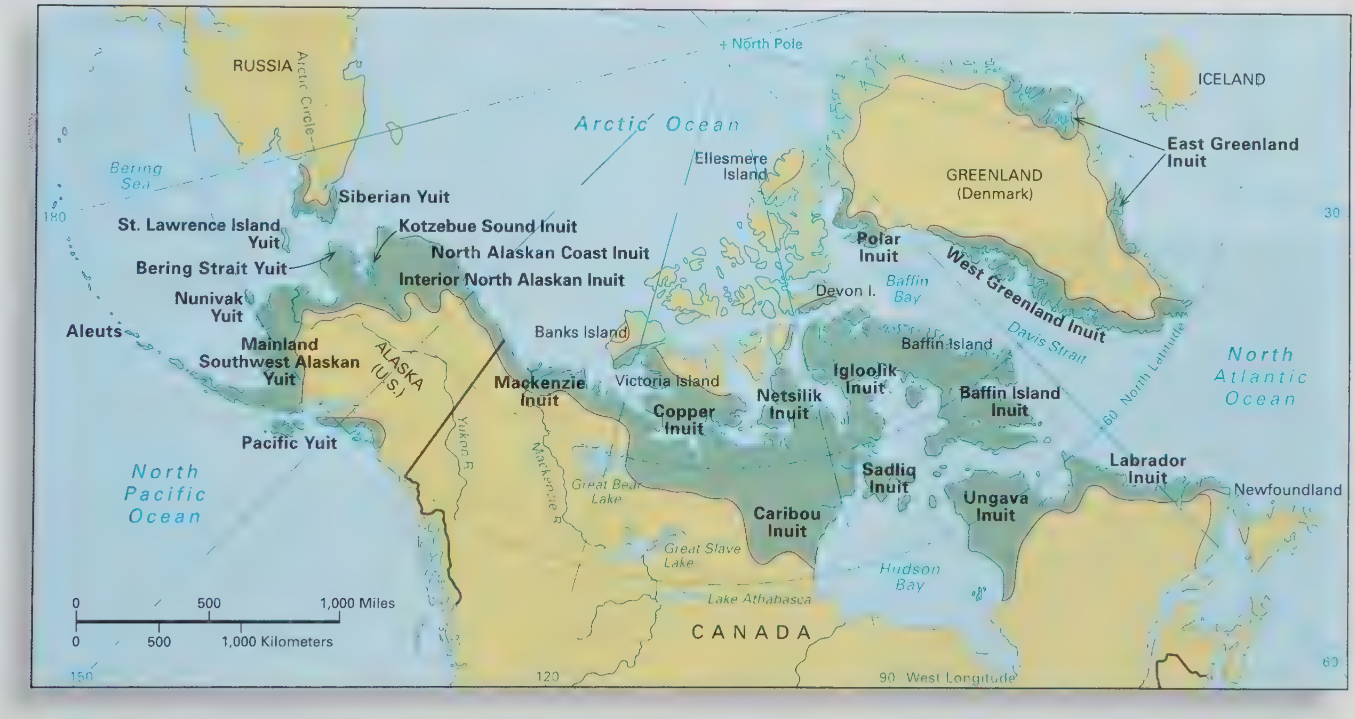
Inuit Today

Of the 100,000 or so Inuit in the world today, more than 25,000 live in Canada. Because they were so isolated in the north, it took until the early 20th century for the Inuit to conveniently trade and live something like a modern life. Today, though, the traditional way of life has ended for most Inuit.

- They live in wooden homes rather than in snowhouses, sod houses, or tents. They wear modern clothing.
- Most Inuit speak English, Russian, or Danish in addition to their native language.

🟩 Land inhabited by the Inuit today

🟨 Land formerly inhabited by the Inuit



North Pole
RUSSIA
Arctic Circle
ICELAND
Arctic Ocean
Bering Sea
Ellesmere Island
GREENLAND (Denmark)
East Greenland Inuit
Siberian Yuit
180
St. Lawrence Island Yuit
Kotzebue Sound Inuit
North Alaskan Coast Inuit
Bering Strait Yuit
Interior North Alaskan Inuit
Polar Inuit
Baffin Bay
Devon I.
West Greenland Inuit
30
Nunivak Yuit
Aleuts
Mainland Southwest Alaskan Yuit
ALASKA (U.S.)
Banks Island
Baffin Island
Davis Strait
North Atlantic Ocean
60 North Latitude
Mackenzie Inuit
Victoria Island
Igloolik Inuit
Netsilik Inuit
Copper Inuit
Baffin Island Inuit
Yukon R.
Mackenzie R.
Pacific Yuit
Great Bear Lake
Sadliq Inuit
Labrador Inuit
Newfoundland
North Pacific Ocean
Ungava Inuit
Caribou Inuit
Great Slave Lake
Hudson Bay
Lake Athabasca
0 500 1,000 Miles
0 500 1,000 Kilometers
CANADA
150
120
90 West Longitude
60

Inuit Lands. This map shows the regions currently and formerly lived in by the Inuit. Land inhabited by the Inuit today includes the northeastern tip of Siberia, the islands of the Bering Sea, and the coastal regions of mainland Alaska. They also include the north coast and islands of the Canadian Arctic and most of the west coast and part of the east coast of Greenland.

First Nations Peoples

Before the Inuit settled in Canada's northern regions, aboriginal groups populated the rugged, forest-filled areas further south, in the Canadian Subarctic region. Their descendants became known as Indians. (See also pages 22–27.)

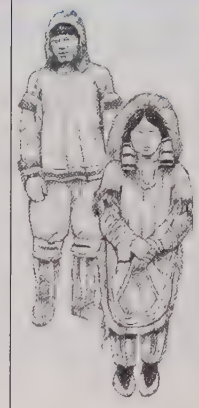
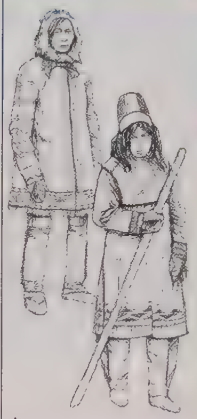
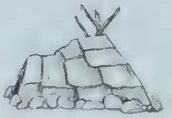
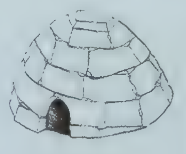
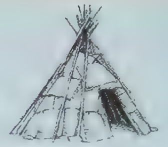
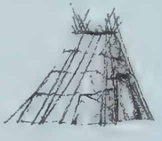
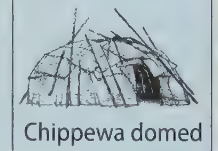
Surviving the Subarctic

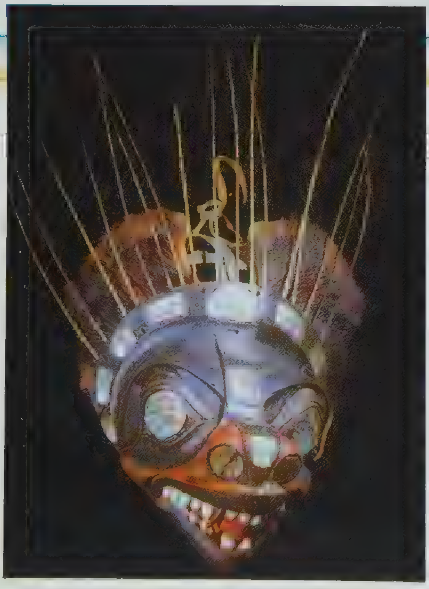
Most of the Indians lived by hunting and fishing. Although there was little agriculture, it was practiced to some extent in the warmer region of southern Ontario and in the valley of the St. Lawrence River.

- Animals, birds, and fish were plentiful, but the Indians had only crude tools and weapons for catching them.
- The great forests provided wood for fuel and housing, while animals provided skins for clothing and shelter.
- The caribou, a large animal that existed in enormous herds, was the chief prize of the hunt for both the Indians and the Inuit.

Finding enough food was a continuing struggle, and when different tribes met while on hunting expeditions, armed conflict over choice areas sometimes resulted.

Ways of Life. This chart shows typical forms of clothing and shelter for the Indians and Inuit of present-day Canada. The groups adapted to the natural resources available to them. ▶

	Arctic	Subarctic
Clothing	 MacKenzie Inuit	 Innu
Buildings and shelters	 Polar Inuit sealskin tent  Iglulik Inuit snowhouse	 Cree bark tipi  Cree bark lodge  Chippewa domed bark lodge



ART AND CULTURE

The aboriginal peoples of present-day Canada developed their own arts and cultures, as well as their own lifestyles.

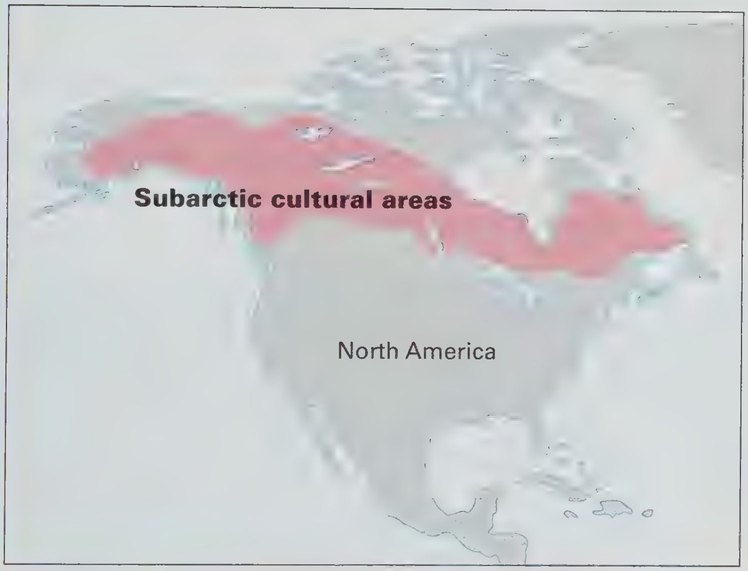
- Inuit artists are noted for their superb work in bone, stone, and ivory carvings of the arctic animal life around them, as well as for ceremonial masks.
- Indians, especially those on the northwest coast, were also carvers, working in wood.

◀ **First Nations Art.** This mask was made by the Nootka of the western coast of British Columbia. The Nootka were also known for their elaborately carved totem poles.

Where They Lived

This chart lists some of the major tribes of the Subarctic and where they were located.

Location	Tribes
East	Micmac, Huron
West	Ottawa, Cree
Interior Prairie Region	Ojibwa (or Chippewa), Nipissing, Assiniboine, Blackfoot, Sioux, Athabascan, Gros Ventre
Far North	Dogrib, Nahani
Pacific Coast	Kwakiutl, Nootka, Tlingit, Haida



◄ **The Subarctic region** includes Alaska's interior and most of Canada.

Contact with Europeans

The first Europeans to make contact with the Indians of Canada and the Inuit were the Vikings, who arrived in the late 900s. Other Europeans arrived in the 1500s. At that point, there probably were about 220,000 Indians and Inuit. It was the mid-1700s before there was much further contact, mostly with trading posts.

Effects. Unlike the Indian groups of the present-day United States, the Indians of present-day Canada had far fewer violent conflicts with the Europeans. They did, however, suffer from the coming of the Europeans.

- They had little resistance to European diseases, so epidemics such as smallpox wiped out half of the members of the tribes.
- As in the United States, Indian groups in Canada also lost much of their lands. Today, many of Canada's Indians live on government reserves.

now you Know!

- ✓ The first people to live in what is now Canada arrived nearly 30,000 years ago from what today are known as Asia and Alaska.
- ✓ These people were the ancestors of the present-day Indians and Inuit.
- ✓ The Inuit traditionally occupied the far northern areas of Canada, which mostly consist of cold, treeless land called tundra.
- ✓ The Indians of the Subarctic lived mostly by hunting and fishing.
- ✓ The first Europeans to make contact with the Indians of Canada were the Vikings, in the late 900s. Other Europeans began to arrive in the 1500s.

The Seven Years' War

The decisive conflict for control of North America was the Seven Years' War, which began in 1754. In the United States, it is known as the French and Indian War.

William Pitt (1759–1806), the British prime minister, planned to devote his war efforts to the defeat of the French Empire—and in particular, New France. He considered the French colonies, with their ability to attack the wealthy English territories, a menace to the British Empire.

Forts Changing Hands

Fort	1755	1756	1757	1758	1759
Ft. Duquesne (Pitt)	France	France	France	Fort destroyed by the French	
Ft. Oswego	England	France (fort taken and destroyed by the French)			
Ft. William Henry	England	England	France (fort taken and destroyed by the French)		
Ft. Carillon (Ticonderoga)	France	France	France	France	Fort destroyed by the French
Ft. Louisbourg	France	France	France	England	England
Ft. Frontenac	France	France	France	England	England
Ft. Niagara	France	France	France	France	England
Ft. Saint-Frédéric (Crown Point)	France	France	France	France	Fort destroyed by the French
By 1760, three of eight major forts were in British hands. The French were in possession of none of them.					

Campaigns and Battles

The Seven Years' War began badly for the British. The French routed them at Fort Duquesne (present-day Pittsburgh, Pennsylvania) in 1755. This influenced the Indians of the Northwest to join the French side and left the English frontier settlements of Pennsylvania, Maryland, and Virginia open to attack.

Early French Victories. Under the leadership of Louis Joseph de Montcalm (1712–1759), the French took the initiative and gained some victories.

- In 1756, French forces captured Fort Oswego, on Lake Ontario, and destroyed it.
- In 1757, the French defeated a British plan to capture Louisbourg and took Fort William Henry in New York (Unfortunately, the Indian allies of the French massacred the British prisoners in spite of Montcalm's attempts to control them.)

Later British Gains. France's fortunes began to decline in 1758 as the British government increased its military strength in America. Although British forces failed in an attempt to capture Fort Carillon (Ticonderoga) in New York, they eventually took Louisbourg after a 7-week siege. Now the French had lost all control of the gulf and Atlantic coast. The English also took Fort Frontenac on the northeast shore of Lake Ontario. They now controlled the gateway to the St. Lawrence River. A final blow came in 1758: the French retreated from Fort Duquesne (present-day Pittsburgh).

The year 1759 was even more disastrous for France.

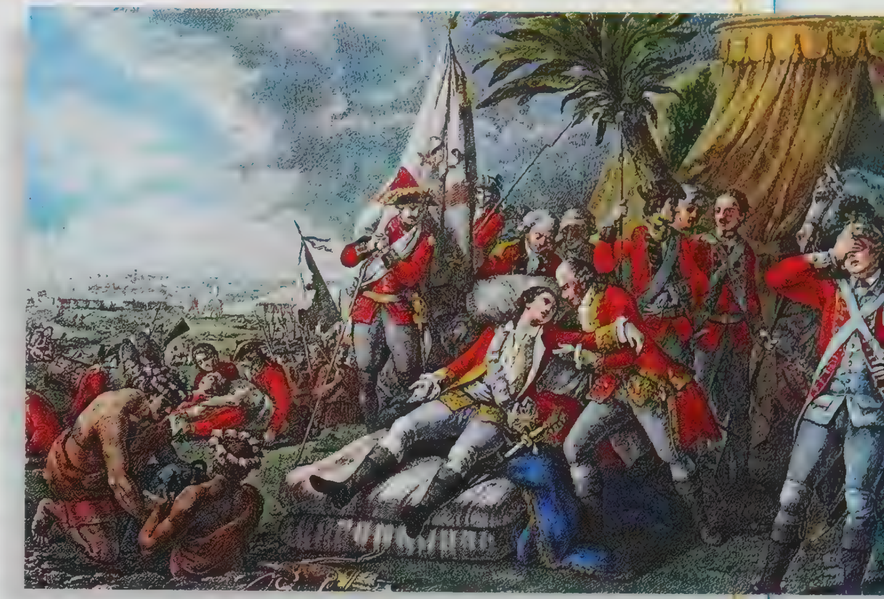
- In July, Fort Niagara, on the south shore of Lake Ontario, was captured by a force led by Sir William Johnson (1715–1774).
- At the same time, the French withdrew from Lake George and Lake Champlain, destroying Fort Carillon (Ticonderoga) and Fort Saint-Frédéric (Crown Point).

The Battle of Quebec

The military climax to a century of struggle between France and England for control of northern North America came in September 1759, with the fall of Quebec City. The British campaign to take this vital stronghold began with a naval blockade that closed the St. Lawrence River both above and below the city.

In June, 9,000 British troops landed opposite Quebec. They were commanded by James Wolfe (1727–1759), a young officer who had been second in command at Louisbourg's capture. Montcalm, meanwhile, had gathered a force of 10,000 to defend Quebec.

Wolfe's forces poured a heavy bombardment into the city, but the general could at first find no way to assault the French forces. At last, his troops reached the Plains of Abraham, outside the walls of the city under cover of darkness on September 12. The next day Montcalm decided to risk a battle. A French attack was quickly driven back. The British were victorious within 15 minutes. Quebec was forced to surrender on September 18.



▲ **The Battle of Quebec** was a swift victory for the British; however, the commanders for both sides—the Marquis de Montcalm of France and James Wolfe of the United Kingdom—were mortally wounded in the battle. This 1759 painting depicts Montcalm's death.

The End of the War. In 1760, the French made a futile attempt to retake Quebec, but they had to retreat to Montreal. In July, the French fleet in the St. Lawrence was destroyed, and on September 8, 1760, Montreal surrendered. After the final defeat of French forces in Newfoundland in 1762, the power of France in the New World was ended.

This defeat was acknowledged in the Treaty of Paris of 1763.

- France gave up Canada and ceded western Louisiana to Spain, its ally.
- Spain handed Florida over to England.
- France kept the islands of St. Pierre and Miquelon, and recovered Guadeloupe and Martinique in the West Indies.
- France ceded Grenada and the Grenadines to Great Britain.

The government of England now had to decide on important matters of policy affecting Canada. Most of the inhabitants were defeated enemies who spoke a different language and who were French Catholics rather than English Protestants.

now you Know!

✓ In the early 1600s, as Britain's settlements expanded, France and Britain began to clash over territory.

✓ In 1689, the first of four wars between France and Britain began. These wars were fought in Europe as well as in North America.

✓ The four wars are known as King William's War, Queen Anne's War, King George's War, and the Seven Years' War.

✓ The Seven Years' War finally ended France's control over Canada.

Select Prime Minister Biographies

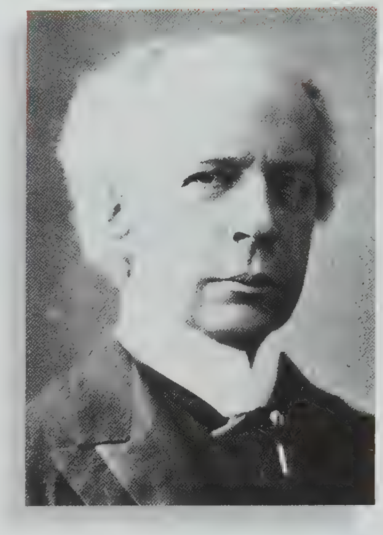
Sir John Alexander Macdonald, Canada's first prime minister, served from 1867 to 1873 and 1878 to 1891. He was instrumental in the formation of the country and in facilitating its rapid expansion.

Macdonald was born January 11, 1815, in Glasgow, Scotland. In 1820, his family moved to what is now Ontario. At the time, Canada did not yet exist; it was a land of fragmented territories ruled by Great Britain.

In 1844, Macdonald, a lawyer, was elected as a government official in the Province of Canada. He quickly worked his way up through the Conservative Party. Increasingly, he recognized the need for centralized government. Macdonald helped draft the plans for confederation, which would bring the disparate provinces together as one country. He thought federal government should be limited and that provinces should govern themselves in most respects.

When Canada was formed by the British North America Act of 1867, Macdonald became its first prime minister. At the time, the country included New Brunswick, Nova Scotia, and the Province of Canada, with the expectation that more provinces would join later. Beginning in 1870, the provinces of Manitoba, British Columbia, and Prince Edward Island were added in quick succession, greatly expanding Canada.

Macdonald's first stint in office ended when he was forced to resign over a bribe scandal. When he regained power 5 years later, he enacted a new domestic agenda called the National Policy, which used steep taxes on foreign imports to build railroads and finance other government projects. He continued as prime minister until he died in 1891.



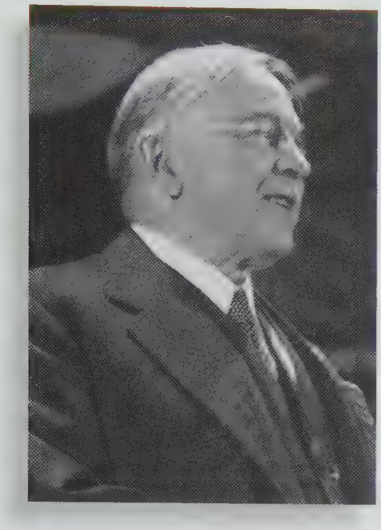
Sir Wilfrid Laurier, the first French-Canadian prime minister, served from 1896 to 1911. His long tenure coincided with a period of intense national growth.

Laurier was born November 20, 1841, in what is now Quebec. (The provinces did not come together to form a unified Canada until 1867.) Throughout his life, there was an informal divide between the French-speaking and English-speaking areas in Canada. As a politician, he tried to bridge that divide.

Laurier's years in office were a time of expansion and growth for the country. The population was on the rise as immigrants poured in, attracted by the prime minister's campaign to attract settlers away from the dwindling American frontier. The country added two new provinces, Saskatchewan and Alberta, in 1905. To accommodate and encourage further growth, Laurier funded more railroads.

His other focus was foreign policy. His main accomplishment was carefully managing Canada's relationship with Great Britain. The British Empire was keen to extend its reach into matters of Canadian government, but Laurier resisted. While he was never antagonistic toward the British government (in fact, he supported it in many ways), Laurier worked hard to maintain Canada's autonomy.

Laurier and his Liberal Party lost power due to his support of a controversial trade agreement with the United States. He remained active in the House of Commons until he died in 1919.



William Lyon Mackenzie King served longer than any other prime minister in Canadian history, totaling more than 21 years of service between 1921 and 1948. He was the country's most important political figure during that stretch, even though his time in office was divided into three distinct time periods (1921–1926, 1926–1930, and 1935–1948).

Born December 17, 1879, in Ontario, King spent his young adulthood immersed in his studies. He attended the University of Toronto and the University of Chicago before receiving his PhD from Harvard University. During his travels, King was exposed to the difficult conditions facing poor laborers. Their plight made a deep impression on him.

King's interest in labor issues led him down the path of government service. In 1900, he helped establish the Canadian government's first department of labor. Eventually, those interests would spill over into his domestic agenda as prime minister, when he promoted social programs such as unemployment insurance and welfare.

Foreign affairs—particularly Canada's relationships with the United States and Great Britain—were another area of focus for King. His administration worked closely with the U.S. government to establish a mutually beneficial trade agreement. As he led Canada through World War II, King managed to support Great Britain while asserting Canada's independence from the British Empire.

King died in 1950, two years after his retirement.

John George Diefenbaker, a Conservative who served as prime minister from 1957 to 1963, dedicated most of his term to promoting social justice.

The Ontario native was born September 18, 1895. After serving in World War I, he worked as a lawyer. He was a gifted orator known for his eloquent speeches. He was also known for his persistence; he lost five elections before he joined the House of Commons in 1940.

After a long career in politics and many years of service in Parliament, Diefenbaker became leader of the Progressive Conservative Party in 1956. His party swept the national election in 1957, gaining control of government after 22 years of Liberal leadership.

Diefenbaker's domestic policy was largely dedicated to social justice. Most importantly, he championed the Canadian Bill of Rights to promote equality and fair treatment. He reorganized federal programs to better help Canada's needy citizens. He also appointed the first woman to a prominent cabinet position and spoke out against apartheid in South Africa.

Foreign policy—particularly the way in which Diefenbaker handled the Cuban Missile Crisis—was the reason the public lost confidence in his administration. He resisted when the United States asked Canada to place its military on high alert in response to the threat of Soviet missiles in Cuba. While Diefenbaker ultimately agreed, his hesitation was widely perceived as tentative and indecisive.

After Diefenbaker lost power, he remained an active politician in the House of Commons until he died in 1979.

Select Prime Minister Biographies

Lester Bowles Pearson served as prime minister from 1963 to 1968. While he had a strong background in foreign relations, he spent much of his time in office developing domestic programs.

Pearson was born in Ontario on April 23, 1897. He left college to serve in the military during World War I and then returned to his studies after he was injured. After a few years working in academia, he joined the foreign service in 1928. He spent the next 20 years in prestigious appointments in Great Britain and the United States.

Pearson was a prominent figure in international diplomacy long before he became prime minister. His most illustrious work was within the United Nations. As a Canadian delegate (1948–1956) and head of the General Assembly (1952–1953), he helped facilitate the end of the Korean War. He won the Nobel Peace Prize in 1957 for his contributions during the Suez Canal Crisis.

Pearson was also a key player in the North Atlantic Treaty Organization (NATO), a military alliance against the Soviet Union that was founded in 1949.

Much of Pearson's time as prime minister was devoted to domestic development, including a wide array of social programs. He broadened federally funded services and benefits, including the implementation of national medical care and a pension system. He also incited a fierce debate over the design of a new national flag, which was adopted in 1965.

After he retired from politics, Pearson resumed teaching until his death in 1972.

Pierre Elliott Trudeau served as prime minister from 1968 to 1984. His tenure was interrupted by a brief interval in 1979, when the Conservative Party controlled the government for less than a year.

Trudeau was born in Montreal on October 18, 1919. He was a lawyer and an academic until he began his career as a politician in 1965. A charismatic leader, he was already hugely popular by the time he took office in 1968.

One of the biggest challenges Trudeau faced was the independence movement championed by separatists who wanted Quebec to secede from Canada. In 1970, a terrorist group kidnapped two government officials. Trudeau, who was committed to a unified Canada, was largely praised for his forceful response to this event, which became known as the October Crisis.

Throughout his tenure, Trudeau continued his efforts to unify the country. He was a champion of multiculturalism and bilingualism, emphasizing that Canadian identity was broad enough to include speakers of both English and French. Under his leadership, Canada officially became bilingual.

Trudeau's other major accomplishments include severing ties with Great Britain by eliminating the British Parliament's influence over Canadian government, and establishing a diplomatic relationship with China.

After he retired from politics in 1984, Trudeau worked as a lawyer. He died in September 2000.

Kim Campbell became Canada's first female prime minister in 1993. She held the office for just over 4 months before power shifted to the Liberal Party.

Campbell was born March 10, 1947, in British Columbia. She was an academic and a lawyer before she turned to politics. After a stint in local government, she joined the House of Commons in 1988. There, she served as minister of State for Indian Affairs and Northern Development, minister of Justice and Attorney General, and minister of National Defense.

When Prime Minister Brian Mulroney retired in 1993, the Progressive Conservative Party chose Campbell to replace him. She assumed the office in June. While she was popular with the Canadian public at first, her party's standing was on shaky ground. (Mulroney had not been well liked.) When a national election was held in November 1993, the Conservatives lost almost all of their seats in Parliament.

Because of her limited time in office, Campbell's record as prime minister is relatively unremarkable. While her legacy as the first woman to serve in that role is important, her most substantial political accomplishments occurred during her time in the House of Commons. Since retiring from politics, Campbell has spent most of her time teaching at Harvard University.

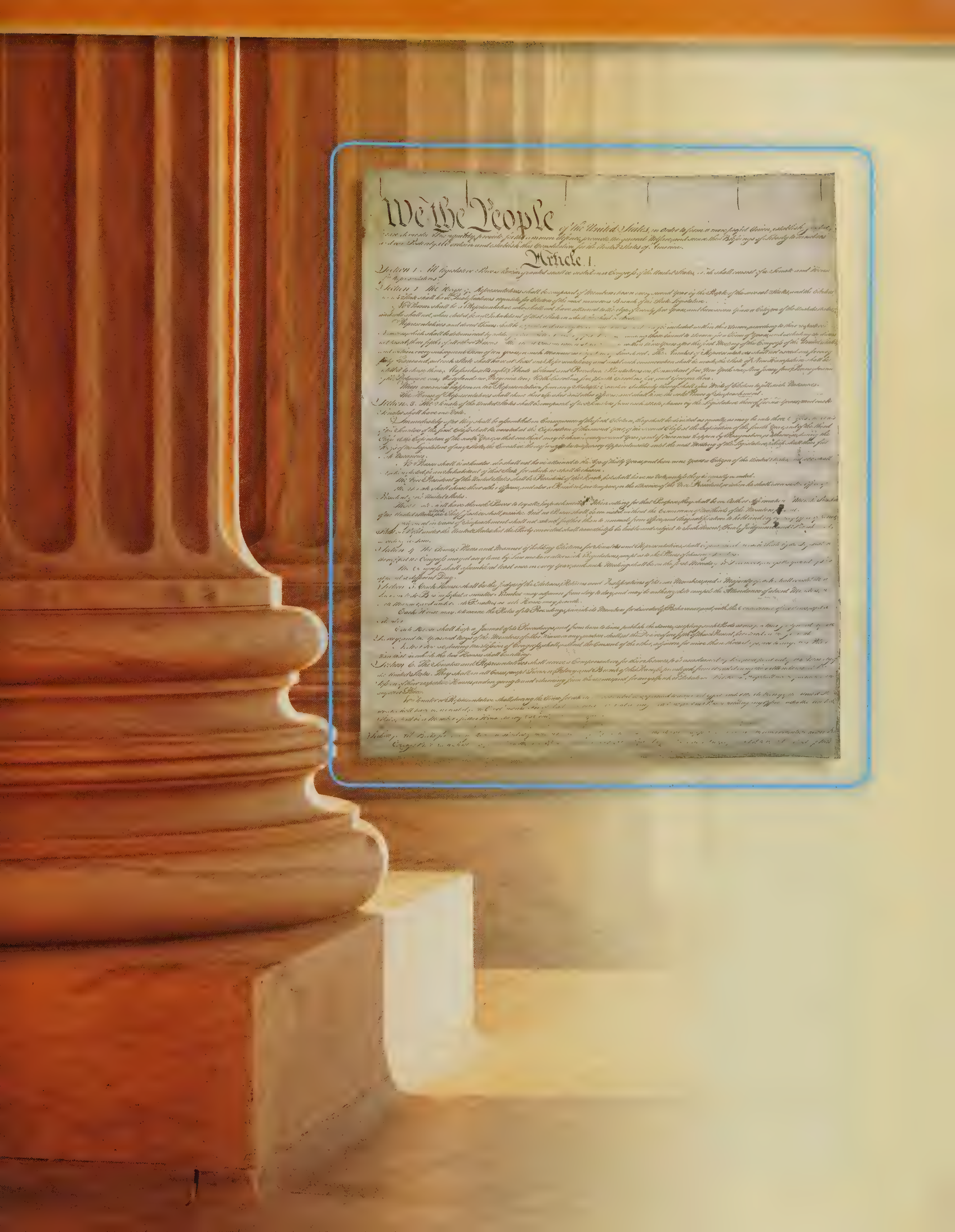
Joseph Jacques Jean Chrétien served three terms from 1993 to 2003. Born January 11, 1934, in Quebec, Chrétien worked for many years as a lawyer and as a politician. He held a wide variety of important cabinet posts during the administrations of Lester Pearson and Pierre Trudeau.

When Chrétien took office, Canada had many financial problems, including a large national deficit. As prime minister, his first priority was to balance the budget. While he had to sacrifice federal programs and services along the way, the government had paid its debts by 1998.

One of the most difficult challenges Chrétien faced on the domestic front was the threat of secession from Quebec. The French-speaking province had long felt isolated from the rest of the country. In 1995, the people of Quebec voted to remain a province of Canada, but the vote was close. This was an embarrassment to Chrétien, a Quebec native who strongly believed in Canadian unity.

While Chrétien faced a number of domestic challenges, foreign policy issues led to his resignation. When the U.S. declared war on Iraq, he felt pressure from many sides to support the effort with troops. His flat refusal to participate was widely perceived as a good decision by the Canadian public, but it was less popular within the prime minister's own political party. As a result, Chrétien stepped down in 2003.

Government



We the People of the United States, in Order to form a more perfect Union, establish

Article. 1.

U.S. GOVERNMENT

Although we seldom think about it, government touches each of our lives every day. When we consider government at all, we think of the president sitting in the Oval Office contemplating national policy or of the local mayor dealing with problems such as budgets or crime. However, the heads of governments are actually tiny parts of much larger institutions.

To understand the governmental system of the United States, it is important to consider the local, state, and federal governments in turn.

Local Government

In small towns, legislative power may be in the hands of citizens at regular town meetings. ▶

Who Runs Local Government?

Form of Local Government	Description	Details
Strong mayor-council form	A miniature of state and federal government	The mayor acts as chief executive, appoints the heads of most major departments, drafts the budget, and has veto power over acts of the city council, which serves as a legislature. The advantages of this system include a division of powers and a system of checks and balances.
Weak mayor-council form	The power of the mayor is severely curtailed.	The council appoints administrators and drafts budgets. The mayor lacks veto power and sits at council meetings only as a moderator.
Commission form	A curious blending of administrative and legislative functions in one body	The commissioners—usually five in number—are elected at large and each assumes executive control of a department (public safety, public works, revenue, parks and recreation, public affairs). The commissioners also serve as a legislature, adopting budgets and ordinances.
Council-manager system	Usually consists of an elected council	The council passes ordinances and sets broad policies. A professional manager, who is appointed by the council, administers the daily business of government and gives advice to the council on matters of policy.
Mayor-manager system	The mayor, as chief executive officer, appoints the city manager to be chief operating officer.	In this model the manager serves at the mayor's pleasure rather than the council's. The voters can get rid of a manager whose actions are unpopular by voting against the mayor.

Functions of Local Government

Despite all the attention showered on national and state politics, local government probably affects more people in more ways than all federal and state government combined. Many local government functions are so familiar that we take them for granted.

Public Works. Perhaps the most visible of local governments' responsibilities lie in the area of public works. Local governments repair streets, remove snow and ice in winter, collect garbage, and dispose of sewage.

Public Safety. One of local governments' most important jobs is promoting public safety through police and fire protection. Not only must local governments prevent crimes, catch lawbreakers, and limit fire damage, they must do so publicly and visibly so that citizens feel secure.

Human Services. Many local governments operate public hospitals and medical clinics. Nearly all have health departments that enforce codes governing the cleanliness of restaurants, proper sewage disposal, inoculation against contagious diseases, and other public health concerns. Cities and counties also administer a variety of welfare services for the poor.

Culture and Recreation. Although many public libraries were built with the help of private contributions and local fund-raising efforts, their basic operating budgets come from local governments. Local governments also develop and maintain parks and other recreational facilities for local residents.

Regulation. Zoning is the determination of how land will be used. Local communities also enforce various building, housing, plumbing, electrical, health, and fire codes.

Education. In many areas, education costs more than all other local government activities combined. Local education is almost always supervised by a board of education that appoints a superintendent of schools, usually a professional educator, and supervises his or her work.

Five Basic Forms of Local Government

All cities with 1 million or more inhabitants use one of the mayor-council systems (see chart left). This form is also prevalent in large cities (500,000 to one million people) and some small towns (5,000 to 10,000). About half the municipalities between 10,000 and 500,000 use one of the city manager systems, about 40 percent employ mayors and councils, and about 10 percent retain commissions.

CANADIAN GOVERNMENT

Although the Canadian and American systems of government stem basically from that of Great Britain, they exhibit certain differences. Canada's governmental structure is almost an exact copy of that of the mother country, with its Parliament, prime minister, elected House of Commons, and appointed Senate, which is similar to the unelected British House of Lords. Nevertheless, over the years the governments of Canada and the United States have developed along similar lines and have faced similar problems.

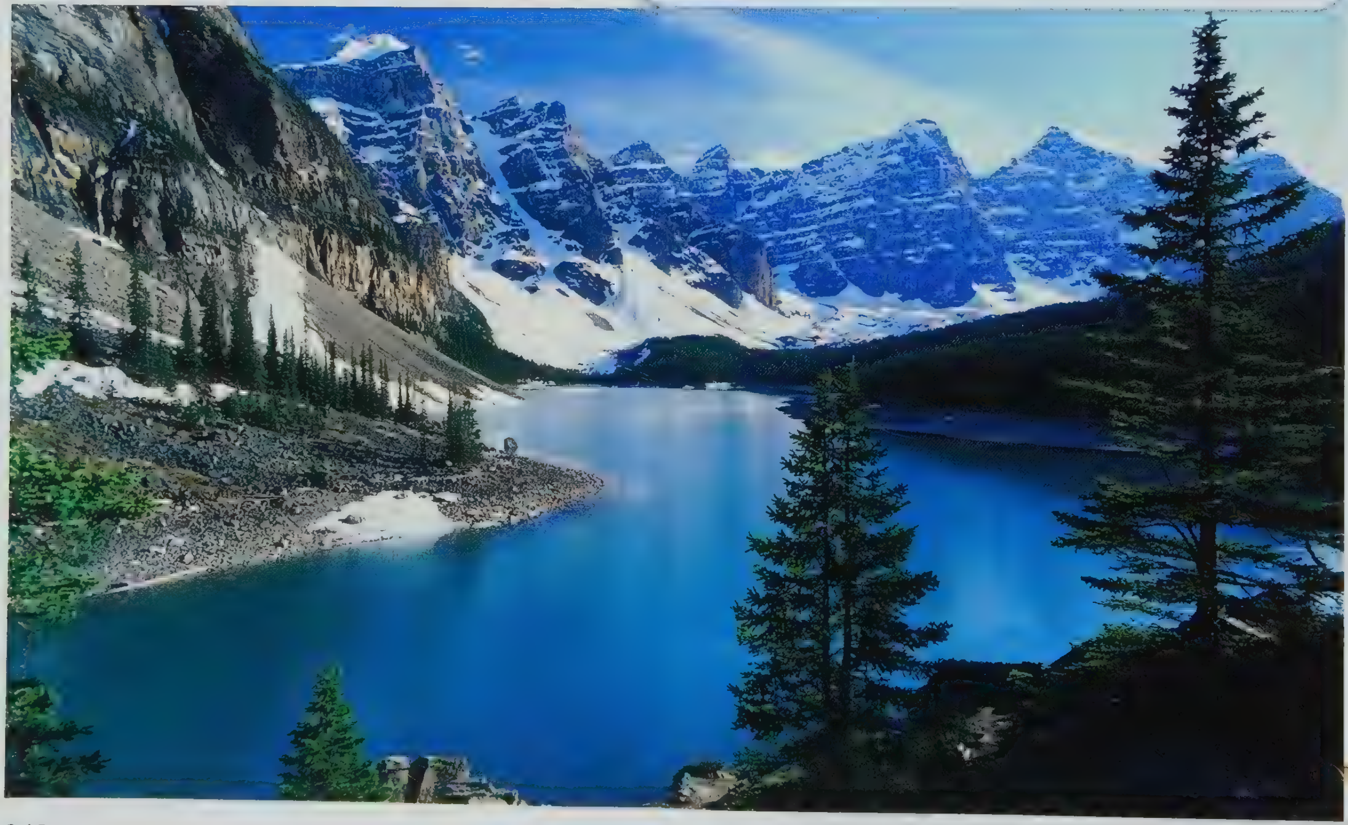
Canada: An Introduction

Canada's Unique History

The history of government in Canada differs from that of any other nation in the Western Hemisphere. South of what became the United States of America, Spanish and Portuguese rule prevailed. Their rule was monarchical, centralized, and bureaucratic. By contrast, the English colonies on the Atlantic seaboard from Georgia to Canada wished to run their own affairs. Their common heritage of language, culture, and government made for unity and consensus.

Two Cultures. England and France fought for control of Canada for nearly 100 years before Great Britain triumphed in 1763. Even then the strength of the French part of Canada was so great that England could not insist on anglicizing its new subjects. This fact was recognized in the Quebec Act of 1774, by which Protestant England allowed French-Canadians to continue their practice of the Roman Catholic religion. The need to recognize French-English differences has continued into the Canadian governmental system to this day.

Banff National Park is part of the Parks Canada Agency of the Ministry of Environment.



Structure and Functions

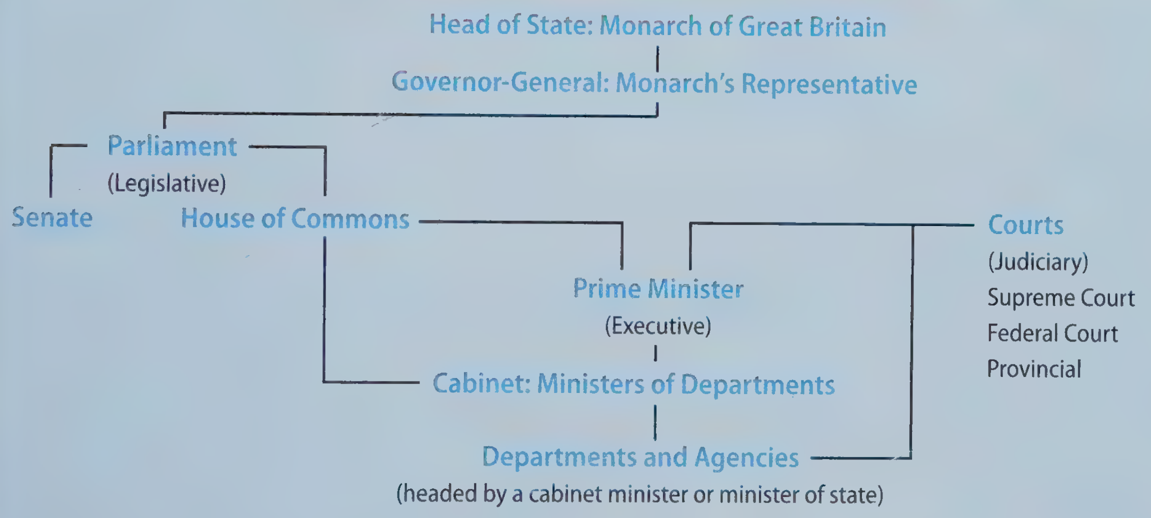
Canada today is a parliamentary democracy made up of 10 provinces and three territories, with its capital in Ottawa. At the formal head of the government, after the British monarch, is the governor-general, who is appointed by the monarch on advice of the Canadian prime minister.

The governor-general's duty is to see to it that the country has a government at all times. When a prime minister resigns, dies, or is voted out in an election, the governor-general fills the office. In practice, that official abides by the principle of majority rule in naming a new prime minister, but there can be complications requiring tact and diplomacy when no political party has a clear majority in the House of Commons.

The Queen's Privy Council. The Queen's Privy Council for Canada includes former cabinet ministers, the chief justice and former chief justices, former speakers of Parliament, and other distinguished persons. Membership is for life. The portion of the Privy Council that actually advises the government is the sitting cabinet.

The Prime Minister. The most powerful member of the government is the prime minister, who is leader of the party holding a majority of seats in the House of Commons. Following a general election the governor-general will ask the leader of the majority party to become prime minister, form a government, and select a cabinet.

STRUCTURE OF THE GOVERNMENT OF CANADA


Head of State: Monarch of Great Britain
Governor-General: Monarch's Representative
Parliament
(Legislative)
Senate
House of Commons
Prime Minister
(Executive)
Cabinet: Ministers of Departments
Departments and Agencies
(headed by a cabinet minister or minister of state)
Courts
(Judiciary)
Supreme Court
Federal Court
Provincial

Aboriginal Affairs and Northern Development
Agriculture
Agriculture and Agri-Food
Arctic Council
Asia-Pacific Gateway
Atlantic Canada Opportunities Agency
Atlantic Gateway
Canadian Heritage and Official Languages
Canadian Northern Economic Development Agency
Canadian Wheat Board
Citizenship, Immigration, and Multiculturalism
Democratic Reform
Economic Development Agency of Canada for the Regions of Quebec
Environment
Federal Economic Development Agency for Southern Ontario
Federal Economic Development Initiative for Northern Ontario
Finance
Fisheries and Oceans
Foreign Affairs
Foreign Affairs (Americas and Consular Affairs)
Health
Human Resources and Skills Development
Industry
Intergovernmental Affairs
International Cooperation
International Trade
Justice
Labour
La Francophonie
Métis and Non-Status Indians
National Defence
National Revenue
Natural Resources
Public Safety
Public Works and Government Services
Queens' Privy Council
Science and Technology
Seniors
Small Business and Tourism
Sport
State
Status of Women
Transport
Transport, Infrastructure, and Communities
Treasury Board
Veterans Affairs
Western Economic Diversification

Economics

上海航空
股份
3.3
8.4
13.0

500 EURO
200
50
100
DOLLARS

ECONOMIC CONCEPTS: SUPPLY AND DEMAND

The supply of a product is the amount of it that businesses are willing and able to offer for sale at alternative prices. The demand for a product is the amount of it that users can and would like to buy at alternative prices. Because the amount that producers actually sell must be the same as the amount that users actually buy, the only price at which everyone can be satisfied is the one for which supply equals demand. This is called the *equilibrium price*.

What Affects Supply?

Factor	Typical Effect	Example
Input costs	Negative	If labor costs rise, producers will reduce the quantity supplied at any given output price to avoid producing some units which were previously profitable but are now unprofitable because of higher labor costs.
Technology advance	Positive	If a technological advance (such as "smarter" equipment) reduces the cost of production, firms will produce more at any given output price.
Prices of related items	Negative	If, with the same or similar resources, a producer can shift production to a more profitable output, the quantity supplied for the original item will fall.

The Meaning of Supply

Supply

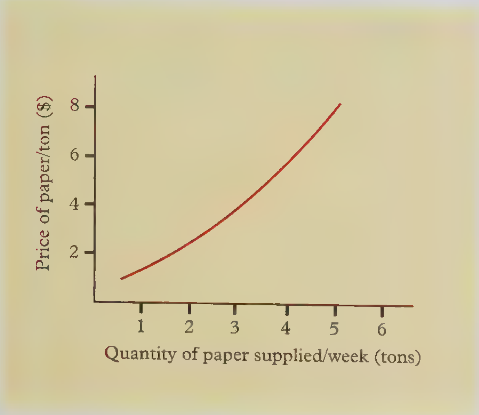
Supply is the relationship between the price of a product and the quantity firms will produce and offer for sale. Given the assumption that a company will produce goods and services for profit, it will typically produce more of an item at a higher price than it will at a lower one. This direct relationship between price and quantity supplied is called the law of supply. It is reflected in the positive, or upward, slope of supply curves.

There are factors other than the selling price of the good or service that affect supply.

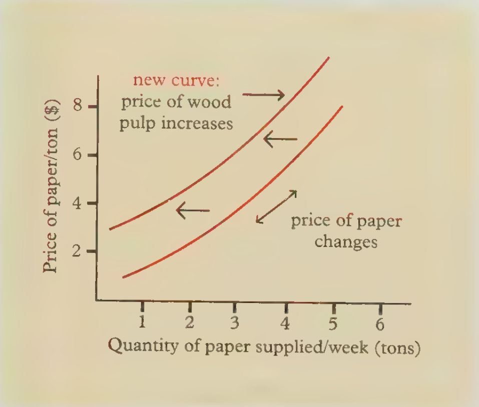
Alternative Outputs. The supply of one item can also be affected by the prices of other goods and services. Since resources can be used in different ways to produce different items, producers may allocate more of their resources toward the more profitable item(s).

For example, if the price of corn rises, farmers will dedicate more acreage to corn production and less to other crops such as wheat. This is true even if the price of wheat hasn't changed. In other words, a rise in the price of one product—corn—can reduce the supply of another product—wheat. It can also increase the supply of by-products—corn oil, for example, in this case.

Consumers make choices in the marketplace by evaluating the tradeoffs between competing products. How much satisfaction will the consumer get from one product versus another? ▶



A graph that portrays all of the combinations of quantity supplied and price for a particular good or service is called a supply curve. Because producers are willing to supply larger quantities at higher prices, the supply curve has a positive, or upward, slope.



When a factor affecting supply other than price changes, it often causes a shift in the entire supply curve. This graph shows how a supply curve might move to the left, given that the price of a production input (wood pulp) increases. Now, at all the same prices as before, the quantity supplied decreases.

Utility

How does a consumer make any choice at all in an entire marketplace of products? To explain how individuals evaluate the tradeoffs among competing products, economists use the term *utility*. Utility describes the amount of satisfaction an individual gains from consuming a product.

Marginal utility is the extra satisfaction that an individual gets from consuming one more unit of an item. For most goods and services, marginal utility decreases as the number of units increases. For example, while the first ice cream sundae a day may be just the thing for a hungry consumer, chances are the fifth or sixth dish will be much less satisfying. As a result, the rational consumer will not be willing to pay as much for the sixth dish as for the first.

This idea—that satisfaction from marginal units decreases as more units are consumed—is called the law of diminishing marginal utility. It is one of the reasons why demand curves slope downward.

fyi!

Changes in factors other than product price represent changes in supply that generate new supply curves. An increase in supply shifts the curve to the right; a decrease shifts the curve to the left. By contrast, when the price of a product changes, we move along the original curve to find the new quantity supplied.

Money

Monetary Policy

In the United States, monetary policy refers to actions taken by the Federal Reserve to control the supply of money. Deliberate expansion and contraction of the money supply are designed to control business cycles, maintain price stability, and contribute to full employment. When an economy is experiencing a recession, an increase in the supply of money can stimulate consumer demand. Such stimulation may result in increased productivity and employment as producers endeavor to meet the higher level of demand. As more people go to work, they spend more money, and the cycle continues. Also, as more people are put to work, tax revenues increase and welfare payments decrease, thus reducing the amount of money the government must borrow in order to operate. However, when the central bank increases the supply of money, inflation may result unless the supply of goods expands quickly enough to meet increased consumer demand.

The Money Supply

In the United States, there are several different measures of money. The two most common measures are *transactions money* or M1, and *broad money* or M2. Within these different measures, the main factor for determining what is and isn't money is liquidity. A liquid asset is one that can be used and accepted immediately as a medium of exchange. Currency (both coins and bills) is the most liquid asset of all.

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One reason some economists use M2 instead of M1 is that it is typically a more stable measure than M1. This is because M2 is not affected by transfers between savings accounts and checking accounts or currency.

THE FOUR ROLES OF MONEY

1. **Medium of exchange.** The primary role of money is to enable exchange of goods and services. Therefore, to be considered money, an item must be widely accepted in trade. The alternative to money as a medium of exchange is the barter system, where goods and services are exchanged directly for one another.
2. **Unit of account.** Money provides a way to meaningfully measure, compare, and add together different types of goods and services.
3. **Store of value.** Money provides a convenient way to hold value for future exchange; it's easier to store than lumber, yogurt, socks, or spaghetti! Because money has a fixed unit value, it is not the best store of value during periods of inflation; in such cases, the real value of money declines. However, during deflationary periods, money increases in real value.
4. **Standard for deferred payment.** Money provides a standard means for identifying the amount of a future payment. For example, if I loan you money today, I can get an I.O.U. for a specific future dollar amount.

Transactions Money. M1 includes currency held outside of banks, as well as checking accounts and traveler's checks. M1 is the most widely used measure of the money supply.

Broad Money. M2 includes everything in M1 plus several close substitutes for transactions money: savings accounts and money market accounts. While neither of these two categories is immediately accepted as a medium of exchange, each is easy to convert into M1.

Interest rates reflect the opportunity cost of holding currency. The relationship between the quantity demanded of money and the interest rate is inverse. For example, when interest rates rise, there is now more incentive for consumers and firms to save their dollars, removing them from the liquid money supply. Thus, when interest rates rise, the quantity demanded of money still in circulation falls.

Federal Reserve Board, Washington, D.C.

The Federal Reserve

In the United States, the money supply is controlled by the Federal Reserve. Founded in 1913, the Federal Reserve is the nation's central bank or "banker's bank." It regulates and supervises commercial banks, lends them money, acts as their clearinghouse, and assists banks in serious financial difficulty.

The Federal Reserve System is divided into 12 geographic districts, each served by a Federal Reserve Bank and subsidiary branch offices. All are governed by a seven-person Board of Governors. The chairperson of the board is appointed by the president for 4 years. The Federal Reserve banks are owned by the member banks in their districts; they are not government agencies.

Federal Reserve Districts

District	Headquarters	District	Headquarters
1	Boston, MA	7	Chicago, IL
2	New York, NY	8	St. Louis, MO
3	Philadelphia, PA	9	Minneapolis, MN
4	Cleveland, OH	10	Kansas City, MO
5	Richmond, VA	11	Dallas, TX
6	Atlanta, GA	12	San Francisco, CA

The Federal Reserve Bank has three methods of controlling the money supply.

- changing reserve requirements
- changing the discount rate
- trading government securities on the open market

Changing Reserve Requirements. All banks are required to hold a percentage of the total value of their deposits with the Federal Reserve. (Deposits are what the banks "owe" their customers, such as savings and checking accounts.) This percentage is called the *reserve requirement.*

For example, if the reserve requirement is 10 percent, a bank with reserves of $100,000 is allowed to have up to $1 million in deposits (10 percent of $1,000,000 is $100,000). Any additional dollars held in reserves are called *excess reserves.* These can be used to cover additional deposits, as long as the required ratio (1:10 in this example) is not violated.

Changing the Discount Rate. Another tool the Federal Reserve has for affecting the money supply is the discount rate—the interest rate banks pay to borrow money from the Federal Reserve. There is an inverse relationship between discount rate and money supply. When the Federal Reserve lowers the discount rate, it expects that banks will borrow more. Adding the loan proceeds to its reserves, the bank can now make new loans of its own. The money supply increases.

Trading in the Securities Market. The tool most often used by the Federal Reserve to control the money supply is open market trading of government securities. The individual or organization that buys the security must pay for it, typically by writing a check on some type of bank demand deposit. That reduces the supply of money.

A similar type of process occurs when the Federal Reserve purchases government securities from the public, only this time it's in reverse. The check that the Federal Reserve writes to purchase the security is deposited in the seller's bank account, increasing the money supply.

GLOBALIZATION

Do you enjoy sprinkling fresh blueberries on your morning cereal? In years past, you could savor those tasty treats during only a few weeks of the year. Blueberries have a short growing season. However, you can now eat those delightful berries all year long—even in the winter. In South America, seasons are the opposite of the seasons in North America. Fall and winter in the United States are spring and summer in Chile and Argentina. Farmers in Chile and Argentina know that millions of Americans want to eat those berries in January, so they began to grow that crop.

Interconnections

What Is Globalization?

Globalization is the interconnection of producers and consumers in today's world economy. A company can buy supplies and parts made in many different countries. They sell those goods to customers in their home country, as well as ship the products to buyers in other nations. In this way, factory workers and farmers in one country are closely linked to consumers in several other nations. Globalization means that a computer company in the United States buys hard drives made in Taiwan, memory chips manufactured in New Mexico, DVD drives made in China, and motherboards produced in South Korea. Then, workers in the company's U.S. factories assemble these parts to make a machine that is shipped to a store to sell.

How Extensive Is Globalization?

Globalization has linked all corners of the world. McDonald's has restaurants in more than 110 countries. Walmart has 225 stores in China alone. The popular sci-fi adventure movie *Avatar* earned more than $2.7 billion in ticket sales around the world—with more than 70 percent of that revenue coming from outside the United States. Meanwhile, Americans enjoy Japanese animé (animated films) and Mexican telenovelas (long-running television serials) and buy cars made by Japanese and German automakers that are built in Tennessee or South Carolina.

Trade is a growing share of the economies of countries around the world. In 1960, trade in goods accounted for only 10 percent of the world's total economic output. That percentage rose until the trade in merchandise equaled more than one-third of the world's economic output.

◀ **Farmers in Chile** saw a demand and began to produce blueberries to meet that demand. They didn't care that this demand originated in another country. They just wanted to take advantage of an opportunity. That's what globalization is all about—taking advantage of opportunities all around the world.

U.S. trade representative Ron Kirk. His responsibilities include advising the president and American citizens about trade issues.

REGIONAL TRADE ASSOCIATIONS

Regional trade associations are blocs of countries that join together to promote free trade among themselves. There are more than two dozen regional trade associations around the world. Among the largest are

- **The North American Free Trade Agreement (NAFTA).** In this treaty, leaders in the United States, Canada, and Mexico agreed to eliminate all trade barriers among their countries over the next several years. (See page 222.)
- **The European Union (EU).** This free trade zone covers most of Europe and includes the people of 27 nations. The nations of the EU account for more than $2.8 trillion in world trade, which is more than any single country except the United States.
- **The Association of Southeast Asian Nations (ASEAN).** This trade bloc includes 10 Southeast Asian nations, embracing more than 500 million people.
- **Mercosur.** This trade bloc links the economies of Argentina, Brazil, and several other nations in South America.

Overseeing the movement is the World Trade Organization (WTO), which helps countries work out free trade agreements. There are 157 member nations in the WTO.

Three Causes of Globalization

Globalization is not new. It has been happening for centuries. Merchants in China were sending caravans of valuable silk across Central Asia to the eastern edges of the Roman Empire 2,000 years ago. From there, the silk was sent to Rome, where the upper classes treasured the fabric for its lightness and vivid colors. Today, globalization takes place on an unprecedented scale. Networks of producers and consumers connect people in all nations.

There are three main causes for the spread of globalization.

Communication and Transportation Technologies. The growth of the Internet allows video game companies to create online games that are played by millions anywhere from Thailand to Bulgaria. Jet air travel means that farmers in Kenya can grow flowers that can be flown overnight to appear the next day in European florist shops.

Interconnected Financial Markets. Investors are now more willing to buy stocks and bonds issued in another country. If they see signs of a decrease in value in those investments, they can sell those investments and move their money to another country. The sell-off of investments can hurt the economy of the country where it occurs. Sometimes the economic bad news spreads around the globe. A sell-off of stocks in the United States can worry investors in other countries, who then sell stocks on their own exchanges.

The Spread of Capitalism. For much of the 20th century, many governments tried to control their economies. In these command economies, government officials decided what products businesses would produce and how they would produce them. But after decades of slow economic growth, the governments in many countries shifted to the market economic system in which individuals make the economic decisions.

World History

o Preste Jioa
Melinde
Angola
Congo
Sofala
CABO BOA ESPERAÇA.

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Renaissance to Modernity, 1500 to 1900. 494

20th Century and Beyond,
1900 to the Present 534

The Kingdoms of Egypt

THE EGYPTIAN PYRAMIDS

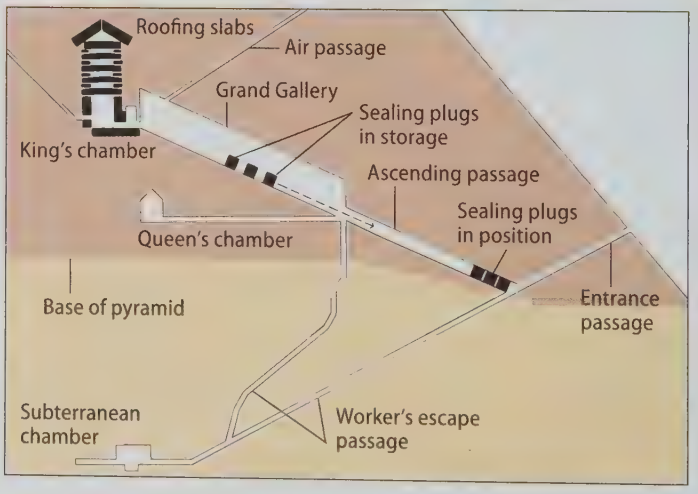
The belief that the Egyptian kings were gods led to one of the Old Kingdom's greatest achievements: the construction of the pyramids. To live their lives after death in proper style, the ancient Egyptians believed they needed to be buried in magnificent tombs with lavish possessions. They also believed that the body needed to be preserved, because the person would need to use it in the next life as well. To do this, the Egyptians mummified—embalmed and dried—the bodies.

- The first pyramid was the Step Pyramid, designed for King Zoser about 2650 BC by the royal architect Imhotep. It stands over 200 feet tall.
- The largest pyramid was the Great Pyramid of Khufu, a king also known as Cheops, located at Giza. From a base that covered 13 acres, it rose to a point that reached 48 stories above the ground.
- It took 100,000 workers over 20 years to complete the Great Pyramid. It was completed in around 2600 BC.

◀ **Because of monuments** such as the great pyramids at Giza (shown here), the Old Kingdom of Egypt is sometimes referred to as the Age of Pyramids.

Building the Pyramids. A study of the Great Pyramid shows how these gigantic structures were built. The ancient Egyptians had no machinery or iron tools. They cut big limestone blocks with copper chisels and saws. Most of the stones came from quarries nearby. But some came from across the Nile River, and others came by boat from distant quarries.

1. Gangs of men dragged the blocks to the pyramid site and pushed the first layer of stones into place.
2. Then they built long ramps of earth and brick, and dragged the stones up the ramps to form the next layer.
3. As they finished each layer, they raised and lengthened the ramps.
4. Finally, they covered the pyramid with an outer coating of white casing stones. They laid these outer stones so exactly that from a distance the pyramid appeared to have been cut out of a single white stone.



▲ **A cross section** of the Great Pyramid in Egypt shows the Grand Gallery, the King's Chamber, the Queen's Chamber, and various passages. Large blocks called sealing plugs were used to close the tomb.

The New Kingdom (c. 1539–c. 1090 BC)

The New Kingdom was a period of nearly 500 years in which ancient Egypt became the world's strongest power. During this period, the Egyptian rulers began to be called pharaohs, from the Egyptian term meaning "great house."

From the start of the New Kingdom, the pharaohs again consolidated their power and devoted their time to empire building. Amenhotep I, Thutmose I, and Thutmose III proved themselves great conquerors. They extended the Egyptian Empire far to the south in Africa and into Asia along the eastern Mediterranean and across to the Euphrates River.

Religious Reforms. Amenhotep IV weakened the empire when he attempted major religious reforms. This led to a bitter struggle with wealthy and powerful Egyptian priests.

- Amenhotep IV devoted himself to a sun god called the Aten.
- He changed his own name to Akhenaten.
- He declared that the Aten had replaced Amun and all other gods except Re.

King Tut. These religious reforms angered many Egyptians. Akhenaten's successor was 9-year-old Tutankhaten, the "boy king." King Tutankhaten removed *-aten* from his name and became Tutankhamun. He restored the old state religion, allowing the worship of the old deities as well as the Aten.

Decline. By the 13th century BC, ancient Egypt was to know its last great period, under the rule of Ramses II, another great monument builder. Over time, the pharaohs' rule weakened, and again Egypt fell prey to invaders.

By 1090 BC, the power of ancient Egypt had collapsed. It would now be fought over and conquered by new empire builders—the Libyans, the Nubians, the Persians, the Macedonian Greeks, and finally the Romans (in about 30 BC).

▲ **Hatshepsut,** the mother of Thutmose III, reigned as pharaoh while Thutmose was a child. She was the first major woman ruler known to history. From 1490 to 1469 BC, she led Egypt through a period of foreign trade and monument building.

fyi!

King Tutankhamun is best remembered today for the treasures of his burial place—"King Tut's tomb"—which would not be unearthed for another 2,500 years.

now you Know!

- ✓ Civilization in the Nile Valley began to develop by about 3500 BC.
- ✓ The ancient Egyptian civilization lasted for 2,000 years. In that time, there were three major periods: the Old Kingdom, the Middle Kingdom, and the New Kingdom.
- ✓ Egyptians developed a writing system called hieroglyphics, which used pictures to convey stories and keep records.
- ✓ Because they believed their kings, or pharaohs, were gods, the Egyptians buried them in enormous tombs called pyramids. The largest of these is the Great Pyramid at Giza.
- ✓ Eventually, Egypt declined and was taken over by a series of invaders, ending with the Romans in 30 BC.

THE INDUS VALLEY

Over 2,000 miles to the east of Egypt, 1,500 miles beyond Mesopotamia, lies the Indus River. The river courses south 1,000 miles from the Himalayas to the Arabian Sea, which is an arm of the Indian Ocean. Its lower valley, located in what is now Pakistan, is one of the most fertile plains in the world. Here, about 2500 BC—roughly the time that the Old Kingdom was thriving in Egypt and Sumer was about to be made part of the Akkadian Empire—a civilization was emerging. In time, the Indus civilization grew to cover most of present-day Pakistan and parts of what are now Afghanistan and northern India.

▲ **Discovering the Indus.** In the 1920s, archaeologists began excavating huge earthen mounds where people had found artifacts and realized that they contained the remains of cities from a previously unknown civilization. Hundreds of Indus sites have been found since.

The Indus and Aryans

Settlement had begun in the Indus Valley, as it had elsewhere, when hunters and gatherers evolved into Neolithic farmers living in villages along the Indus River and its tributaries. By about 2300 BC, two of these villages had grown into the cities of Harappa and Mohenjo-Daro, with close to 40,000 people in each city.

Harappa and Mohenjo-Daro

Both of these cities were marvels of urban planning. At the center of each was a fortress, five stories high. From it, streets ran outward in a grid, making it easy to get around.

- Strict government regulations decreed neat and well-constructed houses, shops, and public buildings, including public baths.
- Many of the buildings were constructed on mud-brick platforms that protected the buildings from seasonal floods.
- Houses were made of baked or sun-dried brick. Beneath the streets, brick-lined sewers carried off waste, a sanitation measure unknown in other cities of the time.

Economy and Culture. Because of its bountiful agriculture, the Indus Valley developed a thriving economy. Here cotton was cultivated for the first time. Cotton cloth, as well as fine pottery and gold jewelry, was traded as far away as Mesopotamia.

Indus artisans also produced a variety of useful and decorative objects. They used copper and bronze to make tools, mirrors, pots, and pans. They also crafted silver and gold utensils and ornaments.

Writing System. Archaeologists have unearthed thousands of small clay figures and stone seals that carry pictures and a form of writing. The pictures indicate that the Indus people probably worshipped certain animals, like sacred bulls, and gods of nature. Unfortunately, scholars have not been able to decipher the writing and perhaps never will.

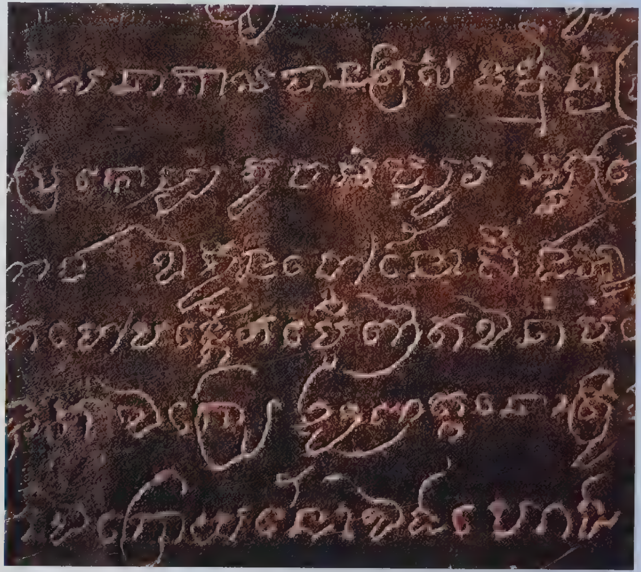
Aryan Conquest

The end of the Indus civilization came around the year 1500 BC, as warlike nomads from central Asia began streaming into the Indus Valley through mountain passes. The invaders called themselves Aryans, their term for "noble ones."

The Aryans quickly overran and looted the Indus towns and cities, as well as the societies of people called Dravidians. Soon they had conquered the entire northern plain of what is today India and Pakistan. By 700 BC, they had established a number of Aryan kingdoms across the Indian subcontinent, each kingdom ruled by a rajah.

SANSKRIT

The Aryans also developed a written language, Sanskrit. In it, they wrote the *Vedas*, books of sacred knowledge describing Aryan religious beliefs, known as Hinduism. The *Vedas* celebrated numerous gods and goddesses who personified natural phenomena, such as the sky, fire, or lightning. The most important Hindu sacred writings, called the *Upanishads*, appeared between 800 and 600 BC. (See also page 454.)

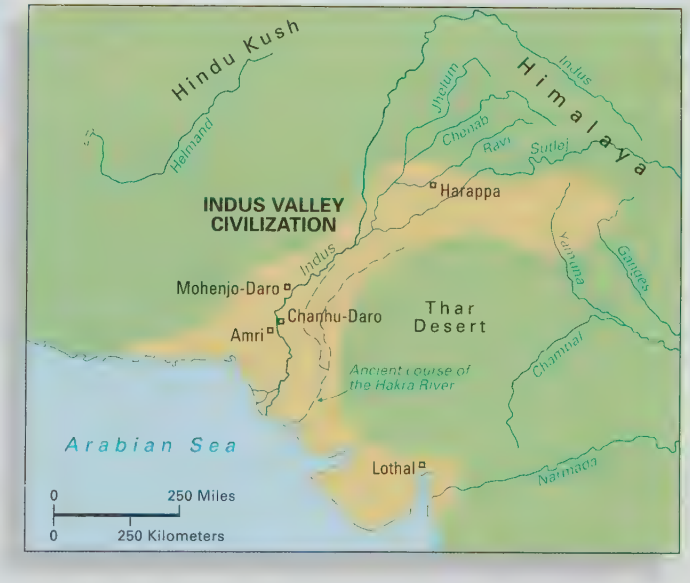


Sanskrit is the basis for many modern Indian languages, including Hindi and Urdu. The name *Sanskrit* means *refined* or *polished*. ▶

▲ **Indus sculptors** made clay figurines of animals and people, probably for use in religious rituals.

Decline. From about 1900 to 1700 BC, the Indus civilization gradually broke up into smaller cultures, called late Harappan cultures and post-Harappan cultures. The breakup was partly caused by changing river patterns that disrupted agricultural and economic systems.

Many people left the cities of the Indus Valley region. However, some aspects of Indus art, agriculture, and possibly social organization continued in the smaller cultures. Some of these aspects became incorporated into another civilization that began developing throughout the region about 600 BC.


Hindu Kush
Himalaya
Indus
Jhelum
Chenab
Ravi
Sutlej
Helmand
Harappa
INDUS VALLEY CIVILIZATION
Indus
Mohenjo-Daro
Chanhu-Daro
Amri
Thar Desert
Yamuna
Ganges
Chambal
Ancient course of the Hakra River
Arabian Sea
Lothal
Narmada
0
250 Miles
0
250 Kilometers

▲ **The Indus River Valley.** This map shows the location of the Indus Valley civilization, which flourished from about 2500 BC to about 1700 BC.

The Roman Empire

More than a century of internal upheavals and civil war had destroyed the Roman Republic. The Roman Empire that took its place brought the civilized world under a single monarchic rule. Still, it used the political terminology of the Roman Republic and preserved the (by then obsolete) Senate. The empire tended toward authoritarianism and military despotism.

Governing the Empire

Although the early emperors of the 1st century AD led scandalous private lives, the empire they ruled actually prospered. During their reigns, Rome developed the centralized bureaucracy that would later make the empire a synonym for the art of government.

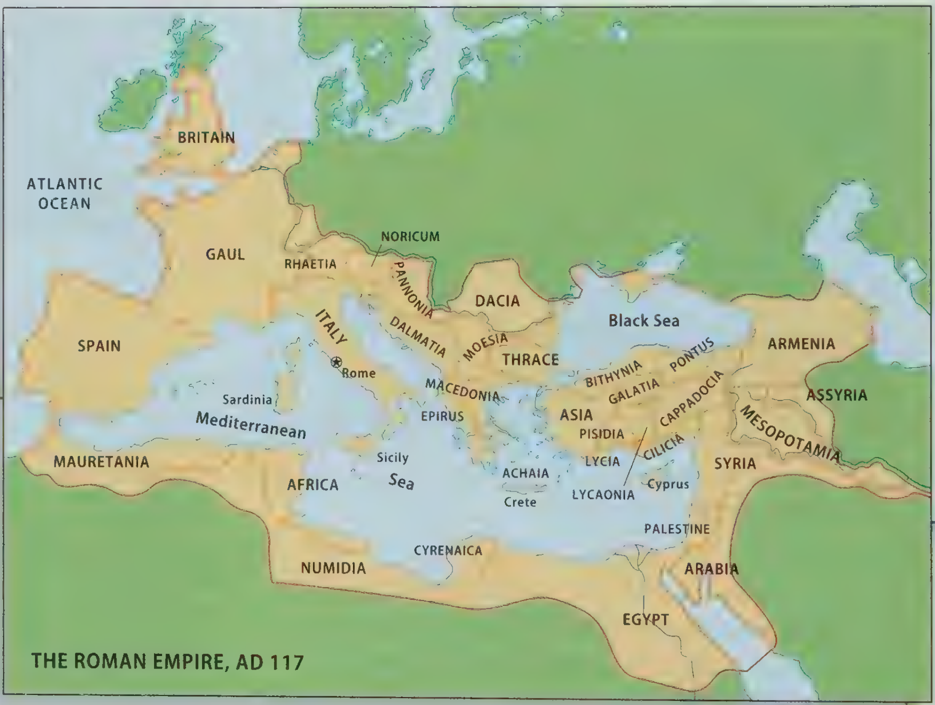
THE PAX ROMANA

The "good emperors" of the first century created the Pax Romana, or Roman Peace. Edward Gibbon, the great English historian of the 1700s, described this era as the period in which "the condition of the human race was the most happy and prosperous."

Civilization spread throughout the vast realm of the Roman Empire. Scores of cities were built where formerly there had been only barbarism. A Greek lecturer who came to Rome in the AD 150s praised Rome's accomplishment. He stated that the new unwalled cities were meant for a world at peace—for the first time in history cities needed no local defense.

The Civil Service. By the middle of the 1st century AD, that government consisted of a permanent, highly organized civil service. Even the emperor considered himself the servant of his demanding office.

This government enabled a vast territory and a population of over 50 million people of different races, languages, cultures, and traditions to enjoy centuries of security, stability, peace, rational and disinterested administration, and almost impartial law.


BRITAIN
ATLANTIC OCEAN
GAUL
RHAETIA
NORICUM
PANNONIA
DACIA
Black Sea
ARMENIA
SPAIN
ITALY
DALMATIA
MOESIA
THRACE
Rome
BITHYNIA
PONTUS
GALATIA
CAPPADOCIA
ASSYRIA
MACEDONIA
Sardinia
Mediterranean
EPIRUS
ASIA
PISIDIA
MESOPOTAMIA
MAURETANIA
Sicily
Sea
CILICIA
LYCIA
SYRIA
ACHAIA
AFRICA
Cyprus
LYCAONIA
Crete
PALESTINE
CYRENAICA
NUMIDIA
ARABIA
EGYPT
THE ROMAN EMPIRE, AD 117

▲ **Spread of the Empire.** The Roman emperor Augustus (63 BC–AD 14) added Egypt and most of southeastern Europe south of the Danube River. The Roman Empire reached its greatest size under Trajan, who ruled until AD 117.

Expansion. Augustus had left strict instructions to his successors not to expand the empire, but this was disregarded.

- Emperor Claudius invaded Britain in 43. He also added Mauretania (now northern Morocco and western Algeria) to the empire.
- Trajan seized Dacia in eastern Europe in 106.

Hadrian returned to the policy of Augustus. He marked the limits of Rome's empire with artificial frontiers on the Danube River, in northern Africa, and elsewhere. In northern England, he constructed Hadrian's Wall, parts of which still stand.

Decline and Fall

Even at its peak, the Roman Empire already contained the seeds of its eventual collapse. The decline and fall of the Roman Empire is one of the most debated issues in history. Historians have offered several different types of explanations for Rome's decay.

- One explanation, called the moral explanation, blames personal failings—the greed of individual emperors, the decadent luxury of the wealthy, indifference to the misery of the masses, and so on.
- Another explanation emphasizes the divisions and conflicts of the empire—between rich and poor, urban and rural, military and civilian.
- A third explanation emphasizes economic issues, such as a devalued currency, growing taxation, and constant importation of luxuries.

Military Issues. The control of the armed forces remained a problem for centuries. There was a constant threat of coups and internal wars for control.

The pressure of two outside powers made a large military force a constant necessity:

- In the East, a revived Persian Empire
- In present-day Europe, a succession of Germanic tribes

Rome eased the threat in Europe in the 200s by admitting whole tribes into the empire.

Divided Empire

The problems in the 200s, and the decreasing power to deal with them, brought about an administrative revision around 395 that divided the empire into an eastern Greek-speaking half and a western Latin-speaking half.

In spite of the efforts of individual emperors, such as Diocletian and Constantine I, to restore unity, the two halves tended to grow further apart. By about 400 they were not only separate, but hostile.

Invasions. Weakened, the Roman Empire fell to a series of invading powers.

- Rome fell to the Visigoths in 410, the first time in 800 years that the city had been taken by foreign invaders.
- The Huns, under Attila, ravaged the East Roman Empire from 434 to 453. Attila reached the outskirts of Rome at one point.
- The Vandals, a Germanic people, sacked Rome in 455.
- Odoacer, a German chieftain, deposed Emperor Romulus Augustulus in 476, marking the end of the West Roman Empire.
- The East Roman Empire became known as the Byzantine Empire.

EMPEROR NERO

One of Rome's most notorious leaders was the emperor Nero (AD 37–68). Nero reigned from AD 54 until his death 14 years later. His scandalous life—he had two of his wives and many senators executed—made him unpopular.

His rule is perhaps best known for a fire that destroyed much of Rome in AD 64. Rumors circulated that Nero had started the fire so that he could build a grand palace on the ruins. However, Nero blamed the Christians, then an unpopular group in Rome, and persecuted them.

now you Know!

- ✓ Rome won its independence from the Etruscans in 509 BC.
- ✓ The period from 509 BC to Rome's total control of the Mediterranean is known as the Roman Republic.
- ✓ Civil war and rebellion led to the rise of the First Triumvirate—Julius Caesar, Pompey, and Crassus. Caesar was assassinated in 44 BC.
- ✓ The Roman Empire began with Emperor Augustus, in 27 BC.
- ✓ At its peak, the Empire controlled half of Europe, much of the Middle East, and the north coast of Africa.
- ✓ The Empire declined in the 200s and 300s. It collapsed finally in AD 476.

WORLD RELIGIONS

For much of human history, religion has been a major driving force. Long before recorded history, ancient burials show evidence of a belief in an afterlife. The Egyptians and ancient Greeks left evidence of complicated religious practices, as did the Romans, Aztecs, Incas, and Hindus. The rise and spread of religions, new and old, brought change, conflict, and learning to the world's cultures. An understanding of the major religions of the world, past and present, can bring insight into the great wars, great art, and great movements of humankind.

Although smaller than the other major religions—with only about 15 million followers worldwide—Judaism is nonetheless considered a major world religion. Its followers are spread over much of the world.

Judaism is the religion of a people called the Jews. The Jews are a people or national group as well as a religion. A few followers of Judaism do not consider themselves to be a part of the Jewish people, and many who consider themselves Jews do not follow or practice the religion of Judaism.

Judaism

History

Judaism was probably the first truly monotheistic religion, devoted to the worship of a single all-powerful God. The Jewish faith holds that God acts through history to change events on Earth. Thus, history is important to Jews in a way that is different from its importance to people of other religions.

Christianity and Islam, both of which developed after Judaism, consider the history of the early Jewish nation to be part of the record of God's actions.

▲ **The Torah,** which is the first part of the Hebrew Bible (and which also makes up the first five books of the Christian Old Testament), reports the acts of God in the history of the Jews.

Ancient Hebrews. The Jews trace their history to a man named Abraham, a seminomadic shepherd who lived about **4,000 years ago.** God told Abraham to take his family and followers (called the Hebrews) to Canaan, a land that today we identify with Israel. (See pages 416–417.)

God made a covenant, or solemn agreement, with Abraham: He would have descendants who would live in Canaan, and they would receive God's care if they followed his laws. With this covenant, the Hebrews (later known as Israelites and then as Jews) became God's chosen people—chosen to represent God to the rest of the world.

Essential Beliefs

The most important teaching of Judaism is that there is one God.

- This God, the creator of heaven and earth, wants people to do what is just and merciful.
- A Jew serves God by studying the Scriptures and fulfilling their divine commandments.
- A Jew who does these things is fulfilling the requirements of the religion.

Judaism allows believers to draw their own conclusions on many religious questions, so long as they fulfill the basic requirements. For example, some Jews believe in eternal life after death, while others do not. Many Jews believe that a messiah, or savior, will one day arrive on Earth.

THE JEWISH SCRIPTURES

Judaism has two major collections of sacred writings: the Hebrew Bible (Tanakh) and the Talmud.

The Hebrew Bible consists of several parts.

- The Torah contains five books—their English names are Genesis, Exodus, Leviticus, Numbers, and Deuteronomy—containing basic Judaic law and the Jewish history.
- The Prophets are books of history and moral teachings.
- The Writings are 11 books of proverbs and other teachings.

The Talmud was assembled in the 6th century AD.

- It is a collection of legal, ritual, and ethical writings, as well as Jewish history and folklore.
- It serves primarily as a guide to Jewish civil and religious laws.

Jewish worship takes place in the home and in a house of worship called a *synagogue*. Important parts of home worship include daily prayers, the lighting of the Sabbath candles, and the blessing of the wine and bread at the Sabbath meal. Jews also observe many holiday rituals, such as Hannukah and Passover (Pesach) at home.

Jewish Denominations	Origins and Founders	Beliefs and Practices
Orthodox	Refers to the original form of Judaism	• Apply all ancient rules of worship • Conduct services in Hebrew, with women and men separate • Observe strict dietary laws (known as kashrut, or kosher) • Requires head coverings (yarmulke) for men and modest dress for women
Hasidic	1700s, Poland Founder: Israel Ball Shem Tov	• Observe the same laws as Orthodox Jews • Consider dancing, singing, and love of nature to be forms of worship • Maintain separate communities, many of them in New York City
Reform	1810, Germany Founders: Abraham Geiger, Samuel Holdheim	• Does not accept traditional interpretation of the Law • Services may be conducted in languages other than Hebrew • Dietary laws may not be observed • Houses of worship are called temples, not synagogues
Conservative	1845, Germany; early 1900s, United States	• Arose from a dispute about the use of Hebrew in services • Maintains Hebrew use • Attempts to balance Orthodox tradition and Reform spirit

Hinduism and Buddhism

Hinduism

Hinduism is the major religion of India. It has about 900 million followers. Although most Hindus live in India, Hindu literature and philosophy have influenced people throughout the world.

Hinduism represents a diverse group of beliefs, practices, and texts that have developed over thousands of years. No single authority exists in Hinduism.

History. Hinduism developed from the beliefs of the Aryan tribes who invaded India around **1500 BC**. The Brahmans, the priestly caste, preserved the sacred writings of the Epic Age and also interpreted them for the people.

The caste, or class, system of the Aryans also became a part of Hinduism. Each caste had its own strict social and religious rules, and living virtuously demanded complete acceptance of, and obedience to, these laws.

According to the Hindu law of karma, persons who live obediently as members of their caste would be reincarnated into a higher caste. If not, they would fall back into a lower caste in the next life, or even be reincarnated as a form of animal life.

Beliefs

In Hinduism there are many gods, but they are all manifestations of one supreme spirit, Brahma, the World Soul. Brahma is the Creator. United in the spirit is a trinity of main Hindu gods.

- Vishnu, the Preserver
- Shiva, the Destroyer
- Shakti, a goddess also known by the names Lakshmi, Kali, Durga, or Parvati

Reincarnation. Hindus believe that everyone has a soul that is part of the World Soul.

- The soul longs to become reunited with the World Soul.
- To do this, it must go through many reincarnations, becoming less worldly and more spiritual in each to earn salvation.
- A person must live a virtuous life to achieve greater spirituality.

▲ **Shiva, the Destroyer,** is one of the main Hindu gods. His destruction allows for things to be reborn. The goddess Shakti, in the forms of Kali or Durga, is also considered a goddess of destruction.

SACRED HINDU TEXTS

Hinduism has no single book that is the source of its doctrines, but it has many sacred writings that have contributed to its beliefs and practices. The great Hindu epics teach traditional Hindu values through their stories.

The *Vedas* are the oldest Hindu scriptures and include sections that are older than the sacred writings of any other major religion.

The *Puranas* are long verse stories that contain many important narratives about deities and the lives of great Hindu heroes.

The *Ramayana* is an epic that tells of Prince Rama and his attempts to rescue his wife, Sita, who has been kidnapped by the demon king Ravana.

The *Mahabharata* is an epic that describes a struggle for a disputed kingdom between the Pandavas and the Kauravas, two families who are cousins.

Buddhism

Buddhism was founded in India about 500 BC, or shortly afterward, by a teacher called the Buddha. Today, Buddhism has about 350 million followers. Most live in Tibet and other regions of China, and in Japan, the Korean peninsula, Sri Lanka, and mainland Southeast Asia.

History. The founder of Buddhism, Siddhartha Gautama, was born in 563 BC, the son of a wealthy rajah. According to legend, he had never known about the suffering of people until the age of 29. Shocked by what he saw, he renounced his life of ease to go into the outside world to learn why there was such suffering and how it might be cured.

After years of wandering and finding no answers, he sat down beneath a sacred fig tree to meditate. While there, he achieved "enlightenment," finding the knowledge he sought. From that experience, he became known as the Buddha, the "Enlightened One," and developed the religious philosophy that became Buddhism.

In some ways, Buddhism was a response to the Hindu caste system. Buddhists believe that anyone can achieve nirvana (peace), regardless of caste or status.

Beliefs

Buddhism is sometimes described as atheistic—believing in no god at all. The followers of Buddhism would dispute this. However, the Buddhist concept of God does not have the "personhood" concept found in other religions.

Over time, Buddhism became divided into two major branches.

- The Hinayana maintained the traditional Buddhist beliefs and honored Buddha as a great teacher.
- The Mahayana proclaimed Buddha a god and made Buddhism a religion of priests and temples.

THE NOBLE EIGHTFOLD PATH

Buddhists believe that eight behaviors will help a person avoid suffering:

1. Knowledge of the truth
2. The intention to resist evil
3. Saying nothing to hurt others
4. Respecting life, morality, and property
5. Holding a job that does not injure others
6. Striving to free one's mind of evil
7. Controlling one's feelings and thoughts
8. Practicing proper forms of concentration

OTHER WORLD RELIGIONS

Thousands of religions are practiced around the world. This chart lists some significant, though smaller, world religions.

Other Major Religions	Origins and Founders	Key Beliefs and Practices
Jainism	Mahavira, 500s BC; India	• Every living thing has a soul; killing any living thing is prohibited.
Shinto	Unknown founders, main texts written c. AD 600s; Japan	• Worship many nature deities called kami
Baha'i	Baha'u'llah, 1863; present-day Iraq	• Believe God wants a world of equality and acceptance; oppose all discrimination
Sikhism	Guru Nanak, c. late 1400s; India	• Monotheistic faith • Believers practice meditation and believe in reincarnation

Christianity

Today the Christian religion has the most followers worldwide, about 2 billion. Christianity is the dominant religion of Europe, both Americas, and Australia, with significant numbers of followers in Asia and Africa as well.

History

The earliest Christians were Jews who came to believe that Jesus of Nazareth was the Messiah the Jews had been awaiting. Jesus was born around 2010 years ago in a village called Bethlehem. He taught in the synagogues and outdoors, healing the sick and welcoming particularly the poor and the outcast. He said he had come to establish a new kingdom, in which people's wrongdoings would be forgiven, allowing them to achieve a new relationship with God.

The Crucifixion. When Jesus was about 33 years old, he was arrested by the official Jewish court and turned over to the Romans, who controlled the country as part of the Roman Empire. The Romans crucified Jesus on a wooden cross.

Three days after he died, according to the Gospels, he rose from the dead and visited many of his followers. Then he went to be with God the Father until the time for his return.

The New Covenant. The Gospels also say

- The night before his arrest, Jesus ate and drank with his closest followers. In this Last Supper, he told them to observe a similar meal to remember him.
- He suggested that his suffering, death, and resurrection would make a new covenant with God, replacing the covenant made between God and Abraham and Moses.
- In this new covenant, God forgives the sins of those who follow Jesus and offers them eternal life with God.

The Early Church. Early Christians thought of themselves as Jews who had recognized Jesus as the Messiah. Some Jews who did not recognize Jesus as the Messiah persecuted the early Christians.

One of the persecutors, Saul of Tarsus, had a vision of Jesus and was converted to Christianity. Under the name of Paul, he began converting non-Jews (Gentiles) in Syria, Greece, and Rome. The church began to separate from its Jewish beginning.

The religion spread rapidly through the Mediterranean. In **AD 325,** the Council of Nicaea affirmed its main beliefs, and named the pope its leader.

Major Christian Denominations	Origins and Founders	Beliefs and Practices
Roman Catholics	Formed by the Council of Nicaea, **325 AD**	• Pope (Bishop of Rome) has ultimate authority • Priesthood open to celibate men only • Seven sacraments: baptism, communion, ordination, confirmation, confession, last rites, and marriage • Transubstantiation: the bread and wine of communion become the literal body and blood of Christ
Eastern Orthodox	Split from the Roman Catholic Church in **1054 AD**	• Accepts the same sacraments as Catholics • Does not require celibate priests • Venerates religious paintings called icons
Lutherans	Martin Luther, **1517**	• The Bible, not the Pope, is sole authority • Salvation through God's grace alone • Two sacraments: communion and baptism • Rejects transubstantiation

▲ **The story of the Last Supper** is the source for the Christian practice of communion, though the exact practice varies among different Christian churches.

THE CHRISTIAN SCRIPTURES

Christianity has three major sets of sacred writings.

The Christian Bible is divided into two parts, which are in turn divided into books:

- The Old Testament (the Jewish Scriptures), which describes God's agreement with the Jewish people
- The New Testament, which describes Jesus' teachings and new covenant

Creeds are short statements of faith that set forth essential Christian beliefs. The oldest in use today are the Apostles' Creed and the Nicene Creed.

Catechisms are writings that present basic religious doctrines, often in a question-and-answer format, for teaching.

Essential Beliefs

Like Judaism, from which it grew, Christianity is monotheistic—its followers believe in only one God.

- Christians also believe that God acts in history and that He is the God of the early Jewish people.
- Early Christians came to believe that the one God expresses Himself through three different natures: God the Father, God the Son (Christ), and God the Holy Spirit. This is called the Trinity.
- The church and its members spread the gospel, or good news, about Christ.
- A few rites are basic to Christianity: baptism (ritual purification with water) and communion.
- Prayer to God is a part of the faith. The main symbol of the Christian religion is the cross, which signifies to Christians that Christ died for their sins.

fyi!

Romans began persecuting Christians in AD 64, under the emperor Nero. This continued with varying degrees of severity for about 250 years. Despite the persecutions, the church grew. Persecution ended in AD 313, when the emperor Constantine made Christianity the state religion of the Roman Empire.

Major Christian Denominations	Origins and Founders	Beliefs and Practices
Calvinists/ Presbyterians	John Calvin, 1533	• God's elect go to heaven; all others go to hell. • The elect cannot fall from grace; others cannot be saved. • Includes the Pilgrims and the Puritans • In Scotland, became the Presbyterian Church
Anglicans	King Henry VIII of England, 1534	• Both Catholic and Protestant • Accepts all sacraments, but only communion and baptism are necessary • Sole authority is the Bible
Baptists/ Mennonites	Menno Simons and others, 1500s	• Baptism must be chosen by an adult • Rejects transubstantiation
Methodists	John Wesley, 1738	• Similar to Anglican beliefs • Focus on Jesus Christ as a guide to living

Islam

The second largest religion in the world, after Christianity, is Islam. Followers of Islam are called Muslims. Worldwide, over 1.3 billion people follow Islam. The largest Muslim populations are in Asia and Africa, but Muslims live in every nation in the world.

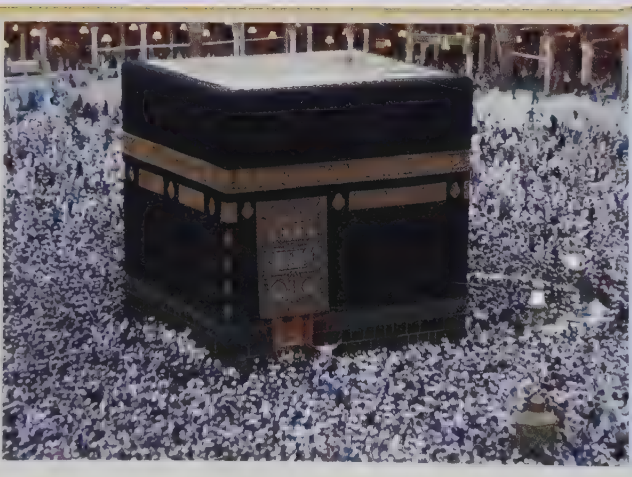
History

The founder of the Islamic religion is Muhammad, who lived in Arabia about 1,400 years ago (AD 570–632). When he was about 40, God began to speak to him. God told Muhammad to transmit His message to the people of Arabia and dictated a book to Muhammad, which was later written down and is known today as the Koran.

Hejira and Empire. In AD 622, the people of Mecca, where Muhammad and his wife and daughters lived, were so upset by Muhammad's teachings that he and his followers had to flee.

- His escape to Medina is known as the Hegira.
- The date of the Hegira is usually considered the beginning of Islam.
- Eight years after the Hegira, Muhammad returned to Mecca. By then, he had so many followers that he took over the city without a struggle.

From then on, the followers of Islam built a mighty empire that at its height stretched from Spain to India. Muhammad was effectively the ruler of Arabia as well as its religious leader when he died. Because the state and the religion were one, a separate church was not needed. (See pages 474–475.)



ISLAM'S SACRED WRITINGS

The Koran and the Sunnah make up the foundation of Islamic law.

The Koran is the collection of revelations Muhammad received from God. Muslims believe it contains God's actual words. The Koran also retells many stories that are familiar to Jews and Christians from the Bible. For example, it tells the stories of Jacob, Joseph, Moses, David, Solomon, and Jesus Christ.

The Sunnah is the example of the words and practices of Muhammad. It presents his sayings and acts in written collections called the Hadith.

From these collections and from the Koran, scholars in the 7th and 8th centuries developed a set of specific rules for worship and for life. These rules are called the Sharia, or Islamic law. The Sharia's relation to the Koran and to Muhammad's Sunnah is similar to that of the Talmud to the Torah.

After Muhammad. When Muhammad died, his followers were in control of Arabia. He had made it clear that the state and worship of God were to be unified but had left no instructions on how to achieve this.

Muhammad had not chosen anyone to take his place. The dispute over the *caliphs*, or successors, who followed Muhammad led to the first divisions of Islam—the Shiites and the Sunnites.

MECCA

Muhammad declared Mecca a holy city. When Muslims pray, they face Mecca. He also declared that the Kaaba, where Arabs had worshiped for many years, was to be a shrine for Allah.

- The Kaaba, shown here, is an empty cube-shaped building in the center of the Great Mosque in Mecca.
- Muslims believe Abraham and Ishmael built the Kaaba as the first house of worship.
- On the Muslim hajj, or pilgrimage, believers walk seven times around the Kaaba.

Beliefs

Like Judaism and Christianity, Islam is also a monotheistic faith. Islam is an Arabic word that means surrender or submission. Its main creed is simple: "There is no God but God, and Muhammad is His prophet." (A prophet is a human chosen by God to relate God's message on Earth.) In Islam, God is called *Allah*.

Muslims believe Muhammad was the last major prophet, but earlier prophets included those who founded Judaism and Jesus Christ. Thus, Muslims believe many of the same things that Jews and Christians do, although not always in exactly the same way.

Five Pillars of Islam. Islam is based on five principles, called the Five Pillars.

1. Faith that there is no God but God (Allah), and Muhammad is His prophet
2. Prayer to God five times daily
3. Charity, shown by helping the poor and providing upkeep for places of prayer
4. Fasting in two ways—by never drinking alcohol or eating pork and certain other foods; and by not eating, drinking, or smoking at all during daylight for 1 month each year (the Muslim month of Ramadan)
5. Pilgrimage to Mecca at least once in each Muslim's life

Muslims believe in a life after death in a paradise that is described in detail in the Koran. Paradise is for all who keep the faith. People who are evil will spend eternity in a fiery hell.

Major Denominations of Islam	Origins	Beliefs and Practices
Sunni	Original form of the religion	• Accept the first three caliphs, or leaders after Muhammad, as genuine • Claim to follow the Sunnah, or example, of Muhammad • Follow a traditional interpretation of Islam
Shia	Formed after **AD 632**	• Rejected the first three caliphs • Believe that Muhammad's son-in-law, Ali, was meant to be caliph • Refer to leaders as imam; imams must be direct descendants of Ali • Divided into two groups: Ismaili and Imami
Imami	Also known as the Ithna Ashari or Twelvers	• See authority as residing in 12 imams, or leaders, starting with Ali • Await the second coming of the "hidden imam," a caliph who disappeared in AD 873 • Believe this imam, Muhammad al-Mahdi, will return at the end of the world and rule the Earth along with Jesus
Ismaili	Broke away from the Imamis in **AD 700**	• Follow the Aga Khan, a religious leader based in France
Kharijites	Rejected Ali in **AD 657**	• Believe that the best Muslim should be caliph, regardless of status • Believe in equality under God • Initially elected their leaders

Transformation of the Workplace

Industrial Revolution

The Industrial Revolution grew from very humble beginnings in Britain, where its first impact was felt in the area of cotton textile manufacture.

During the 1770s and 1780s, a burst of inventive genius revolutionized the textile industry. From hand operations on simple machines, such as spinning a single thread or weaving only one bolt of cloth at a time, the industry progressed to multiple spindles and complicated looms.

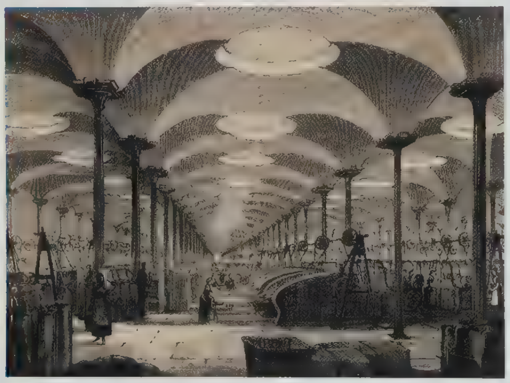
As the machines grew larger and more complicated, they could no longer be run by human power. Instead, they were run by waterpower. Ultimately, even waterpower became insufficient. The development of the steam engine provided a solution to the power problem. Its application in the steam locomotive was to revolutionize transportation.

Areas rich in natural resources but without easy access to water transportation, the only economical method of transport before the railroad, gained new importance. When rail transportation was brought to those areas, they became centers of industrial activity and prosperity.

Social Effects. It was through industrialization that western Europe was transformed from a rural to an urban civilization. Factories drew formerly rural peoples together to one spot for the purpose of production of manufactured goods. Men, women, and children worked long hours for a salary, and labor abuses became prevalent. These abuses, including child labor, unsafe conditions, overwork, and underpayment, led to the growth of socialism.

Cities sprang up and grew in size by leaps and bounds. Urbanization increased political awareness and activity among the masses. The new industrial classes in Britain and France began gradually to undermine the power of the landed aristocracy. This was particularly true in Great Britain, the supreme industrial power of the world, where human labor became an important and valuable resource.

France industrialized much more slowly, and in Prussia, Austria, Russia, and most of the other nations of Europe, the time for industrialization had not yet come. Most nations feared the emergence of a working class that might clamor for equal rights and of subversive democracies resembling those of the tumultuous French Revolution.



▲ **The Industrial Revolution** caused profound changes in the workplace. Used to working on farms or in small shops, people increasingly were employed in large factories.

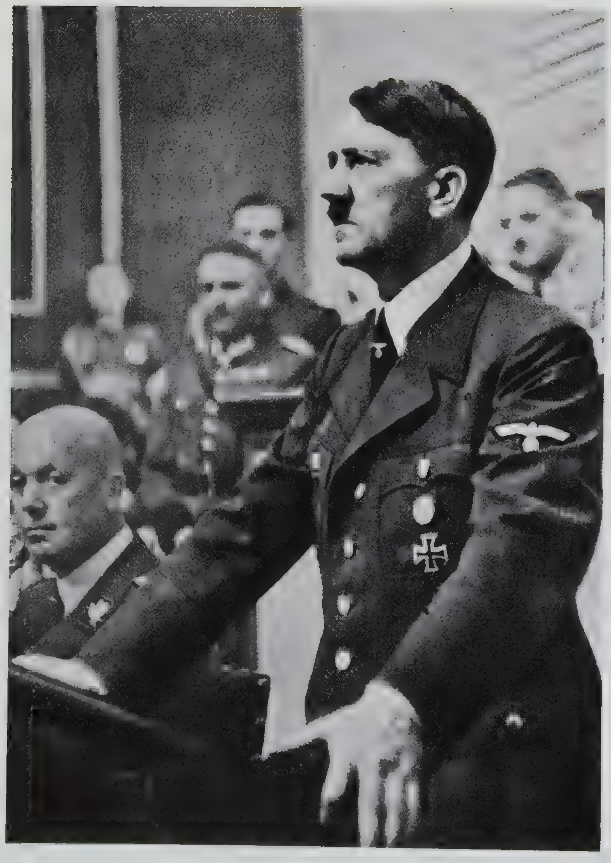
Know *now you* **Know!**

- ✓ The Congress of Vienna (1814–1815) was an effort to ensure a lasting peace in Europe by establishing a balance of power between rival nations. Monarchies in France, Spain, and Austria were restored and a confederation of German states was organized. New kingdoms were set up in the Netherlands and Poland.
- ✓ Peter the Great (ruled 1682–1725) Westernized many Russian institutions and cultural ideas. His modernized, strong, centralized administration enabled Peter to seek "windows to the West."
- ✓ One of the most significant effects of the wave of nationalism and liberalism that swept Europe in the mid-1800s was the forging of two new national states in major geographical-cultural regions—modern Germany and Italy.
- ✓ The Industrial Revolution began when the British textile industry was transformed by increasingly efficient machinery in the late 1700s. As the machines grew larger and more complicated, more power was needed to run them. Ultimately, it was the development of the steam engine that provided a solution to the power problem. Its application in the steam locomotive was to revolutionize transportation.

<table>
<tr><th colspan="2">Key Innovations of the Industrial Revolution</th></tr>
<tr><td colspan="2">The Textile Industry
One of the most spectacular features of the Industrial Revolution was the introduction of power-driven machinery in the textile industries of England and Scotland.</td></tr>
<tr><td>Spinning machines</td><td>Between 1774 and 1779, Samuel Crompton developed the spinning mule, which efficiently spun fine yarn for high-quality cloth. During the 1780s and 1790s, larger spinning mules were built. They had metal rollers and several hundred spindles. These machines ended the home spinning industry.</td></tr>
<tr><td>Weaving machines</td><td>In the mid-1780s, steam-powered looms that made all the movements for weaving were introduced. By 1835, the United Kingdom had more than 120,000 power looms. Most of them were used to weave cotton.</td></tr>
<tr><td colspan="2">The Steam Engine
Many of the most important inventions of the Industrial Revolution required much more power than horses or water wheels could provide. Industry needed a new, cheap, and efficient source of power and found it in the steam engine.</td></tr>
<tr><td>Early steam engines</td><td>In 1698, Thomas Savery patented a pumping engine that used steam. In 1712, Thomas Newcomen created an improved engine that came into general use during the 1720s. Newcomen's steam engine wasted much heat and used a large amount of fuel.</td></tr>
<tr><td>James Watt improves the steam engine</td><td>By 1785, a Scottish engineer named James Watt had eliminated many of the problems of earlier engines. Watt's engine used heat much more efficiently than Newcomen's engine and used less fuel.</td></tr>
<tr><td>Refining tools for precision work</td><td>Because Watt could not find a tool that drilled a perfectly round hole, his engines leaked steam. In 1775, John Wilkinson invented a boring machine that drilled a more precise hole. By 1825, English inventors had developed a planer to smooth the surfaces of steam engine metal parts.</td></tr>
<tr><td colspan="2">Coal and Iron
Coal provided the power to drive the steam engines and was needed to make iron. Iron was used to improve machines and tools and to build bridges and ships. Britain's large deposits of coal and iron ore helped make it the world's first industrial nation.</td></tr>
<tr><td>The revolution in ironmaking</td><td>Between 1709 and 1713, Abraham Darby succeeded in using coke to smelt iron. Coke is made by heating coal in an airtight oven. Smelting with coke was much more economical and efficient than smelting with charcoal. The smelting, puddling, and rolling steps could be combined into a continuous operation near the coal fields.</td></tr>
<tr><td colspan="2">Transportation
The growth of the Industrial Revolution depended on industry's ability to transport raw materials and finished goods over long distances. Thus, the story of the Industrial Revolution is also the story of a revolution in transportation.</td></tr>
<tr><td>Waterways</td><td>Until the early 1800s, waterways provided the only cheap and effective means of hauling coal, iron, and other heavy freight. In 1807, the American inventor Robert Fulton built the first commercially successful steamboat. Within a few years, steamboats became common on British rivers.</td></tr>
<tr><td>Roads</td><td>In the early 1800s, Scottish engineers John Loudon McAdam and Thomas Telford made important advances in road construction: the macadam type of road surface, which consists of crushed rock packed into thin layers; and a technique of using large flat stones for road foundations.</td></tr>
<tr><td>Railroads</td><td>In 1804, Richard Trevithick built the first steam locomotive. However, steam locomotives did not begin to come into general use for passenger and freight transportation until the late 1830s.</td></tr>
</table>

WORLD WAR II

During the 1930s, events in Europe, Asia, and Africa were leading the world toward another world war. Although few people realized it at the time, Hitler's ascent to power in Germany was the decisive moment in the first half of the 20th century. He had spelled out his program in the 1920s in his book *Mein Kampf* (*My Struggle*), which underlined the necessity of a resounding German military victory in Europe to restore the honor and integrity of Germany.



Adolf Hitler speaks to the German Reichstag on September 3, 1939, two days after the German invasion of Poland and the same day that Britain and France declared war.

The War in Europe: Time Lin

September 1
Germany invades Poland. ▲

May 10
Germany invades Belgium and the Netherlands.

1939

September 3
Britain and France declare war on Germany.

September 17
The Soviets (then allies of Germany) invade Poland from the east.

September
Germany and the Soviet Union divide up Polish territories.

October–April 1940
Germany, Britain, and France avoid fighting along the Maginot and Siegfried Lines in what was known as the Sitzkrieg, or Phony War.

1940

May 10
Winston Churchill replaces Neville Chamberlain as British prime minister.

May 12
Germans cross the French frontier using Blitzkrieg (air, tank, and infantry "lightning strike") tactics.

May 26–June 4
German victories force the evacuation of more than 300,000 Allied troops from the French port of Dunkirk.

June 10
Italy declares war on France and Great Britain.

June
France capitulates to German troops, and a puppet government under Marshal Henri Pétain is set up. The armistice between France and Germany is signed on June 22.

July 10
The Battle of Britain begins with German saturation bombings of British military installations, factories, and land and sea transportation sites; the British respond by destroying over 1,500 German Luftwaffe bombers.

March 11
The Lend-Lease Act becomes U.S. law. The act permits President Roosevelt to lend or lease raw materials or supplies to any nation fighting the Axis.

April 6
Germany invades Yugoslavia and Greece.

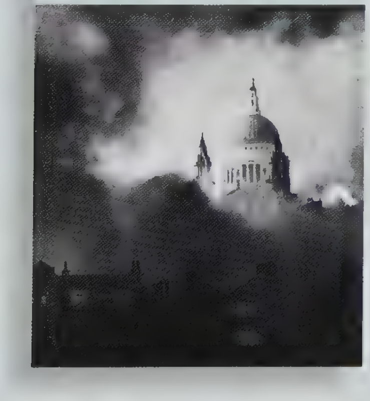
April
German field marshal Erwin Rommel takes over Axis forces in North Africa and begins a rapid German advance there.

August 14
The Atlantic Charter, a joint declaration of war policy, is signed by Roosevelt and Churchill. ▼

1941

September
Italy opens fighting in the North African desert by attacking British Egypt from Libya.

September
Beginning of the Blitz, German bombing campaigns against British cities.



May 20
German paratroopers land on the island of Crete and seize an airfield. The entire island is soon under German control, giving them a critical Mediterranean base.

May 27
German battleship *Bismarck* is sunk in a battle with British naval forces.

June 22
Germany invades the Soviet Union in Operation Barbarossa, ending the German-Soviet alliance and bringing the Soviets into the war against Germany.

September 8
German troops begin the siege of Leningrad (now St. Petersburg), which lasts until January 1944. More than 1 million Soviets die after years of shortages of food, water, and medicine.

September 19
The Soviet city of Kiev surrenders to German forces. Nearly 665,000 Soviet soldiers are captured by the Germans in their greatest single victory.

November 15
German forces attack Moscow. They are exhausted by their slow march through mud, however. The onset of bitter winter cold combined with fresh Soviet reinforcements freezes the German offensive 20 miles (32 kilometers) short of the city.

December 7
Japan bombs the U.S. Pacific Fleet stationed at Pearl Harbor, located on the Hawaiian island of Oahu. The United States, Canada, and the United Kingdom declare war on Japan the next day.

December 11
Germany and Italy declare war on the United States.

The War in Europe: Time Line

June 25
General Dwight Eisenhower becomes head of U.S. operations in Europe.

August–October
German eastward drive in the Soviet Union stalls at Stalingrad.

February 1
The German 6th Army surrenders at Stalingrad, a major turning point in the war in Russia.

April 19–May 16
Jews in the Warsaw ghetto (a section of the city) rise up against the Nazis until the Nazis crush the uprising, killing thousands of Jews and sending most survivors to concentration camps.

July 25
Mussolini is deposed; Badoglio is named premier of Italy.

1942

1943

August 25
Hitler orders his forces to capture Stalingrad.

October 23
British forces attack the Axis powers at El Alamein in Egypt. The tank battle rages until November 5, when German forces withdraw.

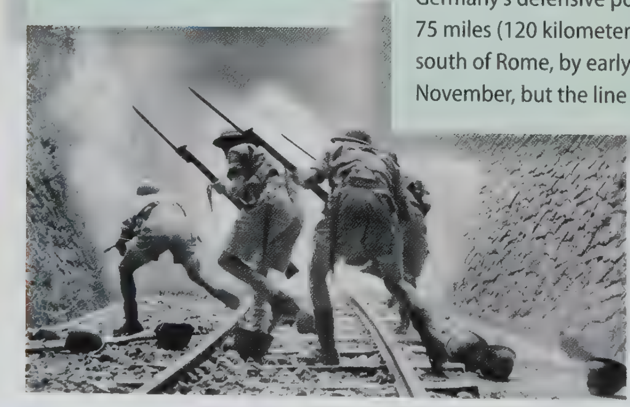
November 8
Allied troops land in Algeria and Morocco, beginning the invasion of North Africa.

November–March 1943
Soviet counterattack at Stalingrad drives Germans into retreat.

May 13
Axis forces in northern Africa surrender.

July 4
Germany opens an assault near the Soviet city of Kursk.

July 10
Allies invade the Italian island of Sicily. ▼



September 3
Italy secretly surrenders to the Allies, but the Germans refuse to surrender the Italian peninsula.

September 9
Allied troops land at Salerno, Italy, and begin a difficult, contested drive north to Rome. They reach the Gustav Line, Germany's defensive position 75 miles (120 kilometers) south of Rome, by early November, but the line holds.

1944

January 22
Allied troops land in Anzio, Italy, but German forces pin them on the beaches for 4 months.

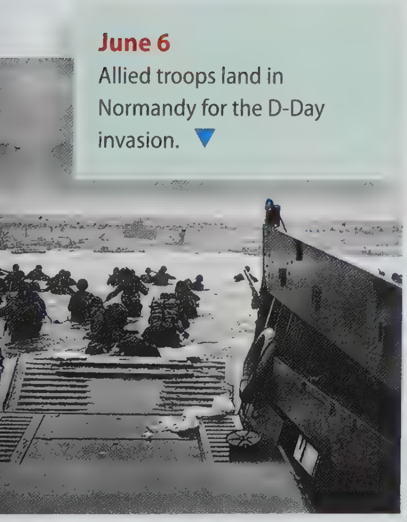
January 27
The siege of Leningrad ends with the retreat of Nazi troops.

February 20
Strategic bombing of Germany begins. Better bombsights, introduced by the U.S. Army Air Forces, allow bombers to target and hit military and industrial sites.

April 2
Russians enter Poland.

June 4
Rome captured by Allies after German defenses are punctured in May.

June 6
Allied troops land in Normandy for the D-Day invasion. ▼



July 20
Plot to assassinate Hitler fails.

August 25
American and Free French forces liberate Paris. Resistance fighters had been battling the Germans street by street since August 19.

October 13
Athens is freed by the Allies.

December 16
Germans strike back at U.S. troops in the Battle of the Bulge—the last major German counteroffensive.

1945

March 7
Allies cross the Rhine in Germany.

April 25
Soviet forces surround Berlin.

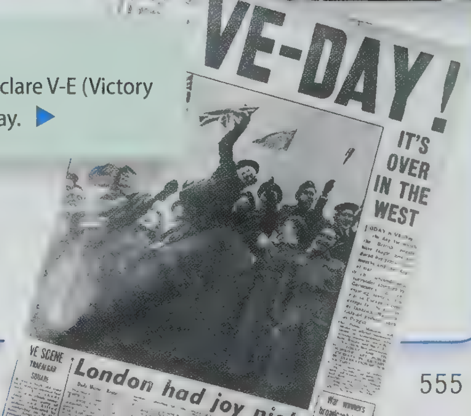
April 28
Benito Mussolini is executed at Lake Como by Italian partisans.

April 30
Hitler takes his life in Berlin.

May 2
German forces in Italy surrender to the Allies. German positions in northern Italy hold during the fall and winter of 1944, but the following spring, the Allies sweep north to the Alps.

May 7
Germany surrenders unconditionally to the Allies in Reims, France, ending World War II in Europe.

May 8
The Allies declare V-E (Victory in Europe) Day. ▶


VE-DAY!
IT'S OVER IN THE WEST
VE SCENE
London had joy ni

July 17–August 2
Potsdam Conference between Allied leaders. Germany is divided into four zones, one each to be occupied by the United States, the Soviet Union, the United Kingdom, and France. The four powers would jointly administer Berlin.

November 20
Nuremberg war crimes trials begin.

WORLD WAR II: EUROPE

Hitler began his expansion in 1936 by sending German troops into a section of western Germany called the Rhineland, in direct violation of the Treaty of Versailles. In that same year, Germany signed a pact with Italy, forming a military alliance called the Rome-Berlin Axis. The year 1936 also marked Italy's conquest of Ethiopia in Africa. In 1938, German troops marched into Austria and annexed it to Germany, another violation of the Versailles treaty. None of these actions met opposition from the Allied powers.

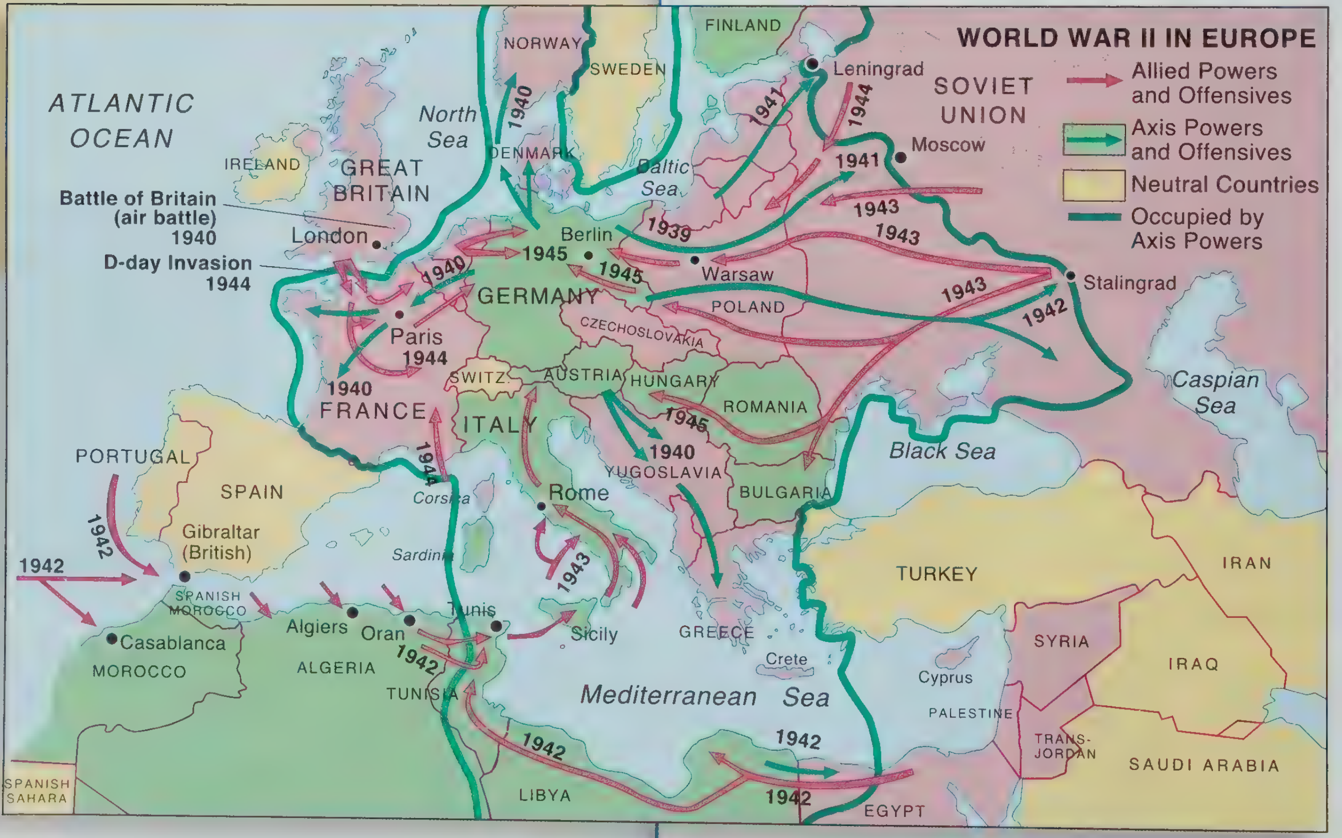
Europe in Peril

War Begins

In 1939, the Nazis occupied all of Czechoslovakia. It became clear that a line had to be drawn somewhere or Hitler would swallow all of Europe. Great Britain and France finally took a stand on the issue of Poland's territorial integrity.

The United Kingdom and France hoped that the Soviet Union would help defend Poland. Stalin recognized that Russia was unprepared for a Nazi assault should Hitler turn farther to the east and so signed a nonaggression pact with Germany that profoundly shocked the world. That pact sealed Poland's fate. On September 1, 1939, German forces began the dismemberment of Poland. On September 3, France and England declared war on Germany.

Overview of World War II in Europe. This map shows major offensives and the greatest extent of Axis domination. ▼


WORLD WAR II IN EUROPE
Allied Powers and Offensives
Axis Powers and Offensives
Neutral Countries
Occupied by Axis Powers
ATLANTIC OCEAN
North Sea
Baltic Sea
IRELAND
GREAT BRITAIN
Battle of Britain (air battle) 1940
D-day Invasion 1944
London
NORWAY
SWEDEN
FINLAND
DENMARK
Leningrad
SOVIET UNION
Moscow
Berlin
Warsaw
Stalingrad
GERMANY
POLAND
CZECHOSLOVAKIA
Paris
FRANCE
SWITZ.
AUSTRIA
HUNGARY
ROMANIA
ITALY
YUGOSLAVIA
BULGARIA
Rome
Corsica
Sardinia
Sicily
PORTUGAL
SPAIN
Gibraltar (British)
SPANISH MOROCCO
Casablanca
MOROCCO
Algiers
Oran
ALGERIA
Tunis
TUNISIA
LIBYA
GREECE
Crete
Mediterranean Sea
Black Sea
Caspian Sea
TURKEY
IRAN
SYRIA
IRAQ
Cyprus
PALESTINE
TRANS-JORDAN
SAUDI ARABIA
EGYPT
SPANISH SAHARA
1939
1940
1941
1942
1943
1944
1945

Fall of France

In May 1940, German tanks and aircraft passed around the Maginot Line as they swept through Luxembourg and Belgium and into northern France. On June 5, the Germans launched a major assault along the Somme River, which had been the scene of brutal slaughter in World War I. The blitzkrieg overwhelmed the French forces, driving them mercilessly backward. Seeing an opportunity to profit from Germany's success, Italy declared war on France and the United Kingdom on June 10. The French government fled from Paris to Bordeaux the same day.

German troops entered Paris on June 14, 1940. Paul Reynaud, the new French premier, wanted to fight on. But many of his generals and cabinet officers believed that the battle for France was lost. Reynaud resigned, and a new French government agreed to an armistice on June 22.

One of the French generals, Charles de Gaulle, escaped to the United Kingdom after France fell. In radio broadcasts to France, he urged the people to carry on the fight against Germany. The troops who rallied around de Gaulle became known as the Free French.

▲ **Paris falls, June 1940.** With the Arc de Triomphe in the background, General Kurt von Briesen (on horseback) salutes his soldiers as they parade down the Avenue Foch.

Battle of Britain

The British were determined to resist the Nazis and fought on alone. Hitler prepared to invade southern England in a military operation with the code name Sea Lion. But before an invasion force could safely cross the English Channel, Hitler had to clear the Royal Air Force from the sky. The Battle of Britain was to become the first battle ever fought solely for air supremacy.

Hitler had originally ordered that no civilian targets should be attacked. But in late August, a lost Luftwaffe pilot violated his orders and bombed central London. An outraged Churchill retaliated with a bombing attack on Berlin. Hitler then unleashed his air force against English cities, beginning what is known as the Blitz. German planes bombed cities from London to Manchester for months. Raids continued throughout the winter and spring until finally, in May 1941, Germany gave up its attempts to defeat the United Kingdom from the air. More than 40,000 British civilians had lost their lives.

Hitler's decision to switch attacks from Royal Air Force (RAF) bases to British cities effectively won the battle for the United Kingdom. It allowed the RAF to constantly replace both planes and pilots. The United Kingdom's survival became immensely important later in the war. The island itself served as a giant airfield and base for the Allied invasion of Europe, and the British Army and High Command made crucial contributions to the ultimate Allied victory.

THE UNITED STATES ENTERS THE WAR

On December 7, 1941, the Japanese launched a surprise attack upon the U.S. Pacific Fleet at Pearl Harbor, Hawaii.

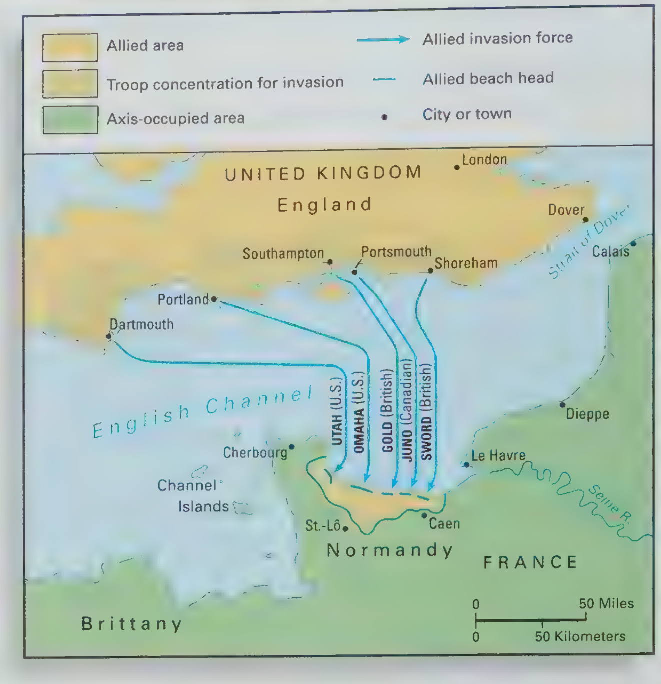
The United States, Canada, and the United Kingdom declared war on Japan on December 8, 1941. Germany and Italy declared war on the United States on December 11. The world was now truly a world at war. (See also pages 184–197.)

A Hard-Won Victory

D-Day

Soon after British forces evacuated Dunkirk in their retreat from the continent in 1940, the United Kingdom started to plan a return to France. In 1942, the United States and the United Kingdom began to discuss a large-scale invasion across the English Channel. Throughout 1943, preparations moved ahead for an invasion of northern France the following year. The invasion plan received the code name Operation Overlord.

General Eisenhower, as supreme commander of the Allied forces, chose Monday, June 5, 1944, as D-Day—the date of the Normandy invasion. However, rough seas forced him to postpone until June 6. During the night, thousands of ships carrying landing craft and more than 130,000 landing troops crossed the channel. Minesweepers had gone ahead to clear the water. In addition, about 23,000 paratroopers and glider troops began dropping behind German lines to capture bridges and railroad tracks. At dawn, battleships opened fire on the beaches. At 6:30 a.m., soldiers from the United States, the United Kingdom, Canada, and France stormed ashore on a 60-mile (100-kilometer) front in the largest seaborne invasion in history.


Allied area
Troop concentration for invasion
Axis-occupied area
Allied invasion force
Allied beach head
City or town
UNITED KINGDOM
England
London
Dover
Strait of Dover
Calais
Southampton
Portsmouth
Shoreham
Portland
Dartmouth
English Channel
UTAH (U.S.)
OMAHA (U.S.)
GOLD (British)
JUNO (Canadian)
SWORD (British)
Dieppe
Cherbourg
Le Havre
Seine R.
Channel Islands
St.-Lô
Caen
Normandy
FRANCE
Brittany
0 50 Miles
0 50 Kilometers

D-Day took the Germans by surprise, but they responded fiercely. At one landing site, code-named Omaha Beach, U.S. troops came under heavy fire and barely managed to stay ashore. Nevertheless, all five Allied landing beaches were secure by the end of D-Day. By the end of June 1944, about 1 million Allied troops had reached France.

The Allied forces advanced slowly. During August, the Allies cleared the Germans out of most of northwestern France. Allied bombers hounded the retreating Germans. More than 50,000 Allied troops and aviators died in the Battle of Normandy.

▲ **Hitting the beach,** Allied infantrymen swarmed ashore along the Normandy coast of northern France on D-Day—June 6, 1944. It was the largest seaborne invasion in history. Hitler had boasted that German defenses along the coast could resist any attack. But he was wrong.

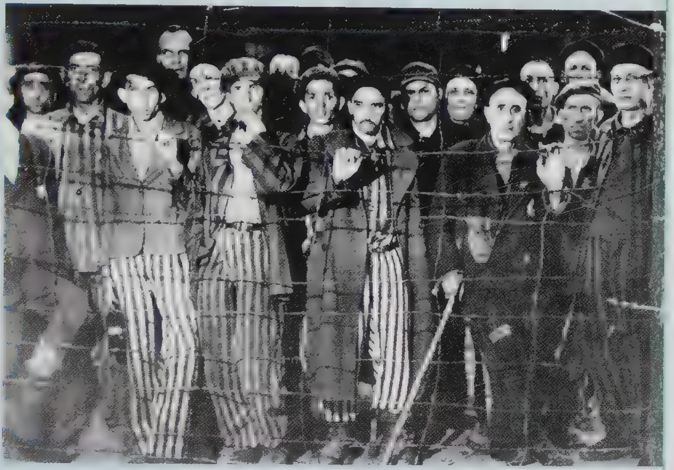
Victory in Europe

The Allies began their final assault on Germany in early 1945. British and Canadian forces cleared the Germans out of the Netherlands and swept into northern Germany. American and French forces raced toward the Elbe River in central Germany. Hitler ordered his soldiers to fight to the death. But seeing the war was over, large numbers of German soldiers surrendered each day.

By April 25, 1945, Soviet troops had surrounded Berlin. From a bunker deep underground, Hitler ordered German soldiers to fight on. On April 30, however, Hitler killed himself rather than face defeat.

The Holocaust

When Adolf Hitler became dictator in 1933, he instituted a crackdown on the Jewish population. Hitler and the Nazis made it impossible for Jews to remain in the professions, businesses, or universities of Germany. In 1942, the Nazi elite gathered in Potsdam to put the finishing touches on a plan called the "Final Solution." This plan foresaw the killing of all Jews in lands controlled by Germany, as well as all physically and mentally handicapped people and all Gypsies. In death camps such as Auschwitz and Buchenwald, victims were led off to gas chambers or killed and then thrown into common ditches for graves. Six million Jews and about 2 million others died in this manner.



▲ **Survivors** of the Nazi concentration camp Buchenwald—some too weak to stand—provided proof of Nazi savagery. The Nazis imprisoned and murdered millions of Jews, Slavs, and members of other groups.

Resistance. Initially, the Jews tried to thwart the Nazis by nonviolent means. Also, it was difficult and dangerous for the Jews to obtain weapons. Anti-Semitism was widespread, and Jewish resistance did not have popular support. Jewish fighters could not disappear among the population because non-Jews might betray them. But many Jews who managed to escape the ghettos joined secret bands of fighters against the Nazis. And some non-Jewish individuals risked their lives to smuggle Jews to safety.

Some Jews in ghettos, slave labor camps, and death camps did fight. In 1943, thousands of Jews revolted in the ghetto in Warsaw, Poland. Although the Jews were surrounded and poorly armed, they held out for about 4 weeks. But the Nazis either killed or sent to death camps all of the 60,000 Jews in the ghetto.

In 1943, uprisings took place at the Treblinka and Sobibor death camps. In 1944, prisoners at Auschwitz revolted and set fire to a crematorium. A few prisoners escaped during each uprising, but most were killed. Such revolts were often acts of desperation. They erupted when the Jews understood Nazi intentions and had abandoned hope of survival. The fighters also hoped to protect Jewish honor and to avenge Jewish death.

The fighting in Berlin claimed the lives of over 70,000 Soviet soldiers. The Germans propped up their ramshackle remaining units with children and elderly reservists. Thousands of them died in that last week of fighting.

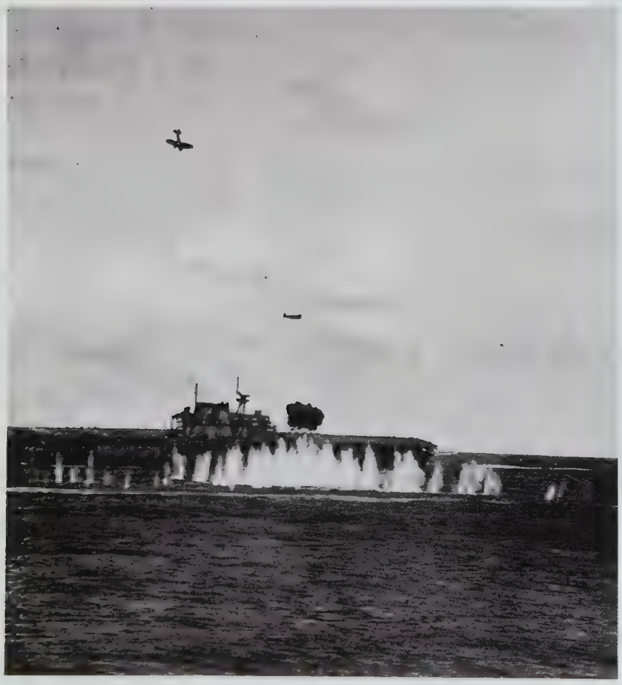
On May 7, 1945, General Alfred Jodl, chief of staff of the German armed forces, signed a statement of unconditional surrender at Eisenhower's headquarters in Reims, France. World War II had ended in Europe. The Allies declared May 8 as V-E Day, or Victory in Europe Day.

Know now you Know!

- ✔ German troops entered Paris on June 14, 1940, forcing the French government to sign an armistice on June 22.
- ✔ Before Hitler would commit troops to an invasion of southern England, he needed to clear the Royal Air Force from the sky. The result was the first battle ever fought solely for air supremacy: The Battle of Britain.
- ✔ The Allies' invasion of France on D-Day was the largest seaborne invasion in history.
- ✔ In 1942, the Nazis put the finishing touches on a plan called the "Final Solution," a plan that called for the killing of all Jews in lands controlled by Germany, as well as all physically and mentally handicapped people and all Gypsies.

WORLD WAR II: THE PACIFIC

The attack on Pearl Harbor on December 7, 1941, left the U.S. Pacific Fleet briefly powerless to halt Japan's expansion. During the next 6 months, Japanese forces swept across Southeast Asia and the western Pacific Ocean. Japan's empire reached its greatest size in August 1942. It stretched northeast to the Aleutian Islands of Alaska, west to Burma (now Myanmar), and south to the Netherlands Indies (now Indonesia). The Allies halted Japan's expansion in the summer of 1942. They hacked away at its empire until Japan surrendered in August 1945.



▲ **A Japanese kamikaze** (suicide pilot) flies toward a United States warship in this photograph. In a last desperate effort to win World War II, kamikazes crashed their planes into Allied ships.

The War in Asia: Time Line

1941

July 21
Japanese occupy French Indochina.

October 18
Hideki Tojo becomes Japanese prime minister.

December 7
Japanese attack Pearl Harbor, beginning a rapid sweep through the Pacific region.

December 8
United States enters the war; Japan lands troops in the Philippines. Britain and Canada declare war on Japan.

1942

February 15 ▲
The British surrender Singapore to the Japanese.

April 9
Bataan falls, ending resistance to the Japanese in the Philippines; U.S. general Douglas MacArthur vows, "I shall return."

April 18
American bombers hit Tokyo in the Doolittle raid.

May 4–8
The Allies stop a Japanese assault in the Battle of the Coral Sea.

May 6
Americans and Filipinos on Corregidor Island in Manila Bay surrender to the Japanese.

June 6
U.S. planes destroy the Japanese fleet at the Battle of Midway.

August 7
U.S. Marines land at Guadalcanal in the first amphibious assault that allows U.S. naval forces to push back the Japanese.

February 9
Japanese resistance on Guadalcanal ends.

March 2–4
U.S. defeats the Japanese in the Battle of Bismarck Sea.

June 19–20
A U.S. naval force defeats the Japanese in the Battle of the Philippine Sea.

July 18
Japan's Prime Minister Tojo resigns.

January 9
The Allies land on the Philippine island of Luzon and drive to Manila.

March 16
U.S. Marines capture Iwo Jima (an island now known as Iwo To).

March 26
U.S. forces land on the Japanese island of Okinawa.

June 21
Allied forces capture Okinawa.

August 8
The Soviet Union declares war on Japan and invades Manchuria.

August 9
A second atomic bomb is dropped, on Nagasaki, Japan.

September 2
Japanese sign surrender terms aboard the battleship *Missouri* as it lay at anchor in Tokyo Bay (V-J Day).

1943 1944 1945

June 1
U.S. starts using submarine warfare against the Japanese.

October 7
Approximately 100 American POWs are executed by the Japanese on Wake Island.

November 20
U.S. forces invade Tarawa in the Gilbert Islands.

December 26
The Allies launch a full assault on New Britain in the Solomon Islands.

October 20
The Allies begin landing in the Philippines.

October 23–26
The Allies defeat Japan's navy in the Battle of Leyte Gulf in the Philippines.

October
General MacArthur returns to the Philippines.

August 6
The United States drops the first atomic bomb, on Hiroshima, Japan; more than 78,000 people are killed in a single minute. ▼

Struggle in the Pacific

Early Japanese Victories

On December 8, 1941, Japanese bombers struck the British colony of Hong Kong on the south coast of China and two U.S. islands in the Pacific Ocean—Guam and Wake. The Japanese invaded Thailand the same day. Thailand surrendered within hours and began cooperating with the Japanese. Japanese troops took Hong Kong, Guam, and Wake Island by the end of the year.

By late January 1942, the Japanese had pushed the British forces back to Singapore, a fortified island off the tip of the Malay peninsula. The Japanese stormed the island on February 8, and Singapore surrendered a week later. Japan captured about 60,000 soldiers, making the fall of Singapore the United Kingdom's worst military defeat ever.

Japan began landing troops in the Philippines on December 8, 1941. American and Philippine forces commanded by U.S. general Douglas MacArthur defended the islands. In late December, MacArthur's forces abandoned Manila, the capital of the Philippines, and withdrew to nearby Bataan Peninsula. Although suffering from malnutrition and disease, they beat back Japanese attacks for over 3 months.

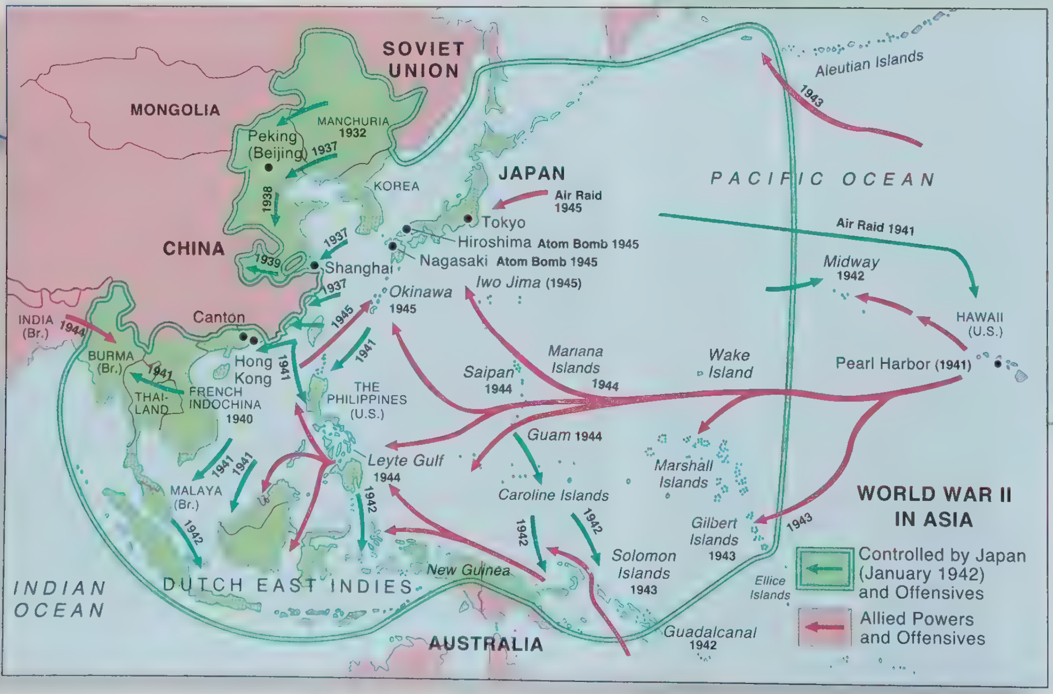
The Battle of Midway

Three events in 1942 helped turn the tide against Japan. The first was the Doolittle raid on April 18, 1942, in which the United States staged a daring bombing raid on Tokyo and other Japanese cities. The second was the Battle of the Coral Sea, in which American warships met the Japanese force northeast of Australia from May 4 to 8.

The third and most significant was the Battle of Midway. Japan sent a large fleet to capture Midway Island at the westernmost tip of the Hawaiian archipelago. The United States had cracked Japan's naval code and thus learned about the coming invasion. Admiral Chester W. Nimitz, commander of the U.S. Pacific Fleet, gathered the ships that had survived the raid on Pearl Harbor and the Battle of the Coral Sea and prepared to ambush the Japanese.

The battle opened on June 4, 1942. During the 3-day battle—also fought entirely by warplanes—the Japanese suffered great losses: 4 aircraft carriers, 275 aircraft, and around 3,000 sailors and aviators.

The Battle of Midway was the first clear Allied victory over Japan in World War II. The battle crippled the Japanese navy and stopped Japan's advances. Aircraft carriers had become the most important weapon in the war in the Pacific.


SOVIET UNION
MONGOLIA
MANCHURIA 1932
Peking (Beijing) 1937
KOREA
JAPAN
Air Raid 1945
Tokyo
Hiroshima Atom Bomb 1945
Nagasaki Atom Bomb 1945
CHINA
1938
1937
1939
Shanghai
Okinawa 1945
Iwo Jima (1945)
INDIA (Br.) 1944
Canton
Hong Kong 1941
BURMA (Br.)
THAILAND
FRENCH INDOCHINA 1940
THE PHILIPPINES (U.S.)
Saipan 1944
Mariana Islands 1944
Wake Island
Pearl Harbor (1941)
HAWAII (U.S.)
Midway 1942
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PACIFIC OCEAN
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1943
MALAYA (Br.)
1941
1942
Leyte Gulf 1944
Guam 1944
Caroline Islands
Marshall Islands
Gilbert Islands 1943
Solomon Islands 1943
Ellice Islands
Guadalcanal 1942
New Guinea
DUTCH EAST INDIES
INDIAN OCEAN
AUSTRALIA
WORLD WAR II IN ASIA
Controlled by Japan (January 1942) and Offensives
Allied Powers and Offensives

◀ **Overview of World War II in the Pacific.** This map shows major offensives and the greatest extent of Japanese control.

The Liberation of the Philippines

The campaigns in the Central Pacific brought the Allies within striking distance of the Philippine Islands. Expecting stiff Japanese resistance, the Allies assembled a landing force of more than 700 ships and pointed them at the island of Leyte in the central Philippines. On October 20, 1944, American troops poured ashore.

During the Battle for Leyte Gulf, the Japanese unleashed a terrifying new weapon—the *kamikaze* (Japanese for "divine wind") suicide pilot. Kamikazes intended to crash their planes filled with explosives into Allied warships, but most were shot down before they reached their targets. The few that got through, however, caused great damage, terror, and loss of life.

On January 9, 1945, the Allies landed on the island of Luzon and fought their way to the capital city of Manila, which was almost completely destroyed. Nearly 100,000 Filipino civilians died in the conquest of Manila. Eventually, the remaining Japanese troops on Luzon—50,000 of them—pulled back to the mountains and went on fighting until the war ended.

American losses in the Philippines were heavy, with 13,700 killed in action. Japanese losses were catastrophic, with at least 200,000 dead. Japan was doomed to defeat after losing the Philippines. But it did not intend to surrender.

The Atomic Bomb and Its Aftermath

On August 6, 1945, an American B-29 bomber called the *Enola Gay* dropped the first atomic bomb used in warfare on the Japanese city of Hiroshima. The explosion killed from 70,000 to 140,000 people, it is estimated, and destroyed about 5 square miles (13 square kilometers) of the city. After Japanese leaders failed to respond to the bombing, the United States dropped another bomb on Nagasaki on August 9. It killed about 40,000 people. Later, thousands more died of injuries and radiation from the two bombings. Meanwhile, on August 8, the Soviet Union declared war on Japan and invaded Manchuria.

Victory in the Pacific. Although Japan's emperors had traditionally stayed out of politics, Hirohito urged the government to surrender. On August 14, Japan agreed to unconditional surrender.

On September 2, 1945, representatives of Japan signed the official statement of surrender aboard the U.S. battleship *Missouri,* which lay at anchor in Tokyo Bay. Representatives of all the Allied nations were present. Truman declared September 2 as V-J Day, or Victory over Japan Day. World War II had ended.

ISLAND HOPPING IN THE CENTRAL PACIFIC

From late 1943 until the fall of 1944, the Allies hopped from island to island across the Central Pacific toward the Philippines. During the island-hopping campaign, the Allies became expert at amphibious invasions—seaborne operations that involve naval, air, and land forces. Each island they captured provided a base from which to strike the next target. But rather than capture every island, the Allies bypassed Japanese strongholds and invaded islands that were weakly held. That strategy, known as leapfrogging, saved time and lives. Leapfrogging carried the Allies across the Gilbert, Marshall, Caroline, and Mariana islands in the Central Pacific.

now you Know!

- ✓ On December 8, 1941, the day after they attacked Pearl Harbor, the Japanese invaded Hong Kong, Thailand, and the islands of Guam and Wake.
- ✓ The Battle of Midway was the first clear Allied victory over Japan. The battle crippled the Japanese navy and stopped Japan's advances in the Pacific.
- ✓ During the Battle for Leyte Gulf in the Philippines, the Japanese unleashed a terrifying new weapon: kamikaze suicide pilots, who intended to crash their planes filled with explosives into Allied warships.
- ✓ The war in the Pacific came to an end only after the United States dropped atomic bombs on Japanese cities: Hiroshima on August 6 and Nagasaki on August 9.

GLOBAL ISSUES

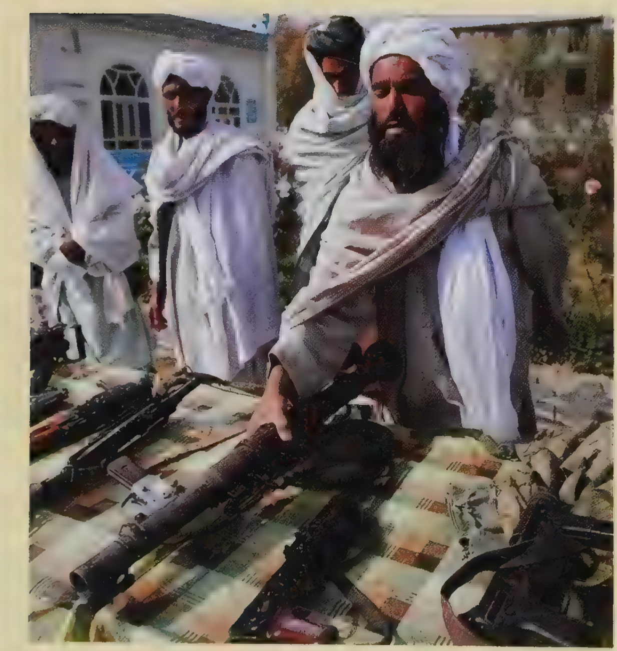
The War Against the West

When the Soviet Union collapsed in 1991, effectively bringing to an end the Cold War that had gripped the world for nearly half a century, many believed that this was a sign that the world was entering a new historical period. For many in Western nations, genuine peace and prosperity seemed to be the inevitable consequence of this momentous event. The reality, as usual, turned out to be more complicated. Three of the most significant trends to emerge in the 1990s and take center stage after the turn of the century were the rise of religious extremism in the Middle East, the threat of global climate change, and great advances in computers and communications technology.

Rise of Extremism

During the last three decades of the 20th century, a religious revival began to sweep through much of the Muslim world. One aspect of this Islamic resurgence has been the growth of an ideology known as Islamism. Islamists believe that the social and political order should be based solely on the sharia (traditional Islamic law). A return to a fully Islamic society, they argue, would bring an end to government corruption, moral decay, and social injustice and would eventually restore the Muslim world to the preeminent position it once held centuries ago, before the rise of the West.

The outside world paid little attention to this until 1979, when a revolution in Iran toppled the Shah and led to the creation of the first modern Islamist state under the leadership of Ayatollah Khomeini. Encouraged by the events in Iran, Islamist groups became more assertive and saw their popularity surge during the 1980s. By 1989, Islamists had taken control of the government of Sudan. Two years later, an Islamist party won a free election in Algeria but was prevented from coming to power by the army. This led to a revolt by radical Islamists, which was suppressed by the government, but only after 6 years of civil war.



THE TALIBAN

The Taliban are a militant Islamic political group that gained control of most of Afghanistan beginning in the mid-1990s. The Taliban enforced strict adherence to their interpretation of Islamic laws.

In 1998, the United States accused the Taliban of harboring Osama bin Laden, wanted in connection with terrorist attacks against two U.S. embassies in Africa. Then, in 2001, the United States accused bin Laden and al-Qaeda of carrying out the September 11 terrorist attacks. The United States demanded that the Taliban hand over bin Laden. The Taliban refused to do so, and the United States and its allies launched a military campaign against the Taliban.

Many Taliban and al-Qaeda members went into hiding across the border in Pakistan. The Taliban began to regroup in Pakistan and clashed frequently with police and government troops.

The Road to 9/11. Many Islamists condemn the United States for its strong support of Israel and for its massive aid to certain Middle Eastern governments, which Islamists believe are corrupt, oppressive, ungodly, and illegitimate. Moreover, American society, in their view, is materialistic and morally lax, the epitome of everything that is wrong with secular Western culture.

The most notorious Islamic extremist was Osama bin Laden. Born to a wealthy family in Saudi Arabia, bin Laden created an international network in the 1980s to raise funds and recruit volunteers for the war against the Soviets in Afghanistan. After the Soviet withdrawal in 1989, he began to use this network, now known as al-Qaeda ("The Base"), to conduct a series of terrorist attacks on American targets at home and abroad. By 1996, bin Laden, now an international outlaw, was living in Afghanistan under the protection of the Taliban, an Islamist militia that had taken control of most of that country.

◀ **Osama bin Laden,** a Saudi-born millionaire and radical Muslim, was the founder and longtime leader of al-Qaeda, a global terrorist organization.

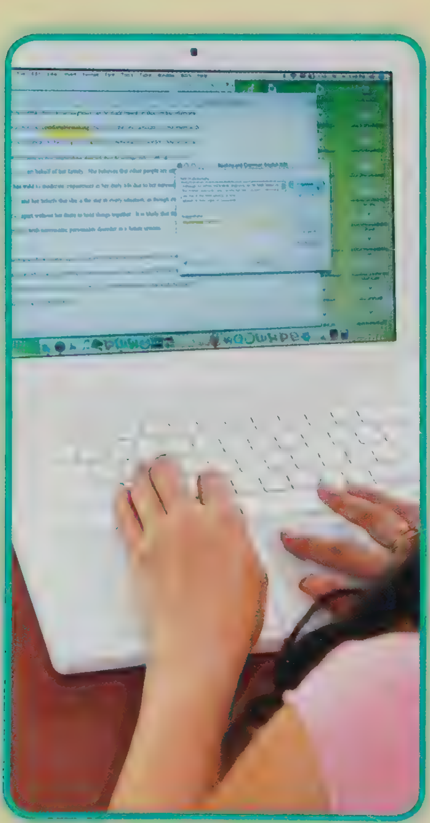
The World Trade Center towers billowed flames and smoke on September 11, 2001, after terrorists crashed hijacked airliners into the two buildings. The towers were the tallest buildings on the New York City skyline until they collapsed a short time after the attack.

On September 11, 2001, 19 al-Qaeda operatives simultaneously hijacked four American airliners and crashed one into the Pentagon and two into the World Trade Center towers in New York, killing nearly 3,000 people, the largest loss of American lives in a single day since the Civil War. A group of passengers on the fourth airliner, attempting to overpower the highjackers, forced the plane to crash in a field in Pennsylvania, far from any intended target.

By the start of 2008, most of the original leadership circle of al-Qaeda had been killed or captured, including Khalid Shaikh Mohammed, who was believed to be the mastermind of the 9/11 attacks, but bin Laden and his chief lieutenant, Ayman al-Zawahiri, remained at large, and the global jihad they had inspired still posed a serious threat to world stability.

America's attention began to shift back to Afghanistan, where the level of violence was increasing thanks to a resurgent Taliban. The situation was also worsening in neighboring Pakistan, where Islamist militants were believed to be responsible for the assassination of popular opposition leader and former prime minister of Pakistan Benazir Bhutto at the end of 2007 and for a series of attacks in the Indian city of Mumbai in November 2008. The possibility that all or part of Pakistan's nuclear arsenal might fall into the hands of Islamic extremists, coupled with North Korea's claim to possess nuclear weapons and Iran's determination to pursue atomic research despite opposition from the UN, the European Union (EU), and the United States, has revived fears that nuclear proliferation might someday provide terrorists with weapons vastly more destructive than any previously employed.

Language Advantage



Grammar

Writing and Research

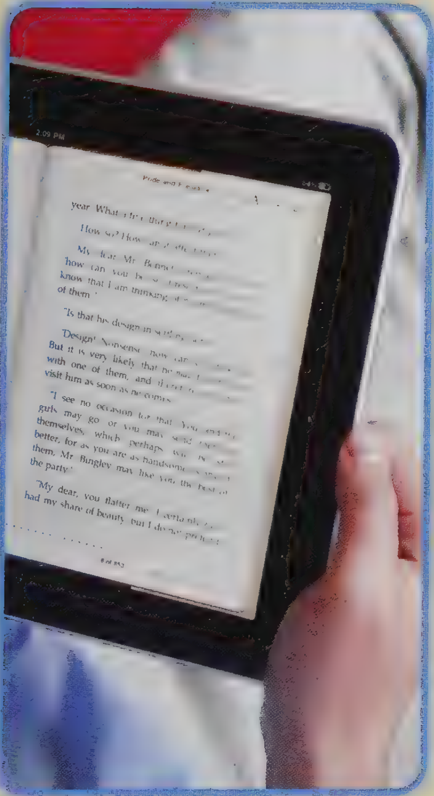
Vocabulary

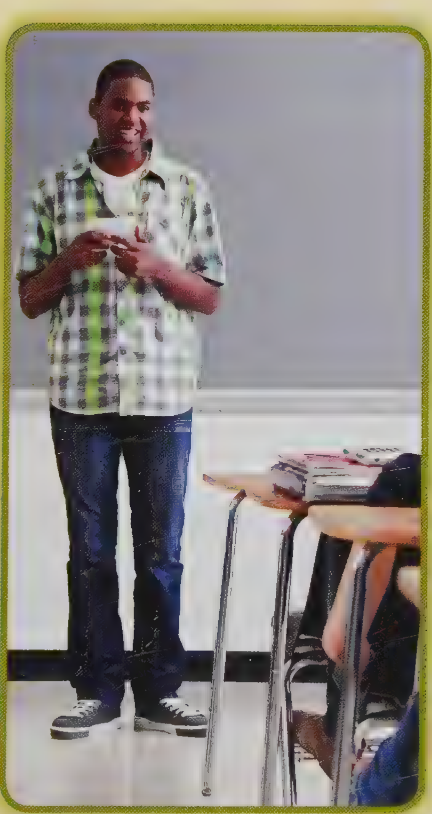
Reading

Speaking

Literature

At a Glance





How to Use Southwestern Advantage

How to Use This Book

These books have been designed so that information can be accessed easily. Social Studies has been divided into six "strands": United States History; Canadian History; Government; Economics; World History; and Geography. Language has likewise been divided into six strands: Grammar; Writing and Research; Vocabulary; Reading; Speaking; and Literature. Each strand is then divided into smaller units.

The first navigational tool is the detailed, color-coded Table of Contents. The contents pages also indicate separately where the special features of the book can be found, such as the Table of Shakespeare's Plays and the history time lines.

UNITED STATES HISTORY

CANADIAN HISTORY

GOVERNMENT

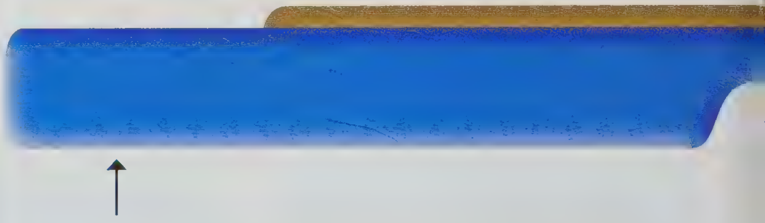
ECONOMICS

WORLD HISTORY

GEOGRAPHY

Next, above the heading on the right-hand text pages, you will see color bars that tell you exactly where you are in the book. The bar that extends all the way to the edge of the page is the color of the unit you are in; the other bar denotes the strand you are in. The strand color is repeated in a tab at the bottom of the page. When the book is closed, you can tell at a glance where each strand and unit begins and ends.

When the book is open, headings on the pages also help to tell you exactly where you are in the book, for example, the Nile Valley section of World History.



Strand color bar

fyi! *Though the Stone Age in the Near East ended in about 3000 BC, not all societies left it behind at once.*

How to Use Southwestern Advantage Online (www.SWadvantage.com)

An integral part of Southwestern Advantage is the accompanying Web site. Organized by subject areas, it is a comprehensive suite of online study helps, additional in-depth subject matter, tips for parents, and coaching for students on how to get better at life.

GRAMMAR

WRITING AND RESEARCH

VOCABULARY

READING

SPEAKING

LITERATURE

SPECIAL ADVANTAGES

Famous Speeches (annotated) 908
Time Line of World Literature 920
Table of Shakespeare's Plays 960

THE ROSETTA STONE

For over 1,000 years, scholars tried but failed to decipher the writing system of ancient Egypt. Then, in 1799, a rock slab with ancient Greek and Egyptian writing was found outside Rosetta, a city near Alexandria.

Features

THE NILE VALLEY **WORLD HISTORY**

Unit within World History

Name of strand World History strand

Unit color bar

Spotlight on...

ALEXANDER THE GREAT

356–323 BC

As one of the greatest generals in history, Alexander the Great was responsible for the spread of Greek culture throughout western Asia and Egypt.

- He was educated by the famous Greek philosopher Aristotle.

THE basics

WORLD WAR II: CAUSES AND BEGINNINGS

✓ Worldwide economic depression during the 1930s helped create military dictatorships in Europe and Asia.

WATCH OUT!

Set a goal for each writing session. Whether it's one paragraph or one page, having a goal will help you focus on that particular section.

!

✓ The end of the Ice Age, about 11,500 years ago, marked the beginning of the Neolithic era.

now you Know!

Strand color tab

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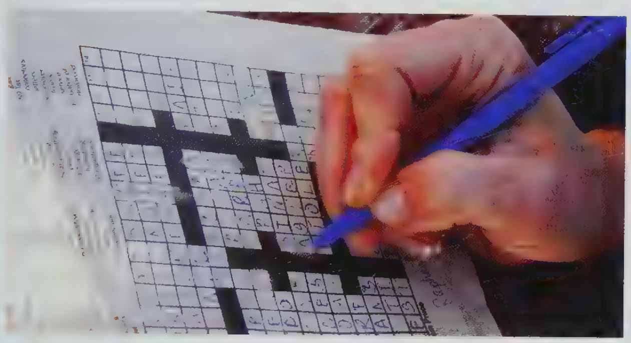

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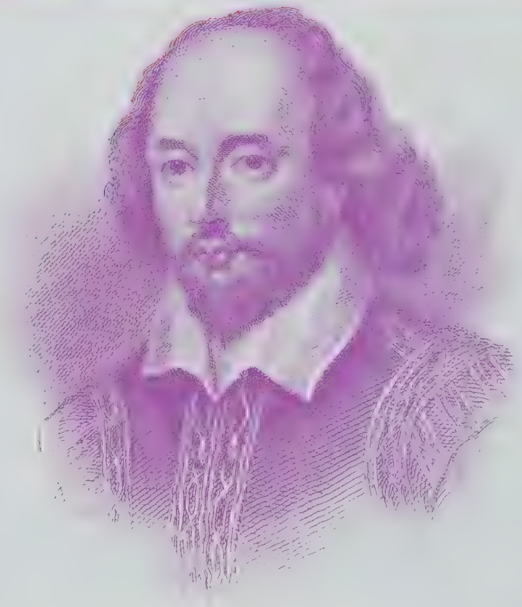
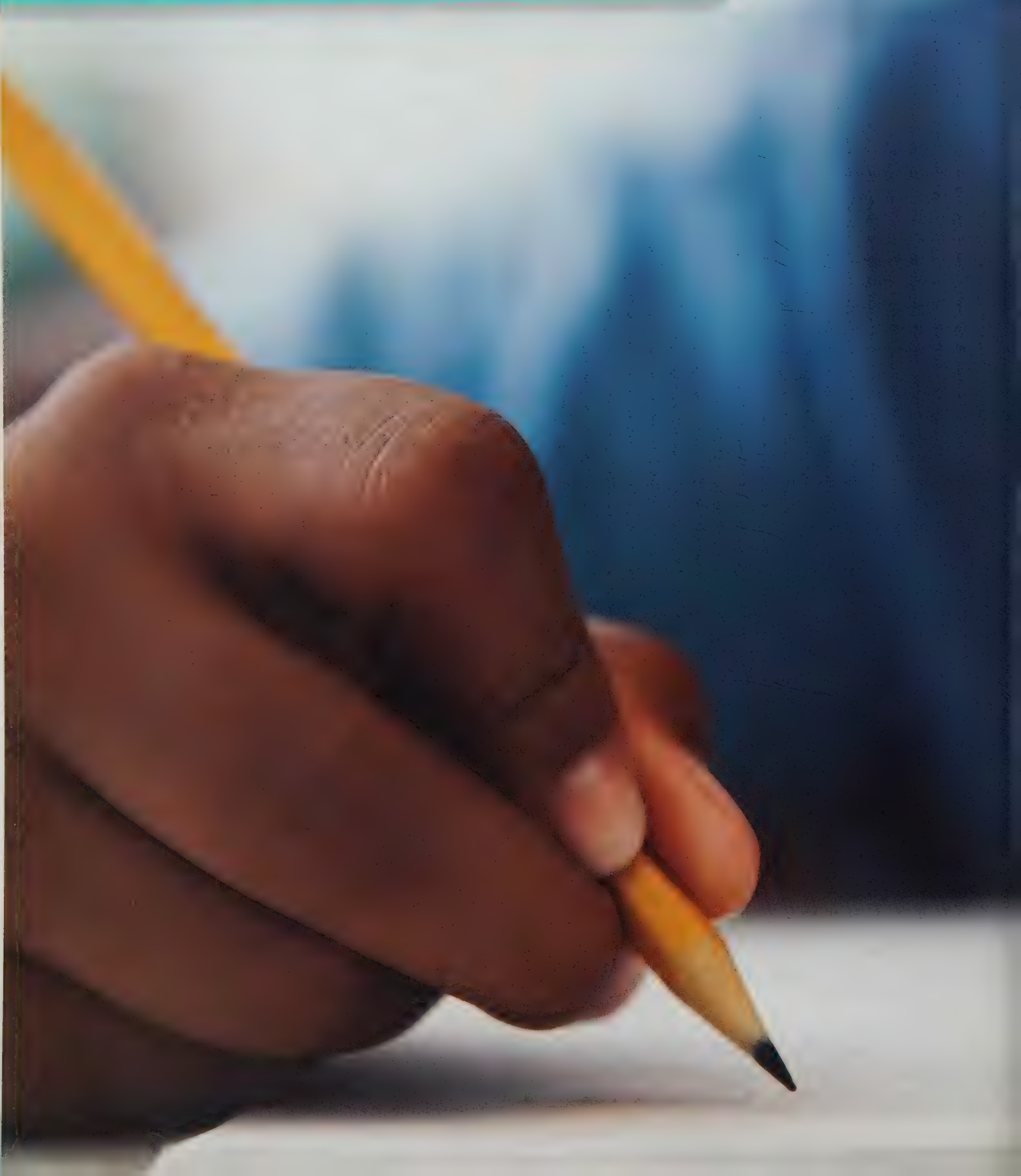
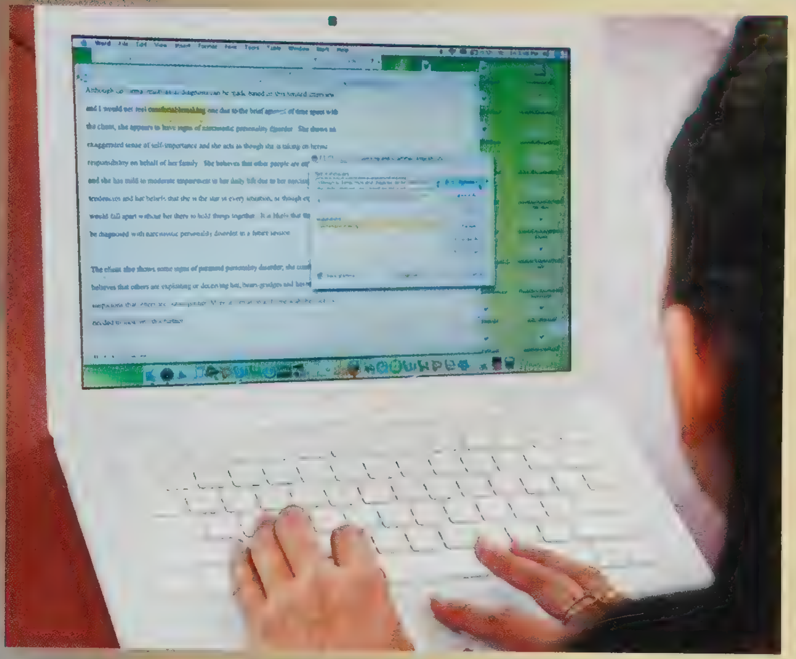


Grammar





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PARTS OF SPEECH

are the word categories of language. Words belong to the same category if they show the same formal features or if they share a common function or position in a sentence.

Introduction

Every word in a sentence has a job. A word can change jobs depending on when, where, and how it's used. We use labels called *parts of speech* to show how words work in different surroundings. There are eight parts of speech: nouns, pronouns, verbs, adjectives, adverbs, prepositions, conjunctions, and interjections.

Once you know a word's job, you can classify it as a particular part of speech. Some words act as only one part of speech. Other words are like lizards that change color to fit their surroundings. For example, the word *down* may function as a **noun,** a **preposition,** or an **adverb** as shown in the examples below.

noun	The quarterback passed on third **down.**
preposition	The ball sailed **down** the field.
adverb	The receiver fell **down.**

Parts of Speech	Jobs	Examples
Nouns	name people, places, things, or ideas	Susan, city, Tuesday, liberty
Pronouns	replace nouns	he, she, it
Verbs	express action or a state of being	flip, draw, read, are, was
Adjectives	describe nouns or pronouns	ugly, tired, peaceful
Adverbs	describe verbs, adjectives, or other adverbs	sadly, nicely, soon
Prepositions	express a relationship between a noun or pronoun and another word in the sentence	of, to, forward
Conjunctions	connect words	and, or, but
Interjections	express strong emotion	Wow! Oh! Gosh!

The parts of speech are like building blocks for every sentence. Many kids enjoy playing with blocks to construct all kinds of things they have seen and learned about. They use blocks of different shapes, sizes, and colors to build houses, forts, castles, towers, bridges, and roads. By trial and error, they learn how to use the blocks correctly so the things they build will stand on a solid foundation and not wobble or fall down.

At first, the structures they build with their blocks are simple, using just a few pieces. But as children grow, so does their understanding of the physical principles involved in building with blocks, and the things they construct become more interesting and complex. They may create an entire city with schools, roadways, airports, office buildings, hospitals, houses, stores, streets, and parks.

When children start to speak, their sentences, like their early block structures, are simple and small and make use of only a few parts of speech. As young people grow, so does their ability to use words in speaking, writing, and reading.

If you want to communicate well, learning how to correctly put together the different parts of speech is as important as choosing the right sizes or shapes of building blocks to use. Learning the principal building blocks of language provides a firm foundation for logical thinking and for clear communicating.

TAKING APART A MAKE-BELIEVE SENTENCE

When you understand how a language functions, you can identify the parts of speech even if most of the words are nonsense.

The plomic basinkers pirked the lampix at the simter ciptically.

Except for **the** and **at,** the sentence is made up of words that have no dictionary meaning. Yet it still reads like a sentence. It begins with a capital letter and ends with a period.

Even though the words are nonsense, they follow a familiar pattern and rhythm. More importantly, the words act like an English sentence. The word **basinkers** is a noun because it has a plural ending, **-s.** It also is marked by the article **the.** In the same way, **lampix** and **simter** are nouns because both are marked by the article **the.**

The word **plomic** is probably an adjective because it appears between an article and a noun. **Pirked** is probably a verb because **-ed** is a characteristic ending of past tense verbs. The word also has a position in the sentence typical of a verb. We know that **pirked** is a transitive verb because it has an object, **lampix.**

At the simter can be identified as a phrase that modifies pirked, telling where. **At** is a preposition often followed by an object. **Ciptically** is probably an adverb because it ends in **-ly.** It may modify **pirked.**

Nouns

Nouns are words that name people, places, things, or ideas. Most nouns are either

- Common or Proper
- Concrete or Abstract
- Singular or Plural

Common or Proper?

All nouns are either common or proper.

Common nouns are people, places, things, or ideas that are general and vague.

veterinarian holiday city friend

Proper nouns are people, places, things, or ideas that are specific and particular.

Dr. Fish Halloween Mumbai Jesse

Notice that proper nouns start with a capital letter.

Concrete or Abstract?

All nouns are either concrete or abstract.

Concrete nouns name things that you can see, touch, feel, taste, or smell.

pickles puppet pony picnic

Abstract nouns name things that cannot be seen or touched, like feelings or ideas.

jealousy justice jaunt joy

Nouns answer the question *who, what,* or *where.*

Sally ate sushi in Sacramento.

who? what? where?

Common names are general	Proper names are specific
puppet(s)	Cookie Monster Pinocchio Punch and Judy
game	Monopoly Charades Boggle
country	Iceland Japan Argentina

COOL COLLECTIVE NOUNS

A collective noun names more than one person or thing, but it's treated as a singular noun in a sentence. Collective nouns are often used in the animal kingdom to describe groups of creatures that live or travel together.

pride of lions
gaggle of geese
swarm of bees
murder of crows
brood of chicks
pack of dogs
flock of seagulls
litter of kittens

Check the dictionary if you're not sure about the plural form of a word.

Singular or Plural?

Most nouns are either singular or plural.

Singular nouns name one person, place, or thing.

cat circus ship match

Plural nouns name more than one person, place, or thing. Most plural nouns are formed by adding *-s* or *-es*.

cats circuses ships matches

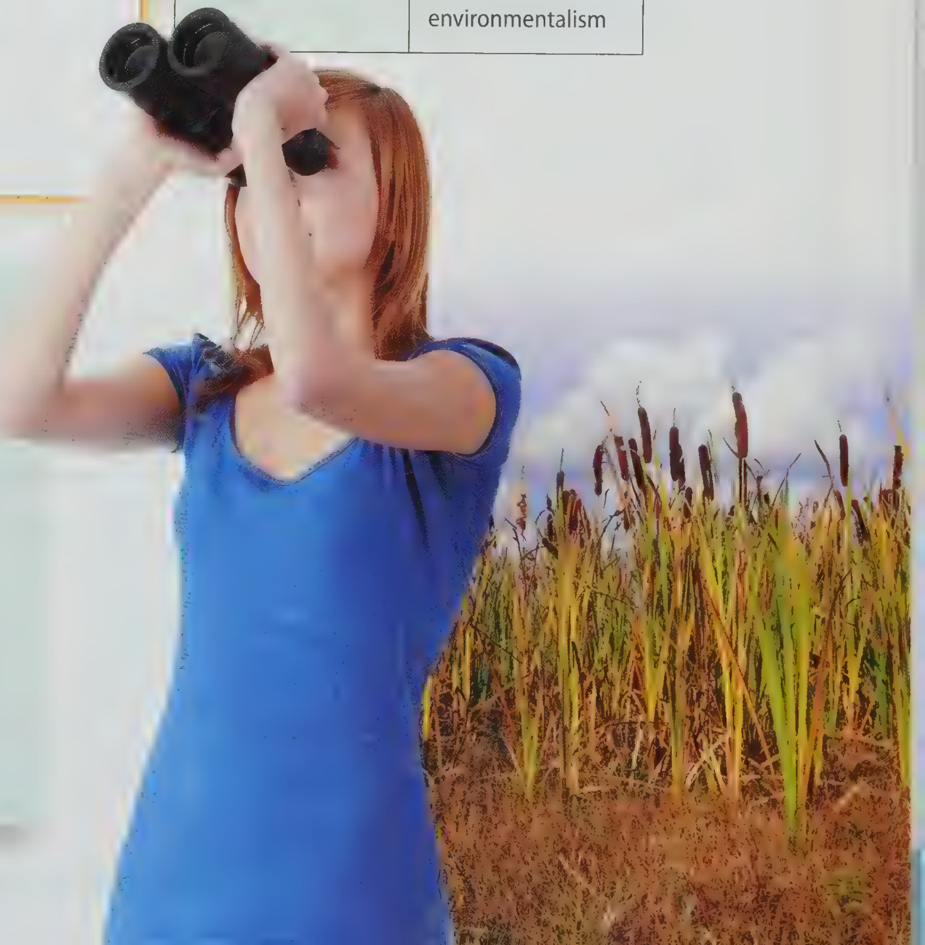
Other words change more in the plural form.

one mouse	three mice
a thief	many thieves
one child	two children

Some words just stay the same in the singular and plural forms.

one sheep	ten sheep
one fish	two fish
one moose	many moose

Nouns	
People	student scientist Jane Goodall
Places	wetland Central Park jungle
Things	binoculars birds tree
Ideas	exploration research environmentalism



WORDS AT WORK

This poem by Frank O'Hara is packed with nouns.

Today

Oh! kangaroos, sequins, chocolate sodas!
You really are beautiful! Pearls,
harmonicas, jujubes, aspirins! all
the stuff they've always talked about

still makes a poem a surprise!
These things are with us every day
even on beachheads and biers. They
do have meaning. They're strong as rocks.

Pronouns

Pronouns are words that take the place of nouns. They are often used to avoid repeating the same noun over and over.

The **ape** used to live at the zoo, but the **ape** recently escaped.

The **ape** used to live at the zoo, but **it** recently escaped.

The noun that a pronoun replaces is called an **antecedent.** In the sentence above, the noun *ape* is the antecedent of the pronoun *it.*

Types

There are five types of pronouns: *personal*, *relative*, *interrogative*, *demonstrative*, and *indefinite*.

COMPOUND SUBJECTS AND OBJECTS

Most people would not say, "Me have a new house." But a surprising number of people would say, "Jane and me have a new house." Whether a sentence has a single subject or a compound subject, the pronoun in the subject should be in the nominative case.

Similarly, to most people, "The rocks hit I" sounds wrong, while "the rocks hit Jane and I" sounds fine.

Whenever you have a compound object or subject including a pronoun, eliminate the other subject to see which case to use. If the pronoun is in the object position, receiving the action of the verb, or serving as the object of a preposition, use the objective case. (See also page 692.)

Personal pronouns refer to specific people or things. They can be

- the person speaking (first person):
  I think it's **my** fault.
- the person spoken to (second person):
  You should make **yourself** some cupcakes.
- the person spoken about (third person):
  Heather wants to teach **her** monkey some tricks.

Personal Pronouns

Number	Person	Nominative	Objective	Possessive
Singular	first	I	me	mine
	second	you	your	yours
	third	he	him	his
		she	her	hers
		it	it	its
Plural	first	we	us	ours
	second	you	you	yours
	third	they	them	theirs

Relative pronouns connect a noun in the main part of the sentence with another group of words.

Mr. Baggins is the artist **who** paints peacocks.

Mr. Baggins is the noun. *Who* is the relative pronoun.

The mystery **book, which** was a gift from Claire's brother, held her spellbound from beginning to end.

The **committee that** formulated this plan has been commended.

Interrogative pronouns are used to ask questions. The words are the same as relative pronouns.

What does this word mean?

Who is going with you?

Relative and Interrogative Pronouns

who
which
whom
that
whose
what

Case

Personal pronouns and the pronouns *who* and *whoever* change their form depending on how they are used in a sentence. These changes show the *case* of a pronoun. There are three cases: *nominative*, *objective*, and *possessive*.

WATCH OUT!

Never use an apostrophe with a possessive pronoun.

Incorrect: your's; it's; who's (*It's* and *who's* are contractions meaning *it is* and *who is*.)

Correct: yours; its; whose

Demonstrative pronouns point out certain people, places, or things.

> That is a huge pumpkin.
>
> Those are my favorite colors.
>
> This will be easy to fix.

Demonstrative Pronouns

- this
- these
- that
- those

Indefinite pronouns refer to people, places, or things in a more general (or vague) way than nouns do. Unlike other pronouns, indefinite pronouns have no antecedents. They are used when the appropriate noun is unknown or unspecified.

> The Lions Club sent invitations to everyone in the community.

Indefinite Pronouns

- all
- any
- anybody
- anyone
- anything
- either
- everybody
- everyone
- everything
- few
- most
- neither
- nobody
- none
- no one
- other
- several
- some
- somebody
- someone

Nominative Case

A pronoun used as the subject of a sentence is in the nominative case.

> I am the only one who knows how to get there.

Nominative Case Pronouns

- I
- we
- you
- he
- she
- it
- they
- who
- whoever

Pronouns used as predicate nominatives are also in the nominative case.

> It was I who called.

Increasingly, people don't use the nominative case in casual conversation. For example, many people say "it's me" instead of "it is I." It's also common to hear "that's him" instead of "that is he."

Objective Case

Use the objective case when a pronoun acts as a direct object, as an indirect object, or as an object of a preposition.

> Kelly saw him at the dance.

Objective Case Pronouns

- me
- us
- you
- him
- her
- it
- them
- whom
- whomever

Possessive Case

The possessive case indicates ownership.

> My favorite number is four.

Possessive Case Pronouns

- my
- mine
- your(s)
- his
- her(s)
- it(s)
- our(s)
- their(s)
- whose

Pronouns

Agreement

All pronouns must agree with their antecedents in number, gender, and person.

Agreement in Number

Use a singular pronoun to refer to or replace a singular antecedent.

Kameko decided that **she** would pack a picnic lunch.

Kameko is the antecedent of the pronoun *she.*

Use a plural pronoun to refer to or replace a plural antecedent.

Kameko's **sisters** decided that **they** would buy something there.

Sisters is the antecedent of the pronoun *they.*

Use a plural pronoun to refer to or replace two or more singular antecedents joined by *and.*

Shannon and Travis have seen **their** favorite movie six times.

Use a singular pronoun to refer to or to replace two or more singular antecedents joined by *or* or *nor.*

Either **Shannon or Travis** will bring **his** photographs to school.

Some indefinite pronouns—including *all, any, more, most, none,* and *some*—can be either singular or plural. Use either singular or plural pronouns to refer to or replace them, depending on the meaning of the sentence.

All of the color in the painting had lost **its** glow.

All refers to *color,* which is singular; *its* refers to *all.*

All of the students are required to bring **their** permission slips.

All refers to *students,* which is plural; *their* refers to *all.*

When an antecedent is a collective noun, decide whether the collective noun is singular or plural. If it is singular, use a singular pronoun to refer to it or replace it.

The **club** voted to change **its** meeting time and location.

The singular is used because the meeting time and location are for the entire club as one unit.

If the collective noun is plural, use a plural pronoun.

The city **council** argued among **themselves.**

The plural is correct in this example because the council members were arguing as individuals.

Agreement in Gender

The gender of a pronoun is either masculine, feminine, or neuter. A neuter pronoun is neither masculine nor feminine.

Use a pronoun that agrees in gender with its antecedent.

Dave leaned **his** bike against the fence.

Myra painted **her** room blue.

The **cat** licked **its** paw.

masculine	he	him	his
feminine	she	her	hers
neuter	it	it	its

Agreement in Person

Use a pronoun that agrees in person with its antecedent. Pronouns can be in the first, second, or third person.

First person: **I** get home before **my** parents do.

Second person: Will **you** get home before **your** parents do?

Third person: **Noel** will get home before **his** parents do.

SEXIST NOUNS AND PRONOUNS

Some writers use *mankind* as a term that includes both men and women. They use *he* as an inclusive pronoun and begin letters to unknown recipients with *Dear Sir.*

It is true that for centuries it was conventional to default to *he* in our language. More recently, however, people have become more aware of the unfairness of that practice. In today's world, traditional gender roles no longer apply. Doctors can be *shes* and nurses can be *hes,* so writers need to be careful about the pronouns they use.

Nonsexist language has evolved so that writers can use more inclusive neutral terms that apply to both sexes. Make sure you're careful to use nonsexist terms that apply to *people* in general.

- Use plural forms. "An engineer must pass **his** certification exam" becomes "Engineers must pass **their** certification exams."
- Add a pronoun. "An engineer must pass **his** certification exam" becomes "An engineer must pass **his or her** certification exam."
- Use another word that is not gendered. For instance, instead of **fireman** use **firefighter.** Instead of **chairman** use **chairperson.** Instead of **mailman,** use **mail carrier.** Instead of **mankind,** use **humanity.**

WHO VS. WHOM

Possibly someday the word **whom** will be obsolete. Already, there are many people who do not bother to distinguish between **who** and **whom.** But the rule that dictates which word to use is actually pretty simple.

Use **who** as a subject.

That is the boy **who** threw the rock.

Use **whom** as an object.

The girl for **whom** I bought the gift was delighted.

CLEAR PRONOUN REFERENCE

Incorrect pronoun usage will confuse your readers. Here are a few guidelines that will help you double-check your writing to make sure that all pronouns match their antecedents.

Do not use a pronoun that can refer to more than one antecedent, even if it means repeating the antecedent.

Unclear: Susan chose Lily to be on her team because she knows the game well.

Clear: Susan chose Lily to be on her team because Lily knows the game well.

In the first example, we are unsure of which of the two girls knows the game well.

Avoid using the pronouns *it, they, you,* or *your* without a clear antecedent.

Unclear: Before you give the baby its bottle, be sure to shake it.

Clear: Before you give the baby its bottle, be sure to shake the bottle.

In the first example, we do not know whether to shake the baby or the bottle.

Avoid unnecessary pronoun shifts that may change the meaning you intended.

Incorrect: I like summer best because **you** can swim in the ocean.

Correct: I like summer best because I can swim in the ocean.

Verbs

Verbs are words that express action or a state of being.

run appear sing seem hope

The verb is sometimes considered the part of speech that is at the heart of a sentence, mostly because it affects meaning more than any other element. It determines the number of nouns needed, and it guides the action that takes place. Verbs can indicate a state of being as well as a physical action. They can even focus a reader's attention on a particular part of a sentence.

Types of Verbs

There are two main types of verbs: **action verbs** and **helping verbs.**

TRANSITIVE OR INTRANSITIVE?

Many verbs can be either transitive or intransitive depending on how they are used in a sentence. If a verb takes a direct object, it is a transitive verb. If it does not take a direct object, it is an intransitive verb.

Transitive: Mary writes a letter.
Intransitive: Mary writes beautifully.
Transitive: John walked his dog.
Intransitive: John walked to the store.
Transitive: I read three books a week.
Intransitive: I read quickly.

Action Verbs

Action verbs describe the behavior, or action, of someone or something and may express either physical action or mental activity.

The band marched down Northfield Avenue.

He believes in working hard.

All action verbs are either **transitive** or **intransitive.**

Transitive verbs express an action that is performed on something. A transitive verb needs an object, called the object of the verb, to complete its meaning.

Michael bought a monkey.

Wally painted his house last summer.

Intransitive verbs have no direct object. (See page 676.)

The subway stopped abruptly.

Jane smiled with pleasure.

Linking verbs such as *be* and *have* are always intransitive.

Common Linking Verbs

- appear
- sound
- feel
- taste
- look
- become
- seem
- smell

Note: Most linking verbs can also be used as action verbs.

Linking verb: The hour grew late.

Action verb: The children grew quickly.

Helping Verbs

Sometimes a verb needs the help of another word called the **helping verb.** Together, a main verb and a helping verb form a *verb phrase.*

Will you be waiting for me after school?

Waiting is the main verb, *will* and *be* are the helping verbs, and *will be waiting* is the verb phrase.

Susan ought to study her grammar more.

To study is the main verb, *ought* is the helping verb, and *ought to study* is the verb phrase.

Common Helping Verbs

- am
- be
- been
- could
- do
- had
- have
- may
- must
- shall
- was
- will
- are
- being
- can
- did
- does
- has
- is
- might
- ought
- should
- were
- would

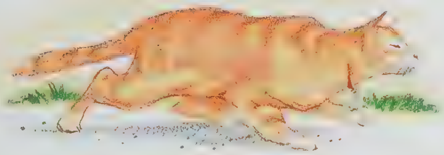
fyi!

Adjectives that follow linking verbs are called predicate adjectives. *Nouns or pronouns that follow linking verbs are called* predicate nominatives *or* predicate nouns.

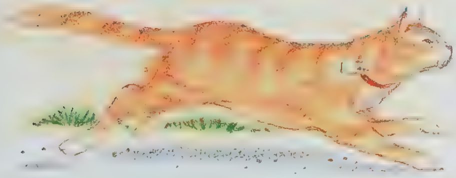
Tense

Verbs have several characteristics, like *tense,* that other parts of speech do not have.

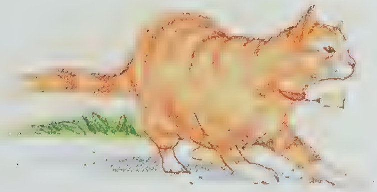
The tense of a verb indicates the time of the verb's action. There are six verb tenses: *present, present perfect, past, past perfect, future,* and *future perfect.*



Future: The cat **will run.**



Present: The cat **runs.**



Past: The cat **ran.**

WHY ARE THERE SIX TENSES?

There are three major divisions of time: past, present, and future. In each of these time frames, the action may be considered as simple (simply occurring at the particular moment) or perfect (the action is completed or "perfected").

Present Tense

The present tense is used when something is happening now, in the present. It also indicates habitual action, which is something that happens regularly.

I **see** at least one movie a week.

Past Tense

The past tense is used when something happened before today, in the past.

I **saw** a good movie two days ago.

Future Tense

The future tense is used if something will happen later, in the future.

I **will see** the new Disney movie this weekend.

Present Perfect Tense

The present perfect tense is used when something was started in the past but completed in the present moment.

I **have seen** two movies so far this month.

Past Perfect Tense

The past perfect tense is used when something happened before another past action or event.

I **had not seen** many movies before last year.

Future Perfect Tense

The future perfect tense is used for actions that will be completed at some future time.

By the end of this year, I **will have seen** more than 50 movies.

POP CORN
POP CORN

Verbs

It's important to understand the difference between the words **was** *and* **were**. *Use* **were** *to express doubt or a wish. Use* **was** *for statements of fact.*

REGULAR OR IRREGULAR?

- Verbs are either regular or irregular.
- Most verbs are regular verbs. They form their past and their past participle the same way, by adding *-ed* or *-d* to the infinitive.
- Irregular verbs do not follow the standard rules for forming the past and the past participle. The only way to master the past and past participial forms of irregular verbs is to memorize them or check the dictionary.

Mood

The mood of a verb suggests your attitude toward the sentence. There are three moods: indicative, imperative, and subjunctive.

Indicative Mood

The indicative mood is used for statements of fact or questions.

I **am coming**. **Are** you **coming**?

Imperative Mood

The imperative mood is used to give commands.

Come here!

Subjunctive Mood

The subjunctive mood is used to express wishes and statements contrary to fact.

I wish you **were** here.

If you **were** here, I would be happy.

Common Irregular Verbs

Present	Past	Past Participle	Present	Past	Past Participle
beat	beat	beat, beaten	leave	left	left
bend	bent	bent	lend	lent	lent
bleed	bled	bled	lie	lay	lain
bring	brought	brought	make	made	made
build	built	built	pay	paid	paid
catch	caught	caught	read	read	read
cost	cost	cost	ring	rang	rung
creep	crept	crept	rise	rose	risen
dig	dug	dug	see	saw	seen
drink	drank	drunk	set	set	set
eat	ate	eaten	shake	shook	shaken
fall	fell	fallen	shut	shut	shut
fight	fought	fought	sit	sat	sat
forget	forgot	forgotten	speak	spoke	spoken
freeze	froze	frozen	spring	sprang	sprung
get	got	gotten	stick	stuck	stuck
grow	grew	grown	swim	swam	swum
hear	heard	heard	take	took	taken
hold	held	held	tear	tore	torn
keep	kept	kept	throw	threw	thrown
know	knew	known	wear	wore	worn

Modals

The auxiliary verbs *can, could, may, might, must, will,* and *should* are known as modals. These verbs are used with main verbs to add emphasis to a sentence or to provide shades of meaning.

Can and *could* express the ability to perform the action of the main verb.

We **can** walk faster if you like.

She **could** have walked with us yesterday if we had called her.

May and *might* express permission or possibility rather than ability.

The doctor said my mother **may** walk now that her ankle has healed.

The test **might** be canceled.

Must expresses a requirement or a certainty.

Sheila **must** call her parents immediately.

Gunnar **must** have been wrong about the date of the meeting.

Should suggests that something ought to happen.

Sheila **should** call home immediately.

Conjugation

To conjugate a verb means to list all of the forms for its six tenses. The conjugation of a verb also shows how the verb forms change for

- the first person, the second person, and the third person
- the singular and the plural

The full conjugation of a verb is the orderly presentation of all its forms.

The infinitive is the basic form of a verb. It usually begins with the word *to.*

to walk	**to** run
to win	**to** lose

CHOOSING EFFECTIVE VERBS

As you write, select specific verbs whenever you can. Strong verbs give your readers precise, clear images.

Dull: The puppy **walked** into the kitchen and **lay** down.

Vivid: The puppy **waddled** into the kitchen and **sprawled** on the floor.

CONJUGATION OF *TO FALL*

Principal parts

Present	Present Participle	Past	Past Participle
fall	(is) falling	fell	(has) fallen

	Singular	Plural
Present tense	I fall you fall he, she, it falls	we fall you fall they fall
Past tense	I fell you fell he, she, it fell	we fell you fell they fell
Future tense	I will fall you will fall he, she, it will fall	we will fall you will fall they will fall
Present perfect	I have fallen you have fallen he, she, it has fallen	we have fallen you have fallen they have fallen
Past perfect	I had fallen you had fallen he, she, it had fallen	we had fallen you had fallen they had fallen
Future perfect	I will have fallen you will have fallen he, she, it will have fallen	we will have fallen you will have fallen they will have fallen

Adjectives

Adjectives are words that make the meaning of a noun or pronoun more specific. Most adjectives are either *descriptive* or *limiting*.

Descriptive adjectives can be

- positive
- comparative
- superlative

Limiting adjectives can be

- numerical
- pronominal
- articles

Adjectives are words that modify nouns and pronouns. *Modify* means to change. An adjective changes the meaning of a noun or pronoun by describing it or making it more specific.

Use adjectives to add detail, to make distinctions, and to be precise when you write.

Adjectives answer the question *how many, what kind,* or *which.*

Two raccoons ate **rotten red** apples from the **trash** can.

how many? *what kind?* *which?*

Adjectives make descriptions sharper and more interesting.

They describe

- color
- smell
- number
- size
- taste
- feel
- shape
- age

Descriptive or Limiting?

Descriptive adjectives indicate a quality or condition of a noun. They provide additional information about the noun or pronoun that is being described.

strange **sad** **stingy** **super** **simple**

Limiting adjectives point out nouns or indicate their number or quantity.

ten **two** **their** **third** **that**

Limiting adjectives can be classified as **numerical, pronominal,** or **articles.**

Numerical adjectives can be cardinal or ordinal.

Cardinal adjectives are numbers that tell how many.

The manuscript contained **ten** pages.

Six people were in the room.

Ordinal adjectives are numbers that explain order.

Our team came in **third** place.

The **second** step is broken.

Pronominal adjectives are pronouns that are used as adjectives.

Welcome to **our** home.

Several questions came up.

Articles—the words **a**, **an**, *and* **the**—*used to be considered a separate part of speech, but now they're thought of as a special type of adjective.* **A** *and* **an** *are called indefinite articles;* **the** *is called a definite article.*

Degrees of Comparison

The degree to which a descriptive adjective indicates a quality or characteristic can be stated as a comparison. There are three degrees of comparison: *positive*, *comparative*, and *superlative*.

The positive degree states the quality or characteristic.

attentive tall good

The comparative degree expresses a degree higher or lower than the positive.

more attentive taller better

The superlative degree expresses the highest or lowest degree of the quality or characteristic.

most attentive tallest best

Downward comparisons use the words *less* or *least*.

Positive	Comparative	Superlative
tall	less tall	least tall
stubborn	less stubborn	least stubborn
reasonable	less reasonable	least reasonable

Upward comparisons use the word endings *–er* or *–est* when the word is one syllable.

Positive	Comparative	Superlative
tall	taller	tallest
high	higher	highest
sweet	sweeter	sweetest

Upward comparisons use the words *more* or *most* when the word is two syllables or more.

Positive	Comparative	Superlative
stubborn	more stubborn	most stubborn
attentive	more attentive	most attentive
reasonable	more reasonable	most reasonable

Occasionally, comparative adjectives are irregular.

Positive	Comparative	Superlative
good	better	best
bad	worse	worst
little	less	least

Placement

Adjectives usually precede the nouns and pronouns they modify.

The skinny young man always ate as much as he wanted.

Sometimes, an adjective is placed after the noun or pronoun for variety or for special emphasis.

The man, skinny and young, always ate as much as he wanted.

WATCH OUT!

Never use *more* or *most* when adding the suffixes *–er* and *–est* to adjectives. This is known as a double comparison.

Incorrect: Sandra was **more smarter** than Caroline.
Correct: Sandra was **smarter** than Caroline.

Incorrect: Harold is the **most tallest** person I know.
Correct: Harold is the **tallest** person I know.

WORDS AT WORK

This poem by Wallace Stevens is enlivened with adjectives.

The Emperor of Ice-Cream

Call the roller of big cigars,
The muscular one, and bid him whip
In kitchen cups concupiscent curds.
Let the wenches dawdle in such dress
As they are used to wear, and let the boys
Bring flowers in last month's newspapers.
Let be be finale of seem.
The only emperor is the emperor of ice-cream.

Take from the dresser of deal,
Lacking the three glass knobs, that sheet
On which she embroidered fantails once
And spread it so as to cover her face.
If her horny feet protrude, they come
To show how cold she is, and dumb.
Let the lamp affix its beam.
The only emperor is the emperor of ice-cream.

More on Modifiers

Adjectives and adverbs expand our experience of language. These parts of speech are often called *modifiers* because they alter, refine, and sharpen our writing.

Sometimes it's hard to distinguish between adjectives and adverbs, which leads to usage problems.

The most important difference between these two parts of speech is the type of words that they modify. Adjectives modify nouns and pronouns, while adverbs modify verbs, adjectives, and other adverbs.

	Parts of Speech Modified	Information Provided
Adjectives	nouns pronouns	which? how many? what kind?
Adverbs	verbs adjectives other adverbs	how? when? where? to what extent?

WATCH OUT!

Don't craft adverbs that sound unnatural. As William Strunk wrote in his influential guide to writing, *The Elements of Style*, "Do not dress words up by adding *–ly* to them, as though putting a hat on a horse."

Awkward: Emily responded **hesitatingly** to her mother's question.

Clear: Emily **hesitated** before responding to her mother's question.

Placement

Putting modifiers in the wrong place can create confusing sentences. To avoid misplacing modifiers, put the adjective or adverb as close as possible to the word you wish to modify.

Unclear: Strolling by the lake, a family of geese walked in front of Jaime.

Clear: Strolling by the lake, Jaime noticed a family of geese in front of him.

Unclear: Grilled over charcoal, we particularly enjoy vegetables like zucchini.

Clear: We particularly enjoy vegetables like zucchini grilled over charcoal.

Consider the difference between these two sentences.

The **daily** newspaper has excellent local sports coverage.

The adjective *daily* modifies the noun *newspaper.*

Arden delivers the newspaper **daily.**

The adverb *daily* modifies the verb *delivers.*

Sometimes adverbs just don't work with nouns.

Jim is a **loudly** guy.

It's a **truly** story.

Choosing the right form can be tricky. Look at these two sentences. Which is right?

The pizza tasted **bad.**

The pizza tasted **badly.**

Bad is an adjective that modifies the noun *pizza*. The adverb *badly* doesn't work because it means the pizza isn't good at tasting.

Adjectives usually follow verbs that relate to the five senses—touch, taste, sound, smell, and sight. That's because sense words modify nouns, not verbs.

Your dog smells **funny.**

not

Your dog smells **funnily.**

Adverb or Adjective?

Many words that end in *–ly* are adverbs.

surely **strongly** **sharply**

However, some words that end in *–ly* are adjectives.

lovely **manly** **friendly** **cowardly**

And some adverbs do not end in *–ly*.

here **there** **far** **soon**

Some words can be used as either adverbs or adjectives.

Word	Adverb	Adjective
deep	Dig *deep* to find water.	We dug a *deep* well.
far	We walked *far* into the forest.	He came from a *far* country.
hard	Mark hit the ball *hard.*	It was a *hard* choice.
little, long	The world will *little* note nor *long* remember . . .	He had *little* feet and *long* legs.
near	The horse came *near.*	It was a *near* escape.
right	Turn *right* at the stop sign.	That was the *right* way to turn.
straight	He drew his lines *straight.*	He walked a *straight* line.

Other words that can be used as either adverbs or adjectives include *close, daily, first, hard, high, late, only,* and *tight.*

In some cases, usually in short commands, the two forms have the same meaning and can be used interchangeably. The shorter form is considered to be more informal.

Go **slow** around that curve. (adjective)

Go **slowly** around that curve. (adverb)

More often, the two forms have different meanings and cannot be used interchangeably.

Sam hit the **hard** ball. (adjective)

Nancy **hardly** had time to catch her breath. (adverb)

COMMONLY CONFUSED ADJECTIVES AND ADVERBS

Bad vs. Badly

Bad is an adjective. Badly is an adverb.

Use *bad* when you want to describe how you feel. If you're modifying a verb, use *badly.*

She sang so **badly** that the audience felt **bad.**

Good vs. Well

Good is an adjective. Well is an adverb.

Never use *good* as a substitute for *well.*

The team played **well,** which made me feel **good.**

Real vs. Really

Real is an adjective. Really is an adverb.

In conversation, *real* is often used for *really.* Be careful in writing, however, to distinguish between the two.

The test was a **real** challenge, but I think I did **really** well.

AVOID WEAK MODIFIERS

Use precise adjectives and adverbs to create colorful, vivid details. Avoid using vague adverbs such as *very, really, only, well, hard,* and *often.* Suppose a friend sent you an e-mail with the sentence:

Mel wore a **very nice** dress.

What does that really tell you about Mel's dress? "Very nice" could mean a lot of things. Was the dress expensive? Attractive? Flattering? Fancy?

Mel wore a **stylish red** dress.

Ah, so *that's* what your friend meant. Why didn't she just say so?

Prepositions

A preposition is a word that combines with a noun to form a phrase, or a small group of words. This phrase shows the relationship between a noun or pronoun and some other word in the sentence.

Prepositions usually state direction, time, or position.

Direction: The plane flew **over** the clouds.

Time: The audience became restless **during** his speech.

Position: Please place the book **on** that table.

Prepositional Phrase

Prepositions rarely stand alone. Together, the preposition, the object of the preposition, and the modifiers of that object form a *prepositional phrase*. A prepositional phrase works as an adjective to modify a noun. (See also page 678.)

The book **with the torn cover** is Henry's.

The prepositional phrase *with the torn cover* modifies the noun *book*.

The object of the preposition is the noun or pronoun that follows the preposition. The preposition describes the object's relationship to another word in the sentence.

The deer are running **toward** the **woods.**

The preposition is *toward*, while *woods* is the object of the preposition.

How Many Prepositions?

When two or more words associated with different prepositions appear before one object, do not delete any of the prepositions.

Charlie was interested **in** and curious **about** local politics.

When two or more words associated with the same preposition appear before one object, all but the last preposition may be deleted.

"Charlie was interested **in** and involved **in** local politics" may be written as, "Charlie was interested and involved **in** local politics."

WORDS AT WORK

Prepositions are important in this poem by William Carlos Williams.

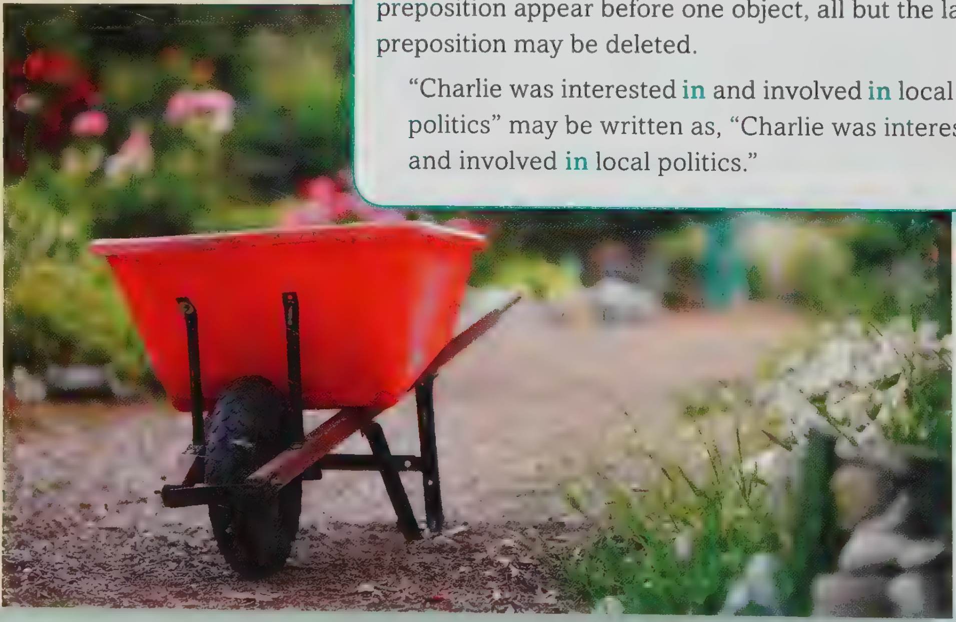
The Red Wheelbarrow

so much depends
upon

a red wheel
barrow

glazed **with** rain
water

beside the white
chickens.



Preposition or Adverb?

Many words that are prepositions can also be adverbs. Sometimes it's hard to tell the difference. *Out* is an example of a word that can be a preposition or an adverb.

Preposition: The cat ran **out** the door.
The preposition *out* takes the object *door*.

Adverb: The cat ran **out.**
The adverb *out* modifies the verb *ran*.

Another example of a word that can do double-duty is *down*.

Preposition: We stumbled **down** the hill.
The preposition *down* takes the object *hill*.

Adverb: We stumbled **down.**
The adverb *down* modifies the verb *stumbled*.

When you're not sure whether the word functions as a preposition or an adverb, check if it takes an object. If it does take an object, it's a preposition. If it doesn't, it's an adverb.

WATCH OUT!

The subject pronouns **I**, **he**, **she**, **we**, and **they** cannot be used with prepositions. Instead, use the object pronouns **me**, **him**, **her**, **us**, and **them**.

Which Preposition to Use?

Specific prepositions are often closely associated with other nouns, adjectives, and verbs.

account for
argue with
capable of
confide in
obedient to
foreign to
happy about
independent of
inseparable from
tamper with
protest against
sensitive to
similar to
sympathize with
envious of

Other times, one word may be associated with several different prepositions. In this case, each preposition provides a slightly different shade of meaning.

angry at
angry with
apply for
apply to
careless about
careless of
concerned for
concerned with
free from
free of
part from
part with

In a good dictionary, the entry for the principal word will explain the subtle differences in meaning created by using different prepositions.

FREQUENTLY USED PREPOSITIONS

about
above
across
after
against
around
as far as
at
because of
before
behind
below
beneath
beside
between
by
down
during
except
for
from
in
in spite of
of
off
on
out
over
throughout
to
toward
under
until
up
with

Conjunctions and Interjections

Conjunctions are connecting words. The three kinds of conjunctions are

- **coordinating**
- **correlative**
- **subordinating**

Conjunctions are like bridges that join words, phrases, or sentences.

Coordinating conjunctions connect words or sentence parts that are similar and equal.

Coordinating conjunctions connect words with words.

It's raining cats *and* dogs.

The conjunction *and* connects the nouns *cats* and *dogs*. *Cats* and *dogs* are considered equal because it doesn't matter which one comes first.

Coordinating conjunctions connect phrases with phrases.

Can you rub your stomach *and* pat your head at the same time?

The conjunction *and* connects the phrases *rub your stomach* and *pat your head*.

Coordinating conjunctions combine sentences.

She wanted a cat, *but* she settled for a hermit crab.

The conjunction *but* connects the sentences *She wanted a cat* and *she settled for a hermit crab*.

Correlative conjunctions are pairs of words that work like coordinating conjunctions.

both/and
not only/but also
either/or
so/as
neither/nor
whether/or

Incorrect: Rita is both talented and makes friends easily.

Correct: Rita is both talented and friendly.

Incorrect: We went not only to the bank but also grocery shopping.

Correct: We went not only to the bank but also to the grocery store.

GOT TO KNOW

The word **FANBOYS** will help you remember the seven coordinating conjunctions.

For **A**nd **N**or **B**ut **O**r **Y**et **S**o

WATCH OUT!

Make sure you use correlative conjunctions to join words and phrases that are similar (nouns with nouns, adjectives with adjectives, etc.).

Incorrect: Kim will take **either** her bike **or** will drive to get down to the beach.

Correct: Kim will take **either** her bike **or** her car to get down to the beach.

Subordinating conjunctions introduce dependent clauses, sometimes called subordinate clauses, which are phrases that cannot stand alone as a complete sentence. (See page 680.)

> We will meet for practice on the field tomorrow **unless it rains.**

The subordinating conjunction *unless* connects the subordinate clause *unless it rains* to the main part of the sentence.

> You must finish your homework **if you want to go out.**

The subordinating conjunction *if* connects the subordinate clause *if you want to go out* to the main part of the sentence.

Some Subordinating Conjunctions

after
although
as
as if
because
before
but
how
if
in order that
since
so that
that
though
unless
until
when
where
while
why

Interjections are words or phrases that express emotions that are strong or sudden.

Interjections that stand alone are usually followed by exclamation points.

> **Wow!** That's the biggest squirrel I've ever seen.

Other interjections are set off with commas within a sentence.

> **Oh,** don't worry about it.

Common Interjections

oh
ouch
bravo
whoops
no
wow
whoa
stop
yikes

Interjections are spoken more often than they are written. Try not to use them often in your writing. Using too many interjections will dull the very impact you are trying to create.

SENTENCES

A sentence is a group of words that expresses a complete thought. Each sentence is a unit that begins with a capital letter and ends with a punctuation mark. Most sentences have several words. *(She was frightened by the rooster.)* Other sentences have just a noun and a verb. *(We ate.)* And occasionally, sentences have just one word. *(Hello!)* By understanding how sentences work, you will learn to choose the best ways to express your ideas.

A complete sentence has a subject and a predicate. These are the most basic units of a sentence.

Subject + Predicate = Sentence

Sentence Types

You can classify sentences according to the speaker's purpose. Most sentences just make a statement. Other sentences ask a question, express strong feelings, make a request, or give a command.

There are four basic types of sentences: *declarative*, *interrogative*, *exclamatory*, and *imperative*.

	Purpose	Punctuation
Declarative	to state something plainly	period (.)
Interrogative	to ask a question	question mark (?)
Exclamatory	to express a strong feeling	exclamation mark (!)
Imperative	to make a request or a command	period (.) or exclamation mark (!)

Sentence Parts

Subject

The **subject** of a sentence includes a noun or pronoun plus its modifiers. (Remember that modifiers are words that add to the meaning of nouns and verbs.) The subject names the person, place, thing, or idea that the sentence is about.

Every subject is built around a noun or a pronoun, which is the **simple subject.** It's easy to find the simple subject in a short sentence.

Jane hates strawberries.

This sentence is about Jane, so that is the simple subject.

Wordy sentences are more likely to have complex subjects.

Mr. Wong's fried dumplings are the best I've tasted.

In the sentence above, *Mr. Wong's fried dumplings* is the **complete subject** (the noun *dumplings* plus its modifiers). *Dumplings* is the noun at the heart of the sentence, so it is the simple subject.

The **stack of dirty dishes in the sink** must be washed.

Stack of dirty dishes in the sink is the complete subject (the simple subject *stack* plus its modifiers).

Direct Objects

A **direct object** is a part of the predicate that follows an action verb. It names the person or thing that receives the action of the verb.

John hits the ball.

The ball receives the action of hitting, so it is the direct object.

Margaret writes letters to her cousin.

The letters receive the action of writing, so they are the direct object.

An easy way to find the direct object in a sentence is to find the subject and verb and then ask "*what?*" or "*whom?*"

Dan likes soccer.

Dan likes what? He likes soccer, which makes that word the direct object.

Rachel called Mitch.

Rachel called whom? She called *Mitch*, which makes him the direct object.

Predicate

The predicate is the part of the sentence that explains what the subject is doing.

Penny went to clown school.

Every predicate is built around a verb, which is the **simple predicate.** It's easy to find the simple predicate in a short sentence.

He cooked dinner.

Long sentences are more likely to have complex predicates.

The eagle rose from the nest with stately grace.

Rose from the nest with stately grace is the **complete predicate** (the verb *rose* plus its modifiers). The verb *rose* is the simple predicate.

The track team ran at a slow pace.

Ran at a slow pace (the simple predicate *ran* plus its modifiers) is the complete predicate.

Compound Subjects and Predicates

A **compound subject** consists of two or more nouns or pronouns that are the subject of the same verb.

Sandra and Jim went to the movies.

Sandra and *Jim* are subjects of equal importance, so they form a compound subject.

A **compound predicate** consists of two or more verbs that have the same subject.

The soda bubbled and fizzed in the glass.

The verbs *bubbled* and *fizzed* form a compound predicate because they refer back to the subject *soda.*

Indirect Objects

An **indirect object** names the person or thing to whom or for whom the action of the verb is being performed. It comes immediately after the verb and before the direct object.

John threw the dog a bone.

To whom was the bone thrown? It was thrown to the dog, which makes *dog* the indirect object.

Jennifer sent her friend a letter.

To whom was the letter sent? It was sent to Jennifer's friend, which makes *friend* the indirect object.

An easy way to find the indirect object in a sentence is to find the direct object and then ask *to whom?* or *to what?*

Compound Objects

Just as there are compound subjects and predicates, there are compound objects. A compound object consists of two or more objects that complete the same predicate.

Jake read several books and articles about icebergs.

Books and *articles*—the things that are being read—form a compound direct object.

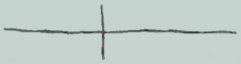
Marlene offered Carol and her brother tickets to the game.

The words *Carol* and *brother*—the people who receive the tickets—form a compound indirect object.

Diagrams

Diagramming is one way to grasp all the parts of a sentence and their relationship to each other. A sentence diagram is like a map of a sentence. It arranges the words in a way that makes it easy to see their relationships.

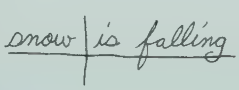
Always begin your diagram with a horizontal line crossed with a vertical line.



Simple Sentences

On the left side of the horizontal line, write the simple subject of the sentence. The simple subject is either the main noun in a noun phrase or a pronoun. On the right side of the line, write the verb, including any auxiliaries.

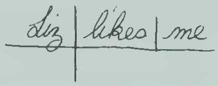
Snow is falling.


snow
is falling

Direct Objects

When the verb is transitive, draw a vertical line to the right of the verb. The vertical line should meet but not cross the base line. To the right of this line, write the main noun or pronoun of the direct object.

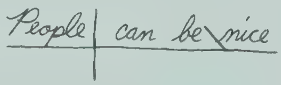
Liz likes me.


Liz
likes
me

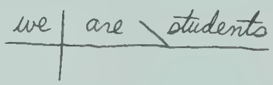
Linking Verbs

If the sentence has a linking verb, draw a diagonal line to the right of the verb. The line should meet but not cross the base line. To the right of this line, write the adjective complement or the predicate nominative.

People can be nice.


People
can be
nice

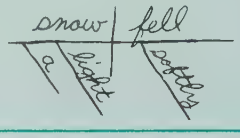
We are students.


we
are
students

Modifiers

If there are adjectives (including determiners) modifying a noun, put them on a diagonal line beneath the noun. Do the same with adverbs modifying the verb.

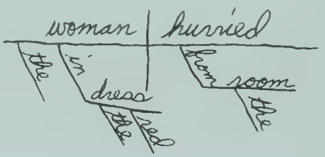
A light snow fell softly.


snow
fell
a
light
softly

Prepositions

When diagramming a prepositional phrase, write the preposition on a diagonal line beneath the word that the phrase modifies. Then write the noun or pronoun on a horizontal line, and put any adjective modifiers on diagonal lines below.

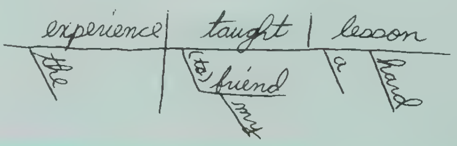
The woman in the red dress hurried from the room.


woman
hurried
the
in
dress
the
red
from
room
the

Indirect Objects

Diagram an indirect object as if it were a prepositional phrase with the preposition *to* understood. Put *to* in parentheses.

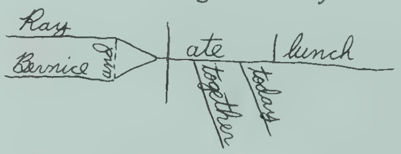
The experience taught my friend a hard lesson.


experience
taught
lesson
the
(to)
friend
my
a
hard

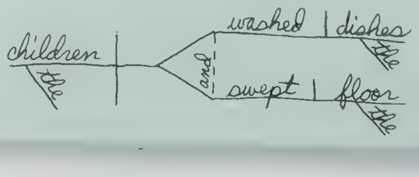
Compound Subjects or Predicates

To diagram compound sentence elements, place them on parallel lines. Connect the two lines with a dotted vertical line, and write the coordinating conjunction on it.

Ray and Bernice ate lunch together today.


Ray
Bernice
and
ate
lunch
together
today

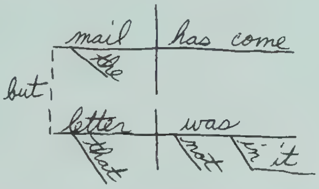
The children washed the dishes and swept the floor.


children
the
and
washed
dishes
the
swept
floor
the

Compound Sentences

To diagram a compound sentence—that is, two simple sentences joined by a coordinating conjunction—diagram each of the simple sentences, one above the other. Then connect them with a vertical dotted line and write the conjunction on that line.

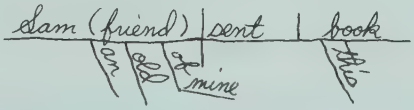
The mail has come, but that letter was not in it.


mail
has come
the
but
letter
was
that
not
in
it

Appositives

To diagram an appositive, put the appositive noun or pronoun in parentheses after the noun that it renames or identifies.

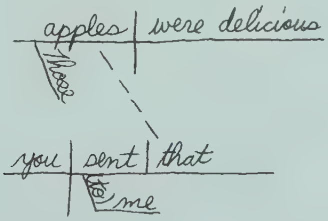
Sam, an old friend of mine, sent this book.


Sam (friend)
sent
book
an
old
of
mine
this

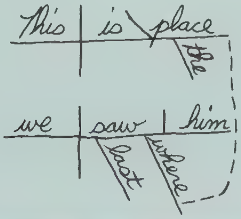
Relative Clauses

To diagram a relative clause, diagram the clause on its own line below the main line. With a dotted line, connect the noun that's being modified to the relative pronoun or relative adverb.

Those apples that you sent me were delicious.


apples
were delicious
those
you
sent
that
to
me

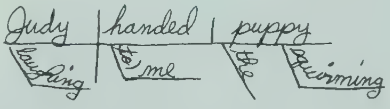
This is the place where we saw him last.


This
is
place
the
we
saw
him
last
where

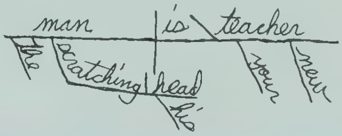
Participles and Participial Phrases

A participle is diagrammed in a curve along a diagonal and a horizontal line. The complement of the participle, if any, is to the right on the horizontal line.

Laughing, Judy handed me the squirming puppy.


Judy
handed
puppy
laughing
to
me
the
squirming

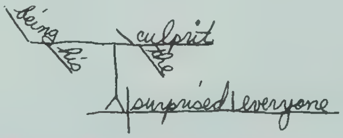
The man scratching his head is your new teacher.


man
is
teacher
the
scratching
head
his
your
new

Gerunds

A gerund is diagrammed the same way as a participle, except that it is placed on a pedestal. The bottom of the pedestal rests on the base line in a noun's position.

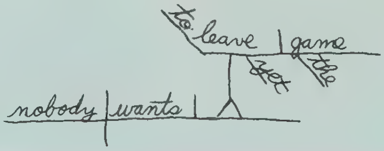
His being the culprit surprised everyone.


being
his
culprit
the
surprised
everyone

Infinitive Phrases

An infinitive phrase occupying a noun position is diagrammed the same way as a gerund.

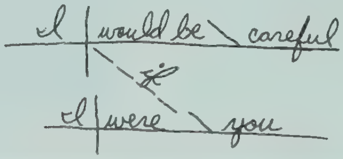
Nobody wants to leave the game yet.


to
leave
game
yet
the
nobody
wants

Adverbial Clauses

An adverbial clause is diagrammed on a line below the base line. The subordinating conjunction is written along a diagonal dotted line that connects the word being modified to the clause's verb.

If I were you, I would be careful.


I
would be
careful
if
I
were
you

PUNCTUATION is the use of certain marks in writing. It has one purpose: to make writing clear and easy to read. Like traffic signs, punctuation tells readers when to slow down, when go carefully, and when to pause or stop. This section covers 13 types of punctuation.

- Periods
- Question marks
- Exclamation points
- Commas
- Semicolons
- Colons
- Dashes
- Hyphens
- Ellipses
- Parentheses
- Brackets
- Quotation marks
- Apostrophes

End Punctuation

There are three kinds of end punctuation: the period, the question mark, and the exclamation point. End punctuation brings the reader to a full stop. Every sentence must end with one of these marks.

Periods are at the end of an ordinary sentence.

Ben is a good dog.

We should leave.

Question marks ask for information instead of stating it.

Is Ben a good dog?

When should we leave?

Exclamation points give a sentence extra "oomph."

Ben is a good dog!

We have to leave now!

Punctuation marks are signals that tell readers to slow down, speed up, or take a break. ▼

Punctuation Mark	Meaning
Period (.)	Signals a strong pause
Comma (,)	Signals a weak pause
Semicolon (;)	Signals a medium-strength pause
Colon (:)	Introduces additional material
Parentheses (( ))	Sets aside information from the rest of the sentence
Brackets ([ ])	Sets aside information within parentheses
Dash (–)	Signals an interruption
Ellipsis points (...)	Signals an omission or a pause

Periods

Periods are plain punctuation placed at the end of sentences that are statements or commands.

Let's see a movie.

My cat caught a canary.

Dad sold three pies at the bake sale.

Periods signal the end of a thought, telling the reader to pause before moving on to the next sentence.

They are also used in certain abbreviations, including initials, months, countries, titles, and other commonly abbreviated forms.

T. S. Eliot

Feb.

Dr. and Mr. Green

Finally, periods are used after numbers and letters on outlines and lists.

Eating Contests

I. Hot dog eating contests

A. Dates

B. Winners

Question Marks

Question marks are placed at the end of sentences that ask something. Use question marks whenever you need more information.

Were there lions at the circus?

What's your favorite book?

Is there time to stop for snacks?

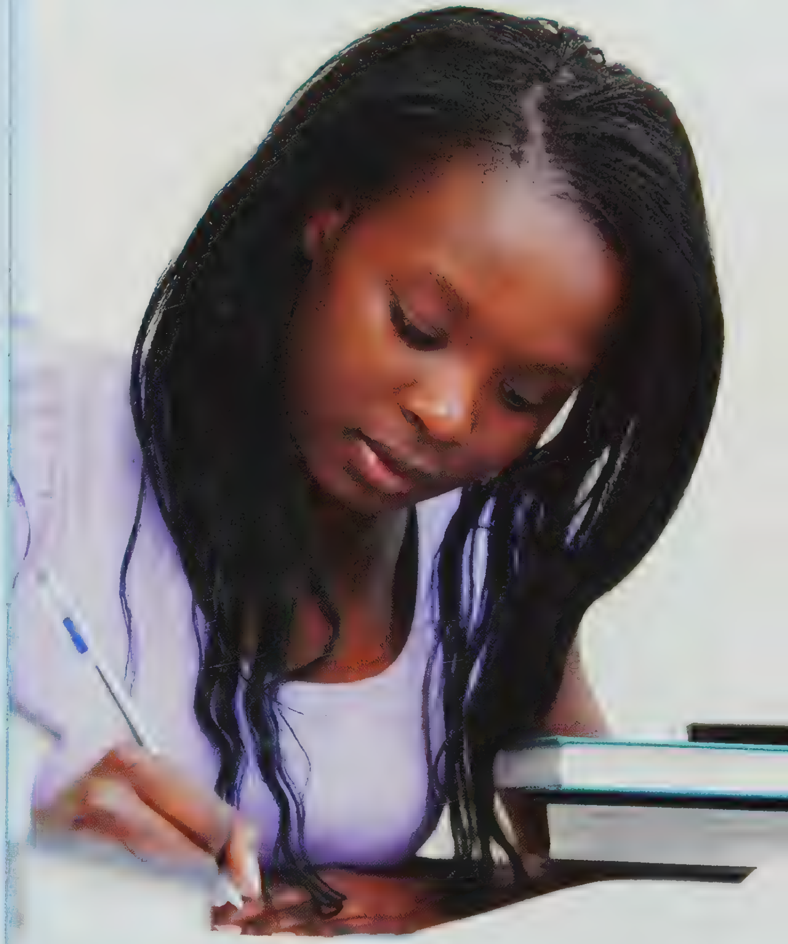
Exclamation Points

Exclamation points are used at the end of sentences that show strong feelings. Use exclamation points for emphasis whenever you have a strong point of view.

I don't want to go camping!

Go away!

What a great party!



RIGHT AND WRONG PUNCTUATION

There are two issues in punctuation. The first is a simple question of right and wrong. A writer who ends each sentence with a colon rather than a period is just wrong because sentences cannot end with a colon. It's that simple.

The second issue is less clear-cut; it's more a matter of style. Sometimes, a writer is free to choose punctuation that suits his or her taste. For instance, one writer might choose to use a semicolon to separate two closely related thoughts, while another writer might choose to use a period. With practice, you'll learn which choices make your meaning most clear.

Pauses

Some punctuation marks stand in for the natural pauses or changes in inflection you would use if you were speaking instead of writing. Punctuation that asks readers to pause includes

- Commas
- Semicolons
- Colons
- Dashes
- Hyphens
- Ellipsis points

Commas

Commas ask readers to pause briefly as they read a sentence. They separate certain parts of the sentence without interrupting the overall flow.

Commas are the most frequently used punctuation mark. You can use them in many different ways.

You can use commas to separate items in a series.

> Penguins, polar bears, and peacocks are my favorite zoo animals.

Without commas, the same sentence would be much harder to read.

> Penguins polar bears and peacocks are my favorite zoo animals.

You can also use commas to separate compound sentences joined by conjunctions such as *and, or,* or *but.*

> Peacocks like popcorn, but they don't care for peas.

Additionally, commas set off parenthetical information and appositives.

> The penguins' pen, which is north of the food court, is big and clean.

> The new polar bear, Percy, likes to play hide and seek.

Use commas to set off an introductory modifier.

> Feeling hot from the sun, Percy went for a swim.

Finally, use commas before or after the name of a person being spoken to.

> Percy, please save some fish for me.

fyi!

A good rule of thumb is to use a comma anywhere you might pause if you were saying a sentence out loud.

WORDS AT WORK

In this sentence from her short story "Average Waves in Unprotected Waters," Anne Tyler demonstrates how to use commas to organize details.

> Most men came behind them, dressed in work clothes, carrying folding chairs, black trunklike boxes with silver hinges, microphones, a wooden lectern, and an armload of bunting.

Semicolons

Semicolons join two sentences without using a conjunction. They create a pause that is stronger than a comma and weaker than a period.

Use a semicolon to separate sentences that are, in your judgment, too closely related to be separated by a period.

I want to finish the article now; I'll go to lunch later.

A semicolon can be used when two sentences are joined by a transitional word like *however* or *therefore*.

Today is a holiday; therefore, the mail will not be delivered.

Semicolons are also used to separate items in a list when commas are used within the items.

Speakers at the town meeting included Mrs. Sloan, the butcher; Mr. Bates, the baker; and Mr. White, the candlestick maker.

Colons

Colons are used to introduce lists, examples, questions, or long quotes. Whereas semicolons tell readers to stop and separate, the colon says, "Go on; look ahead. Here comes something."

You will need the following items: a compass, a rope, and a water bottle.

The question is this: What should we do next?

The world's three largest countries in area are as follows: Russia, Canada, China.

It is also used after the salutation in a business letter.

Gentlemen:

Dear Mrs. Rogers:

To Whom It May Concern:

Another use for a colon is to separate a title from its subtitle and to separate hours from minutes.

Thanks a Lot: Mastering the Art of Faint Praise

6:30 p.m.

MARCEL PROUST

Marcel Proust, a French writer, was a wordy fellow. His famous novel *Remembrance of Things Past* was so long that it had to be published in seven parts. He is also known for having written one of the longest sentences in all of literature. Clocking in at 958 words in English, Proust had to use a lot of semicolons to make his supersized sentence hang together.

WORDS AT WORK

This excerpt from William Faulkner's short story "Barn Burning" uses two colons.

Hit's big as a courthouse, he thought quietly, with a surge of peace and joy whose reason he could not have thought into words, being too young for that: *They are safe from him. People whose lives are part of this peace and dignity are beyond his touch, he no more to them than a buzzing wasp: capable of stinging for a little moment, but that's all.*

Pauses

Dashes

Dashes ask readers to pause before a sudden break in thought or an interruption.

The best way to finish that—but no, you don't want my opinion.

Dashes are also used to emphasize or define part of a sentence.

Margie Miller—the best writer at the paper—got a promotion.

To form a dash, you may need to type two hyphens together. A single hyphen is not a dash.

WORDS AT WORK

Emily Dickinson is famous for using many dashes in her poems. (See pages 946–947.)

After great pain, a formal feeling comes—
The Nerves sit ceremonious, like Tombs—
The stiff Heart questions 'was it He, that bore,'
And 'Yesterday, or Centuries before'?

The Feet, mechanical, go round—
A Wooden way
Of Ground, or Air, or Ought—
Regardless grown,
A Quartz contentment, like a stone—

This is the Hour of Lead—
Remembered, if outlived,
As Freezing persons, recollect the Snow—
First— Chill—then Stupor—then the letting go—

Hyphens

Use hyphens to form compound adjectives. A compound adjective is two or more words that modify a noun.

I prefer **up-to-date** Web sites.

A **well-known** painter will be there.

Note that compound adjectives are hyphenated only when they appear BEFORE the noun.

The Web sites I prefer are **up to date.**

The painter who will be there is **well known.**

Hyphens are also used to form compound nouns. Compound nouns are hyphenated regardless of their position in a sentence.

father-in-law **great-grandmother**
jack-in-the-box

Hyphens are used to spell out compound numbers between twenty-one (21) and ninety-nine (99).

twenty-four **sixty-one** **twenty-ninth**

Hyphens are used to spell out fractions that are used as adjectives.

We saw a **one-third** increase in earnings this year.

Note that fractions that are used as nouns are NOT hyphenated.

Two thirds were counted as present.

You should also hyphenate prefixes and suffixes to avoid confusion of words that are spelled alike.

re-cover the sofa/**recover** from the loss
re-count money/**recount** a story
re-creation of the scene/**recreation** area

Ellipsis Points

An ellipsis can be used to indicate a pause or an unfinished thought.

Can this . . . really be true?

I'm not sure. . . .

The main use of ellipses, though, is to indicate that words have been deleted from a direct quote. You do not need to use an ellipsis if it is obvious you are using a partial quote.

William Price Fox told the interviewer that he "didn't like to think very much before writing."

An ellipsis can be three dots (. . .) or four dots (. . . .) depending on where it's placed in a sentence. If the omission occurs at the beginning of a quotation, or within a quotation, use three dots.

William Price Fox told the interviewer that he "didn't like to think . . . before writing."

Note that there is a space on both sides of the three dots.

When the omission occurs at the end of a quotation, use four dots. The extra dot is the period at the end of the sentence.

William Price Fox told the interviewer that he "didn't like to think. . . ."

WATCH OUT!

Make sure that you don't change the meaning of a sentence when you omit words.

For example, imagine you are going to use the following quote:

Mayor: "The budget committee is doing all it can to avoid having to raise taxes again for at least two years."

Incorrect: The mayor's last point about "… having to raise taxes again…" seemed to draw the most attention from the crowd.

Compound nouns can be two words joined by a hyphen (bull's-eye), two words smashed together (stepfather), or two separate words (vice president). Check the dictionary if you're not sure how a compound noun should look.

Writing and Research

Writing 706

Research. 782

THE SEVEN-STEP SYSTEM is a process you can use for any writing project you're assigned, whether it's an informal essay or a long research report. The steps will help guide you through the process, including picking a good topic, constructing a strong thesis, and putting together your ideas in a logical order.

Every time you write for school, organize your work according to this system. Eventually, the steps will become second nature.

Where Am I in the Seven-Step System?

THE SEVEN-STEP SYSTEM

1. **Choosing a Topic**
2. Gathering Information
3. Constructing a Thesis
4. Outlining
5. Drafting
6. Revising
7. Proofreading

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Choosing a Topic

The first step is to choose a good topic. Spending extra time to set up a good writing project at the front end will help the later stages move faster.

Most writing assignments demand a narrow topic so you have enough room for all the detail you need. For example, if you try to write a paper on a broad topic like the circulatory system, you will find yourself overwhelmed with information. A more manageable topic might be, say, how white blood cells fight disease.

Paper Topics

Too Broad	More Manageable
the circulatory system	how white blood cells fight disease
life in colonial America	tools used by colonial farmers
space travel	life aboard a space station

Background Reading

If you're having a hard time homing in on a topic, do some background reading to become more familiar with your subject. The *Southwestern Advantage Topic Source* is designed for this specific purpose. As you read, think about what kinds of information you would like to include in your report and what can be left out. It's natural to find some sections that interest you more than others. This will help you pinpoint the aspect of the subject you would most like to write about.

Make a list of points as you go to serve as a preliminary outline and research guide. Your list might just be a series of questions you hope to answer in your report. If you're reading about life aboard space stations, you might ask questions like

- What does the inside of the station look like?
- How big are the crew's quarters?
- How do astronauts eat?
- How do they spend most of their time?

Generating Ideas

Once you've settled on a topic, it's time to think things through. Jot down some notes as you go. At this point, you're just writing for yourself, so no one has to see.

Techniques you can use to guide your thinking include *brainstorming, freewriting,* and *cubing.* Give yourself about 15 minutes for each technique.

Brainstorming

Make a list of words and phrases that relate to the topic as they come to mind. Don't censor yourself. Just write down ideas without thinking about them too much.

Freewriting

Freewriting is another good way to generate ideas. The technique is based on the belief that we often limit or block our creativity by focusing on details like grammar and word choice. Let your ideas flow freely by concentrating on your subject and allowing yourself to write as fast as you can, without editing or evaluating your words as you go.

When your time is up, read over what you've written. You'll find that some ideas seem more useful than others. Circle or underline the words and phrases that seem to be headed in the right direction and ignore the rest.

Cubing

Cubing is like freewriting, but it's more directed. It allows you to explore a topic from six perspectives.

1. Describe it
2. Compare it to another topic
3. Associate it with another topic
4. Analyze it
5. Explain how it can be used
6. Evaluate it (argue for or against it)

Each perspective brings different questions into play. Freewrite on each perspective for three to five minutes. Don't worry if you find yourself writing more questions than answers. The idea is to generate lines of inquiry, which will give you different ways to approach your topic.

Keep in mind that there are other questions you can use to form the six sides of your cube. Journalists, for instance, often ask the questions *who, what, when, where, why,* and *how.*

Brainstorming is a great way to generate ideas as you try to choose a good topic. ▶

Gathering Information

Step 2 in the seven-step system is gathering information. This step is important enough to have its own section in this volume. (For more information, see the research section beginning on page 782.)

Finding Sources

Not all writing projects require research. For example, simple book reports rarely require you to consult outside sources. If you're not sure what your project requires, check with your teacher.

Most of the sources you use for your writing projects will be in the library or on the Internet. Common types of sources include

- books
- magazines
- newspapers
- encyclopedias
- Web sites
- dictionaries
- almanacs
- reference librarians
- original research (interviews, experiments, etc.)

Taking Notes

As you gather information, take notes about the different sources you consult. Make sure you differentiate between the ideas that belong to other people and the ideas you've had yourself. Also, be careful to mark direct quotes so you don't accidentally use someone else's words as your own.

Think about the kinds of evidence you might want to use to support your claim, which might include

- examples
- definitions
- statistics
- descriptions
- stories
- explanation

Where Am I in the Seven-Step System?

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Process and Evaluate

Approach your research materials with a critical eye. As you read, digest the information by thinking about how you might use it in your writing project.

Questions you can ask yourself include

Does the author have a bias? Everyone has a point of view that informs their writing. Think about the perspective the author has on the material and how that perspective might have influenced his or her writing.

What kind of expertise does the author have? Does the writer have training or other credentials that make you trust (or distrust) his or her claims? Has the author written anything else on a similar topic?

How does the source relate to my topic? Think about how the source relates to your writing project. If the connection doesn't seem strong, drop what you're reading and move to the next thing.

How current is the source? Check the publication date. If it's recent, you're in good shape. If it's older, ask yourself how much the topic has changed over time. Some topics, like literature, probably haven't changed much. Others, like science, change so fast that an older source might contain outdated information.

Get Rid of Anything You Don't Need

Don't assume you can use every source you consult. It's important that you use good judgment to choose only the best and most relevant sources. Put aside any research material that isn't up to snuff; it will do more harm than good.

Document Your Sources

It's important that you show your readers where to look for information that you use in your paper. As you take notes for your writing project, make sure to take down the details that will be required on your works cited page. Typically, this includes author, title, publication date, publisher, and page numbers. Note the url, page/topic name, and author (if listed) for Internet sources.

GATHERING INFORMATION

The gathering information phase of your writing project should include the following steps.

1. Find sources.
2. Take notes.
3. Process and evaluate the information.
4. Get rid of anything you don't need.
5. Document the sources you think you'll use.

WATCH OUT!

For research projects, your teacher might base part of your grade on the sources you choose.

Constructing a Thesis

Writing the Thesis

As you read through the information you have assembled, you will begin to form certain opinions on your topic. You can use these opinions to develop a tentative thesis, which is a statement that summarizes the main thrust of the ideas in your paper.

Think of your thesis as a work in progress. It can grow and change as your paper develops.

The most important thing about a thesis is that it needs to make an argument. It would be difficult, if not impossible, to argue that the middle class pays 85 percent of the taxes in the United States. That statement is either correct or incorrect, so it is an ineffective thesis.

Changing the thesis to assert that the middle class should pay more in taxes would be an arguable point.

> *The most important thing about a thesis is that it needs to make an argument.*

Where Am I in the Seven-Step System?

THE SEVEN STEP SYSTEM

1. Choosing a Topic
2. Gathering Information
3. **Constructing a Thesis**
4. Outlining
5. Drafting
6. Revising
7. Proofreading

Testing Your Thesis

There are three conditions that a good thesis should satisfy.

The thesis should be news. Your thesis should not be accepted as common knowledge. Obviously, the audience in this regard is crucial. It would seem entirely unnecessary to argue, for instance, that Earth circles the sun unless your audience consisted of primitive tribesmen, the Flat Earth Society, or thirteenth-century popes.

A thesis must be understandable. Keep in mind that your readers probably don't know the topic as well as you do. Make sure you explain your ideas in terms that people will understand.

A thesis must be supportable. You already know you must support your thesis with evidence. But keep in mind that what seems like good evidence to one group of readers might not seem strong to another group. If your thesis depends upon support from data compiled by the American Veal Association, for instance, and your audience is People for the Ethical Treatment of Animals (PETA), you really do not have a supportable thesis because the veal industry, for that audience, is not an authority. (See pages 798–803.)

WRITING A STRONG THESIS

- Never write your thesis as a question. Instead, use a declarative sentence with a period (.) at the end. While many theses start with a question, by the time you're writing you should be able to frame it as an answer.
- Remember that your thesis cannot be a statement of fact. It should be an arguable statement *about* the facts.
- Make sure you can support your thesis with evidence.
- Be precise and specific. Your thesis is not the place for vague language.
- Make sure your thesis limits the range of topics and presents an idea manageable enough to guide the length and scope of your research paper. Remember you can tweak your thesis (or even make a major change) as you work your way through the seven-step process.

EVALUATING THREE THESIS STATEMENTS

The good, the bad, and the ugly

Thesis: Satan is the real hero of Milton's *Paradise Lost*.

Rating: *Good.* This argument works because it is arguable. Some people will disagree with it, naming another character as the hero.

Thesis: Lincoln's role in the Civil War was crucial.

Rating: *Bad.* No controversy here. Everyone knows that Lincoln was a crucial figure in the Civil War, so there's no point in arguing it. To improve this thesis, go against the flow by arguing that his role was less important or more important than most people think.

Thesis: I like chocolate.

Rating: *Ugly.* It's hard to argue with a personal preference. How can anyone else say that I don't like chocolate? Instead, try to make a universal statement, such as "Chocolate is beneficial to the diet as an aid in combating depression." Now you've got an argument because there is an opposing point of view.

WATCH OUT!

Remember the thesis statement should summarize the purpose of your paper. Make sure you have it right before you go any further.

MAKE A PLAN

The front end of your project is a good time to make a plan. Here are five steps you can follow to come up with a good one.

1. **Break the project down into small steps.**
   You already know that you need to write a paper by a certain date. Breaking up your project into chunks will make it easier to get started. It will also help you track your progress, which makes it easier to adjust your pace if you fall behind.
2. **Gather everything you need.**
   Make a list of the materials you need and gather them so you have everything in one place.
3. **Organize your time.**
   It's human nature to postpone working on dull or difficult assignments. Make yourself start early so time doesn't get away from you. Look at the list of steps you created in step one. Estimate how much time you need for each one. Try to be realistic!
4. **Coordinate your plan with your other activities.**
   Make a list of any other projects you have going on. This includes ongoing projects, such as music lessons and practice, or one-time-only events, like a test. When you have a complete list, assign each entry a number: 1 for most important, 2 for second-most important, and so on. This is called setting priorities, and it's a very important part of planning.
5. **Stick to your plan.**
   Just because you make a plan doesn't mean you're going to stick to it. Plenty of people make plans on paper but never follow through with them. Put your head down and work through the steps you listed.

   If you finish part of the project earlier than expected, go ahead and get a head start on the next phase. That way, if you fall behind later, you have time to get back on track.

TYPES OF WRITING

There are three basic kinds of writing: *argumentative, expository,* and *narrative.* Many writing projects will combine all three forms. It's important to understand these types so you have them at your fingertips, whether you're working on an essay, a test, a research paper, or another type of analysis.

Argumentative

Argumentative writing, also called persuasive writing, attempts to convince the reader to agree with the writer's ideas, opinions, or attitudes. Argumentative writing may also attempt to convince the reader to follow a particular course of action.

Arguing a Position

Use a variety of the following techniques when constructing an argument: give facts and/or examples, relate incidents, and present opposing views. An effective argument must be supported by evidence. (See pages 810–811.) It is not enough to simply give an opinion.

TIPS FOR ARGUMENTATIVE WRITING

- Choose a topic that has at least two clear sides.
- State your position clearly at the beginning of your text.
- Make sure you differentiate between facts and opinions.
- Be reasonable and maintain an even tone. If you sound angry or defensive, you may put readers on edge and make them less receptive to your ideas.
- Make sure you support your ideas. It's not enough to explain *what* you think; you need to explain *why* you think it.
- Don't get carried away with your claims. Try to be as specific as possible.
- Give some thought to the other side's position. It will help you explain why you disagree with it.
- Choose a topic you know something about or that you can research adequately in the amount of time you have.

Arguing for a Course of Action

Give special consideration to the audience when organizing an argument that tries to persuade people to take a course of action. Persuading people to do something is usually more difficult than persuading them to agree with an idea or an opinion. When arguing for a course of action, use the kinds of evidence used in arguing for a position. In addition, follow the guidelines below.

1. Avoid offending the readers with insults and abusive language.
2. Use language that readers can understand. Do not use language that is condescending or confusing.
3. Use facts to support your opinion. Facts are convincing evidence.
4. Organize your argument by giving the least important facts first and the most important facts last. That way, you will conclude with your most convincing evidence.
5. Summarize your argument at the end by stating the course of action you feel the readers should take.

WATCH OUT!

Make sure you can defend your argument. If you're having trouble thinking of or finding evidence to support your opinions, that's a good sign you need to rethink the issue.

WEAK ARGUMENT

The following argument is not convincing because it does not offer proof to support the writer's opinions.

Cultural exchange programs between countries are, in my opinion, of little value. This sending of a lot of singers, dancers, and musicians back and forth is not going to solve any problems. Maybe some good feeling is created, but the stage cannot take the place of the conference table. And just try to sing away a stockpile of atomic bombs. When foreigners see our entertainers—and our sports stars, too—they realize how much better we are, and this makes them jealous.

STRONG ARGUMENT

The following argument is convincing because it uses reasons and facts to support the writer's opinion.

Employers should be allowed to pay teenagers less than the minimum wage. Minimum wage laws were passed to protect workers trying to support themselves and a family. However, most teenagers are not self-supporting. Many employers are willing to hire teenagers but cannot afford to pay them minimum wage because of their inexperience. But how can young people get job experience if no one is willing to hire them? Permitting teenagers to work for less than minimum wage would allow them to gain valuable job experience. It would also reduce the high level of youth unemployment.

Expository

Expository writing is writing that explains something. You will probably work more with exposition than any other kind of writing. Expository writing answers questions such as

- Who or what is the person or thing under discussion?
- Why is that person or thing important?
- What does the person or thing do?
- How does the thing work?
- What is its origin?
- How did it develop?

fyi! *Avoid using personal pronouns such as "I" and "my" in expository writing. Instead, focus on the question you're trying to answer for the reader: the who, what, how, or why.*

Elements of Expository Writing

There are a number of different elements in expository writing. Ways of supporting the thesis statement will vary depending on your content. Not every technique will be used in every piece of exposition.

Thesis. The thesis states the main idea of the composition. All sentences in a piece of writing should support the thesis statement. Expository writing always requires a thesis statement.

Examples. Examples are a great way to support your thesis. They help illustrate abstract concepts for readers.

Comparisons and Contrasts. Another good way to support the thesis is to compare and/or contrast one thing with another. Explaining the similarities and the differences helps readers see how concepts relate to one another.

Cause and Effect. Often, some aspect of your thesis can be explained by using a cause-and-effect relationship. A cause is something that brings about some action or result. An effect is what happens as a result of the cause.

Definitions. You can use definitions to support the thesis statement. A definition explains something by telling what it is or how it works.

Processes. Support your thesis statement by explaining how something operates or works. Process explanations usually take the reader through a process step by step.

TIPS FOR EXPOSITORY WRITING

- Begin writing with a clear goal in mind.
- Try to stick with one idea per paragraph.
- The thesis is the only element of expository writing that is required. Don't feel like you have to cram all of the others into one essay.
- Instead, choose a few of the elements that work best with your material.

EXPOSITORY WRITING SAMPLE

1 Most Paris neighborhoods these days are a striking blend of the old and the new. Narrow nineteenth-century buildings still dot the residential streets, but they now coexist with towering modern apartment complexes. Down in the commercial areas, 2 small decades-old shops specializing in one commodity—cheese, perhaps, or fish or fruit or baked goods—stand side by side with huge new supermarkets carrying every food imaginable.

Like the surreal paintings of the French masters, contemporary Parisian neighborhoods present a striking juxtaposition of contrasts: modern chrome and mottled wood, poured concrete and ancient stone. But, unlike the Surrealists' creations, Paris's transformation is far from fanciful. It is based on cold, hard economic facts. 3

Real estate prices are soaring in Paris, just as they are in most modernized countries. A lot that ten years ago would have sold for the equivalent of $500,000 in U.S. dollars now goes for at least three times that price. Rents, too, have increased accordingly, echoing the rising property values.

The increase in price of land has affected both residential and commercial patterns. Owners of older, smaller residences, tempted by the high prices offered by developers, sell their properties to the highest bidders. Often, neighbors sell en masse, thus opening up the large lots necessary for development of high-rise apartments. 4

1. THESIS

Notice how the author states the thesis up front, so it's hard to miss.

2. EXAMPLES

The author uses specific examples of old and new structures to support the thesis.

3. COMPARE AND CONTRAST

The author explains how the neighborhoods are like (and unlike) French surrealist paintings.

4. CAUSE AND EFFECT

In the final paragraph, the author explains the causes behind new development in Paris neighborhoods.

Narrative

Narrative writing tells about events that happen. Nonfiction narrative writing usually tells about personal events, incidents, or experiences.

Using Chronological Order

Chronological order is often the organizing principle in narrative writing. Chronological order starts at the beginning of a sequence of events and follows those events step by step until the conclusion.

Using Descriptive Writing

Descriptive writing paints word pictures of particular people, places, or things. It appeals to the reader's senses: sight, sound, touch, taste, and hearing. (See pages 762–763.)

Selecting Details. Limit your subject before choosing details. Since effective description calls for a selection of the most significant details, it is often better to tell less about a subject but to tell it well.

Concrete details are specific, precise details that create an impression. They are not general or abstract.

The concrete details in the following example are in **bold** type.

> I saw him standing outside the train station, clutching his **white cane** and rattling the coins in his **battered tin cup**. His clothes were **old and tattered**, and he wore a **thin coat** even though it was the dead of winter. The **tempting odors of bacon and eggs** drifted from a restaurant a few doors away, where customers sat eating their fill in **comfortable booths** near the window.

TIPS FOR NARRATIVE WRITING

- **Appeal to readers' senses.** Using vivid details helps readers form strong images as they read.
- **Keep the reader oriented in time.** Make it clear to the reader when you are talking about the past, present, or future.
- **Pretend that you're telling a story.** Is your story funny? Suspenseful? Sad? Or something else?
- **Stay focused.** The key to a good narrative is to stay on track and keep the story moving.
- **Make an emotional appeal.** Choose a subject you have strong feelings about. Your writing is more likely to be compelling if it has some heart.
- **Don't lose track of the reason you're writing the narrative.** Are you merely sharing information? Or is there some other goal?

WATCH OUT!

If you don't understand what you're describing, chances are that your reader won't understand, either.

fyi!

Short stories are sometimes considered fictional narrative writing.

NARRATIVE WRITING EXAMPLE

Notice how the descriptive language in the passage below appeals to three of the five senses.

TOUCH

TOUCH

SMELL

SIGHT

We reached the top of the hill, dropped our backpacks, and stretched out on the **soft** grass that covered the **cool** earth. The air was filled with the **fragrance** of hundreds of brilliant wild flowers, their dense pattern and colors—**purple, pink, red, orange, and blue**—spread out like Oriental carpets displayed at a bazaar.

Using Flash-Forwards and Flashbacks

Use flash-forwards and flashbacks to create a special emphasis. A flash-forward is a device that a writer uses to tell about an event before it happens in straight chronological order. A flashback tells about an event that happened at an earlier time in the narrative.

When I stepped on the stage at the age of six to give my first violin recital, my heart was racing and the blood was pounding in my throat. I walked timidly to the center of the stage, made a stiff bow, brought the instrument up to my shoulder, and started to play.

Had I anticipated the thunderous applause that would greet me at the end of my performance, I would have been less nervous. My mother and father leapt to their feet and cheered. There were even calls for an encore.

But all that possible adulation was far from my mind as I made those first scratchy noises on my half-size violin. At the start of the recital, all I wished for was its conclusion.

Writing Under Pressure

An essay exam or timed essay can be thought of as a particular kind of expository writing. The form usually requires you to recall information quickly, to present it in a logical, orderly fashion, and to draw conclusions. As you read an essay exam question, it is important that you know what you are being asked to do. Look for the strategy term or phrase, which tells you how to approach the task of writing.

Key words in the questions will dictate the way you answer the question. Words to watch for include

- analyze
- define
- compare
- contrast
- describe
- explain
- summarize

WATCH OUT!

Make sure you always answer the question that is being asked. Remember to look back at the question as you write to make sure you're on the right track.

Five-Step Process

Use a five-step process to write your exam essay.

Step 1: Identify Your Topic

Before you begin to write, examine the question to find your topic. Be sure you understand what you are being asked to define, analyze, discuss, evaluate, or explain. You may want to circle the key word.

Step 2: Plan Your Essay

Once you understand what you are being asked to do, spend about 10 percent of your time planning your essay. Once you have determined the strategy for your essay, you will begin the second component of your exam—content. Prepare a quick outline by jotting down important points.

Step 3: Develop Your Outline

One approach to beginning your outline is to make a list of the main points you plan to cover in your essay. Fill in the list with supporting examples. The examples you plan to include should be noted in your outline by jotting down important points and subpoints.

Step 4: Formulate Your Thesis

After you have developed an outline, you can formulate your thesis. This approach is different from the way you normally write an essay, in which you develop your thesis first. Because you are in a time-pressured situation, writing the outline first and developing your thesis from it saves time and keeps you from forgetting a key piece of information.

Step 5: Writing the Essay

Once you are ready to write, keep the following points in mind.

- Clearly state your thesis and establish the essay's structure.
- Be direct and to the point.
- Provide examples and clear explanations.
- Avoid digressions and generalizations.
- Do not restate the question in the essay.
- Follow your outline closely.
- Use transitional phrases to move from one point to another.
- Write neatly and leave room for editing corrections.

How to Take a Test

Test-taking in general requires a strategy. These four steps can help you organize your limited time.

1. Take a moment or two to read through the whole test. Read all the instructions carefully. Notice how many questions there are and try to get a sense of how much time you will have for each question. Decide how much time you'll spend on different sections of the test.

2. Read each question carefully. If you misunderstand the question, you will answer it incorrectly no matter how much you know about the subject. Some tests have questions with tricky wording just to test your understanding. If a question seems complicated, read it a second time.

3. Skip questions you aren't sure about. It's best to answer all the questions that seem easy first. Then you can use all the remaining time to consider the more difficult questions. This will keep you from getting a mental block and will assure that you answer everything you know best before time runs out.

4. Take the last two or three minutes to read through your answers. This is especially important on essay tests. When you write quickly, you may leave out an important word. In multiple-choice tests, make sure that you have put your answers in the right space or column and (if there's time) that each answer seems reasonable.

EDITING YOUR ESSAY EXAM

Remember that you need to save some time for revising and editing your answers. Follow the same guidelines as you would for any paper. Helpful questions include

- Does the thesis answer the question on the exam?
- Is the thesis clearly stated?
- Have all the major points been covered?
- Are the details specific?
- Is each sentence complete?
- Are the spelling and grammar correct?
- Is the writing legible and neat—even with cross-outs and inserts?

BEFORE THE TEST

- Take good notes during class.
- Take good notes as you read assignments.
- Review and outline your notes periodically.
- When the time for the test approaches, make detailed outlines based on your notes.
- Try to determine which points have been stressed and repeated most often in class. Have you noticed any recurring themes? Is there a thread that ties all the major points together?
- Write down several possible test questions, and try to answer at least one of them before the exam as practice.

EXAM TIPS

- Begin by reading all the questions.
- During the exam, take time to prepare each answer.
- If you have several questions from which to choose, answer only the required number.
- Devote the appropriate amount of exam time to your answers according to the point value of the questions. For example, on a 100-point exam, a 30-point question should be allotted one-third of the exam time since it is worth approximately one-third of the grade.

BOOK REPORTS

The book report is a standard school assignment from junior high on up. Just because it's common doesn't mean it has to be a dull, pointless exercise. Book reports can be surprisingly enjoyable to write. The process of writing book reports sharpens your ability to analyze books and recognize their strengths and weaknesses. They will also help you broaden and refine your writing skills. Remember, the deeper you explore a good book, the more interesting it becomes.

Reading the Book

A book report is not simply a point-by-point retelling of the book, nor is it a free-flowing presentation of your thoughts and opinions. Instead, it should have a definite form and structure.

Good book reports

- present a concise (yet thorough) description of the book.
- analyze the book's strengths and weaknesses.
- describe key plot points in general terms.
- avoid spoiling the ending for someone who hasn't yet read the book.

Choosing the Right Book

Perhaps the most common difficulty shared by book report writers is selecting the right book.

Consider Your Assignment. Are you required to write about a particular genre (such as fiction or nonfiction) or topic? If not, just choose something you think you'll enjoy.

Check the Book's Length. A 1,200-page novel may be so rich in characters and subplots that you can't treat it adequately in the limited space of a short book report. (If you're not a rapid reader, you may not even be able to finish the book in time.) On the other hand, a light novel may not provide enough material for any but the shortest book report.

Check the Cover. Sometimes you *can* judge a book by its cover. Read the blurb on the inside leaf of the dust jacket or the back cover of the book to see if it looks interesting.

Read a Little. Are the first few pages of the book interesting? Do they make you want to read more? Do the characters seem intriguing? If the answers are yes, consider using the book for your report.

Reading Tips

Read the Entire Book. You may be tempted to read the first few chapters and then skim the rest of the book to find out how it ends. Don't do it. You might think you'll save time, but the truth is that it is actually much more difficult to write a coherent, interesting book report about a book you aren't really familiar with.

Give the Book Your Full Attention. When the writer was laboring over the book you have selected, you can be sure that his or her attention was focused completely on the work at hand. When you settle down to read a book, try to do the same.

Give Yourself Plenty of Time to Read the Book. Once you've chosen your book, it's best to start reading right away. Estimate how long it will take you to read so you leave enough time to write the report.

Take Notes. One of the most useful things you can do while you read is to write down important ideas and good quotes as you come across them. Don't let the notes become a distraction. Write just enough to help you focus your attention on the material you're reading. Make sure you write down page numbers, especially for quotes.

Reread. Don't try to report on a book you read months or years ago unless you reread it. As you write the report, allow yourself time to skim or reread sections of the book as needed.

fyi! *Always use the present tense when you write about characters and events in a work of fiction. Use the past tense when you write about the people and events in a nonfiction book.*

Writing the Report

Ready, Set, Write!

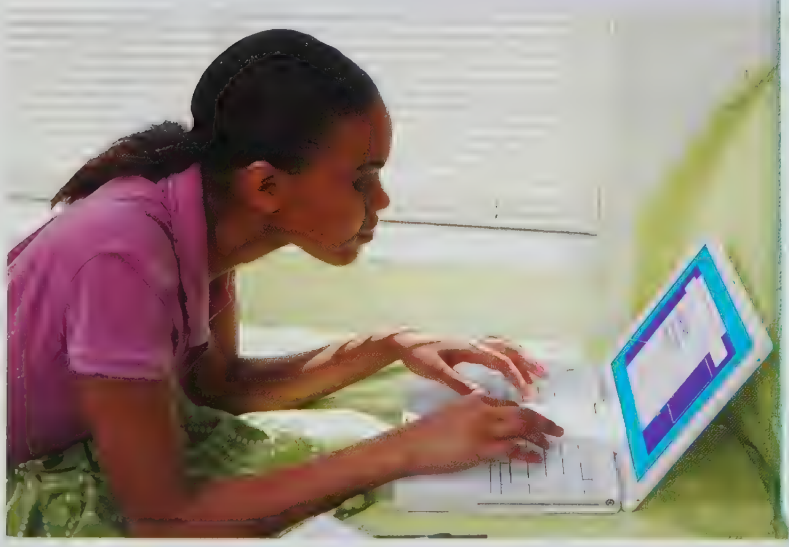
When you finish reading, you may be tempted to sit down and immediately begin writing your paper. But you will find the writing task much easier if you first spend some time thinking about the book you have just read.

Collect Your Thoughts. Consider how much material the book offers for thought. A fast-paced suspense novel may not contain many deep thoughts to ponder. A book about American foreign policy in the nuclear age may contain so many ideas that it will take you some time to sort through them.

Refresh Your Memory. You may want to reread a paragraph or a page or two here and there. Review your notes. Soon you should begin to get a good idea of the important points you want to include in your report.

Ask Yourself Questions. There are three main questions you should consider when thinking about the book for your report: What is the book about? What is the author trying to say? What do you think about it, and why?

Write an Outline. If your book report is relatively short (say, five to seven paragraphs), you can just make a simple list of the facts and ideas you want to present.



Sample Book Report

1. OPENING PARAGRAPH

The first paragraph gives a general overview of the book you have read. Include the complete title of the work, the name of the author, what genre it is, and a general description of the book's contents, theme, or story line.

2. BODY (PARAGRAPHS 2–7)

Use the middle part of the book report to give a more fully developed description of the book's contents. Try to answer the basic questions, such as:

- Who?
- What?
- When?
- Where?
- Why?

Discuss the main characters and give a general but thorough synopsis of the plot and setting, telling what happens and when the story takes place. Also, discuss any important facts about the book's structure or the author's writing style and how these elements help advance the plot or the central theme.

3. CONCLUSION

Use the final paragraph to tell what you think about the book. Explain what you liked, what you disliked, and why. Decide whether the author accomplished whatever he or she set out to do.

WATCH OUT!

Think of your book report as a means of expressing your reasoned opinions about the book. Then, supply details to support your opinions. As you write, avoid using "I" statements. The elimination of the first person will make your sentences seem more direct and forceful.

1 When Stephen Crane wrote what was to become his best-known novel, *The Red Badge of Courage*, he had never personally witnessed the horrors of battle. Yet this psychological study of the effects of combat on a young recruit during the Civil War has become an American classic. Even today, a century after the novel was first published, it is noted for its realistic depiction of battle and its insight into the effects of warfare on human thoughts and actions.

2 All the events related in the novel center on and are seen through the eyes, thoughts, and emotions of Henry Fleming, a young Union volunteer, during a single, unnamed battle of the Civil War.

It could be any battle of any war, because Crane's real focus is on another battle: the hidden one raging within Henry Fleming as his romantic notions of war are crushed by the realities of fear, suffering, and death. Through Fleming's thoughts and actions, we see that the external conflict, the struggle between the Union and Confederate forces, is the foundation for his internal struggle to come to grips with his own fear and self-doubt.

Henry Fleming had dreamed of the glory of battle, but his first months in the army are filled with marches, encampments, and endless tedium. He spends much of his time wondering whether he will run from battle when the time comes.

The day finally comes when Fleming's regiment is sent into battle. The regiment repulses a Confederate attack, but when a second attack threatens to overwhelm the Union ranks, Fleming and some other soldiers panic and run. Separated from his regiment, Fleming feels burning shame when he learns that the Union line has held. He is ashamed again when he joins a group of wounded soldiers, one of whom keeps asking him, "Where yeh hit?" His shame burns yet again when he meets a mortally wounded friend, a tall soldier named Jim Conklin, who asks him, "Where yeh been, Henry?" Jim dies soon after this, beneath a highly symbolic "red sun . . . pasted in the sky like a wafer" that makes the scene one of blood sacrifice. Jim's death fills Fleming with silent rage.

Fleming meets a group of retreating soldiers, one of whom fells him with the blow of a rifle butt. That night a kind soldier, whose face Fleming never sees, helps him get back to his regiment. Fleming tells his comrades that he has been shot, and he accepts their warm approval and friendship.

The next day the Confederates attack again. Fleming, filled with rage and hatred, fights with an almost inhuman ferocity and shows great courage. When the fighting ends, Henry Fleming comes to terms with the personal shortcomings that the battle has revealed to him; he finally feels "a quiet manhood, nonassertive but of sturdy and strong blood."

3 *The Red Badge of Courage* is definitely not for the squeamish. Crane describes such gruesome details of war as ants swarming over the gray face of a dead soldier, and Jim Conklin's mortal wound, which looks "as if it had been chewed by wolves." Yet Crane's intriguing use of symbols and the sympathetic portrayal of Fleming's inner turmoil and ultimate triumph more than balance out the novel's grimness and make the book well worth reading.

LITERARY ANALYSIS

When you analyze something, you break it down into parts. By looking at its components, you hope to develop a better understanding of the whole. In literary analysis, your goal is to help your reader achieve a better understanding of a particular work of literature. To achieve this goal, you break a literary work into its components and choose the ones you want to discuss in depth.

Approaches

The act of interpreting a literary work is called literary analysis. The process involves

- reading a piece of literature carefully.
- drawing your own conclusions about the author's intent and message in the work.
- looking at the various elements of the work.
- determining which element would best help clarify that message for your reader.
- locating the references from the work itself that will justify your conclusions.

Elements of Literature

Before you can analyze a work of literature, you need to know the elements that go into the making of that work. You need to be able to identify its basic components.

LITERARY ELEMENTS

The elements of literature include these concepts.

- Tone
- Mood
- Plot
- Character
- Setting
- Theme
- Imagery
- Style
- Symbols
- Point of view

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Sometimes, your assignment may require that you use outside sources to support the connections you make. If so, use those sources to confirm or support your own ideas.

Analysis

Literary analysis—sometimes called an explication—is basically an *explanation*. It involves breaking down a work into various elements and analyzing those elements one at a time for the benefit of the reader. In most cases, the writing is organized around a thesis.

An explication is based on your analysis of the characteristics and meaning of the work. But, it is important to remain focused on the work you are explicating rather than straying to your own experience or opinions.

Although you will want to stay focused on the work you are explicating, this doesn't mean you will just be paraphrasing or rewording the original piece. Nor will you simply summarize the plot. Instead, you will need to logically analyze the various elements that contribute to the overall effect of the work.

For additional analysis, you can include the comments of a person who is an expert in the field. As with any paper, you will need to document the sources you use, in the proper format.

TIPS FOR LITERARY ANALYSIS PAPERS

- Write your literary analysis with the assumption that your reader is familiar with the work being analyzed. Don't provide extensive summaries of the work. Instead, use excerpts to support your points.
- Provide page numbers for your references whenever you're discussing a short story or a novel. For plays, provide the act and scene numbers. For poems, use stanzas and line numbers. Include these numerical references as part of the sentence introducing your textual example, or place them in parentheses after the example.
- For secondary sources, give the full textual citation for your source on the works cited page.

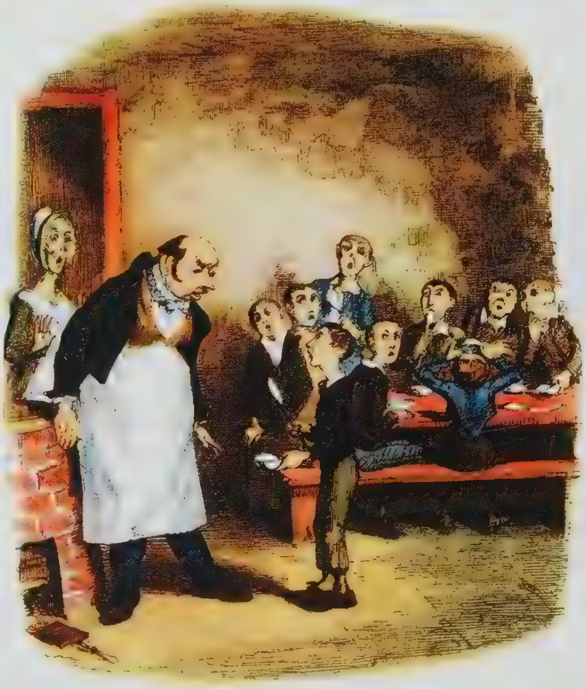
Three Ways to Understand Literature

Great works of literature can be understood in at least three ways.

Biographical: Literature gives us insight into the author's head and heart. In this sense, it can be approached as the study of great minds in human history.

Historical: Literature gives us insight into other times and places. In this sense, it can be approached as the study of the great movements in human history.

Interpretive: Literature gives us insight into enduring human problems. This approach focuses on the work itself, rather than biographical or historical distractions. The power of great literature lies in its ability to tell a reader what authors from other times and places understand about life's possibilities.



Orphaned Oliver Twist audaciously asks for another helping of gruel in the Charles Dickens classic *Oliver Twist*. (See also page 944.)

Tone and Mood

In writing, the terms *tone* and *mood* mean different things. The concepts are often related, but they are not interchangeable.

Tone	Mood
attitude	atmosphere
author	reader
voice	feeling

Tone

Often, in conversation, people communicate using their tone of voice. Sometimes we interpret whether someone is happy or angry or sad not by the words that they say, but by the way in which the words are said. In literature, too, writers communicate using tone.

Tone involves two main elements.

- Author's attitude toward the subject
- Author's attitude toward the reader

Words you might use to describe tone include serious, silly, formal, informal, sarcastic, humorous, wry, frustrated, academic, confident, confused, enthusiastic, and bored.

LITERARY ANALYSIS CORNER

To write about tone, follow these steps.

- Set aside your personal reaction.
- Focus on the author's attitude toward his or her topic.
- Pretend as if you're describing it to a friend. Think of at least three adjectives you would use in your description.
- Consider how the author's tone aligns with his or her subject matter. Does the tone seem like what you would expect, given the topic? Or is it surprising?
- Think about how the tone furthers the author's purpose. What kind of message does it help create?
- Now focus on the author's attitude toward the reader. Does it seem friendly? Hostile? Angry? Again, think of three adjectives you would use to describe it to a friend.

In his essay "A Modest Proposal," Jonathan Swift proposes that the disadvantaged people of Ireland fatten up their children and sell them as food to the rich. He uses a **mock serious tone** to underline how ridiculous the idea is.

> I have been assured by a very knowing American of my acquaintance in London, that a young healthy child well nursed is at a year old a most delicious, nourishing, and wholesome food, whether stewed, roasted, baked, or boiled; and I make no doubt that it will equally serve in a fricassee or a ragout.

While *Moby-Dick* is a serious, dark novel, Herman Melville often uses a **light tone,** as in this excerpt. While the narrator is describing his depression, he uses humor to keep things from getting too dark.

> Whenever I find myself growing grim about the mouth; whenever it is a damp, drizzly November in my soul; whenever I find myself involuntarily pausing before coffin warehouses, and bringing up the rear of every funeral I meet; and especially whenever my hypos get such an upper hand of me, that it requires a strong moral principle to prevent me from deliberately stepping into the street, and methodically knocking people's hats off—then, I account it high time to get to sea as soon as I can.

Mood

Mood is the emotional atmosphere of a story or poem. While tone focuses on the author's attitude, mood focuses on the reader's feelings about what he or she is reading.

Just as many different factors affect your mood on a given day, different literary elements contribute to the mood of a text, including

- Tone
- Characters
- Setting
- Theme
- Plot
- Imagery
- Setting

LITERARY ANALYSIS CORNER

To write about mood, focus on your own feelings and reactions to the story or poem. Then think about all the elements that contribute to those feelings.

- Characters: What are the characters' personalities? What are their moods?
- Setting: What kind of emotional charge do you get from the setting? Does it seem happy? Sad? Or something else?
- Theme: What is the message of the story or poem? Is it positive or negative?
- Plot: How does the end of the story make you feel? Was it satisfying? Upsetting? Or something else?
- How does the author's tone help create the atmosphere of the story or poem?
- What kind of images does the story or poem contain? What sort of emotional associations do those images call up?

The **mood** of this passage from Edgar Allan Poe's poem "Annabel Lee" (1849) is **dark and somber.**

The angels, not half so happy in heaven,
Went envying her and me—
Yes!—that was the reason (as all men know,
In this kingdom by the sea)
That the wind came out of the cloud by night,
Chilling and killing my ANNABEL LEE.

In sharp contrast, the **mood** of this passage from Lewis Carroll's *Through the Looking Glass* (1871) is **whimsical and silly.**

"The time has come," the Walrus said,
"To talk of many things:
Of shoes—and ships—and sealing wax—
Of cabbages—and kings—
And why the sea is boiling hot—
And whether pigs have wings."

"But wait a bit," the Oysters cried,
"Before we have our chat;
For some of us are out of breath,
And all of us are fat!"
"No hurry!" said the Carpenter.
They thanked him much for that.

Plot

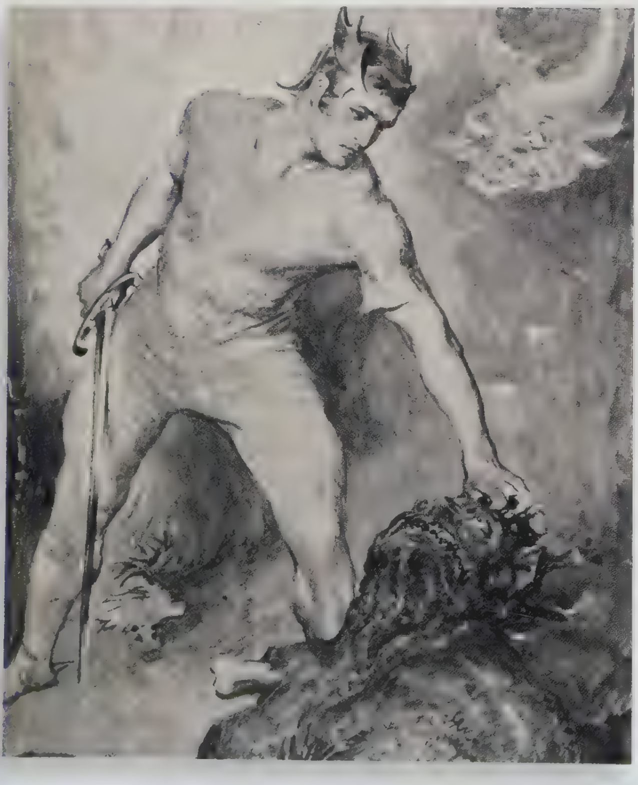
Plot is the way that events are organized or ordered in a story. Traditionally, plots have a clear beginning, middle, and end that take place within a certain timeframe. Authors typically present a story's events to the reader in chronological order.

There are exceptions. William Faulkner's short story "A Rose for Emily" is a famous example of a story that jumps around in time, shaping the reader's response by rearranging the order of what happened.

Conflict

At the heart of every plot, there is a **conflict** between two forces. There are many types of conflict in fiction, including

- human vs. human
- human vs. society
- human vs. self
- human vs. nature
- human vs. animal
- human vs. supernatural
- human vs. technology



The central conflict of *Beowulf,* the Old English epic poem, is human vs. supernatural.

LITERARY ANALYSIS CORNER

In literary analysis, writing about plot in a work of fiction means analyzing the conflict. Explain what the conflict is. Discuss the major events that illustrate the conflict. Describe the climax and its resolution. Since your goal is to help your reader understand the whole work, you must explain what the conflict means in terms of the theme or the message of the work.

OTHER PLOT-RELATED TERMS

Anticlimax:
a conclusion to the rising action that seems disappointing, sudden, or silly

Deus ex machina:
a plot resolution (usually considered "cheating") in which the conflict ends due to a new concept or character

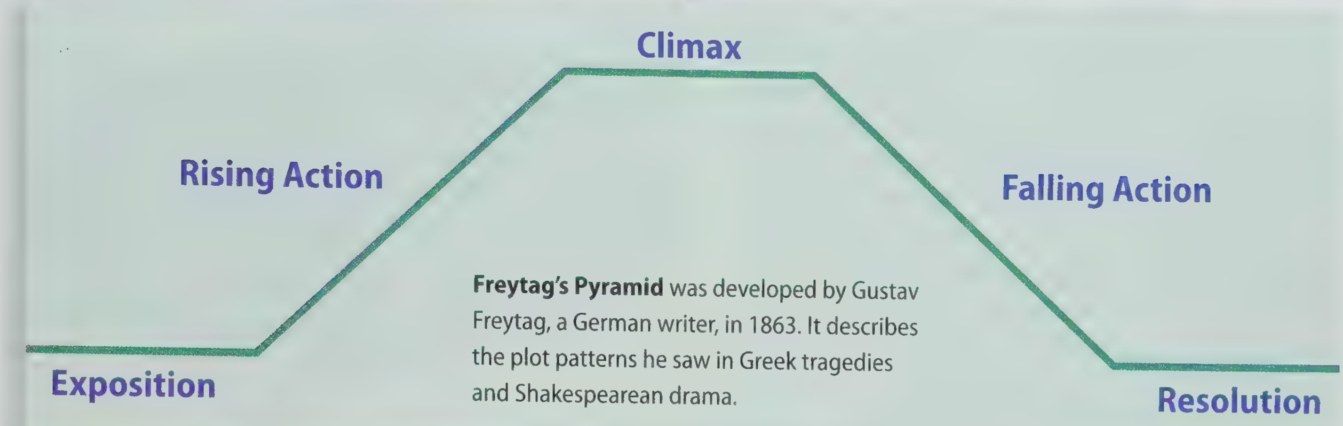
Flashback:
when a story leaps backward to a scene that takes place earlier in time

Foreshadowing:
a clue in the plot that seems to predict or point toward an event that happens later in the story

In media res:
when a story begins in the middle of things instead of at the beginning

Subplot:
a side story that may involve supporting characters or weave in and out of the main plot

Macguffin:
a coveted object that drives a story's plot



Freytag's Pyramid was developed by Gustav Freytag, a German writer, in 1863. It describes the plot patterns he saw in Greek tragedies and Shakespearean drama.

Exposition

The exposition reveals the background information that readers need to understand the story. In an artful exposition, this information seems to be a natural part of telling the story. Readers are often introduced to the setting and main characters.

Rising Action

The rising action describes conflict that arises when forces of some sort work against, or oppose, each other. It is a series of events that thicken, or develop, the story. A problem arises or comes to the forefront.

Climax

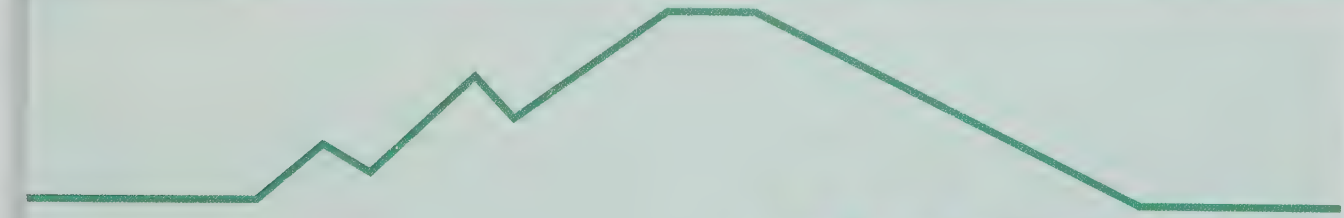
The climax is a moment of tension in the story when some decisive action is taken. It is the turning point for the main character, or protagonist.

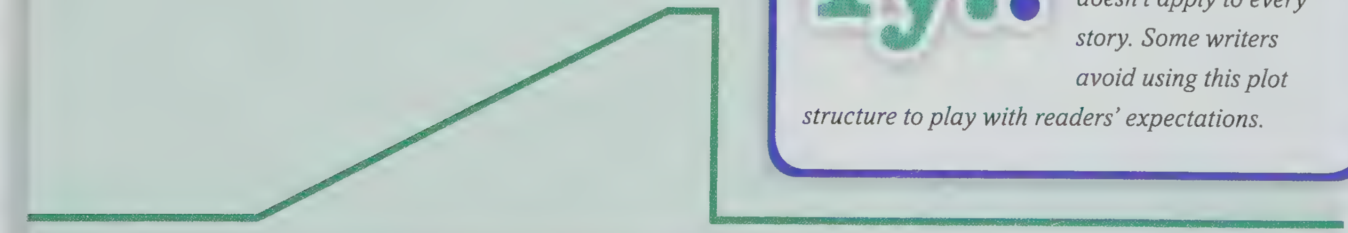
Falling Action

The falling action shows the consequences, or results, of the climax.

Resolution

The resolution occurs when the conflict is resolved and the story draws to a close.





fyi! *Freytag's Pyramid doesn't apply to every story. Some writers avoid using this plot structure to play with readers' expectations.*

Character

Think about the people in your life and how well you know them. You could probably rattle off a lot of details about your family and close friends, such as their likes, dislikes, skills, and accomplishments. You know less about the lives of your acquaintances—people like your dentist or your neighbor. Then there are strangers that you cross paths with, like the people you pass in the street, about whom you know next to nothing.

Just as you have to pick and choose the people you want to get to know, authors must focus their attention on certain characters. In most stories, there are only a handful of characters you "get to know."

The terms "round character" and "flat character" were coined by E. M. Forster, the English novelist who wrote Howards End.

Round Characters

The most complex people in a story are called round characters. Round characters are the objects of the author's (and the reader's) attention. They are the people we know the most about. Round characters usually change in some way over the course of the story.

Flat Characters

The simplest people in a story are called flat characters. They are the people we know the least about. Often, a flat character is defined by one major personality trait or feature. Flat characters rarely change over the course of the story.

	Round Character	Flat Character
also known as	main character	secondary character
characteristics	complex dynamic	simple static

LITERARY ANALYSIS CORNER

Ask yourself the following questions as you read and write to learn more about a character.

- How old is the character?
- What does he or she look like?
- What is his or her living situation?
- How would you describe the character's personality?
- Does the author tell you about the character using description, action, or dialogue?
- Is the character predictable?
- Did the character seem realistic? Why or why not?
- Do you recognize the character as a type that you've encountered in other works of fiction?
- What details brought the character to life?
- Do you like the character? Why or why not?
- What is the character motivated by? What does he or she care most about?
- What kind of relationships does he or she have with other characters?
- What do you know about the character's thoughts?

Trickster

The trickster is a type of character that has been around for a long time. When a certain type of character crops up in different forms across many cultures, it becomes known as an *archetype*.

All tricksters share similar characteristics. They live off their wits, getting through tough situations by tricking others. They're clever and foolish at the same time. They're cunning, prideful, and generally very entertaining.

By taking a closer look at three prominent tricksters—*Loki, Puck,* and *Br'er Rabbit*—you can get an idea of how that archetype has evolved over time and across cultures.

Loki is a shape-shifting trickster god in Nordic mythology. In addition to frequent cameos across the history of Scandinavian literature, Loki was featured as a character in Neil Gaiman's *Sandman* comic books.

Puck got his start in old English folklore, when he was a hobgoblin. Later, in medieval times, he became known as the fairy Robin Goodfellow. But the best-known Puck is probably the trickster in Shakespeare's *A Midsummer Night's Dream,* in which he appears as the mischievous servant of King Oberon. (See pages 964–965.)

Br'er Rabbit was immortalized by Joel Chandler Harris in the Uncle Remus stories, but long before that, the trickster was a folk hero in African-American storytelling. But the rabbit trickster's roots can be traced back even further than that to Africa, where the hare has long been part of oral tradition.

fyi!

Fiction writers often invest meaning in their characters' names. For instance, in To Kill A Mockingbird, *the town eccentric was called Boo Radley, a name that implies he's scary.*

Setting

Setting refers to the location and the time period of the series of events. It can include

- Time of day
- Place
- Historical period
- Social environment
- Cultural environment
- Weather
- Neighborhood
- Geographic location
- Landscape
- Building
- Room
- Season

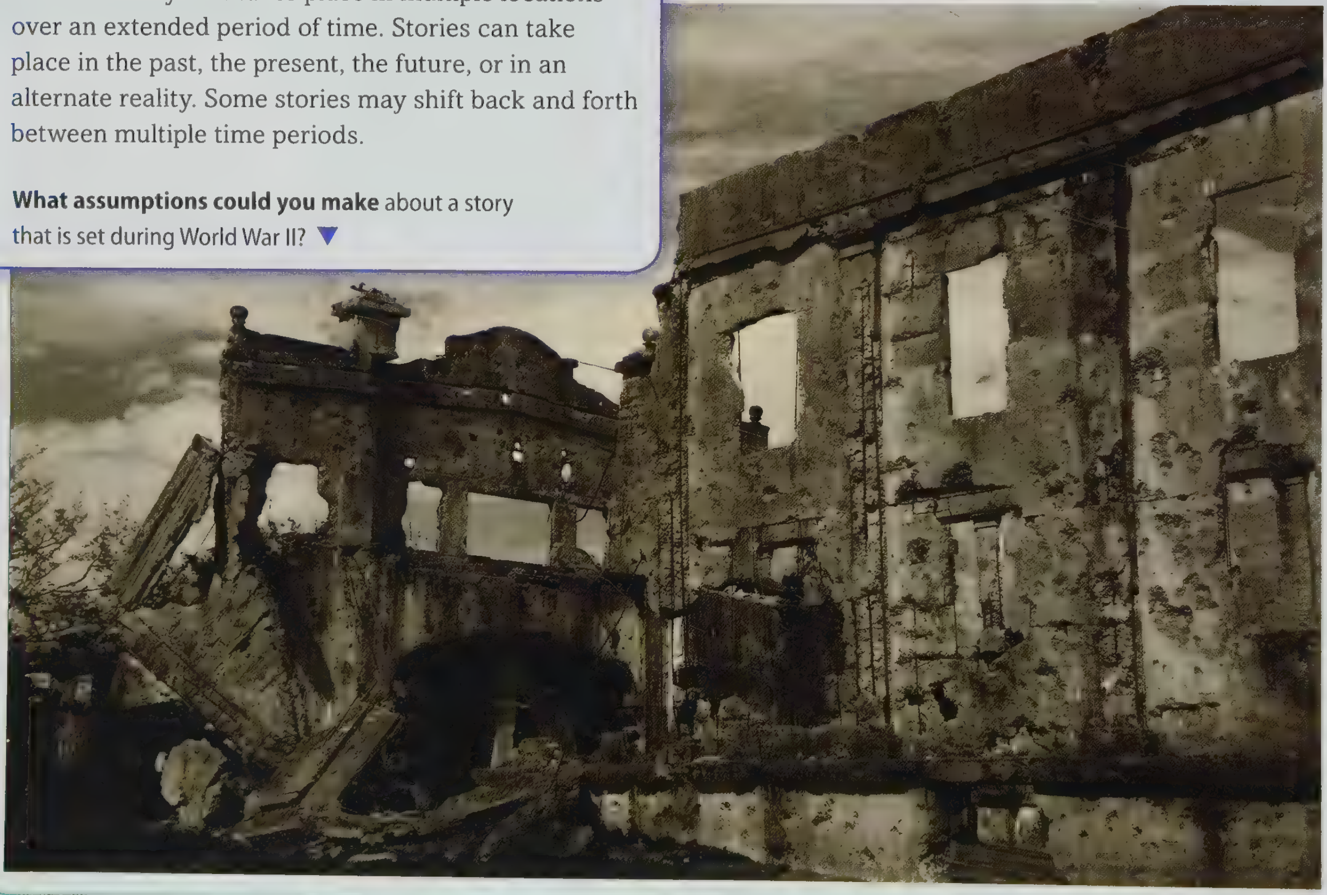
You may read a story that takes place in one particular location over a relatively short period of time or a story that takes place in multiple locations over an extended period of time. Stories can take place in the past, the present, the future, or in an alternate reality. Some stories may shift back and forth between multiple time periods.

What assumptions could you make about a story that is set during World War II? ▼

WORDS AT WORK

Writers are always looking for ways to convey information to readers without stating it directly. An author can accomplish many different goals by using a story's setting as shorthand. For example, setting can

- influence characters' thoughts, feelings, and actions.
- establish a particular mood or tone.
- reflect a character's emotions (for example, a story about someone who's down in the dumps might be set on a rainy day).
- foreshadow events that haven't happened yet.
- provide a framework for what happens.
- provide a historical context that creates a backstory.



Setting is an important element in "The Fall of the House of Usher," a short story by Edgar Allan Poe. Consider how the weather, season, landscape, time of day, and the house itself are described in this passage.

> During the whole of a dull, dark, and soundless day in the autumn[1] of the year, when the clouds hung oppressively low [2]in the heavens, I had been passing along, on horseback, through a singularly dreary tract of country; [3]and at length found myself, as the shades of the evening[4] drew on, within view of the melancholy House of Usher. I know not how it was—but, with the first glimpse of the building, a sense of insufferable gloom pervaded my spirit.

1 SEASON
2 WEATHER
3 LANDSCAPE
4 TIME OF DAY

LITERARY ANALYSIS CORNER

Whatever the location or timeframe, when you write a literary analysis based on setting, you are discussing how the setting has an impact on the plot or on the characters.

Here are some questions you can ask about setting as you read.

- Have you been to the place where the story is set? If so, what do you know about that place?
- Is the story set in the past? If so, what do you know about that time in history?
- What kind of language does the author use to describe the setting? Does it seem positive or negative?
- Is the story set in the future? If so, what has changed? What has stayed the same?
- Is the setting realistic?
- What is the atmosphere of the story?
- How does that atmosphere affect (directly or indirectly) the series of events that occurs?
- How does the setting affect the behavior of the characters?
- How does the setting reflect the theme of the story?
- How does the author seem to feel about the setting? What about the narrator?

fyi!

A shift to a scene in the past from the present is called a flashback. *(See page 737.)*

Setting

Setting refers to the location and the time period of the series of events. It can include

- Time of day
- Place
- Historical period
- Social environment
- Cultural environment
- Weather
- Neighborhood
- Geographic location
- Landscape
- Building
- Room
- Season

You may read a story that takes place in one particular location over a relatively short period of time or a story that takes place in multiple locations over an extended period of time. Stories can take place in the past, the present, the future, or in an alternate reality. Some stories may shift back and forth between multiple time periods.

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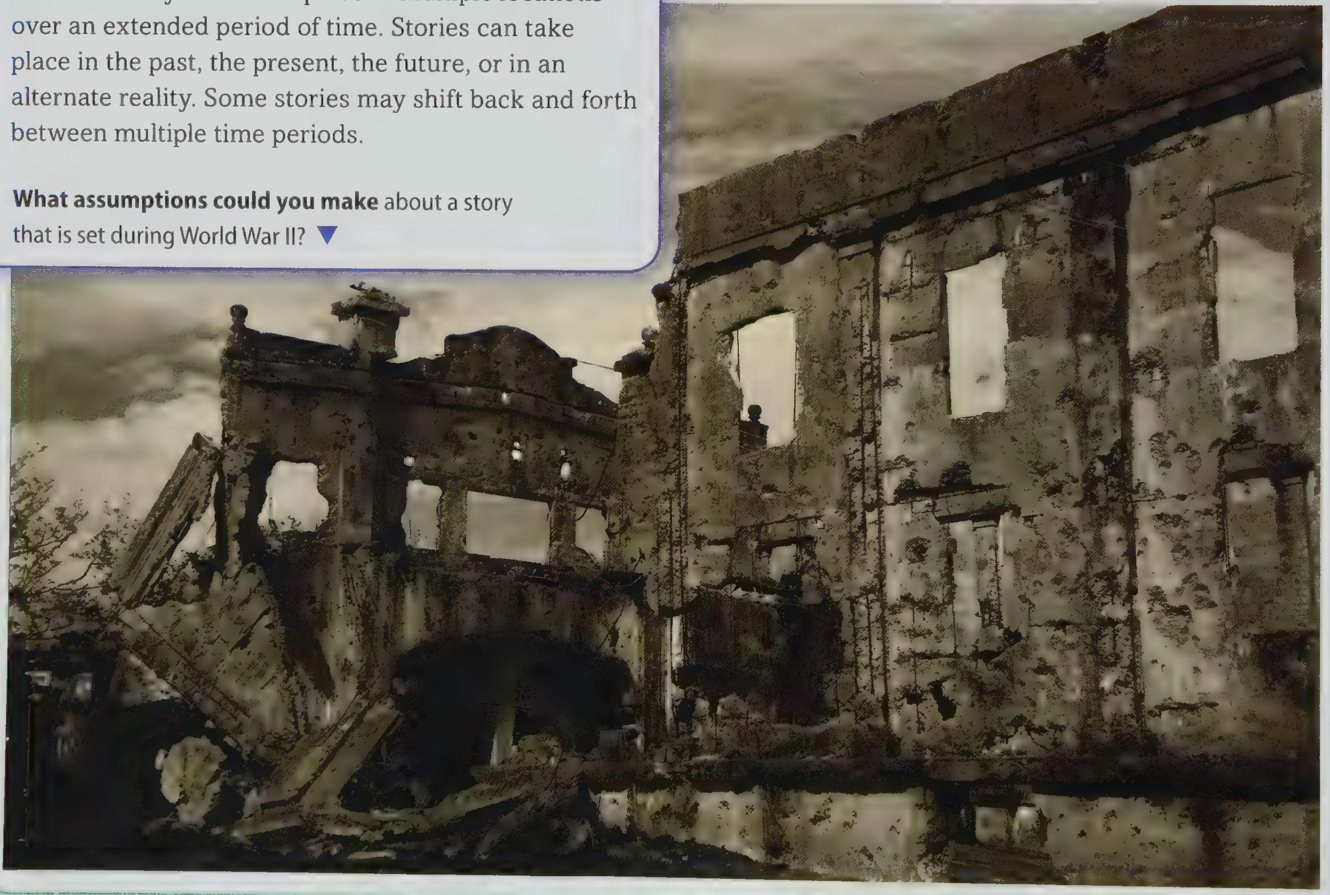

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fyi!

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Theme

The theme is the main point or the central idea that the author is trying to convey to the reader. Every literary work has one, but some themes are more obvious than others.

Sometimes, you will walk away from a literary work with a strong sense of the author's message. You might finish a novel and feel like the writer banged you over the head with a message like, say, "crime doesn't pay" or "honesty is the best policy."

Other times, themes are subtle. A short story may illustrate a social injustice, for instance, without directly commenting on it. The theme may be an exploration of some part of the human life cycle, such as the end of a relationship or the transition into adulthood. Or the theme might just be a snapshot of a human characteristic, such as courage or pride.

While theme is an important element in literature, it doesn't exist in a vacuum; many different parts of a story or poem come together to help create it.

The most basic formula for theme is to consider a work's subject plus the author's attitude toward that topic.

Theme = Subject + Author's Attitude

Theme can be specific to a particular time and place. For instance, Charles Dickens often wrote about the mistreatment of children, a practice that clearly disgusted him, to illustrate his belief in social reform in Victorian England.

Often, enduring literature expresses a universal theme that resonates with readers, no matter who they are, or when they live(d).

UNIVERSAL THEMES

Some of the themes that have been explored by different authors over time include

- Alienation
- Loss of innocence
- Fate versus free will
- Futility of war
- Love and loss

LITERARY ANALYSIS CORNER

When you write a literary analysis paper on theme, your goal is to help readers achieve a better understanding of the overarching idea in a literary work. To analyze a theme, ask yourself the following questions.

- What is the main idea that the author wants his or her readers to walk away with?
- What elements of the story or poem contribute to that idea?
- What excerpts can you use to support your ideas about theme?

THEME-RELATED ELEMENTS

Other literary elements that contribute to theme include, but aren't limited to

- Characters
- Setting
- Plot
- Tone and Mood
- Symbols
- Imagery
- Style

When I Heard the Learn'd Astronomer

by Walt Whitman

1 When I heard the learn'd astronomer,

When the proofs, the figures, were ranged in columns before me,

2 When I was shown the charts and diagrams, to add, divide, and measure them,

When I sitting heard the astronomer where he lectured with much applause in the
lecture-room,

3 How soon unaccountable I became tired and sick,

Till rising and gliding out I wander'd off by myself,

4 In the mystical moist night-air, and from time to time,

Look'd up in perfect silence at the stars.

The theme of this poem by Walt Whitman (see pages 946–947) is that the universe can't be adequately described by mere facts and figures. Different elements of the poem come together to create that theme.

1. **Form.** The poem is written in free verse, a type of poetry that isn't bound by rules. The form echoes the speaker's rejection of the proofs and figures of the astronomer.

2. **Long lines** are used to discuss the astronomer, suggesting that his lectures were tedious and boring.

3. **Vivid language** emphasizes the speaker's boredom. The phrase "tired and sick" appeals to the senses more than a simple statement of boredom.

4. **Alliteration** in the phrase "by myself in the mystical moist night-air" lends the end of the poem musical quality, which supports the speaker's preference of mystery over science.

RESEARCH PAPERS are a great way to learn not only about the particular topic you're investigating, but also about the process of investigation itself. You must become a kind of detective to find the facts and theories needed to come to an informed conclusion. Putting the collected information together with personal ideas is the way all investigators solve mysteries. Unlike mysteries, however, research topics may have more than one possible conclusion.

Overview

Research paper: the very words fill some students with dread. Preparing a research paper is a big job. But if you go about it the right way, it is a manageable task. It helps to start out with a confident attitude.

Think of your research paper as a chance to learn a lot more about something that interests you. It's an opportunity to develop skills in information gathering, organization, and writing. These skills will make you better equipped to handle many other school assignments and even job responsibilities later in life.

A good research paper integrates information gathered from various sources with your own ideas. It demonstrates your ability to think and write clearly, as well as your skills in using the library and other sources to track information.

In writing a good paper, you learn how to argue your point fairly, yet persuasively.

Misconceptions

There are several common misconceptions about research papers. One is that a research paper is a collection of strung-together quotes and paraphrases from published sources with a lot of footnotes attached so no one can accuse you of plagiarism. Another is that it's a collection of uninformed opinions on a given topic.

In reality, a research paper is a combination of those two concepts—your informed opinions and ideas based on your investigation of published, and occasionally unpublished, information.

Another common misconception is to think of the research paper as strictly a writing project. While it's true that the final product you hand in will be a written paper, much work goes into the process before you ever pick up a pen or sit down at a keyboard to start writing.

Use interesting comparisons to bring statistics to life. For example, a figure for the population of India is much more interesting if you add the fact that more people live in India than in all countries of North and South America combined.

WATCH OUT!

Don't just regurgitate facts, figures, and other writers' opinions when you write a research paper. Show off your critical thinking skills by demonstrating that you've carefully considered the topic.

Making a Schedule

Planning ahead is a good idea for any writing assignment. For a research paper, it's a must. It's a good idea to go ahead and make a schedule as soon as you get the assignment.

Mark the due date on a calendar and count backwards to plan blocks of time for finishing the final draft, including

- research and note taking.
- preparing an outline.
- organizing note cards.
- writing the first draft.
- revising and proofreading.

The exact amount of time that goes into each of those blocks depends on the assignment and the speed at which you work.

Information Versus Evidence

Whenever you write a research paper, you will have to sort through and digest a large amount of information.

Your first task is to decide what information is relevant to your project. Facts, statistics, examples, and quotes are common types of information you will find. (See also pages 810–811.)

To write a successful research paper, your challenge will be to turn all that information into evidence or proof. Evidence is what information becomes when you use it to support your ideas.

If a piece of information does not help you make your point, think twice before you use it.

fyi!

Plagiarism is presenting someone else's ideas as if they were your own. Changing words around is still plagiarism if you don't give the original author credit for the idea.

Using Electronic Sources

The Internet, CD-ROMs, and DVDs have drastically changed the way most people perform research inside and outside of the classroom. In order to research a subject effectively, you need to know how to find information electronically.

- The Internet is a network of computers that spans the globe. Government agencies, businesses, nonprofit organizations, and individuals provide free or fee-based access to their Web sites.
- CD-ROMs hold a lot of information and offer fast, interactive searches. Periodical indexes on CD-ROM allow you to quickly find recent articles by simply entering a keyword search.
- DVDs can hold the same information as CD-ROMs, but because their storage capacity is greater, they can also hold audio and video.

Search Engines

Web sites offer information on practically every subject. Unlike the knowledge in an encyclopedia, however, this information is not neatly organized. When you want information on a particular topic, you have to use a search engine such as Google to help locate that material.

To find what you need, search engines use indexes that summarize the contents of Web sites. You start by typing keywords or search terms that describe your topic or question. The search engine then returns a list of links to Web sites that relate to those words.

For example, say you need to research the Great Chicago Fire of 1871. If you begin with the keyword *Chicago,* the search engine will return millions of matches. If you add the keywords *fire* and *1871,* far fewer matches will result, and the information will probably be more relevant.

BOOLEAN SEARCH TERMS

AND (or +)

When you combine two words using AND, you will narrow your search to articles that contain both of these words. For instance, if you searched for “schools,” you might get thousands of articles dealing with schools. A second search for “uniforms” would find similar, overwhelming results. However, if you combined the words with *and* to search for “uniforms AND schools,” your choices would be narrowed down to articles that deal with the idea you wish to explore.

The more words you add, the more specific you can make your search. You could search for “uniforms AND schools AND mandatory policies” to get an even more specific search result.

NOT (or –)

Adding NOT will narrow your search, too, but by excluding words. For instance, if you found that too many articles for “uniforms AND schools” were about private schools, you could change your search to “uniforms AND schools NOT private.”

OR

Using OR in the search will broaden your results if your search finds too few sources. It is best used with terms that have common synonyms, such as “elementary OR primary.” OR is usually used in parentheses, so a search might look like this: “uniforms AND schools AND (elementary OR primary).

Many search engines will assume the OR when you enter keywords. This results in the search engine finding any site that has any of the words you entered.

fyi!

The Boolean search is named after the logician George Boole, who founded the idea of symbolic logic.

Commercial Web Sites

Web sites that can be accessed for free are called general Web sites. Web sites that charge a fee for use are commercial Web sites. Many libraries purchase subscriptions to such sites, which members can then access for free. Commercial databases include periodical indexes as well as reference books such as encyclopedias, biographical directories, literary criticisms, and more.

If you're using the Internet for a research paper, the periodical index databases provided by your library should generally be your first stop. You can usually find them on your school or library's Web page. Some of the most popular databases include FirstSearch, InfoTrac, ProQuest, LexisNexis Academic Service, and EBSCOhost.

These resources allow you to enter key terms to find articles from well-known journals, magazines, government documents, and other reference works. Using these sources, you can be sure you're getting more reliable and timely information than you will find on most general Web sites.

EFFICIENT INTERNET SEARCHES

- Before you begin, think about your search strategy. There is a lot of information on the Internet, but much of it is unreliable or of poor quality. (See pages 802–803.)
- Start with a simple search. Each word you use limits the results the search engine finds. If you're too wordy, the search engine will filter out a lot of Web sites that might have been useful. You can always add more keywords if the results are too broad.
- Don't worry too much about getting the wording just right. There might be times when you can't remember how something is spelled or the full name of whatever you're searching for. Just take a shot—a good search engine like Google can often automatically correct your mistakes.
- Be as specific as possible. While your goal should be to use as few keywords as possible, your results will be better if you use descriptive words that link to your particular topic. Try to visualize the words that a Web site on your topic might contain.
- Keep in mind you can filter your search to return results for particular types of Web sites (for example, only sites that end with .edu) or file types (such as .pdf).

SEARCH ENGINE TIPS

Searches are case insensitive, so don't worry about capital letters. On the Web, *New York* is the same as *new york*.

Boolean search terms (AND, OR, NOT) DO need to be in all capital letters, though. Otherwise, common words like *a, an,* and *the* are usually ignored by search engines.

Punctuation is rarely important. The same goes for special symbols.

Note Cards

As you take notes, remember that you are a filter for the information you read. With the exception of direct quotes that you want to include in your paper, your notes should be in your own words. You need to digest new concepts in order to write about them. You can't do this if you simply copy straight from the sources you're reading.

Write your notes on index cards, making sure to have only one note per card, even if there are two pieces of important information on the same page of your source.

There are four main types of note cards.

- Direct Quotes
- Paraphrases
- Summaries
- Personal Notes

It's important that you label each note card according to its type to prevent confusion.

WATCH OUT!

Too many direct quotes can work against your paper by causing you, the author, to have no voice. The paper is YOUR argument. The sources are there to back up and further your points.

Direct Quotes

Direct quotes add credibility to the information you present. (See pages 730–731.) Use them when you want to make sure the meaning of the author's words isn't lost. When you find a quote you might want to use in your paper, make sure you record

- the exact quote, word for word (copy it carefully).
- its page number.
- quotation marks (so you don't mistake the quote for a paraphrase).

Paraphrases

When you paraphrase, you put the author's ideas into your own words. Paraphrasing can be difficult because you have to really understand the source before you can explain it well.

Make sure that you write any comments or reactions to the material on a separate note card.

Summaries

Summaries are similar to paraphrases in that they put someone else's ideas in your own words. However, when you summarize, you condense that information, picking out the basics and leaving behind the rest of the detail. (Think the inside of a book jacket or the synopsis of a movie.) This means that an entire paragraph—or even a full-length book—could be summarized in one sentence.

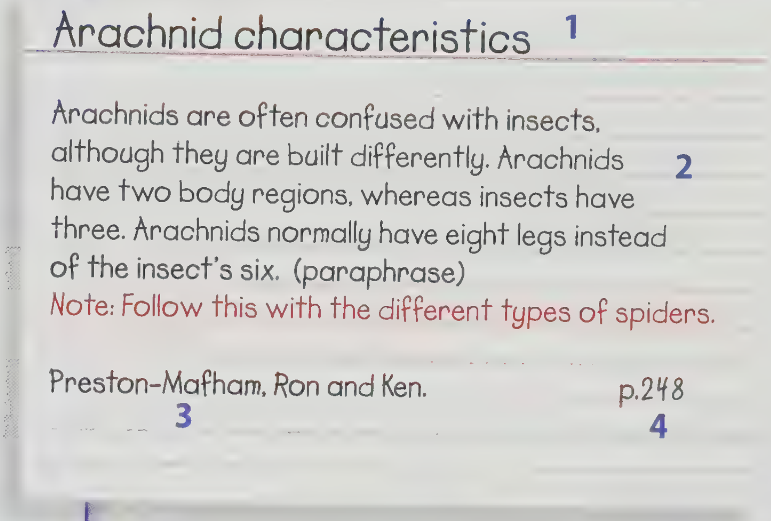
Personal Comments

Personal notes help you remember your insights and impressions as you read. You might write, for example, a note about why you disagree with an author. It may seem silly for you to take the time to write down your thoughts, but if you don't write down those ideas as they occur to you, you may lose them.

Mark personal comments clearly to keep from confusing them later with the author's point of view.

Parts of a Note Card

1. **The heading** is a short description of the note that tells what kind of information is on the card.
2. **The body** of the note is the quote, paraphrase, summary, or personal comment that you might use in your paper.
3. **The source** is a way to indicate the origins of the information. It can be a number corresponding to a source card or the source's author.
4. **The page number** shows you exactly where to go if you need to return to the source.



Arachnid characteristics

Arachnids are often confused with insects, although they are built differently. Arachnids have two body regions, whereas insects have three. Arachnids normally have eight legs instead of the insect's six. (paraphrase)

Note: Follow this with the different types of spiders.

Preston-Mafham, Ron and Ken. p.248

Writing a Note Card

- Limit yourself to one fact, idea, or quote per index card.
- Use only the front side of the index card.
- Double-check the source to make sure you included all the necessary information.
- Make sure you have stated the meaning completely in your own words (unless you're using a direct quote).
- Make sure to write down the page number on which the material was found.

fyi!

When you finish taking notes, shuffle your cards around according to topic. The paper could quickly order itself into a logical outline.

Evaluating Web Sources

Internet sources can be difficult to sift through because there is such a large amount of material. There are countless Web sites, so you must be thoughtful about the ones you use in your research. You also have to be careful; anyone can put anything on the Web, so you have to view every Web site with a critical eye.

When you go to the library for information, you know that librarians have carefully selected the books and journals inside. With online sources, there are no such safeguards. The Internet is like the Wild West of information. There are no rules. This often makes it difficult to verify the accuracy of a given Web site.

While you should carefully evaluate any source that you plan to use, it's especially critical to carefully evaluate online sources. While the same criteria—authority, relevance, bias, and publication date—apply to both print and electronic sources, the way you think about those categories is a little different.

A balanced approach to research involves different kinds of sources. Don't rely on the Internet for all your information. Make sure you review print sources, too.

Authority

Authority can be difficult to pin down on the Internet. When you read a book or an article, it's almost always obvious who the author is. On a Web site, that is often not the case. Often, no author is listed. And even when an author is listed, that person may fudge his or her credentials or other information.

As you browse the Web, keep an eye out for authoritative sources. The Web site of *The New York Times*, for example, is widely known as a reliable information source. Someone's personal blog, on the other hand, probably hasn't been established as a credible source, so facts and figures should be double-checked whenever possible.

You can ask questions to help determine whether or not a Web site is authoritative.

- Is the author of the page identified?
- What are his or her credentials?
- Who sponsors (or pays for) the Web site?
- What is that organization's (or author's) reputation?

Relevance

It can be difficult to stay on-task as you search for Web sources. There are a lot of distractions online, which makes it easy to lose time. You might become absorbed by a site that's interesting, but off-topic. Here are a few questions you can ask yourself to determine if a source is worthy of your valuable research time.

- What topics are covered?
- Are the topics explored in-depth?
- How does the page compare to print sources on the same topic?

STATIC AND DYNAMIC SOURCES

Print sources are static—they will always stay the same. A book will say the same thing today as it said 20 years ago. When you list a book on your works cited page, your reader has all the information she needs to find a source and read exactly the information that you read.

In contrast, Web sources are dynamic. While some sites are relatively stable, others change from moment to moment. Something you read on a Web site today might be different if you read that Web site tomorrow. Sometimes, the content—or even the Web site itself—will disappear. When you list a Web site on your works cited page, your reader has no guarantee that the source looks the same as it did when you reviewed it.

WATCH OUT!

When you use Web sites in your paper, make sure that you don't plagiarize. Web sites are copyrighted just like printed words, so be very careful that you cite the source accurately in the paper and include the source in your works cited list.

Bias

Just as the writers of an advertisement have a different goal or purpose than the author of an encyclopedia article, different Web sites have different goals and purposes and points of view.

To determine if a site contains objective, unbiased information, you need to learn about the different types of Web sites. You can often learn something about a Web site by examining the last extension of its address.

Often, the most trustworthy and objective sites end in .edu or .gov. Their main purpose is to deliver information, and they are held accountable for the information that is disseminated.

Sites ending in .org are organizations, which tend to have bias on a subject. While an organization is most likely legitimate, you need to be aware of the organization's agenda in providing the information.

Sites ending in .com are commercial sites, and their main purpose is to sell products.

Extension	Type of Site	Example
.com	commercial	www.apple.com
.edu	educational	www.uchicago.edu
.gov	governmental	www.whitehouse.gov
.org	organizational	www.redcross.org
.mil	military	www.navy.mil

STRENGTHS OF THE WEB

- The Web is often more current than print sources because material can be published instantly. Print materials require more planning and lead time. Newspapers like *The New York Times* can publish breaking news stories in real time.
- The Internet is a great place to find factual information such as public documents and contact information for organizations and public figures.
- It can make you aware of other people who are doing similar kinds of research.

WEAKNESSES OF THE WEB

- The Internet is not as comprehensive as a library. Online research should be used to supplement (not replace) library research.
- Sources are not vetted or reviewed, so it can be difficult to confirm accuracy.
- Most copyrighted materials are not available for free. Some books and journals allow partial access or fee-based access.

Publication Date

One of the Web's strengths is its ability to publish up-to-the-minute information, but that doesn't mean that all Web sites are current. The date that you visit a Web site is one thing, but the date that the Web site was created or updated is quite another. All of those dates are important, but they aren't always readily available.

fyi!

Wikipedia can be a vital research tool, but you should avoid citing it as a source. Instead, you can use it as a starting point to get an overview of your topic or to find links to other valuable Web sites.

MLA Works Cited Entries

Sample Works Cited Page

1

Works Cited

2 Adler, Alfred, et al. *Co-Operation between the Sexes: Writings on Women and Men, Love and Marriage, and Sexuality.* W.W. Norton, 1982.

3 —. *Superiority and Social Interest: A Collection of Later Writings.* W.W. Norton, 19 79.

Biden, Joe. 'We Veeps Stick Together. Jill and I, and All of the Bidens, Are with You, Julia. Pic.twitter.com/JP0c2wtrJ6."*Twitter,* Twitter, 28 Sept. 2017, twitter.com/JoeBiden/status/913531470001602565.

4 Catlovers. "This Tiger Steps Gently." *www.flickr.com/Photos/90389S46@N00/3S12034897.*

Coomaraswamy, Ananda K. "Sir Gawain and the Green Knight: Indra and Namuci." 5 *Speculum,* vol. 19, no. 1, 1944, pp. 104-125., doi:10.2307/2856858. 6

7

"Rhino." *WWF,* World Wildlife Fund, www.worldwildlife.org/species/rhino. 8

9 Turteltaub, Jon, director. *Cool Runnings.* Walt Disney Pictures, 1993.

1. Center the title of your bibliography at the top of the first page.

2. The author's name is always listed last name first. Entries should be arranged in alphabetical order.

3. If you list more than one work by the same author, use three dashes instead of writing the name out again.

4. Use of online handles and screen names is allowed.

5. Titles of long works like books and magazines should be italicized. Titles of shorter works like articles and poems should be in quotation marks.

6. DOIs should be used instead of URLs if they are known.

7. Double space the entire works cited list. Make sure the second line of each entry (and each subsequent line) is indented.

8. Website URLs should be included in your citation.

9. Use alphabetical order throughout your bibliography.

Sample Works Cited Entries

Book (One Author)

Chabon, Michael. *The Amazing Adventures of Kavalier & Clay: a Novel.* Random House Trade Paperbacks, 2012.

Book (Multiple Authors)

Strunk, William. *The Elements of Style.* Rockland Press, 2017.

Edited Book

Kane, Thomas S., et al., editors. *Writing Prose: Techniques and Purposes.* Oxford University Press, 2003.

Information from an Encyclopedia

Beller, Steven. "Vienna." *World Book.*

Website

Beasley, Maurine H. "Eleanor Roosevelt." *World Book,* worldbookonline.com/student/home.

Magazine Article, Print

Zakaria, Fa reed. "A New Afghanistan Stratgy." *Newsweek,* 9 Feb. 2009, pp. 36-37.

Scholarly Journal Article, Online Database

Piper, Andrew. "Rethinking the Print Object: Goethe and the Book of Everything." *Pmia,* vol. 121, no. 1, 2006, pp. 124-138., doi:10.1632/003081206x96131.

Newspaper Article

Wronski, Richard. "Prairie Parkway Project on a Road to Nowhere?" *Chicago Tribune,* 4 May 2009.

Microfiche Article

Smith, John. "Welfare Reform Suffers Blow." *Spokane Journal.* 25 January 1990. *Microform. NewsBank: Welfare and Social Problems,* 5 (1990): fiche 2, grids A9011.

Map or Chart

"Southwest USA." *Rand McNally Road Atlas,* Rand McNally , 2017.

Television Program

"State of Denial." *60 Minutes,* CBS, 1 Oct. 2006.

Government Publication

US Census Bureau. "Statistical Abstract of the United States: 2012." *Census.gov,* 1 Aug. 2011, www.census.gov/library/publications/2011/compendia/statab/131ed.html.

Interview

Meyers, Davin. "Personal Interview." 4 May 2009.

Lecture or Speech

Mason, Nathaniel. "Eliminating World Hunger." World Bank Forum. WBH Convention, 14 Aug. 2010, Denver, Livingston Hotel.

Film

Kenner, Robert, director. *America's Endangered Species: Don't Say Good-Bye.* National Geographic Society, 2008.

MAKE THE GRADE

Your teacher will look at your works cited list to see how thorough your research was. The works cited page identifies how many sources you consulted, the types of sources you consulted, where those sources can be found, how up-to-date those sources are.

If your works cited list shows that you relied on only one book, periodical, database or website for all your sources, your teacher may conclude that you didn't spend enough time researching the topic.

APA Works Cited Entries

Books

Single-Author Book

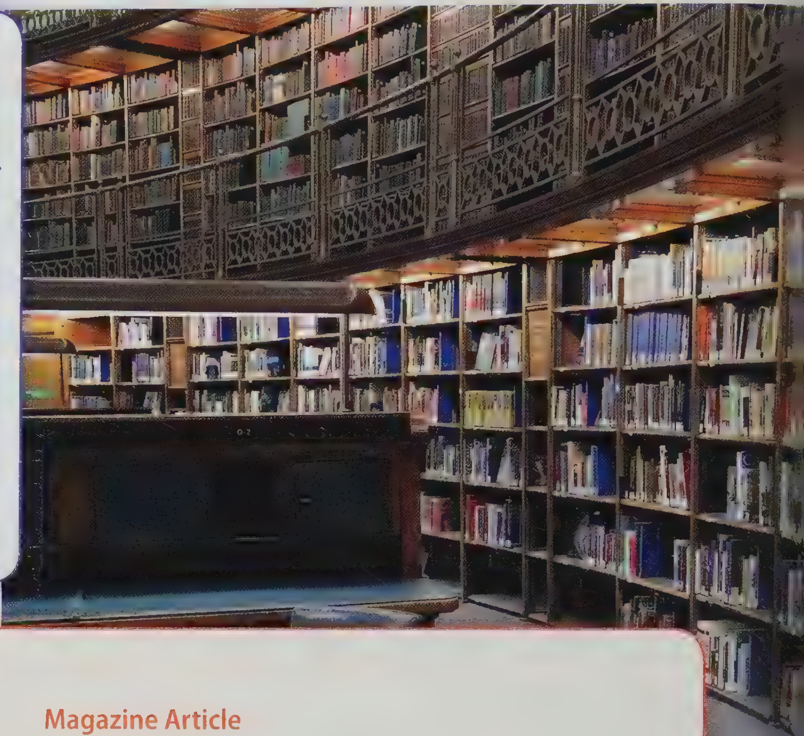
Author's last name, first initials (Year). *Title of book* (Edition number). City of publication, state: Name of publisher.

Note: For first editions, no edition number is needed.

Turner, H. R. (1995). *Science in Medieval Islam* (3rd ed.). Austin, TX: University of Texas Press.

Book with More than One Author

Author's last name, first initials, & Author's last name, first initials (Year). *Title of Book* (Edition number). City of publication, state: Name of publisher.



Articles

Journal Article

Author's last name, first initials (Year). Title of article. *Name of Journal, volume number* (issue number), page numbers.

Note: Issue number is needed only when the journal is paginated by issue.

Journal Article with a Digital Object Identifier (DOI)

Author's last name, first initials (Year). Title of article. *Name of Journal, volume number* (issue number), page numbers. doi number.

Herbs-Damm, K. L. (2005). Volunteer support, marital status, and the survival times of terminally ill patients. *Health Psychology, 24*, 225-229. doi: 10.1037/0278-6133.24.2.22.

Online Journal Article Without a Digital Object Identifier (DOI)

Author's last name, first initials (Year). Title of article. *Name of Journal, volume number* (issue number), page numbers. Retrieved from URL.

Chandra, A., & Acosta, J. D. (2010). Disaster recovery also involves human recovery. *JAMA, 304*, 1608-1609. Retrieved from http://jama.ama-assn.org/cgi/content/extract/304/14/1608.

Magazine Article

Author's last name, first initials (Year published, month and day). Title of article. *Name of Magazine, volume number*, page numbers.

Newsletter Article

Author's last name, first initials (Year published, month and day). Title of article. *Title of the Publication, volume number,* page numbers.

Newspaper Article

Author's last name, first initials (Year published, month and day). Title of article. *Title of Newspaper*, page numbers.

Note: Page numbers should be preceded with p. or pp.

Jones, P. H. (1999, February 16). In forecasting emotions, most people flunk out. *New York Times*, pp. A20-23.

Newspaper Article, No Author

Title of article. (Year published, month and day). *Title of Newspaper*, page numbers.

Amazing Amazon region. (1989, January 12). *New York Times*, pp. D11, D14.

WORKING WITH APA GUIDELINES

Papers in the social sciences are written according to American Psychological Association (APA) guidelines. The works cited page is quite similar to MLA style, but there are subtle differences. These differences are captured in the examples shown on these pages.

- For citations for online resources, include the same citation elements and in the same order as you would for print, or other fixed-media, resources. The only addition is the retrieval information, such as the URL, that is needed to locate the resource.
- Access dates are not required for online resources.

Other

Encyclopedia Entry

Author's last name, first initials (Year). Title of article. *Title of encyclopedia* (Edition, volume number, page numbers). City of publication, state: Name of publisher.

Note: If the author is not provided, begin the reference with the entry title and date of publication.

Isenberg, G. I. & Netravali, A. N. (2009). Television. *The world book encyclopedia* (2009 ed., Vol. 26, pp. 110-128). Chicago, IL: World Book.

Technical and Research Reports

Author's last name, first initials (Year). *Title of work* (Report No. xxxx). City of publication, state: Name of publisher.

Published Doctoral Dissertation and Master's Thesis

Author's last name, first initials (Year). *Title of doctoral dissertation or master's thesis* (Doctoral dissertation or master's thesis). Retrieved from Name of database. (Accession or Order No.).

Unpublished Doctoral Dissertation and Master's Thesis

Author's last name, first initials. (Year). *Title of doctoral dissertation or master's thesis* (Unpublished doctoral dissertation or master's thesis). Name of institution, Location.

Motion Picture

Producer's last name, first initials. (Producer), & Director's last name, first initials (Director). Year of release. *Title of motion picture* [Motion picture]. Country of origin: Name of studio.

Musical Recording

Writer's last name, first initials. (Copyright year). Title of song [Recorded by Name of artist if different from writer]. On *Title of album* [Medium of recording: CD, record, cassette, etc.]. Location: Label (date of recording if different from song copyright date).

lang, k. d. (2008). Shadow and the frame. On *Watershed* [CD]. New York, NY: Nonesuch Records.

Meaning

How to Read a Poem

One of the joys and frustrations of reading poetry is that a poem does not necessarily reveal its meaning to you on the first read. If you don't understand a poem the first time you read it, relax; it doesn't mean you're doing something wrong.

Often, you'll need to read a poem at least four or five times—maybe more—to understand it well. Take your time.

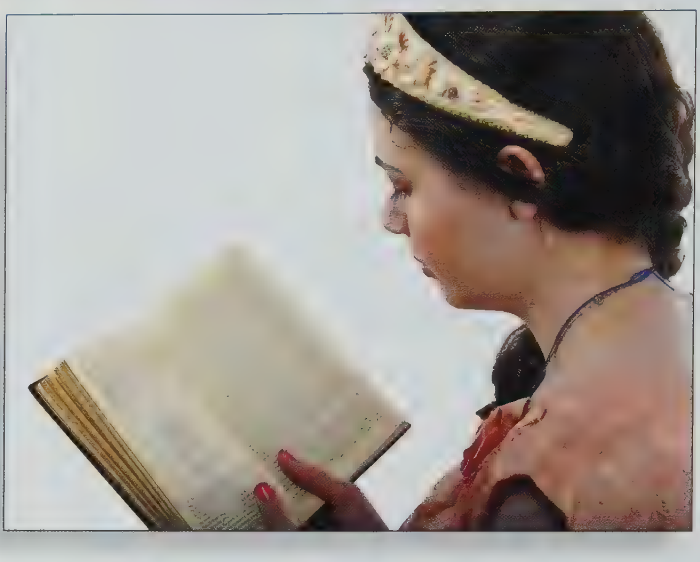
First Read. Read the poem through once at a normal rate. Pay attention to your first impressions. What makes sense? What seems confusing? You will want to target the confusing bits in the next rounds of reading.

Second Read. Read the poem again more carefully. Watch for words or allusions you don't understand. Look up unfamiliar words and write down their meanings. Check allusions by looking them up in an encyclopedia or using some other resource (like your teacher, librarian, friends, or parents).

Third Read. Read the poem again. This time, try to translate it into your own words. Write the meaning of each line. Recognize that your paraphrase cannot mean the same thing as the poem itself—just as reading a description of a hamburger does not have the same taste as eating the hamburger.

Fourth Read. Read the poem again. Assume there is something wrong with your translation. Try to explain on paper what that problem might be. How is your paraphrase different? What are the problems with the poem that the paraphrase does not solve? This strategy may help you see the poem in a different way.

Fifth Read. Finally, read the poem again. This time, relax and enjoy the experience of moving through it. Write out (or underline) any words or phrases you find particularly effective or important. Now you're ready to analyze the poem.



Maya Angelou, a celebrated American poet, reads her piece "On the Pulse of Morning" at President Clinton's inauguration, in 1993.

How to Analyze a Poem, Step by Step

Here are some guidelines you can use to get to know a poem better. If you can't answer a question, move on to the next step. You can always ask for help later.

See the Obvious

1. Look at the title. What does it tell you about the poem?
2. Find out a little about the poet, such as when and where he or she lived.
3. Count the number of lines in each verse. This will help you determine what kind of poem it is.
4. Read the poem.
5. Look up words you don't know in the dictionary.
6. Write a one-sentence summary of what the poem is about.

Now you should know the subject, the historical context, the form, the words, and the theme of the poem.

Listen Carefully

7. Read the poem out loud. Listen for sound effects.
8. Assign letters to each end rhyme. Each word that rhymes gets the same letter. Each new sound gets a new letter.
9. What form does the rhyme scheme support?
10. Circle any example of alliteration.
11. Underline examples of consonance.
12. Note internal rhymes.
13. Find any instances of onomatopoeia.
14. Think about how these sounds support the poem's meaning.

Feel the Rhythm

15. Count the number of syllables in each line.
16. Mark stressed and unstressed syllables.
17. Draw a slash between feet.
18. Note any additional feet, such as a double stress or single syllable.
19. Name the feet and meter.

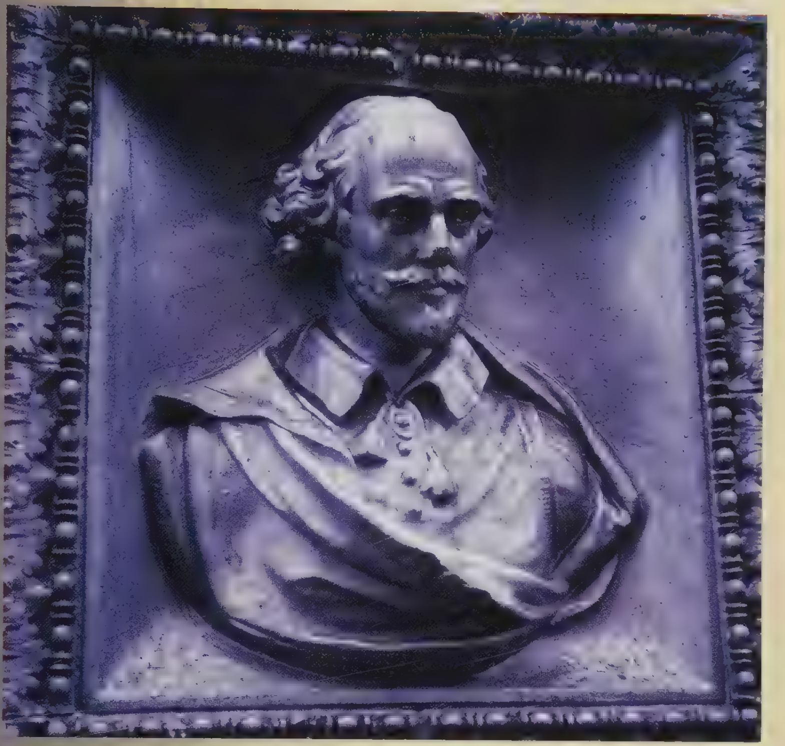
Dig for Meaning

20. Reread the poem.
21. Identify images and figures of speech in each verse.
22. Explain how those images and figures of speech support the meaning of the poem.
23. Paraphrase each verse or section of the poem in your own words.

fyi!

The key to analyzing a poem is to take your time.

Literature



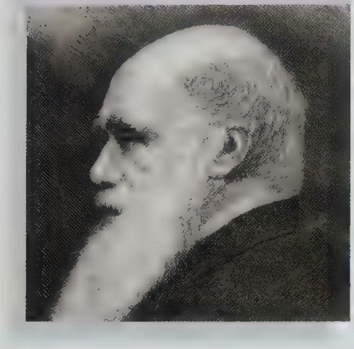
World Literature Time Line 920

Shakespeare 960

Literature: 1858 to 1880

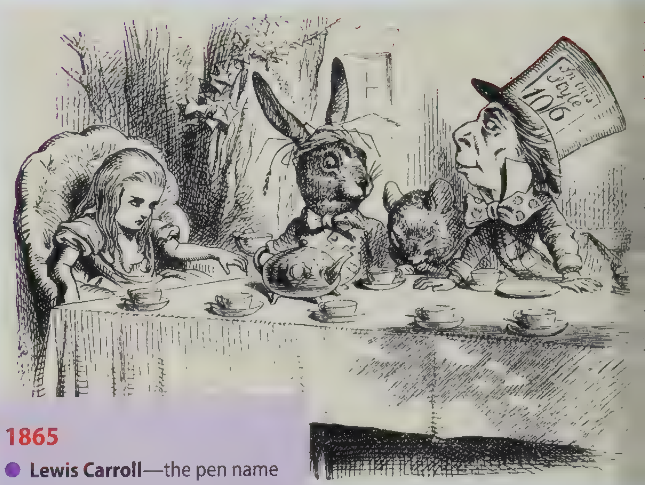
1859

● **Naturalist Charles Darwin** publishes his theories of evolution in *On the Origin of Species*, marking a general shift from Romanticism to Realism.



1863

● **Edward Everett Hale** writes "The Man Without a Country," a short story about a young Army officer. Hale's story caused such a sensation that many people failed to realize that it was fiction.



1865

● **Lewis Carroll**—the pen name of Charles Lutwidge Dodgson, a matematician—writes *Alice's Adventures in Wonderland*.

1858

1865

1860

● **Charles Dickens** begins publishing *Great Expectations*. (Most of his novels were published in serial form.) Dickens was a keen observer of life and had a great understanding of humanity, especially of young people. He sympathized with the poor and helpless, and mocked and criticized the selfish, the greedy, and the cruel.

1861

● **The U.S. Civil War** begins when Confederate troops fire on Fort Sumter, a U.S. military post in Charleston, South Carolina. (See pages 118–131.)

1862

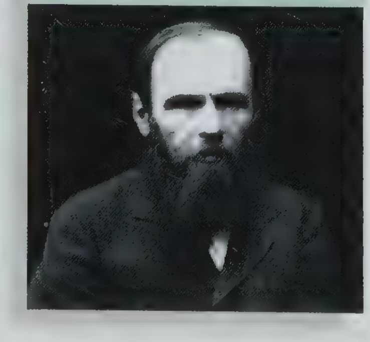
● **Ivan Turgenev** writes *Fathers and Sons*, a novel that depicts Russian society on the eve of emancipation of the serfs. He was the first Russian writer to achieve substantial recognition in the West.

1864

● **John Greenleaf Whittier** publishes "Barbara Frietchie," a poem. His best-known poems fall into two groups—those attacking slavery, and those praising the charms of New England country life.

1864

● **Fyodor Dostoevsky** writes *Notes from the Underground*. The underlying theme in his books is the struggle between good and evil for dominance of the human soul. Dostoevsky attempts to resolve this struggle by leading his characters to salvation through purifying suffering.



1866

● **Dostoevsky** writes *Crime and Punishment*, a novel about redemption through suffering.

1867

● **Matthew Arnold,** the most important literary critic of his time, publishes his poem "Dover Beach." It voiced the anxieties people felt about the industrial, political, and social upheaval that characterized Victorian England.

1867

● **Henrik Ibsen,** a Norwegian playwright who is recognized as the father of modern drama, writes *Peer Gynt*. Over the course of his career, Ibsen would write 26 plays with complex characters, lifelike dialogue, believable social and psychological motivation, and stories that addressed important issues.

Legend

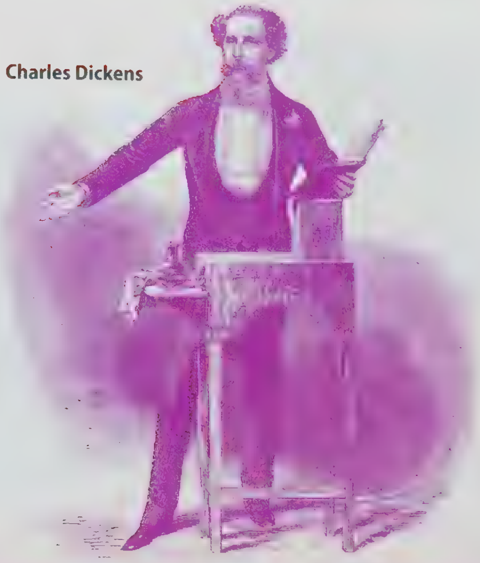
- ● American
- ● Canada
- ● Britain
- ● Europe
- ● World
- Period Facts

1870

● **Jules Verne,** a French novelist, writes *Twenty Thousand Leagues Under the Sea*. Verne wrote some of the first science-fiction stories.

1873

● **French poet Arthur Rimbaud** publishes *A Season in Hell*. In this tortured, allegorical autobiography, the writer celebrates himself as an outcast. Rimbaud wrote his major verse between the ages of 15 and 20. He then abandoned his literary career and became a trader in what is now Ethiopia.



Charles Dickens

1868

1875

1869

● **American novelist Louisa May Alcott** publishes the second part of *Little Women*, a loosely autobiographical story that traces the lives of four sisters. G. K. Chesterton later wrote that Alcott "anticipated realism by 20 or 30 years." Alcott also worked to gain voting rights for women and was active in the temperance movement.

1868

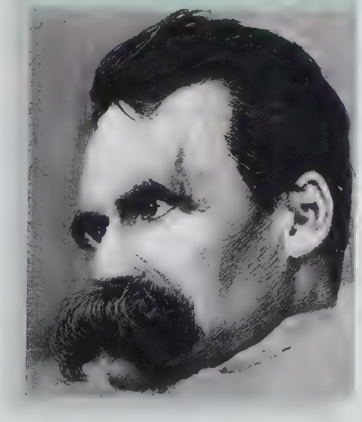
● **Robert Browning** starts publishing his long poem, *The Ring and the Book*, in four installments. The 12 books, each spoken by a different narrator involved in a murder trial in 17th-century Italy, epitomize his dramatic monologue style.

1872

● **George Eliot** finishes publishing her serialized novel *Middlemarch: A Study of Provincial Life*. It explores the moral and social problems surrounding life in the English countryside.

1872

● **German philosopher Friedrich Nietzsche** writes *The Birth of Tragedy*. It presented a new theory of the origins of classical Greek culture. His work would deeply influence many philosophers, artists, and psychologists of the 1900s.



1875

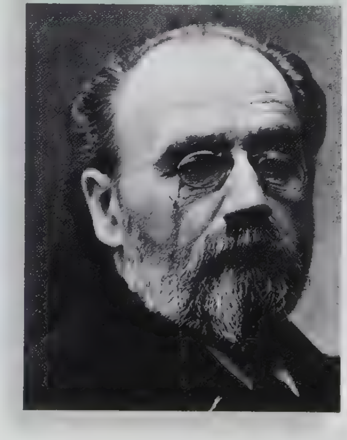
● **Leo Tolstoy** begins publishing *Anna Karenina*. The novel explores broad social, moral, and philosophical issues of Russia and its aristocracy in the 1870s. These issues include the hypocritical attitude of the upper class toward adultery and the role of religious faith in a person's life.

● 1875

Writer and editor William Ernest Henley publishes *Book of Verses*, a collection of poetry that contains his popular poem "Invictus." It was written while he was in a tuberculosis hospital.

1880

● **French writer Émile Zola** publishes *Nana*, a novel that's part of his long series of novels, *The Rougon-Macquart*. He made Naturalism the leading form of literature in France in the late 1800s.



Literature: 1929 to 1940

1929

● **Stephen Vincent Benét** writes *John Brown's Body*, a modern epic poem. This book-length narrative of the Civil War wins him the Pulitzer Prize.

1929

The stock market crash marks the beginning of the Great Depression.

1929

● **Ernest Hemingway** writes *A Farewell to Arms*. Hemingway's style, which has been widely imitated, used a plain, forceful prose style characterized by simple sentences and few adjectives or adverbs. He wrote crisp, accurate dialogue and exact descriptions of places and things.

1932

● **Aldous Huxley** writes *Brave New World*. It reflects Huxley's concern over the impact of science and technology on society.

1932

● **James T. Farrell** writes the first of his *Studs Lonigan* trilogy. They explore the impact of urban industrial life on a boy growing up in a poor Chicago neighborhood.

1929 1930 1933

1929

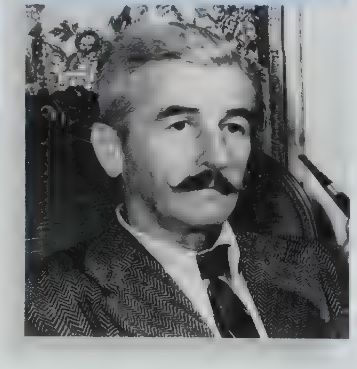
● **Erich Maria Remarque,** a German-American author, writes *All Quiet on the Western Front*. This antiwar story relates the shattering experiences of a group of German soldiers in World War I. It shocked the world because it dealt with the everyday horrors in the trenches in a cool, matter-of-fact style.

1929

● **Thomas Wolfe** publishes his autobiographical novel, *Look Homeward, Angel*. Wolfe claimed that all great art was necessarily autobiographical, so the story of his childhood and youth assumes a symbolic significance in his novels.

1929

● **William Faulkner** publishes *The Sound and the Fury*, a novel that recounts the last stages in the decline of one proud Southern family, the Compsons. It is told in an involved fashion that skips back and forth in time through the complex interior monologues of the three Compson brothers. Faulkner's work is characterized by a remarkable range of technique, theme, and tone.



1930

● **Hart Crane** publishes *The Bridge*, his long poem. The mystical work explores modern American consciousness through images of the subway, the airplane, and most importantly, the Brooklyn Bridge. Historical and legendary characters such as Rip Van Winkle, Emily Dickinson, and Walt Whitman are united in this effort to understand a national, democratic spirit.

1930

● **Dashiell Hammet** publishes his classic detective novel, *The Maltese Falcon*. The work was influential in removing the mystery story from the genteel country house to a realistic underworld setting.

1931

● **Pearl Buck** writes *The Good Earth*. Based on the author's observations during nearly 40 years of study of China, this story of a Chinese peasant family's struggles for survival became the most famous of the author's many works on that country.

1933

● **Nathanael West,** an American novelist noted for a brilliant but bitter view of modern American life, writes *Miss Lonelyhearts*.

1933

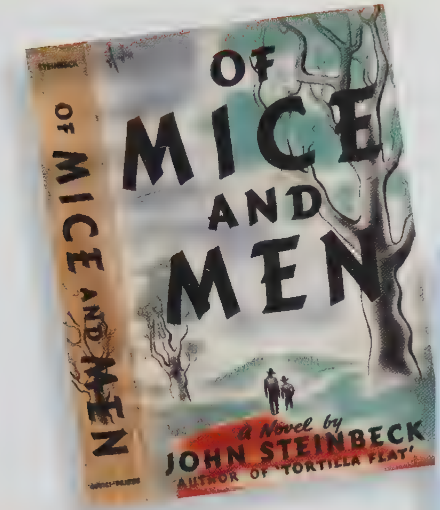
● **Franklin D. Roosevelt** becomes president of the U.S. and begins to implement the New Deal. (See also pages 178–181.)

1933

● **Spanish poet and playwright Federico García Lorca** writes *Blood Wedding*, a rural tragedy. His plays are filled with violent passion and poetic symbolism.

1933

● **Gertrude Stein** writes *The Autobiography of Alice B. Toklas*. She introduced a unique style of writing that influenced writers such as Sherwood Anderson and Ernest Hemingway.



1937

● **John Steinbeck** publishes *Of Mice and Men*. The story portrays the friendship between Lennie Small and George Milton, two migrant workers in California during the Great Depression.

1939

World War II begins when Nazi Germany invades Poland. (See pages 552–563.)

1939

● **Dorothy Parker,** an American poet and short-story writer, publishes *Here Lies*, a collection of stories. She also won fame for her witty conversation and literary criticism.

1934 1936 1939

1934

● **Dylan Thomas** publishes his first book, *Eighteen Poems*. The volume bewildered and fascinated readers with its extraordinary verbal and musical energy and with its exploration of emotional extremes.

1934

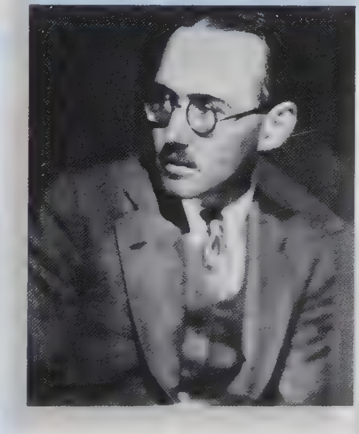
● **F. Scott Fitzgerald** writes *Tender Is the Night*, a novel about the marriage of a young psychiatrist, Dick Diver, to one of his mental patients, Nicole. The book failed because readers during the Great Depression were not interested in Jazz Age frivolity.

1934

● **Evelyn Waugh** writes *A Handful of Dust*, a biting commentary on social mores among England's upper classes.

1936

● **Stephen Vincent Benét** writes "The Devil and Daniel Webster," a short story that combines a folktale with New England history. Benét was very interested in American history and folklore.



Legend

- ● American
- ● Canada
- ● Britain
- ● Europe
- ● World
- Period Facts

1938

● **Thornton Wilder** writes *Our Town*, a Pulitzer Prize–winning play that deals with the cycle of life in a small New England town.

1938

● **Marjorie Kinnan Rawlings** writes *The Yearling*. Rawlings often wrote about the conflict between people and nature in the Florida backwoods.

1938

● **Jean-Paul Sartre,** a French philosopher, publishes his first novel, *Nausea*. The bare existence of things fascinated and horrified Sartre, because there seems to be no reason why anything should exist.

1939

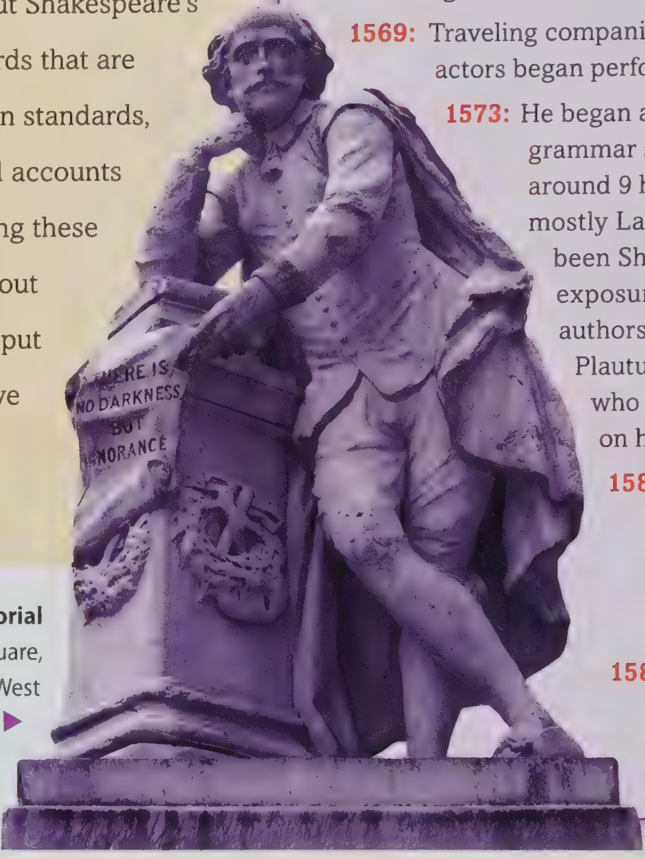
● **Steinbeck** writes *The Grapes of Wrath*. He effectively demonstrates how the struggles of one family mirror the hardships of the entire nation during the Great Depression.

1939

● **James Joyce** publishes *Finnegans Wake*. Joyce portrayed one family and at the same time all families, everywhere, at all times in history. In the story, Dublin symbolizes all cities. Joyce crammed the book with topical and historical names, events, myths, songs, jokes, and gossip. His goal was to make all people, places, things, and times repeat and resemble each other.

THE FIRST PERIOD

In Shakespeare's time, the English didn't care about keeping careful records about matters that didn't relate to the affairs of the Church or State. The information we have about Shakespeare's life has been culled from records that are few and incomplete by modern standards, including church registers and accounts of business dealings. By relating these records to what they know about English history, scholars have put together a fairly comprehensive account of his life. Still, gaps remain.



The Shakespeare Memorial Fountain rests in Leicester Square, the epicenter of London's West End theater district. ▶

Shakespeare's Life

1564: William Shakespeare was born in the small market town of Stratford-upon-Avon, the third of eight children.

1569: Traveling companies of professional actors began performing in Stratford.

1573: He began attending the Stratford grammar school. Students spent around 9 hours a day studying mostly Latin. This might have been Shakespeare's first exposure to ancient Roman authors like Cicero, Ovid, Plautus, Seneca, and Virgil, who were major influences on his work later in life.

1582: Shakespeare received a license to marry Anne Hathaway, the daughter of a local farmer.

1583: Anne gave birth to the couple's first child, Susanna.

Shakespeare's birthplace is located on Henley Street in Stratford-upon-Avon, Warwickshire. ▼

Play	Type	First Performed	First Published
The Comedy of Errors	Comedy	1589–1594	1623
Richard III	History	1592–1594	1597

1585: Anne gave birth to twins, Judith and Hamnet.

1585–1592: The so-called lost years. Scholars have proposed a number of theories about Shakespeare's activities during this time, but no one knows exactly what he was up to.

1592: Shakespeare arrived in London and began to work in the theater.

1593: Turning to poetry because of the frequently closed theaters, Shakespeare published *Venus and Adonis*, a volume of poetry that drew on the Ovid's *Metamorphoses*. It was so popular that it was reprinted at least 15 times during his life.

1594: Shakespeare became a shareholder in the Lord Chamberlain's Men, a theater company in London. His position as a shareholder helped him achieve a level of financial success unmatched by the dramatists of the age, many of whom lived in poverty.

1594: He published *The Rape of Lucrece*, another volume based on the works of Ovid. Despite the commercial success of these early publications, Shakespeare made no effort to make a career as a poet.

1596: Shakespeare's son, Hamnet, died.

1597: The playwright purchased New Place, one of Stratford's two largest townhouses.

1599: Shakespeare became one of six shareholders who signed The Globe Theatre's lease.

1603: Queen Elizabeth I died. She was succeeded by her cousin King James I, who actively supported the theater. He issued a royal license to Shakespeare and his fellow players that allowed the company to call itself The King's Men. In return, the actors regularly entertained the king at court.

1609: *Shakespeare's Sonnets*, a collection of more than 150 poems, was published. In the late 1500s, it was fashionable for English gentlemen authors to write sequences of sonnets.

1612: Shakespeare became widely known as England's most successful playwright.

1613: The writer bought a house in London.

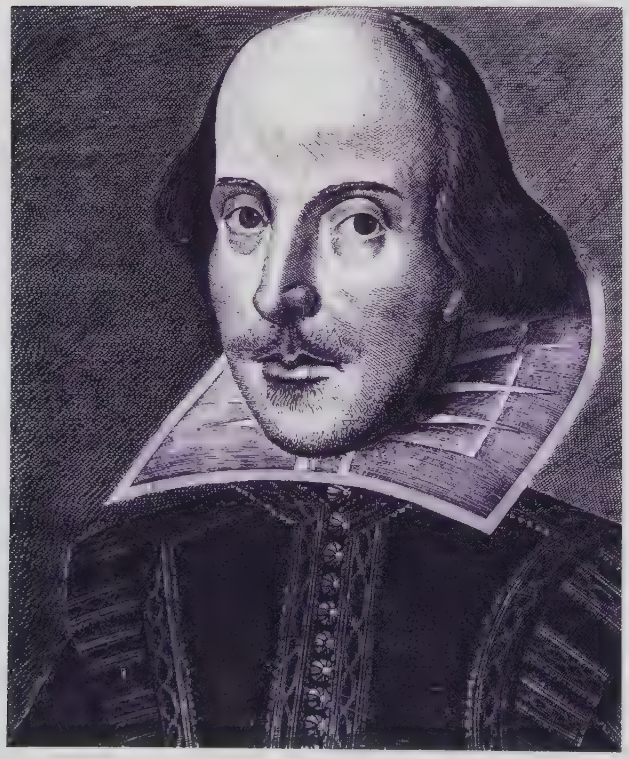
1616: Shakespeare died.

Source Material	Plot	Notes
Menaechmi, a play by Plautus	The action in *The Comedy of Errors* takes place in the ancient Greek city of Ephesus. The plot deals with identical twin brothers, both named Antipholus. Each brother has a servant named Dromio, who also happen to be twin brothers. The twins of each set were separated as children, and neither twin knows where his brother is living. After a series of mistaken identities and comical mix-ups, the twin brothers are reunited.	*The Comedy of Errors* focuses on intrigue, broad humor, and physical comedy instead of characterization or fine poetry.
Partly based on Hall's *The Union of the Two Noble and Illustrious Families of Lancaster and York* (1548); Holinshed's *Chronicles* (1577)	The play deals with the end of the Wars of the Roses. It opens with the hunchbacked Richard, Duke of Gloucester, confiding his villainous plans to the audience. As King Edward IV lies on his deathbed, Richard plots a way to secure the throne for himself. To eliminate any competition, Richard has his brother, the Duke of Clarence, killed. When Edward dies, Richard sends the dead king's sons to the Tower of London, where they are executed. Shortly after seizing the throne, Richard's allies turn against him. Richard's army is defeated at the Battle of Bosworth Field.	Shakespeare's portrayal of Richard as a villain whose character is as twisted as his back was very popular in Tudor England. It gave Richard a reputation for evil that can't be wholly supported by historical fact. While Richard is thoroughly wicked, his soliloquies give his character depth and his frequent asides engage the audience.

1590–1591

The plays of William Shakespeare's first period tend to follow their sources more closely than the plots of Shakespeare's later works. Generally, the plots consist of a series of loosely related episodes, rather than a tightly integrated dramatic structure. Generally, the plays emphasize events more than the portrayal of character.

In his first period, Shakespeare's use of language indicates that he was still struggling to develop his own flexible poetic style. For example, his descriptive poetry in this period is more flowery than directly related to the development of the characters of the story. Speeches often use highly patterned schemes that involve word and sound repetitions.



The First Period

Play	Type	First Performed	First Published
The Taming of the Shrew	Comedy	1593	1623
King John	History	1594	1623
Titus Andronicus	Revenge tragedy	1594	1594
The Two Gentlemen of Verona	Comedy	1594	1623

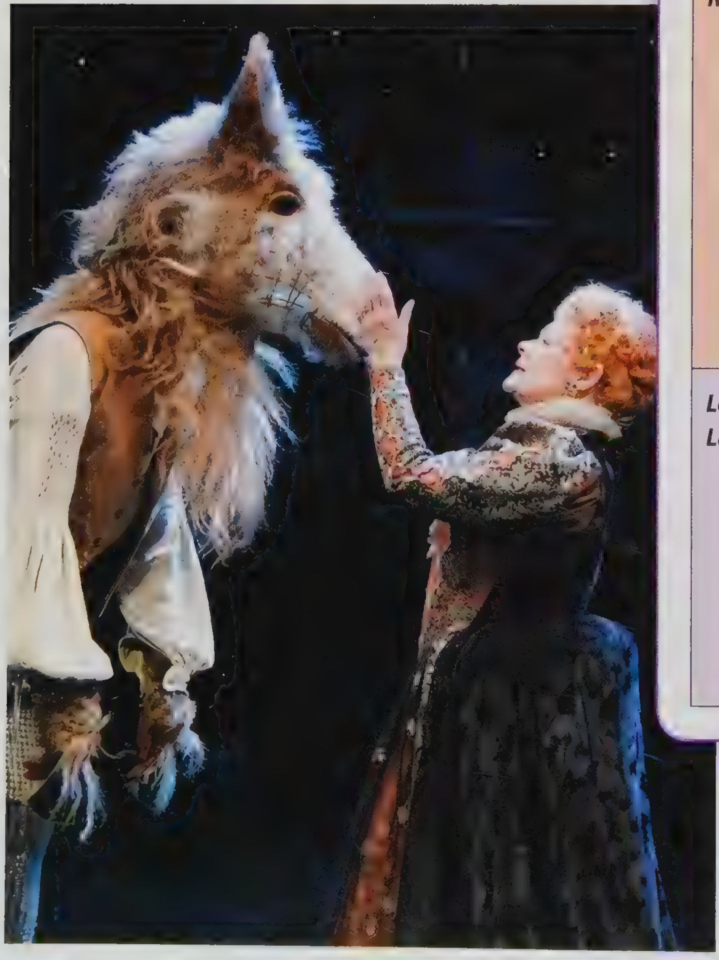
Source Material	Plot	Notes
The Taming of the Shrew, unknown English playwright; *Supposes* (1566), a comedy by the English author George Gascoigne	This play dramatizes how Petruchio, an Italian gentleman, woos the beautiful but bad-tempered Katherine, whose biting tongue has discouraged other suitors. Petruchio marries her. Before and after the wedding, he systematically humiliates Katherine to cure her of her temper. After many comical clashes between the two, Petruchio's strategy succeeds and Katherine becomes an obedient wife. At this point, Petruchio reveals himself to be genuinely fond of Katherine.	*The Taming of the Shrew* is a broad and vigorous comedy that provides two outstanding roles in the characters of the battling lovers. The parts of Petruchio and Katherine have been a showcase for generations of gifted actors.
The Troublesome Reign of John, King of England (1591), a play by an unknown English author	The story concerns the efforts of England's King John to defend his throne against the claims of his older brother's son, Arthur, the young Duke of Brittany. John defeats and captures Arthur, who is supported by the king of France. When the young prince dies under suspicious circumstances, many of John's nobles abandon him and join an invading French force. The rebellious English lords only return to John when they learn that the French, if victorious, will execute their English supporters. A long war is avoided by the intervention of Pandulph, the papal representative, just as King John dies either from poison or illness.	Shakespeare focuses less on aesthetics in this work and chooses, rather, to retell the political events in a pragmatic tone. Though the play is not entirely historically accurate, it is based on some actual events in King John's reign. Philip Faulconbridge, the illegitimate son of Richard I, is arguably the moral center of the play. His witty comments on the action orient the audience's response to the play, which is deeply concerned with loyalty, allegiance, and legitimacy.
Possibly based in part on *The History of Titus Andronicus*, a story by an unknown English author	The action takes place in and around ancient Rome and involves a series of violent acts. The central conflict is between Tamora, the captured queen of the Goths, and Titus Andronicus, a Roman general. The exchange of insults and injuries reaches its climax at a feast in which Titus serves Tamora a pie containing the remains of her two sons.	In spite of the play's emphasis on spectacular violence, it does have moments of highly charged poetry. The most complex character is Aaron the Moor, Tamora's love interest and a self-declared villain in the mold of Richard III. Aaron's plotting drives much of the action, but when the child he has fathered with Tamora is threatened with death, he displays an unexpected warmth and humanity.
Diana (c. 1559) by Jorge de Montemayor; *The Book of the Governor* (1531) by Sir Thomas Elyot	The play is a witty comedy of love and friendship set in Italy. Two friends from Verona, Valentine and Proteus, meet in Milan. They soon become rivals for the love of Silvia, the daughter of the Duke of Milan. Valentine then tells his friend that he can have Silvia, but his generosity becomes unnecessary. Proteus learns that Julia, his former mistress, has followed him to Milan disguised as a page. Proteus realizes that he really loves Julia. He marries her at the end of the play, and Valentine marries Silvia.	Shakespeare introduced several features and devices that he later used so effectively in the great romantic comedies of his second period. For example, he included beautiful songs, such as "Who Is Silvia?"; scenes in a peaceful, idealized forest; and a young woman, disguised as a page, braving the dangers of the world.

1595–1600

The focus of Shakespeare's second period was historical drama and Elizabethan romantic comedy. Particularly in his histories and comedies of this period, Shakespeare demonstrated his genius for weaving various dramatic actions into a unified plot, rather than writing a series of loosely connected episodes. Throughout the second period, Shakespeare steadily developed the matchless gift for characterization that marks the great tragedies he produced in the early 1600s.

The Second Period

Play	Type	First Performed	First Published
A Midsummer Night's Dream	Comedy	1599	1600
Richard II	History	1595	1597
Love's Labour's Lost	Comedy	1596	1598



◀ **A Queen Loves a Donkey.** Shakespeare's plays are constantly being performed. This production of *A Midsummer Night's Dream*, with Dame Judi Dench as Titania, was staged in 2010.

Source Material	Plot	Notes
No chief sources	In an enchanted forest outside of Athens, two young men, Lysander and Demetrius, and two young women, Hermia and Helena, wander about together after they become lost. Lysander and Demetrius both love Hermia and ignore Helena, who loves Demetrius. Oberon, king of the fairies, orders the mischievous elf Puck to anoint Demetrius's eyes with magic drops that will make him love Helena. However, Puck mistakenly anoints Lysander's eyes, creating much comic confusion. Puck finally straightens out the mix-up. In a subplot, Oberon quarrels with Titania, his queen. He anoints Titania's eyes with the magic drops while she sleeps so that when she awakens, she will love the first living thing she sees. At this time, Nick Bottom, a weaver, and his comical friends are rehearsing a play. When Titania awakens, she sees Bottom and falls in love with him. Aided by her fairy attendants, Titania woos Bottom until Oberon takes pity on her and has Puck remove the spell.	Shakespeare wrote some of his most richly lyrical poetry for this play. He balanced the romantic fantasy with the rough humor of Bottom and his friends. The self-absorbed Bottom ranks as one of Shakespeare's finest comic figures. While the comedy has a serious side, it also makes fun of romantic love.
Holinshed's *Chronicles*	As the play begins, King Richard exiles his cousin Bolingbroke from England. Later, Richard seizes Bolingbroke's property. While Richard fights rebels in Ireland, Bolingbroke returns to England and demands his property. Richard hurries back to England to find his cousin leading a force of nobles who are unhappy with Richard's rule. Richard gives up his crown to Bolingbroke without a fight. Bolingbroke then orders that Richard be put in prison. After Bolingbroke is crowned Henry IV, the imprisoned Richard is killed by a knight who mistakenly believed that the new king wanted Richard murdered. At the end of the play, Henry vows to make a journey to the Holy Land to pay for Richard's death.	In *Richard II,* Shakespeare seriously explored for the first time the idea that a person's character determines his fate. The play is a study of a weak, self-centered man. Richard becomes so out of touch with reality that his only defense of his kingdom is the hope that his "master, God omnipotent, / Is mustering in his clouds on our behalf / Armies of pestilence." When he faces the certain loss of his crown, Richard compares himself to Christ, who "in twelve, / Found truth in all but one; I, in twelve thousand none."
No chief sources	King Ferdinand of Navarre and his friends, Berowne, Longaville, and Dumain, vow to live in seclusion without the company of women for 3 years to pursue philosophical study. But the princess of France unexpectedly arrives at the king's court with three female companions. The comedy centers on the efforts of the men to woo the women while pretending to keep their vow. At the play's end, the men propose to their visitors, who promise to give their answer in a year and a day.	This witty comedy has more references to events of the day than do any of Shakespeare's other plays. Many of these references have lost their meaning for modern audiences, which makes some passages difficult to understand. In addition, much of the language is elaborate and artificial. But Shakespeare included two simple songs—"When Daisies Pied and Violets Blue" and "When Icicles Hang by the Wall."

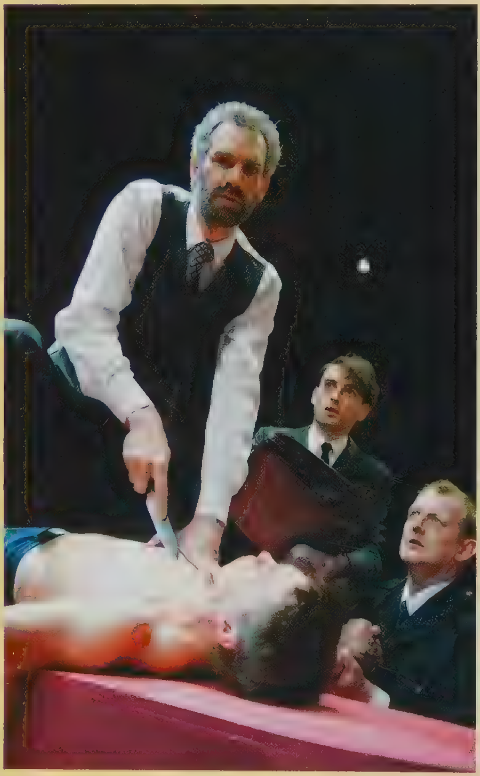
fyi!

In Shakespeare's time, most playwrights were freelancers who were paid a one-time fee for their plays. They usually worked for several companies. After 1594, Shakespeare maintained a relationship with a single company.

The Second Period

Theater Companies

After arriving in London, Shakespeare began an association with one of the city's repertory theater companies. These companies consisted of a permanent cast of actors who presented a variety of plays week after week. The companies had aristocratic patrons, and the players were technically servants of the nobles who sponsored them. But the companies were commercial operations that depended on selling tickets to the general public for their income.



The Royal Shakespeare Company's ensemble actors perform in a production of *The Merchant of Venice*. This company, with stages in London and Shakespeare's birthplace, Stratford-upon-Avon, is perhaps the world's best-known theater company.

Play	Type	First Performed	First Published
Romeo and Juliet	Tragedy	1596	1597
The Merchant of Venice	Comedy	1597	1600
Henry IV (Parts I and II)	History	1597 (Part I) 1598 (Part II)	1598 (Part I) 1600 (Part II)

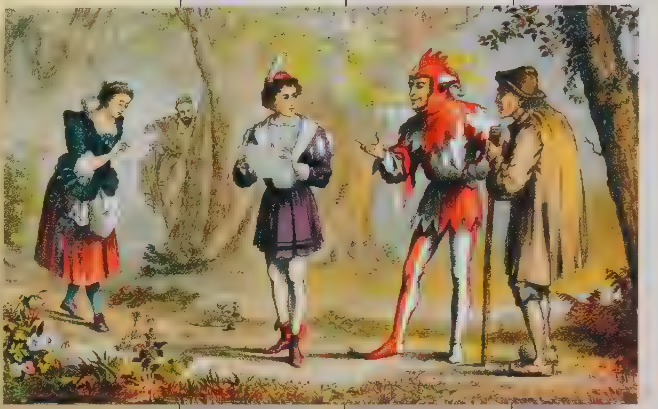
Source Material	Plot	Notes
The Tragicall Historye of Romeus and Juliet (1562), a poem by Author Brooke	*Romeo and Juliet* deals with two teenage lovers in Verona, Italy, who are caught in a bitter feud between their families, the Montagues and the Capulets. Romeo, a Montague, and his friends come uninvited to a masked ball given by the Capulets. At the ball, Romeo meets Juliet, a Capulet, and they fall in love. The next day, the couple are secretly married by Friar Laurence. Juliet's father, unaware that she is already married, tries to force her to marry a kinsman named Paris. To allow Juliet to escape from her father's demand, Friar Laurence gives Juliet a drug that puts her into a deathlike sleep for 42 hours. The friar sends a messenger to the exiled Romeo to tell him of the drug, but the messenger is delayed. Romeo hears that Juliet is dead and hurries to the tomb where she has been placed. There, he takes poison and dies by Juliet's side. Juliet awakens to find her husband dead and stabs herself. The discovery of the dead lovers convinces the two families that they must end their feud.	The popularity of *Romeo and Juliet* owes much to Shakespeare's sympathy for the young people in the play. Although the play does suggest that the boldness of young love is dangerous, Shakespeare does not present Romeo and Juliet as responsible for their fate. Instead, the play draws attention to the violence and aggressiveness that shapes the adult world. The success of the play also comes from effective characterizations and intensely lyrical poetry. Shakespeare's language shows signs of the simpler, more direct style he would use in his later tragedies.
A story in *Il Pecorone,* a collection of tales written circa 1378 by the Italian author Giovanni Fiorentino	Antonio, a merchant in Venice, Italy, borrows money from the Jewish moneylender Shylock to help his friend Bassanio. Antonio has promised Shylock a pound of his flesh if he does not repay the loan in 3 months. Three months later, when Antonio can't pay, Shylock demands the pound of flesh. Meanwhile, Bassanio has courted and married the beautiful and gifted heiress Portia. In court, disguised as a lawyer, she asks Shylock to reconsider. He remains firm. Portia then explains that he can, according to the contract, take one pound of flesh but not a drop of blood. If Shylock spills any blood, he will not only forfeit his own property but his life as well. Shylock drops his demand, and Antonio is saved.	In *The Merchant of Venice,* Shakespeare combined comic intrigue with a vivid portrait of hatred and greed. Although the play ends happily for everyone except Shylock and the melancholy Bassanio, it is not a light-hearted comedy. In Shakespeare's time, both the Church and the State considered lending money with a high interest rate a crime. Shylock was thus a natural object of scorn. On the surface, Shakespeare's view of him reflected the attitudes of the day. But the dramatist treated him with sympathy.
Two related histories: Holinshed's *Chronicles* and *The Famous Victories of Henry the Fifth* (1598), a play by an unknown English author	The two parts of *Henry IV* dramatize events that follow the murder of England's King Richard II. In Part I, the guilt-ridden Henry IV wants to go to the Holy Land to repent for Richard's death. But political unrest in England prevents him. At the same time, Prince Hal, his son, leads an apparently irresponsible life with his brawling friends, led by the fat, jolly knight Sir John Falstaff. The king quarrels with Henry Percy, known as Hotspur, the fiery young son of the powerful Earl of Northumberland. As a result of the quarrel, the Percy family revolts. At the Battle of Shrewsbury, Hal reveals himself to be a brave warrior and kills Hotspur. Part II of *Henry IV* also has many scenes of Falstaff's clowning. These scenes are set against the background of the continuing Percy rebellion and the approaching death of Henry IV, who is ill. Hal's brother, Prince John, finally defeats the rebels. The king dies, and Hal, as Henry V, takes the throne. He quickly reveals his royal qualities and rejects Falstaff and his friends, telling them to stay away until they have abandoned their wild living.	Of the two plays, Part I is more memorable. It introduces Falstaff, a bragging, lying, and thievish drunkard whose clowning provides most of the play's humor. Falstaff's faults are balanced by his clever sense of humor, his contagious love of life, and his refusal to take either himself or the world seriously.

The Second Period

Shakespeare's England

- During most of Shakespeare's lifetime, England was ruled by Queen Elizabeth I. Her reign is often called the Elizabethan Age.
- Shakespeare's works reflect the cultural, social, and political conditions of the Elizabethan Age. Knowledge of these conditions can provide greater knowledge of Shakespeare's plays and poems.
- Elizabethans believed in ghosts, witches, and magicians. No biographical evidence exists that Shakespeare had such beliefs, but he used them effectively in his works. Ghosts play an important part in *Hamlet, Julius Caesar, Macbeth,* and *Richard III.* Witches are major characters in *Macbeth.* Prospero, the hero of *The Tempest,* is a magician.
- Elizabethans were keenly aware of death and the brevity of life. They lived in constant fear of plague. When an epidemic struck, they saw victims carted off to common graves.
- Yet death and violence also fascinated many Elizabethans. Londoners flocked to public beheadings of traitors, whose heads were exhibited on poles. They also watched as criminals were hanged, and they saw the corpses dangle from the gallows for days.
- Elizabethan literature mirrored the violence and death characteristic of English life. Shakespeare's tragedies, like other Elizabethan tragedies, involve the murder or suicide of many of the leading characters.
- In spite of their tolerance of cruelty, Elizabethans were extremely sensitive to beauty and grace. They loved many forms of literature, including poetic drama, narrative and lyric poetry, prose fiction, and essays. People of all classes enjoyed music, and English composers rivaled the finest composers in all of Europe.

Play	Type	First Performed	First Published
As You Like It	Comedy	1599	1623
Henry V	History	1599	1600
Julius Caesar	Tragedy	1599	1623



▲ **In the Forest of Arden.** An 1890 book of Shakespeare plays included this drawing of a scene from *As You Like It.*

Source Material	Plot	Notes
Partly based on *Rosalynde* (1590), a novel by the English author Thomas Lodge	Rosalind and her cousin Celia leave the court of Celia's father, Duke Frederick, after he unjustly banishes Rosalind. Accompanied by Touchstone, the court jester, the two young women take refuge in the Forest of Arden. Also in the forest are Orlando, who loves Rosalind; Jaques, a melancholy philosopher; Audrey, a goatherd; Silvius, a shepherd; and Phebe, a shepherdess. Duke Frederick's brother, who is Rosalind's father and the rightful ruler of the land, also lives in the forest with a band of merry outlaws. Rosalind, disguised as a young shepherd named Ganymede, meets Orlando in the forest. Not recognizing the young woman in disguise, Orlando agrees to pretend that Ganymede is Rosalind so he can practice his declarations of love. Rosalind finally reveals her identity and marries Orlando. Oliver, Orlando's formerly wicked brother, marries Celia, Touchstone marries Audrey, and Silvius marries Phebe. The news that Rosalind's father has been restored to his dukedom completes the comedy's happy ending.	Like many other Elizabethan romantic comedies, *As You Like It* concerns young lovers who pursue their happy destiny in a world seemingly far removed from reality. Although evil threatens, it never harms. Shakespeare consistently balanced the merry laughter of *As You Like It* with notes of seriousness and even sadness. Touchstone's wit and Jaques's remarks question the nature of love and the values of society. Jaques adds a strong note of melancholy to the play with his famous description of the seven ages of man.
Partly based on Holinshed's *Chronicles* and *The Famous Victories of Henry the Fifth*	The king decides to press a claim he believes he has to the French throne. He heads an army that lands in France. Inspired by Henry's leadership, the outnumbered English troops defeat the French at the town of Harfleur. The two armies then meet in battle near the village of Agincourt. Against overwhelming odds, the English win a great victory. The triumphant Henry is received at the French court. There he is promised the throne and the hand of Katherine, the French princess.	Henry claims to hate war in general. Yet he finds himself carried away by the glamour and glory of the French campaign. Although the play occasionally seems to glorify war, Shakespeare sets the heroics against a background of political treachery and empty honor. Comic scenes mock the vanity of the royal court. These scenes remind audiences that monarchs and their councils plan wars, but ordinary people must fight and die in them.
Parallel Lives of Illustrious Greeks and Romans by the ancient Greek biographer Plutarch	The play takes place in ancient Rome. In spite of its title, the play's central character is Brutus, a Roman senator and Caesar's best friend. Brutus reluctantly joins a plot to murder Caesar because he believes Rome's preservation requires Caesar's death. Brutus defends the assassination to a crowd of Romans. But he unwisely allows the clever and eloquent Mark Antony to deliver a funeral speech over Caesar's body. Antony tells the people, "I come to bury Caesar, not to praise him." He then describes the plotters with heavy sarcasm as "honorable men." At the same time, Antony points out Caesar's virtues and thus gradually turns the crowd into a mob ready to avenge Caesar's death. The conspirators are forced to flee Rome. Mark Antony leads an army that defeats the conspirators at the Battle of Philippi. At the end of the battle, Brutus commits suicide.	*Julius Caesar* has become a popular play because of its innovative use of language and sharp character portraits. (For example, Caesar describes the plotter Cassius as having a "lean and hungry look.") But the play's real interest centers on the character of Brutus. A thoughtful, withdrawn man, he is torn between his affection for Caesar and his strong sense of duty to the Roman Republic.

The Second Period

Shakespeare's Style

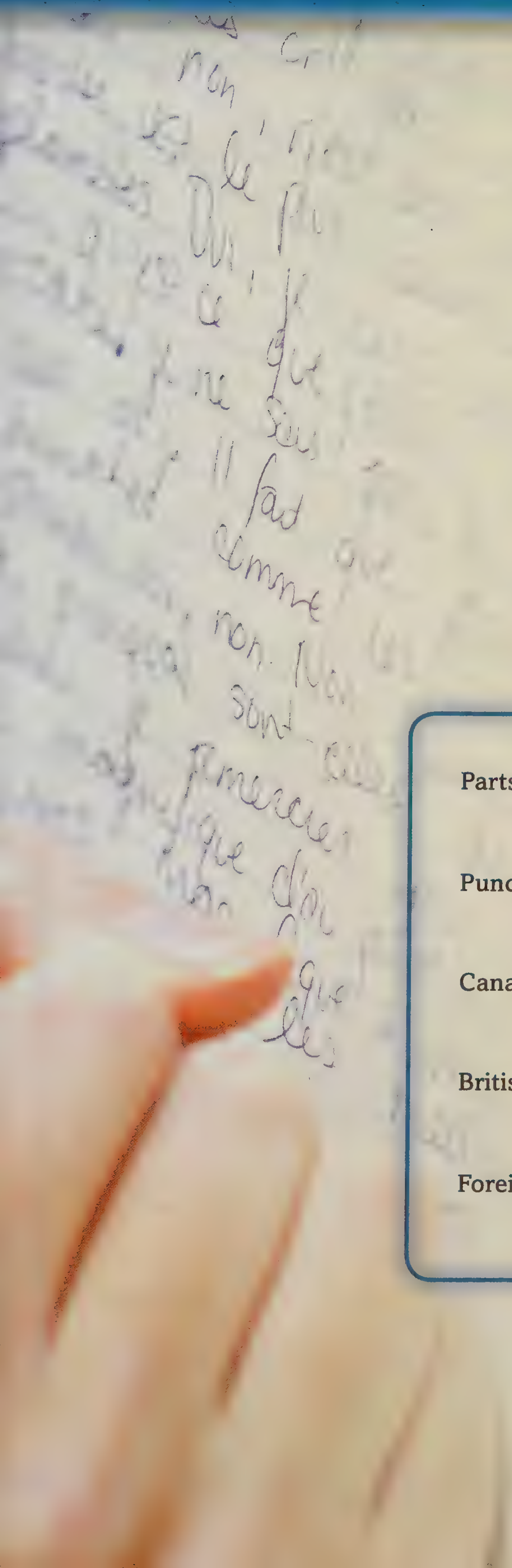
- In Shakespeare's time, early modern English was still assuming its fully modern form.
- Shakespeare and other Elizabethan writers looked upon the English language as alive and changing. They did not consider it fixed for all time in a set of correct, unbreakable rules.
- Shakespeare experimented freely with sentence structure and vocabulary to create special effects. He also used various literary devices to present information.
- His style is probably best known for its brilliant use of language to create vivid pictures in the mind.
- Since his death, Shakespeare's style has helped shape the language of all English-speaking countries. Many later writers in English have accepted the Elizabethan style as their model. As a result, a large chunk of English and American literature reflects certain characteristics of Elizabethan writing.
- Shakespeare's vocabulary of about 29,000 words is remarkably rich. Like his fellow writers, he put old words to new uses, borrowed from other languages, and invented new terms.
- What sets apart Shakespeare's verbal creativity is that so many of his innovations were adopted by English speakers. Thomas Nashe, a contemporary of Shakespeare's, also freely invented words, but most of them are now forgotten.
- The richness of Shakespeare's vocabulary sometimes raises difficulties for modern readers. Not all the words and meanings used by Shakespeare remain current.

Play	Type	First Performed	First Published
Much Ado About Nothing	Comedy	1599	1600
Twelfth Night	Comedy	1600	1623
The Merry Wives of Windsor	Comedy	1600	1602

▲ **Patrick Stewart is Malvolio.** Stars of television and screen often test their mettle in a Shakespeare production.

Source Material	Plot	Notes
Partly based on *Orlando Furioso* (1516), an epic poem by the Italian author Ludovico Ariosto; and *Novelle*, a story in a collection of tales by the Italian author Matteo Bandello	The plot concerns the attempts by the villainous Don John to slander the virtue of Hero, the daughter of the governor of Messina, Italy. Hero is about to be married to Claudio, a young lord from Venice. Don John manufactures an accusation of infidelity that causes Claudio to jilt Hero at the altar. After much intrigue, Don John's plot is exposed and the couple happily marry. Much of the interest in the play centers on the relationships between Beatrice, Hero's cousin, and Benedick, a lord of Padua. These two witty characters trade insults for much of the play, but they come together in an attempt to restore Hero's damaged honor and soon realize that they are themselves in love.	The play's combination of sharp intelligence and lack of self-knowledge produces rich comedy. Broad humor is supplied by the talkative village constable, Dogberry, and his assistant, Verges.
Partly based on a story in *Barnabe Riche: His Farewell to Military Profession* (1581), a collection of tales by the English author Barnabe Riche	Viola and Sebastian, who are twins, become separated during a shipwreck. Stranded in the country of Illyria, Viola disguises herself as Cesario, a page, and enters the service of Duke Orsino. The duke sends the page to woo Countess Olivia for him. But the countess falls in love with Cesario. Meanwhile, Viola falls in love with the duke. The romantic action alternates with scenes of realistic comedy involving the fat knight Sir Toby Belch and his friends. Maria, Countess Olivia's lady-in-waiting, tricks the countess's steward, Malvolio, into thinking that Olivia loves him. The plot becomes increasingly tangled when Sebastian, Viola's twin brother, appears and agrees to marry Olivia. In the final scene, Viola, still disguised, is confronted by Olivia, who is confused by the youth's refusal to acknowledge their recent marriage. Duke Orsino is enraged by the treachery of "Cesario" and threatens violence. But all is resolved when Sebastian reappears and Viola reveals her identity. Viola and Orsino then declare their mutual love, and the play concludes anticipating their marriage. Only Malvolio is left unhappy.	*Twelfth Night* offers a blend of sentiment and humor. Feste, Olivia's clown, makes witty comments about the foolish ways of people. Feste's songs contribute both gaiety and sadness to the mood of the play. Only Malvolio, who thinks he is more moral than other people, spoils the gentle mood of the play.
Unknown	According to a popular (if unproven) story, Queen Elizabeth requested the play. She so enjoyed the comic character Sir John Falstaff in the *Henry IV* plays that she asked Shakespeare to write a comedy portraying Falstaff in love. The comedy dramatizes Falstaff's efforts to make love to Mistress Ford and Mistress Page, two honest housewives in the town of Windsor. Instead of winning their love, Falstaff ends up the victim of a number of comical tricks invented by the women.	Although *The Merry Wives of Windsor* lacks the romantic poetry of most Shakespearean comedies, the play is highly entertaining. The Falstaff in this work has less imagination and wit than the Falstaff in the *Henry IV* plays. But the character remains theatrically effective, even though the audience laughs at him rather than with him, as in the earlier plays.

At a Glance



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Parts of Speech

The English language has eight basic parts of speech, which are listed here alphabetically. Each part of speech serves a distinct grammatical function, although occasionally one part of speech may be altered to serve a different use. (For example, some verb forms can act as nouns or modifiers.) The table below provides quick definitions and examples of usage for the basic parts of speech.

Adjectives modify nouns or pronouns.

- **Descriptive adjectives** provide additional information about the object or person being described.

  Degrees of comparison are usually indicated by the suffixes *-er* for the comparative degree and *-est* for the superlative (fat, fatter, fattest), although some adjectives require the use of *more* and *most* (beautiful, more beautiful, most beautiful).

  A few adjectives have special comparative forms (good, better, best; bad, worse, worst).
- **Intensifying adjectives,** which emphasize the related noun or pronoun, are related to the personal pronouns.
- **Possessive adjectives,** which show ownership, are related to personal pronouns, as can be seen in the chart below.
- **Demonstrative adjectives** (this, that, etc.) are used to indicate specific nouns. (Have you read *this* book?)

Number	Person	Intensifying Adjective	Possessive Adjective
Singular	first	myself	my
	second	yourself	your
	third	himself	his
		herself	her
		itself	its
Plural	first	ourselves	our
	second	yourselves	your
	third	themselves	their

Adverbs modify verbs, other adverbs, adjectives, phrases, and clauses. Degrees of comparison are usually indicated by the use of more and most (quickly, more quickly, most quickly).

Conjunctions are used to connect individual words or groups of words. When conjunctions are used to connect clauses, either coordinating or subordinating conjunctions are used.

- **Coordinating conjunctions** (such as *and*, *but*, and *or*) connect two independent clauses.
- **Subordinating conjunctions** (such as *as*, *because*, and *until*) connect an independent clause to one that is dependent on it.

Interjections are exclamatory expressions with no grammatical relation to the sentence in which they occur. (*Gosh*, is it really that late?)

Nouns are words that are used to represent a person, animal, place, thing, idea, or quality. Nouns may be singular (referring to one) or plural (referring to more than one).

- **Collective nouns** refer to groups of people or animals (family, herd). When the group is being emphasized, a collective noun is treated as singular, but when the collective noun refers to the members in the group, it is treated as plural.
- **Common nouns** refer to any person, animal, or place (a relative, a pet, a town).
- **Proper nouns** refer to a particular person, animal, or place (Uncle Charlie, Fido, New York City).

Prepositions show the relationship between a noun (or pronoun) and another part of the sentence. Common prepositions include at, between, for, of, and with. A pronoun following a preposition is always in the objective case.

Pronouns are words that can be used instead of nouns. The noun replaced by a pronoun is called the antecedent.

- **Indefinite pronouns** (each, all, etc.) do not have a clear antecedent. (*Each* tried to succeed, but none did.)
- **Interrogative pronouns** (who, whom, whose, what, which) are used to ask questions about unknown nouns. (*Who* came? *What* did she say?)
- **Personal pronouns** are listed in the chart at right.
- **Reflexive pronouns** (myself, yourself, himself, herself, itself, ourselves, yourselves, themselves) refer back to the person performing an action. (He taught *himself*.)
- **Relative pronouns** (who, whom, whose, that, what, which) introduce subordinate clauses and replace nouns or pronouns in the main clause. (He is the man *who* tried to help me.)

Personal Pronouns

		Case		
Number	**Person**	**Nominative**	**Objective**	**Possessive**
Singular	first	I	me	mine
	second	you	you	yours
	third	he	him	his
		she	her	hers
		it	it	its
Plural	first	we	us	ours
	second	you	you	yours
	third	they	them	theirs

Verbs are used to express an action or a state of being. There are four types of verbs: *transitive*, *intransitive*, *linking*, and *auxiliary*. All verbs have four forms: the *infinitive* (to sing), the *present* (sing), the *past* (sang), and the *past participle* (sung).

Types of Verbs

- **Transitive verbs** show an action that has an effect on an object. (John *sang* the song.)
- **Intransitive verbs** do not have an effect on an object. (John *swam*.)
- **Linking verbs** equate a subject and its predicate. (John *is* good.)
- **Auxiliary verbs** are used with other verbs to indicate tense. (John *has* sung the ballad many times. John *will* swim tomorrow.)

Verbs vary according to tense, mood, and voice.

Tense. The tense of a verb indicates time. *Past*, *present*, and *future* are the three main tenses in English. In addition, English uses the *present perfect*, the *past perfect*, and the *future perfect*. The present and past perfect tenses are used to give more information about the past. The present perfect indicates that something has occurred at some indefinite time in the past, while the past perfect indicates that something has occurred at a particular time in the past. Similarly, the future perfect indicates that something will occur at some particular time in the future. (She has taken a nap. By teatime, she had taken a nap. By dinnertime, she will have taken her nap.) These tenses are formed by combining auxiliary verbs with the past participle.

Mood. English has three moods: *indicative*, *imperative*, and *subjunctive*. The indicative is used for statements of fact or questions (I am coming. Are you coming?); the imperative to give commands (Come here!); and the subjunctive mainly to express wishes (I wish you were here) and statements contrary to fact (If you were here, I would be happy).

Voice. Verbs may either be *active* or *passive*. A verb in the active voice shows that the subject has performed the action, while a verb in the passive voice shows that the subject has received the action. (I threw the ball. The ball was thrown.)

Punctuation Review

Questions of punctuation are always a serious concern in formal writing. The following pages provide a brief review of major types of punctuation.

Good style in punctuating, especially in the use of commas, often depends on a grasp of English grammar.

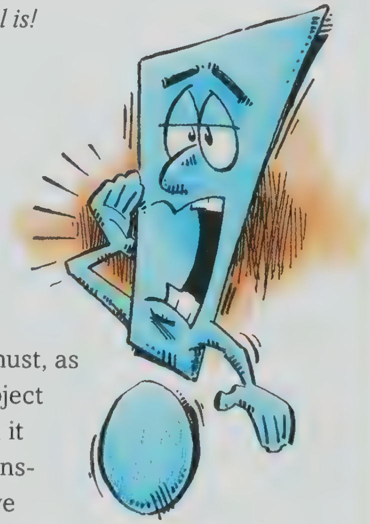
There are two issues in punctuating. The first is a simple question of right and wrong. A writer who ends each sentence with a colon rather than a period is simply wrong. Sentences cannot end with a colon.

The second issue is less clear-cut. For example, in some cases, the writer may choose punctuation to suit his or her taste: a semicolon to separate two closely related thoughts, or a period. Writers learn with practice which possibility to choose to make their meaning clearest.

End punctuation. There are three kinds of end punctuation: the *period*, the *question mark*, and the *exclamation point.* Every sentence must end with one of these marks.

The exclamation point ends an exclamatory sentence, and the question mark is at the end of a question. All other sentences end with a period.

What a good boy Paul is!
Is Paul a good boy?
Paul is a good boy.



The most common mistake in end punctuation usage is to punctuate as a sentence a group of words that is not a sentence. A sentence must, as a minimum, have a subject and a verb. In addition, it must not have been transformed into an adjective clause by the addition of a subordinate conjunction, nor into a relative clause by the use of a relative pronoun. Such clauses should not be punctuated as separate sentences, but only as parts of some other, longer sentence.

Commas. The most common mistakes in comma usage are using too many and using commas instead of semicolons to join two independent clauses.

- **Compound sentences.** When two simple sentences are joined together by a coordinating conjunction, put a comma before the conjunction. (This comma is often omitted when the clauses being joined are especially short.) Do not use commas when only a part of the sentence has been compounded.

  She got up to close the window, but he asked her to sit down again.

- **Adverbial clauses.** An adverbial clause at the beginning of a sentence is set off from the rest of the sentence by a comma. Such a clause at the end of a sentence is not set off.

  When we got back from the beach, we were too tired to eat dinner.

  We were too tired to eat dinner when we got back from the beach.

- **Clarity.** Sometimes—but very rarely—a comma is needed to avoid confusion and to make a sentence clearer:

  To John, Matilda would always be a mystery.

Sentence Fragment
Although I like the design.

Corrections
(1) *I like the design.*
(2) *Although I like the design, I don't care to use it in a living room.*

Periods. Besides its use at the end of a sentence, the period is used after initials (*T. S. Eliot*) and after many other abbreviations, including those of months (*Feb.*), countries (*U.S.A.*), states (*Tenn.*), and other commonly abbreviated forms (*St.*, *Ave.*, *Dr.*, *Mr.*, and so on).

- **Parenthetical expressions.** Parenthetical expressions are set off by commas. These include yes, no, and mild interjections (those not followed by an exclamation point), such as *well* and *oh*,

  Well, it's time to leave

  nouns of address,

  How is your garden growing, Mary?

  and such expressions as *of course* and *however*.

  We will, of course, be ready; others, however, may not be.
- **Series.** Use commas after all but the last item in a series.

  Her Russian was barely passable, but she was fluent in English, French, Italian, and Mandarin.
- **Nonrestrictive relative clauses.** A nonrestrictive relative clause is set off from the rest of the sentence by commas. So, too, are expressions that derive from nonrestrictive clauses, such as appositives and nonrestrictive participles and participial phrases.

  Mr. Jackson, standing on the makeshift platform, gave a rousing campaign speech.

Semicolons. Use a semicolon to join two sentences without using a conjunction. The semicolon is also used when two sentences are joined by such an expression as therefore or however.

Two plus two equals four; therefore, five is not an acceptable answer.

The semicolon is also used to separate the items in a series when there are already commas within individual elements of the series:

He has lived in Moline, Illinois; Boulder, Colorado; and Seattle, Washington.

Colons. The main use of a colon is to introduce a list, an example, a question, or a long quotation.

The question is this: What should we do next?

A colon used in this way should always follow a noun or a pronoun, never a verb or a preposition. It may also come after the expressions *as follows* and *the following*.

Dashes. The dash indicates a sudden break or change of emphasis in a sentence:

I have here a—now, where did I put that thing?

Dashes can also be used to set off an appositive when the appositive is to be emphasized or when it contains commas within it.

Parentheses. Parentheses set off material in a sentence that is separate or apart from the main thought.

She traveled through Davenport (a city in which she once lived) and on toward Chicago.

Quotations. A direct quotation—the exact words that someone has said or written—is enclosed in quotation marks. If the quotation is included within another sentence, it is set off from the rest of the sentence by commas:

"I think," he said, "that you are on the right track."

When more than one person is being quoted, as in a conversation, begin a new paragraph for each change of speaker.

Question marks and exclamation points are placed inside the quotation marks if they are part of the quote

"Who are you?" she asked

and outside if they apply to the sentence as a whole

Who was it that said, "I shall return"?

Periods and commas are always placed inside quotation marks, while the semicolon and colon are always placed outside.

Do not use quotation marks for an indirect quotation, that is, one that does not report someone's exact words:

He said that it was raining.

Punctuation Summary

Italics. Words to be set in italics are indicated in typed or handwritten material by an underline. Italics are used to single out words, phrases, or even sentences for special emphasis. Titles of books, plays, magazines, and newspapers, and the names of ships, trains, and airplanes, are italicized. (Shorter works like poems and stories are put in quotation marks.)

The Adventures of Tom Sawyer
Hamlet
National Geographic
the *Titanic*
"Annabel Lee"
"The Masque of the Red Death"

Capitalization. All proper nouns (names of specific persons, beings, places, or things) are capitalized. In addition, capitalize the first word of every sentence; the first word of a direct quotation embedded in another sentence; the names of groups, associations, and businesses; the letters of some abbreviations; and all historic events, buildings, monuments, and documents:

General Motors *Grant's Tomb*
NASA *Declaration of*
Independence
World War II

Titles used with proper nouns are capitalized:

Dr. Brown
Senator Douglas

Also capitalized are the first, last, and important words in titles of printed texts. Prepositions, articles, and conjunctions are not capitalized unless they are the first or last word of the title.

Symbol Name	Use
. **period**	Placed at end of statements.
	Used after initials.
	Used after abbreviations.
? **question mark**	Placed at end of sentences that ask something.
	Used in parentheses to express doubt.
! **exclamation point**	Placed at end of statements that show strong feelings.
, **comma**	Used to separate items in a series.
	Used to separate two clauses of a compound sentence joined by a coordinating conjunction.
	Used to set off parenthetical information.
	Used to set off appositives.
	Used to set off an introductory modifier.
	Placed after or before the name of someone being addressed.
	Used before an exact quotation or to conclude a statement in quotation marks.
	Placed after *yes, no, oh,* and *well,* and also after interjections.
	Used in dates to separate the day and year.
	Used to separate the name of a city and state or province.
" " **quotation marks**	Used to set off someone's exact words.
' ' **single quotation marks**	Used for quoted speech within a quoted sentence.
; **semicolon**	Used to separate independent clauses closely related to each other.
	Used to separate items in a series when the items include commas.
: **colon**	Used before a list.
	Used to separate independent clauses when the second clause provides additional information about the first.
	Used before a character's lines in a play.
	Used to separate the hour from the minutes when writing the time.
— **dash**	Placed before a sudden break in thought or an interruption.
	Used to add emphasis to part of a sentence.
' **apostrophe**	Used to indicate a missing letter in contractions.
	Used to show possession.
( ) **parentheses**	Used to set off additional information from the rest of a sentence.
	Used to set off sentences that are separate from the main sentence.
[ ] **brackets**	Used to set off information inserted to clarify quoted material.

Example
Grandfather is old.
J. F. K., T. S. Eliot
Ph.D.
Is your grandfather very old?
Chaucer, born in 1340(?), died in 1400.
Wow! Grandfather is old!
My mom is young, witty, and attractive.
My dad is old, but he is in good health.
Fred, who is quite old, is still in good health.
Grandfather, a most generous man, is well regarded in our town.
Despite all her hardships, she remains optimistic. Hurt by her taunts, he left abruptly. Politically, his future is not rosy.
Jane, please come here. We'll miss you, Emma.
He said, "We'll miss you." "We'll miss you," he said.
Well, the food finally arrived. Yes, food is what we need.
My grandfather was born on May 7, 1910.
Duluth, Minnesota Toronto, Ontario
"Grandfather is old," Dad said.
"I use the words attributed to Caesar, *'Et tu, Brute,'* to express my dismay," said John.
Grandfather is old; his face is very wrinkled. Grandfather is old; however, I'm not sure when he was born.
He has traveled to London, England; Paris, France; and Munich, Germany.
You will need the following items: a pen, a pencil, and an eraser.
I now have time to travel: I visit Italy to see Renaissance art. I visit England to see my grandchildren.
Peter: We'll fly away. *Wendy:* But, Peter, I can't fly.
6:30
I want to finish—but first I must tell you how it all began.
Anne was influential in our community— she was the leading physician and an inspiring speaker—and could be counted on to lead us to victory.
He's no friend of mine. She's running for office.
Peter's father treats us badly. The Smiths' station wagon was damaged.
He has traveled to two cities (London and Paris) and enjoyed them both.
The president campaigned vigorously. (In fact, he made appearances in all 50 states. According to most reports, he was well received.)
"Nothing was left for him [the president] to do but await word from his ambassador."

The Canadian Constitution

Address to Her Majesty

THAT, WHEREAS in the past certain amendments to the Constitution of Canada have been made by the Parliament of the United Kingdom at the request and with the consent of Canada;

AND WHEREAS it is in accord with the status of Canada as an independent state that Canadians be able to amend their Constitution in Canada in all respects;

AND WHEREAS it is also desirable to provide in the Constitution of Canada for the recognition of certain fundamental rights and freedoms and to make other amendments to that Constitution;

A respectful address be presented to Her Majesty the Queen in the following words:

To the Queen's Most Excellent Majesty: Most Gracious Sovereign:

We, Your Majesty's loyal subjects, the House of Commons of Canada in Parliament assembled, respectfully approach Your Majesty, requesting that you may graciously be pleased to cause to be laid before the Parliament of the United Kingdom a measure containing the recitals and clauses hereinafter set forth:

An Act to give effect to a request by the Senate and House of Commons of Canada

Whereas Canada has requested and consented to the enactment of an Act of the Parliament of the United Kingdom to give effect to the provisions hereinafter set forth and the Senate and the House of Commons of Canada in Parliament assembled have submitted an address to Her Majesty requesting that Her Majesty may graciously be pleased to cause a Bill to be laid before the Parliament of the United Kingdom for that purpose.

Be it therefore enacted by the Queen's Most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows:

Constitution Act, 1982 enacted

1. *The Constitution Act, 1981* set out in Schedule B to this Act is hereby enacted for and shall have the force of law in Canada and shall come into force as provided in that Act.

Termination of power to legislate for Canada

2. No Act of the Parliament of the United Kingdom passed after the *Constitution Act, 1981* comes into force shall extend to Canada as part of its law.

French version

3. So far as it is not contained in Schedule B, the French version of this Act is set out in Schedule A to this Act and has the same authority in Canada as the English version thereof.

Short title

4. This Act may be cited as the *Canada Act.*

CONSTITUTION ACT, 1982

Part 1, Schedule B

Canadian Charter of Rights and Freedoms

Whereas Canada is founded upon principles that recognize the supremacy of God and the rule of law:

Guarantee of Rights and Freedoms

Rights and freedoms in Canada

1. The *Canadian Charter of Rights and Freedoms* guarantees the rights and freedoms set out in it subject only to such reasonable limits prescribed by law as can be demonstrably justified in a free and democratic society.

Fundamental Freedoms

Fundamental freedoms

2. Everyone has the following fundamental freedoms:
- *(a)* freedom of conscience and religion;
- *(b)* freedom of thought, belief, opinion and expression, including freedom of the press and other media of communication;
- *(c)* freedom of peaceful assembly; and
- *(d)* freedom of association.

Democratic Rights

Democratic rights of citizens

3. Every citizen of Canada has the right to vote in an election of members of the House of Commons or of a legislative assembly and to be qualified for membership therein.

Maximum duration of legislative bodies

4. (1) No House of Commons and no legislative assembly shall continue for longer than five years from the date fixed for the return of the writs of a general election of its members.

Continuation in special circumstances

(2) In time of real or apprehended war, invasion or insurrection, a House of Commons may be continued by Parliament and a legislative assembly may be continued by the legislature beyond five years if such continuation is not opposed by the votes of more than one-third of the members of the House of Commons or the legislative assembly, as the case may be.

Annual sitting of legislative bodies

5. There shall be a sitting of Parliament and of each legislature at least once every twelve months.

Mobility Rights

Mobility of citizens

6. (1) Every citizen of Canada has the right to enter, remain in and leave Canada.

Rights to move and gain livelihood

(2) Every citizen of Canada and every person who has the status of a permanent resident of Canada has the right
- *(a)* to move to and take up residence in any province; and
- *(b)* to pursue the gaining of a livelihood in any province.

Limitation

(3) The rights specified in subsection (2) are subject to
  (a) any laws or practices of general application in force in a province other than those that discriminate among persons primarily on the basis of province of present or previous residence; and
  (b) any laws providing for reasonable residency requirements as a qualification for the receipt of publicly provided social services.

Affirmative action programs

(4) Subsections (2) and (3) do not preclude any law, program or activity that has as its object the amelioration in a province of conditions of individuals in that province who are socially or economically disadvantaged if the rate of employment in that province is below the rate of employment in Canada.

Legal Rights

Life, liberty and security of person

7. Everyone has the right to life, liberty and security of the person and the right not to be deprived thereof except in accordance with the principles of fundamental justice.

Search or seizure

8. Everyone has the right to be secure against unreasonable search or seizure.

Detention or imprisonment

9. Everyone has the right not to be arbitrarily detained or imprisoned.

Arrest or detention

10. Everyone has the right on arrest or detention
  (a) to be informed promptly of the reasons therefor;
  (b) to retain and instruct counsel without delay and to be informed of that right; and
  (c) to have the validity of the detention determined by way of *habeas corpus* and to be released if the detention is not lawful.

Proceedings in criminal and penal matters

11. Any person charged with an offence has the right
  (a) to be informed without unreasonable delay of the specific offence;
  (b) to be tried within a reasonable time;
  (c) not to be compelled to be a witness in proceedings against that person in respect of the offence;
  (d) to be presumed innocent until proven guilty according to law in a fair and public hearing by an independent and impartial tribunal;
  (e) not to be denied reasonable bail without just cause;
  (f) except in the case of an offence under military law tried before a military tribunal, to the benefit of trial by jury where the maximum punishment for the offence is imprisonment for five years or a more severe punishment;
  (g) not to be found guilty on account of any act or omission unless, at the time of the act or omission, it constituted an offence under Canadian or international law or was criminal according to the general principles of law recognized by the community of nations;
  (h) if finally acquitted of the offence, not to be tried for it again and, if finally found guilty and punished for the offence, not to be tried or punished for it again; and
  (i) if found guilty of the offence and if the punishment for the offence

has been varied between the time of commission and the time of sentencing, to the benefit of the lesser punishment.

Treatment or punishment

12. Everyone has the right not to be subjected to any cruel and unusual treatment or punishment.

Self-incrimination

13. A witness who testifies in any proceedings has the right not to have any incriminating evidence so given used to incriminate that witness in any other proceedings, except in a prosecution for perjury or for the giving of contradictory evidence.

Interpreter

14. A party or witness in any proceedings who does not understand or speak the language in which the proceedings are conducted or who is deaf has the right to the assistance of an interpreter.

Equality Rights

Equality before and under law and equal protection and benefit of law

15. (1) Every individual is equal before and under the law and has the right to the equal protection and equal benefit of the law without discrimination and, in particular, without discrimination based on race, national or ethnic origin, colour, religion, sex, age or mental or physical disability.

Affirmative action programs

(2) Subsection (1) does not preclude any law, program or activity that has as its object the amelioration of conditions of disadvantaged individuals or groups including those that are disadvantaged because of race, national or ethnic origin, colour, religion, sex, age or mental or physical disability.

Official Languages of Canada

Official languages of Canada

16. (1) English and French are the official languages of Canada and have equality of status and equal rights and privileges as to their use in all institutions of the Parliament and government of Canada.

Official languages of New Brunswick

(2) English and French are the official languages of New Brunswick and have equality of status and equal rights and privileges as to their use in all institutions of the legislature and government of New Brunswick.

Advancement of status and use

(3) Nothing in this Charter limits the authority of Parliament or a legislature to advance the equality of status or use of English and French.

Proceedings of Parliament

17. (1) Everyone has the right to use English or French in any debates and other proceedings of Parliament.

Proceedings of New Brunswick legislature

(2) Everyone has the right to use English or French in any debates and other proceedings of the legislature of New Brunswick.

Parliamentary statutes and records

18. (1) The statutes, records and journals of Parliament shall be printed and published in English and French and both language versions are equally authoritative.

New Brunswick statutes and records

(2) The statutes, records and journals of the legislature of New Brunswick shall be printed and published in English and French and both language versions are equally authoritative.

Proceedings in courts established by Parliament

19. (1) Either English or French may be used by any person in, or in any pleading in or process issuing from, any court established by Parliament.

Proceedings in New Brunswick courts

(2) Either English or French may be used by any person in, or in any pleading in or process issuing from, any court of New Brunswick.

Communications by public with federal institutions

20. (1) Any member of the public in Canada has the right to communicate with, and to receive available services from, any head or central office of an institution of the Parliament or government of Canada in English or French, and has the same right with respect to any other office of any such institution where

(a) there is a significant demand for communications with and services from that office in such language; or

(b) due to the nature of the office, it is reasonable that communications with and services from that office be available in both English and French.

Communications by public with New Brunswick institutions

(2) Any member of the public in New Brunswick has the right to communicate with, and to receive available services from, any office of an institution of the legislature or government of New Brunswick in English and French.

Continuation of existing constitutional provisions

21. Nothing in sections 16 to 20 abrogates or derogates from any right, privilege or obligation with respect to the English and French languages, or either of them, that exists or is continued by virtue of any other provision of the Constitution of Canada.

Rights and privileges preserved

22. Nothing in sections 16 to 20 abrogates or derogates from any legal or customary right or privilege acquired or enjoyed either before or after the coming into force of this Charter with respect to any language that is not English or French.

Minority Language Educational Rights

Language of instruction

23. (1) Citizens of Canada

(a) whose first language learned and still understood is that of the English or French linguistic minority population of the province in which they reside, or

(b) who have received their primary school instruction in Canada in English or French and reside in a province where the language in which they received that instruction is the language of the English or French linguistic minority population of the province

have the right to have their children receive primary and secondary school instruction in that language in that province.

Continuity of language instruction

(2) Citizens of Canada of whom any child has received or is receiving primary or secondary school instruction in English or French in Canada, have the right to have all their children receive primary and secondary school instruction in the same language.

(3) The right of citizens of Canada under subsections (1) and (2) to have their children receive primary and secondary school instruction in the language of the English or French linguistic minority population of a province

(a) applies wherever in the province the number of children of citizens who have such a right is sufficient to warrant the provision to them out of public funds of minority language instruction; and

(b) includes, where the number of those children so warrants, the right to have them receive that instruction in minority language educational facilities provided out of public funds.

Application where numbers warrant

Enforcement

24. (1) Anyone whose rights or freedoms, as guaranteed by this Charter, have been infringed or denied may apply to a court of competent jurisdiction to obtain such remedy as the court considers appropriate and just in the circumstances.

Enforcement of guaranteed rights and freedoms

(2) Where, in proceedings under subsection (1), a court concludes that evidence was obtained in a manner that infringed or denied any rights or freedoms guaranteed by this Charter, the evidence shall be excluded if it is established that, having regard to all the circumstances, the admission of it in the proceedings would bring the administration of justice into disrepute.

Exclusion of evidence bringing administration of justice into disrepute

General

25. The guarantee in this Charter of certain rights and freedoms shall not be construed so as to abrogate or derogate from any aboriginal, treaty or other rights or freedoms that pertain to the aboriginal peoples of Canada including

(a) any rights or freedoms that have been recognized by the Royal Proclamation of October 7, 1763; and

(b) any rights or freedoms that now exist by way of land claims agreements or may be so acquired.

Aboriginal rights and freedoms not affected by Charter

26. The guarantee in this Charter of certain rights and freedoms shall not be construed as denying the existence of any other rights or freedoms that exist in Canada.

Other rights and freedoms not affected by Charter

27. This Charter shall be interpreted in a manner consistent with the preservation and enhancement of the multicultural heritage of Canadians.

Multicultural heritage

28. Notwithstanding anything in this Charter, the rights and freedoms referred to in it are guaranteed equally to male and female persons.

Rights guaranteed equally to both sexes

29. Nothing in this Charter abrogates or derogates from any rights or privileges guaranteed by or under the Constitution of Canada in respect of denominational, separate or dissentient schools.

Rights respecting certain schools preserved

Application to territories and territorial authorities

30. A reference in this Charter to a province or to the legislative assembly or legislature of a province shall be deemed to include a reference to the Yukon Territory and the Northwest Territories, or to the appropriate legislative authority thereof, as the case may be.

Legislative powers not extended

31. Nothing in this Charter extends the legislative powers of any body or authority.

Application of Charter

Application of Charter

32. (1) This Charter applies

(a) to the Parliament and government of Canada in respect of all matters within the authority of Parliament including all matters relating to the Yukon Territory and Northwest Territories; and

(b) to the legislature and government of each province in respect of all matters within the authority of the legislature of each province.

Exception

(2) Notwithstanding subsection (1), section 15 shall not have effect until three years after this section comes into force.

Exception where express declaration

33. (1) Parliament or the legislature of a province may expressly declare in an Act of Parliament or of the legislature, as the case may be, that the Act or a provision thereof shall operate notwithstanding a provision included in section 2 or sections 7 to 15 of this Charter.

Operation of exception

(2) An Act or a provision of an Act in respect of which a declaration made under this section is in effect shall have such operation as it would have but for the provision of this Charter referred to in the declaration.

Five-year limitation

(3) A declaration made under subsection (1) shall cease to have effect five years after it comes into force or on such earlier date as may be specified in the declaration.

Re-enactment

(4) Parliament or the legislature of a province may re-enact a declaration made under subsection (1).

Five-year limitation

(5) Subsection (3) applies in respect of a re-enactment made under subsection (4).

Citation

Citation

34. This Part may be cited as the *Canadian Charter of Rights and Freedoms*.

Part II Rights of the Aboriginal Peoples of Canada

Recognition of existing aboriginal and treaty rights

35. (1) The existing aboriginal and treaty rights of the aboriginal peoples of Canada are hereby recognized and affirmed.

Definition of "aboriginal peoples of Canada"

(2) In this Act, "aboriginal peoples of Canada" includes the Indian, Inuit and Métis peoples of Canada.

Part III Equalization and Regional Disparities

Commitment to promote equal opportunities

36. (1) Without altering the legislative authority of Parliament or of the provincial legislatures, or the rights of any of them with respect to the exercise of their legislative authority, Parliament and the legislatures, together with the government of Canada and the provincial governments, are committed to

(a) promoting equal opportunities for the well-being of Canadians;

(b) furthering economic development to reduce disparity in opportunities; and

(c) providing essential public services of reasonable quality to all Canadians.

Commitment respecting public services

(2) Parliament and the government of Canada are committed to the principle of making equalization payments to ensure that provincial governments have sufficient revenues to provide reasonably comparable levels of public services at reasonably comparable levels of taxation.

Part IV Constitutional Conference

Constitutional conference

37. (1) A constitutional conference composed of the Prime Minister of Canada and the first ministers of the provinces shall be convened by the Prime Minister of Canada within one year after this Part comes into force.

Participation of aboriginal peoples

(2) The conference convened under subsection (1) shall have included in its agenda an item respecting constitutional matters that directly affect the aboriginal peoples of Canada, including the identification and definition of the rights of those peoples to be included in the Constitution of Canada, and the Prime Minister of Canada shall invite representatives of those peoples to participate in the discussions on that item.

Participation of territories

(3) The Prime Minister of Canada shall invite elected representatives of the governments of the Yukon Territory and the Northwest Territories to participate in the discussions on any item on the agenda of the conference convened under subsection (1) that, in the opinion of the Prime Minister, directly affects the Yukon Territory and the Northwest Territories.

Part V Procedure for Amending Constitution of Canada

General procedure for amending Constitution of Canada

38. (1) An amendment to the Constitution of Canada may be made by proclamation issued by the Governor General under the Great Seal of Canada where so authorized by

(a) resolutions of the Senate and House of Commons; and

(b) resolutions of the legislative assemblies of at least two-thirds of the provinces that have, in the aggregate, according to the then latest general census, at least fifty per cent of the population of all the provinces.

Majority of members

(2) An amendment made under subsection (1) that derogates from the legislative powers, the proprietary rights or any other rights or privileges of the legislature or government of a province shall require a resolution supported by a majority of the members of

each of the Senate, the House of Commons and the legislative assemblies required under subsection (1).

Expression of dissent

(3) An amendment referred to in subsection (2) shall not have effect in a province the legislative assembly of which has expressed its dissent thereto by resolution supported by a majority of its members prior to the issue of the proclamation to which the amendment relates unless that legislative assembly, subsequently, by resolution supported by a majority of its members, revokes its dissent and authorizes the amendment.

Revocation of dissent

(4) A resolution of dissent made for the purposes of subsection (3) may be revoked at any time before or after the issue of the proclamation to which it relates.

Restriction on proclamation

39. (1) A proclamation shall not be issued under subsection 38(1) before the expiration of one year from the adoption of the resolution initiating the amendment procedure thereunder, unless the legislative assembly of each province has previously adopted a resolution of assent or dissent.

Idem

(2) A proclamation shall not be issued under subsection 38(1) after the expiration of three years from the adoption of the resolution initiating the amendment procedure thereunder.

Compensation

40. Where an amendment is made under subsection 38(1) that transfers provincial legislative powers relating to education or other cultural matters from provincial legislatures to Parliament, Canada shall provide reasonable compensation to any province to which the amendment does not apply.

Amendment by unanimous consent

41. An amendment to the Constitution of Canada in relation to the following matters may be made by proclamation issued by the Governor General under the Great Seal of Canada only where authorized by resolutions of the Senate and House of Commons and of the legislative assembly of each province:

(a) the office of the Queen, the Governor General and the Lieutenant Governor of a province;

(b) the right of a province to a number of members in the House of Commons not less than the number of Senators by which the province is entitled to be represented at the time this Part comes into force;

(c) subject to section 43, the use of the English or the French language;

(d) the composition of the Supreme Court of Canada; and

(e) an amendment to this Part.

Amendment by general procedure

42. (1) An amendment to the Constitution of Canada in relation to the following matters may be made only in accordance with subsection 38(1):

(a) the principle of proportionate representation of the provinces in the House of Commons prescribed by the Constitution of Canada;

(b) the powers of the Senate and the method of selecting Senators;
(c) the number of members by which a province is entitled to be represented in the Senate and the residence qualifications of Senators;
(d) subject to paragraph 41*(d)*, the Supreme Court of Canada;
(e) the extension of existing provinces into the territories; and
(f) notwithstanding any other law or practice, the establishment of new provinces.

Exception

(2) Subsections 38(2) to (4) do not apply in respect of amendments in relation to matters referred to in subsection (1).
(a) the office of the Queen, the Governor General and the Lieutenant Governor of a province;
(b) the right of a province to a number of members in the House of Commons not less than the number of Senators by which the province is entitled to be represented at the time this Part comes into force;
(c) subject to section 43, the use of the English or the French language;
(d) the composition of the Supreme Court of Canada; and
(e) an amendment to this Part.

Amendment of provisions relating to some but not all provinces

43. An amendment to the Constitution of Canada in relation to any provision that applies to one or more, but not all, provinces, including
(a) any alteration to boundaries between provinces, and
(b) any amendment to any provision that relates to the use of the English or the French language within a province,
may be made by proclamation issued by the Governor General under the Great Seal of Canada only where so authorized by resolutions of the Senate and House of Commons and of the legislative assembly of each province to which the amendment applies.

Amendments by Parliament

44. Subject to sections 41 and 42, Parliament may exclusively make laws amending the Constitution of Canada in relation to the executive government of Canada or the Senate and House of Commons.

Amendments by provincial legislatures

45. Subject to section 41, the legislature of each province may exclusively make laws amending the constitution of the province.

Initiation of amendment procedures

46. (1) The procedures for amendment under sections 38, 41, 42 and 43 may be initiated either by the Senate or the House of Commons or by the legislative assembly of a province.

Revocation of authorization

(2) A resolution of assent made for the purposes of this Part may be revoked at any time before the issue of a proclamation authorized by it.

Amendments without Senate resolution

47. (1) An amendment to the Constitution of Canada made by proclamation under section 38, 41, 42 or 43 may be made without a resolution of the Senate authorizing the issue of the proclamation if, within one hundred and eighty days after the adoption by the House of Commons of a resolution authorizing its issue, the Senate has not adopted such a resolution and if, at any time after the expiration of that period, the House of Commons again adopts the resolution.

Computation of period

(2) Any period when Parliament is prorogued or dissolved shall not be counted in computing the one hundred and eighty day period referred to in subsection (1).

Advice to issue proclamation

48. The Queen's Privy Council for Canada shall advise the Governor General to issue a proclamation under this Part forthwith on the adoption of the resolutions required for an amendment made by proclamation under this Part.

Constitutional conference

49. A constitutional conference composed of the Prime Minister of Canada and the first ministers of the provinces shall be convened by the Prime Minister of Canada within fifteen years after this Part comes into force to review the provisions of this Part.

Part VI Amendment to the Constitution Act, 1867

Amendment to Constitution Act, 1867

50. The *Constitution Act, 1867* (formerly named the *British North America Act, 1867*) is amended by adding thereto, immediately after section 92 thereof, the following heading and section:

"Non-Renewable Natural Resources, Forestry Resources and Electrical Energy

Laws respecting non-renewable natural resources, forestry resources and electrical energy

92A.(1) In each province, the legislature may exclusively make laws in relation to

(a) exploration for non-renewable natural resources in the province;

(b) development, conservation and management of non-renewable natural resources and forestry resources in the province, including laws in relation to the rate of primary production therefrom; and

(c) development, conservation and management of sites and facilities in the province for the generation and production of electrical energy.

Export from provinces of resources

(2) In each province, the legislature may make laws in relation to the export from the province to another part of Canada of the primary production from non-renewable natural resources and forestry resources in the province and the production from facilities in the province for the generation of electrical energy, but such laws may not authorize or provide for discrimination in prices or in supplies exported to another part of Canada.

Authority of Parliament

(3) Nothing in subsection (2) derogates from the authority of Parliament to enact laws in relation to the matters referred to in that subsection and, where such a law of Parliament and a law of a province conflict, the law of Parliament prevails to the extent of the conflict.

Taxation of resources

(4) In each province, the legislature may make laws in relation to the raising of money by any mode or system of taxation in respect of

(a) non-renewable natural resources and forestry resources in the province and the primary production therefrom, and

(b) sites and facilities in the province for the generation of electrical energy and the production therefrom,

whether or not such production is exported in whole or in part from the province, but such laws may not authorize or provide for taxation that differentiates between production exported to another part of Canada and production not exported from the province.

"Primary production"

(5) The expression "primary production" has the meaning assigned by the Sixth Schedule.

Existing powers or rights

(6) Nothing in subsections (1) to (5) derogates from any powers or rights that a legislature or government of a province had immediately before the coming into force of this section.

Idem

51. The said Act is further amended by adding thereto the following Schedule:

"The Sixth Schedule Primary Production from Non-Renewable Natural Resources and Forestry Resources

1. For the purposes of section 92A of this Act,

(a) production from a non-renewable natural resource is primary production therefrom if

(i) it is in the form in which it exists upon its recovery or severance from its natural state, or

(ii) it is a product resulting from processing or refining the resource, and is not a manufactured product or a product resulting from refining crude oil, refining upgraded heavy crude oil, refining gases or liquids derived from coal or refining a synthetic equivalent of crude oil; and

(b) production from a forestry resource is primary production therefrom if it consists of sawings, poles, lumber, wood chips, sawdust or any other primary wood product, or wood pulp, and is not a product manufactured from wood."

Part VII General

Primacy of Constitution of Canada

52. (1) The Constitution of Canada is the supreme law of Canada, and any law that is inconsistent with the provisions of the Constitution is, to the extent of the inconsistency, of no force or effect.

Constitution of Canada

(2) The Constitution of Canada includes

(a) the *Canada Act 1982*, including this Act;

(b) the Acts and orders referred to in Schedule; and

(c) any amendment to any Act or order referred to in paragraph *(a)* or *(b)*.

Amendments to Constitution of Canada

(3) Amendments to the Constitution of Canada shall be made only in accordance with the authority contained in the Constitution of Canada.

Repeals and new names

53. (1) The enactments referred to in Column I of the Schedule are hereby repealed or amended to the extent indicated in Column II thereof and, unless repealed, shall continue as law in Canada under the names set out in Column III thereof.

Consequential amendments

(2) Every enactment, except the *Canada Act 1982*, that refers to an enactment referred to in the Schedule by the name in Column I thereof is hereby amended by substituting for that name the corresponding name in Column III thereof, and any British North America Act not referred to in the Schedule may be cited as the *Constitution Act* followed by the year and number, if any, of its enactment.

Repeal and consequential amendments

54. Part IV is repealed on the day that is one year after this Part comes into force and this section may be repealed and this Act renumbered, consequential upon the repeal of Part IV and this section, by proclamation issued by the Governor General under the Great Seal of Canada.

French version of Constitution of Canada

55. A French version of the portions of the Constitution of Canada referred to in the Schedule shall be prepared by the Minister of Justice of Canada as expeditiously as possible and, when any portion thereof sufficient to warrant action being taken has been so prepared, it shall be put forward for enactment by proclamation issued by the Governor General under the Great Seal of Canada pursuant to the procedure then applicable to an amendment of the same provisions of the Constitution of Canada.

English and French versions of certain constitutional texts

56. Where any portion of the Constitution of Canada has been or is enacted in English and French or where a French version of any portion of the Constitution is enacted pursuant to section 55, the English and French versions of that portion of the Constitution are equally authoritative.

English and French versions of this Act

57. The English and French versions of this Act are equally authoritative.

Commencement

58. Subject to section 59, this Act shall come into force on a day to be fixed by proclamation issued by the Queen or the Governor General under the Great Seal of Canada.

Commencement of paragraph 23(1)(a) in respect of Quebec

59. (1) Paragraph 23(1)*(a)* shall come into force in respect of Quebec on a day to be fixed by proclamation issued by the Queen or the Governor General under the Great Seal of Canada.

Authorization of Quebec

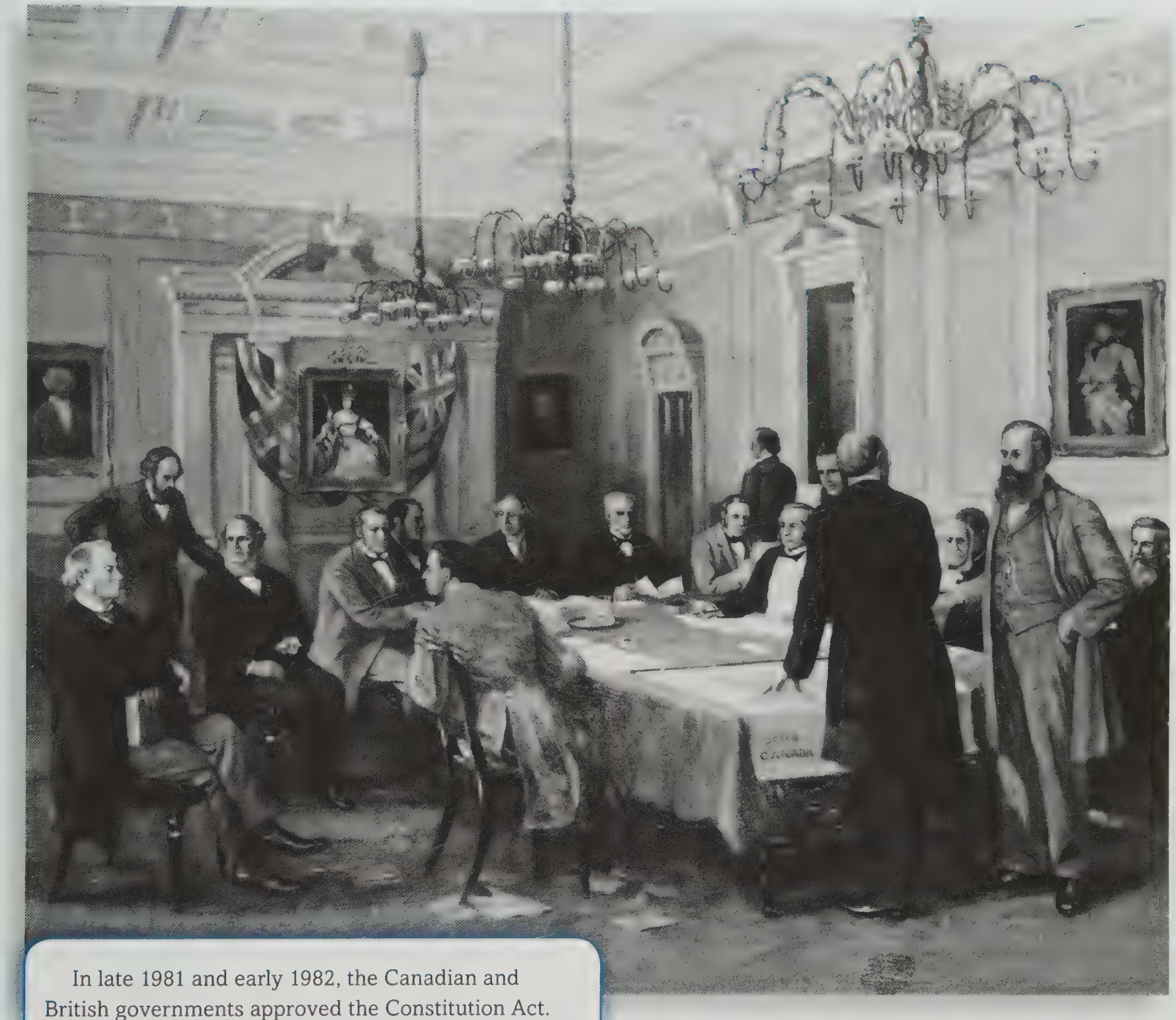
(2) A proclamation under subsection (1) shall be issued only where authorized by the legislative assembly or government of Quebec.

Repeal of this section

(3) This section may be repealed on the day paragraph 23(1)*(a)* comes into force in respect of Quebec and this Act amended and renumbered, consequentially upon the repeal of this section, by proclamation issued by the Queen or the Governor General under the Great Seal of Canada.

Short title and citations

60. This Act may be cited as the *Constitution Act, 1981*, and the Constitution Acts 1867 to 1975 (No. 2) and this Act may be cited together as the *Constitution Acts, 1867 to 1982.*



In late 1981 and early 1982, the Canadian and British governments approved the Constitution Act. The constitutional package consisted of 18 acts of the British Parliament, 8 acts of the Canadian Parliament, and 4 British orders-in-council. Elizabeth II remained queen of Canada and head of state, and Canada remained in the Commonwealth of Nations.

As part of these actions, the British North America Act, 1867 was renamed the Constitution Act, 1867.

▲ **This London** conference was one of several meetings held to frame the British North America Act and established the Dominion of Canada

British North America Act, 1867

Excerpts of the British North America Act, 1867 (renamed the Constitution Act, 1867, in 1982)

Whereas the Provinces of Canada, Nova Scotia, and New Brunswick have expressed their Desire to be federally united into One Dominion under the Crown of the United Kingdom of Great Britain and Ireland, with a Constitution similar in Principle to that of the United Kingdom:

And whereas such a Union would conduce to the Welfare of the Provinces and promote the Interests of the British Empire:

And whereas on the Establishment of Union by Authority of Parliament it is expedient, not only that the Constitution of the Legislative authority in the Dominion be provided for, but also that the Nature of the Executive Government herein be declared:

And whereas it is expedient that Provision be made for the eventual admission into the Union of other Parts of British North America:

Be it therefore enacted and declared by the Queen's most Excellent Majesty, by and with the Advice and Consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the Authority of the same, as follows: . . .

II. UNION

3. It shall be lawful for the Queen, by and with the advice of Her Majesty's Most Honourable Privy Council, to declare by Proclamation that, on and after a Day therein appointed, not being more than Six Months after the passing of this Act, the Provinces of Canada, Nova Scotia, and New Brunswick shall form and be One Dominion under the Name of Canada; and on and after that day these Three Provinces shall form and be One Dominion under the name accordingly. . . .

Canada shall be divided into Four Provinces, named Ontario, Quebec, Nova Scotia, and New Brunswick.

6. The Parts of the Province of Canada (as it exists at the passing of this Act) which formerly constituted respectively the Provinces of Upper Canada and Lower Canada shall be deemed to be severed, and shall form two separate Provinces. The Part which formerly constituted the Province of Upper Canada shall constitute the Province of Ontario; and the Part which formerly constituted the Province of Lower Canada shall constitute the Province of Quebec. . . .

9. The Executive Government and Authority of and over Canada is hereby declared to continue and be vested in the Queen. . . .

12. All Powers, Authorities, and Functions which under any Act of the Parliament of Great Britain, or of the Parliament of the United Kingdom of Great Britain and Ireland, or of the Legislature of Upper Canada, Lower Canada, Nova Scotia, or New Brunswick, are at the Union vested in or exercisable by the respective Governors or Lieutenant-Governors of those Provinces, with the advice, or with the Advice and Consent, of the respective Executive Councils thereof, or in conjunction with those Councils, or with any Number of Members thereof, or by those Governors or Lieutenant-Governors individually, shall, as far as the same continue in existence and capable of being exercised after the Union in relation to the Government of Canada, be vested in and exercisable by the Governor General, with the Advice or with the Advice and Consent of or in conjunction with the Queen's Privy Council for Canada, or any Members thereof, or by the Governor General individually, as the Case requires, subject nevertheless (except with respect to such as exist under Acts of the Parliament of Great Britain or of the Parliament of the United Kingdom of Great Britain and Ireland) to be abolished or altered by the Parliament of Canada. . . .

17. There shall be One Parliament for Canada, consisting of the Queen, an Upper House styled the Senate, and the House of Commons.

18. The Privileges, Immunities, and Powers to be held, enjoyed and exercised by the Senate and by the House of Commons and by the members thereof respectively shall be such as are from Time to Time defined by Act of the Parliament of Canada, but so that the same shall never exceed those at the passing of this Act held, enjoyed, and exercised by the Commons House of Parliament of the United Kingdom of Great Britain and Ireland and by the Members thereof.

19. The Parliament of Canada shall be called together not later than six Months after the Union.

20. There shall be a Session of the Parliament of Canada once at least in every Year, so that Twelve Months shall not intervene between the last sitting of the Parliament of one Session and its first Sitting in the next Session.

21. The Senate shall, subject to the Provisions of this Act, consist of Seventy-two Members, who shall be styled Senators. . . .

37. The House of Commons shall, subject to the Provisions of this Act, consist of One hundred and eighty-one Members, of whom Eighty-two shall be elected for Ontario, Sixty-five for Quebec, Nineteen for Nova Scotia, and Fifteen for New Brunswick. . . .

58. For each Province there shall be an Officer, styled the Lieutenant-Governor, appointed by the Governor General in Council by Instrument under the Great Seal of Canada. . . . .

91. It shall be lawful for the Queen by and with the Advice and Consent of the Senate and House of Commons, to make Laws for the Peace, Order and good Government of Canada, in relation to all Matters not coming within the Classes of Subjects by this Act assigned exclusively to the Legislatures of the Provinces: . . .

93. In and for each Province the Legislature may exclusively make Laws in relation to Education, . .

95. In each Province the Legislature may make Laws in relation to Agriculture in the Province, and to Immigration into the Province; and it is hereby declared that the Parliament of Canada may from Time to Time make Laws in relation to Agriculture in all or any of the Provinces, and to Immigration into all or any of the Provinces; and any Law of the Legislature of a Province relative to Agriculture or to Immigration shall have effect in and for the Province as long and as far only as it is not repugnant to any Act of the Parliament of Canada. . . .

102. All Duties and Revenues over which the respective Legislatures of Canada, Nova Scotia, and New Brunswick before and at the Union had and have Power of Appropriation, except such portions thereof as are by this Act reserved to the respective Legislatures of the Provinces, or are raised by them in accordance with the special Powers conferred on them by this act, shall form One Consolidated Revenue Fund, to be appropriated for the public Service of Canada in the Manner and subject to the Charges in this Act provided. . . .

145. Inasmuch as the Provinces of Canada, Nova Scotia, and New Brunswick have joined in a Declartion that the Construction of the Intercolonial Railway is essential to the Consolidation of the Union of British North America, and to the assent thereto of Nova Scotia and New Brunswick, and have consequently agreed that provision should be made for its immediate Construction by the Government of Canada: Therefore, in order to give effect to that Agreement, it shall be the duty of the Government and Parliament of Canada to provide for the Commencement within Six Months after the Union, of a Railway connecting the River St. Lawrence with the City of Halifax in Nova Scotia, and for the Construction thereof without Intermission, and the Completion thereof with all practiable Speed.

146. It shall be lawful for the Queen, by and with the Advice of Her Majesty's Most Honourable Privy Council, on Addresses from the Houses of the Parliament of Canada, and from the Houses of the respective Legislatures of the Colonies or Provinces of Newfoundland, Prince Edward Island, and British Columbia, to admit those Colonies or Provinces, or any of them, into the Union, and on Address from the Houses of Parliament of Canada to admit Rupert's Land and the Northwestern Territory, or either of them, into the Union, on such Terms and Conditions in each case as are in the Addresses expressed and as the Queen thinks fit to approve, subject to the Provisions of this Act; and the Provisions of any Orders of Council in that Behalf shall have effect as if they had been enacted by the Parliament of the United Kingdom of Great Britain and Ireland. . . .

Verbal Spelling Irregularities

Spelling changes often occur in regular verbs. These changes appear to be needed to retain the original sounds within the infinitive forms. As you will shortly see, for example, by adding a cedilla to ***c***, the soft ***c*** of verbs ending in ***cer*** is retained before vowels that normally would make the ***c*** hard.

Verb ending	Change	Present Participle	Present Indicative	Imperfect Indicative	Past Definite	Imperfect Subjunctive
-cer avancer (*to advance*)	requires the soft *c* sound throughout the conjugation	avançant	avance avançons	avançais avancions	avançai avançâmes	avançasse avançassions
-ger manger (*to eat*)	requires the soft *g* so *g* followed by *a* or *o* becomes *ge*	mangeant	mange mangeons	mangeais mangions	mangeai mangeâmes	mangeasse mangeassions
		Present Indicative	**Future**	**Conditional**	**Present Subjunctive**	
-oyer **-uyer** nettoyer (*to clean*)	*y* changes to *i* before *e* in conjugation, but not elsewhere	nettoie, etc.	nettoierai	nettoierais	nettoie, etc.	
-ayer **-eyer** payer (*to pay*)	may either retain *y* throughout, or change to *i* before *e*	paye paie	payerai paierai	payerais paierais	paye paie	
-er when unaccented and followed by a single consonant mener (*to lead*)	*e* changes to *è* when the ending begins with an unstressed *e*	mène mènes mène menons menez mènent	mènerai mèneras mènera mènerons mènerez mèneront	mènerais mènerais mènerait mènerions mèneriez mèneraient	mène mènes mène menions meniez mènent	
-eter **-eler** (most verbs) appeler (*to call*) jeter (*to throw*)	the *l* or *t* is doubled to cause the first *e* to be stressed	appelle appelles appelle appelons appelez appellent jette	appellerai appelleras appellera appellerons appellerez appelleront jetterai	appellerais appellerais appellerait appellerions appelleriez appelleraient jetterais	appelle appelles appelle appelions appeliez appellent jette	
-eter **-eler** (some verbs) acheter (*to buy*)	follows the rule for unaccented *e*, as in *mener* above	achète	achèterai	achèterais	achète	

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
aller (*to go*)	je	vais	allais	irai	allai	aille	
allant	tu	vas	allais	iras	allas	ailles	va
allé	il/elle	va	allait	ira	alla	aille	
	nous	allons	allions	irons	allâmes	allions	allons
	vous	allez	alliez	irez	allâtes	alliez	allez
	ils/elles	vont	allaient	iront	allèrent	aillent	
asseoir (*to seat*)	je	assieds	asseyais	assiérai	assis	asseye	
asseyant	tu	assieds	asseyais	assiéras	assis	asseyes	assieds
assis	il/elle	assied	asseyait	assiéra	assit	asseye	
	nous	asseyons	asseyions	assiérons	assîmes	asseyions	asseyons
	vous	asseyez	asseyiez	assiérez	assîtes	asseyiez	asseyez
	ils/elles	asseyent	asseyaient	assiéront	assirent	asseyent	
asseoir (*to sit*)	je	assois	assoyais	assoirai	assis	assoie	
assoyant	tu	assois	assoyais	assoiras	assis	assoies	assois
assis	il/elle	assoit	assoyait	assoira	assit	assoie	
	nous	assoyons	assoyions	assoirons	assîmes	assoyions	assoyons
	vous	assoyez	assoyiez	assoirez	assîtes	assoyiez	assoyez
	ils/elles	assoient	assoyaient	assoiront	assirent	assoient	
avoir (*to have*)	je	ai	avais	aurai	eus	aie	
ayant	tu	as	avais	auras	eus	aies	aie
eu	il/elle	a	avaint	aura	eut	ait	
	nous	avons	avions	aurons	eûmes	ayons	ayons
	vous	avez	aviez	aurez	eûtes	ayez	ayez
	ils/elles	ont	avaient	auront	eurent	aient	
battre (*to beat*)	je	bats	battais	battrai	battis	batte	
battant	tu	bats	battais	battras	battis	battes	bats
battu	il/elle	bat	battait	battra	battit	batte	
	nous	battons	battions	battrons	battîmes	battions	battons
	vous	battez	battiez	battrez	battîtes	battiez	battez
	ils/elles	battent	battaient	battront	battirent	battent	
boire (*to drink*)	je	bois	buvais	boirai	bus	boive	
buvant	tu	bois	buvais	boiras	bus	boives	bois
bu	il/elle	boit	buvait	boira	but	boive	
	nous	buvons	buvions	boirons	bûmes	buvions	buvons
	vous	buvez	buviez	boirez	bûtes	buviez	buvez
	ils/elles	boivent	buvaient	boiront	burent	boivent	

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
conduire (*to lead; to take; to drive*) **conduisant** **conduit**	je	conduis	conduisais	conduirai	conduisis	conduise	
	tu	conduis	conduisais	conduiras	conduisis	conduises	conduis
	il/elle	conduit	conduisait	conduira	conduisit	conduise	
	nous	conduisons	conduisions	conduirons	conduisîmes	conduirons	conduisons
	vous	conduisez	conduisiez	conduirez	conduisîtes	conduirez	conduisez
	ils/elles	conduisent	conduisaient	conduiront	conduisirent	conduiront	
connaître (*to know*) **connaissant** **connu**	je	connais	connaissais	connaîtrai	connus	connaisse	
	tu	connais	connaissais	connaîtras	connus	connaisses	connais
	il/elle	connaît	connaissait	connaîtra	connut	connaisse	
	nous	connaissons	connaissions	connaîtrons	connûmes	connaissions	connaissons
	vous	connaissez	connaissiez	connaîtrez	connûtes	connaissiez	connaissez
	ils/elles	connaissent	connaissaient	connaîtront	connurent	connaissent	
coudre (*to sew*) **cousant** **cousu**	je	couds	cousais	coudrai	cousis	couse	
	tu	couds	cousais	coudras	cousis	couses	couds
	il/elle	coud	cousait	coudra	cousit	couse	
	nous	cousons	cousions	coudrons	cousîmes	cousions	cousons
	vous	cousez	cousiez	coudrez	cousîtes	cousiez	cousez
	ils/elles	cousent	cousaient	coudront	cousirent	cousent	
courir (*to run*) **courant** **couru**	je	cours	courais	courrai	courus	coure	
	tu	cours	courais	courras	courus	coures	cours
	il/elle	court	courait	courra	courut	coure	
	nous	courons	courions	courrons	courûmes	courions	courons
	vous	courez	couriez	courrez	courûtes	couriez	courez
	ils/elles	courent	couraient	courront	coururent	courent	
couvir (*to cover*) **couvrant** **couvert**	je	couvre	couvrais	couvrirai	couvris	couvre	
	tu	couvres	couvrais	couvriras	couvris	couvres	couvre
	il/elle	couvre	couvrait	couvrira	couvrit	couvre	
	nous	couvrons	couvrions	couvrirons	couvrîmes	couvrions	couvrons
	vous	couvrez	couvriez	couvrirez	couvrîtes	couvriez	couvrez
	ils/elles	couvrent	covraient	couvriront	couvrirent	couvrent	
craindre (*to fear*) **craignant** **craint**	je	crains	craignais	craindrai	craignis	craigne	
	tu	crains	craignais	craindras	craignis	craignes	crains
	il/elle	craint	craignait	craindra	craignit	craigne	
	nous	craignons	craignions	craindrons	craignîmes	craignions	craignons
	vous	craignez	craigniez	craindrez	craignîtes	craigniez	craignez
	ils/elles	craignent	craignaient	craindront	craignirent	craignent	

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
croire (*to think, to believe*) **croyant** **cru**	je	crois	croyais	croirai	crus	croie	
	tu	crois	croyais	croiras	crus	croies	crois
	il/elle	croit	croyait	croira	crut	croie	
	nous	croyons	croyions	croirons	crûmes	croyions	croyons
	vous	croyez	croyiez	croirez	crûtes	croyiez	croyez
	ils/elles	croient	croyaient	croiront	crurent	croient	
devoir (*to owe; to have to*) **devant** **dû**	je	dois	devais	devrai	dus	doive	
	tu	dois	devais	devras	dus	doives	dois
	il/elle	doit	devait	devra	dut	doive	
	nous	devons	devions	devrons	dûmes	devions	devons
	vous	devez	deviez	devrez	dûtes	deviez	devez
	ils/elles	doivent	devaient	devront	durent	doivent	
dire (*to say; to tell*) **disant** **dit**	je	dis	disais	dirai	dis	dise	
	tu	dis	disais	diras	dis	dises	dis
	il/elle	dit	disait	dira	dit	dise	
	nous	disons	disions	dirons	dîmes	disions	disons
	vous	dites	disiez	direz	dîtes	disiez	dites
	ils/elles	disent	disaient	diront	dirent	disent	
dormir (*to sleep*) **dormant** **dormir**	je	dors	dormais	dormirai	dormis	dorme	
	tu	dors	dormais	dormiras	dormis	dormes	dors
	il/elle	dort	dormait	dormira	dormit	dorme	
	nous	dormons	dormions	dormirons	dormîmes	dormions	dormons
	vous	dormez	dormiez	dormirez	dormîtes	dormiez	dormez
	ils/elles	dorment	dormaient	dormiront	dormirent	dorment	
écrire (*to write*) **écrivant** **écrit**	je	écris	écrivais	écrirai	écrivis	écrive	
	tu	écris	écrivais	écriras	écrivis	écrives	écris
	il/elle	écrit	écrivait	écrira	écrivit	écrive	
	nous	écrivons	écrivions	écrirons	écrivîmes	écrivions	écrivons
	vous	écrivez	écriviez	écrirez	écrivîtes	écriviez	écrivez
	ils/elles	écrivent	écrivaient	écriront	écrivirent	écrivent	
être (*to be*) **étant** **été**	je	suis	étais	serai	fus	sois	
	tu	es	étais	seras	fus	sois	sois
	il/elle	est	était	sera	fut	soit	
	nous	sommes	étions	serons	fûmes	soyons	soyons
	vous	êtes	étiez	serez	fûtes	soyez	soyez
	ils/elles	sont	étaient	seront	furent	soient	

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
faire (*to do; to make*)	je	fais	faisais	ferai	fis	fasse	
faisant	tu	fais	faisais	feras	fis	fasses	fais
fait	il/elle	fait	faisait	fera	fit	fasse	
	nous	faisons	faisions	ferons	fîmes	fassions	fasions
	vous	faites	faisiez	ferez	fîtes	fassiez	faites
	ils/elles	font	faisaient	feront	firent	fassent	
lire (*to read*)	je	lis	lisais	lirai	lus	lise	
lisant	tu	lis	lisais	liras	lus	lises	lis
lu	il/elle	lit	lisait	lira	lut	lise	
	nous	lisons	lisions	lirons	lûmes	lisions	lisons
	vous	lisez	lisiez	lirez	lûtes	lisiez	lisez
	ils/elles	lisent	lisaient	liront	lurent	lisent	
mentir (*to lie*)	je	mens	mentais	mentirai	mentis	mente	
mentant	tu	mens	mentais	mentiras	mentis	mentes	mens
menti	il/elle	ment	mentait	mentira	mentit	mente	
	nous	mentons	mentions	mentirons	mentîmes	mentions	mentons
	vous	mentez	mentiez	mentirez	mentîtes	mentiez	mentez
	ils/elles	mentent	mentaient	mentiront	mentirent	mentent	
mettre (*to put; to set*)	je	mets	mettais	mettrai	mis	mette	
mettant	tu	mets	mettais	mettras	mis	mettes	mets
mis	il/elle	met	mettait	mettra	mit	mette	
	nous	mettons	mettions	mettrons	mîmes	mettions	mettons
	vous	mettez	mettiez	mettrez	mîtes	mettiez	mettez
	ils/elles	mettent	mettaient	mettront	mirent	mettent	
mourir (*to die*)	je	meurs	mourais	mourrai	mourus	meure	
mourant	tu	meurs	mourais	mourras	mourus	meures	meurs
mort	il/elle	meurt	mourait	mourra	mourut	meure	
	nous	mourons	mourions	mourrons	mourûmes	mourions	mourons
	vous	mourez	mouriez	mourrez	mourûtes	mouriez	mourez
	ils/elles	meurent	mouraient	mourront	moururent	meurent	
offrir (*to offer; to present*)	je	offre	offrais	offrirai	offris	offre	
offrant	tu	offres	offrais	offriras	offris	offres	offre
offert	il/elle	offre	offrait	offrira	offrit	offre	
	nous	offrons	offrions	offrirons	offrîmes	offrions	offrons
	vous	offrez	offriez	offrirez	offrîtes	offriez	offrez
	ils/elles	offrent	offraient	offriront	offrirent	offrent	

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
ouvrir (*to open*)	je	ouvre	ouvrais	ouvrirai	ouvris	ouvre	
ouvrant	tu	ouvres	ouvrais	ouvriras	ouvris	ouvres	ouvre
ouvert	il/elle	ouvre	ouvrait	ouvrira	ouvrit	ouvre	
	nous	ouvrons	ouvrions	ouvrirons	ouvrîmes	ouvrions	ouvrons
	vous	ouvrez	ouvriez	ouvrirez	ouvrîtes	ouvriez	ouvrez
	ils/elles	ouvrent	ouvraient	ouvriront	ouvrirent	ouvrent	
partir (*to leave;*	je	pars	partais	partirai	partis	parte	
to go away)	tu	pars	partais	partiras	partis	partes	pars
partant	il/elle	part	partait	partira	partit	parte	
parti	nous	partons	partions	partirons	partîmes	partions	partons
	vous	partez	partiez	partirez	partîtes	partiez	partez
	ils/elles	partent	partaient	partiront	partirent	partent	
pleuvoir (*to rain*)	je	—	—	—	—	—	
pleuvant	tu	—	—	—	—	—	—
plu	il/elle	pleut	pleuvait	pleuvra	plut	pleuve	
	nous	—	—	—	—	—	—
	vous	—	—	—	—	—	—
	ils/elles	pleuvent	pleuvaient	pleuvront	—	pleuvent	
pouvoir (*to be able*)	je	peux, puis	pouvais	pourrai	pus	puisse	
pouvant	tu	peux	pouvais	pourras	pus	puisses	—
pu	il/elle	peut	pouvait	pourra	put	puisse	
	nous	pouvons	pouvions	pourrons	pûmes	puissions	—
	vous	pouvez	pouviez	pourrez	pûtes	puissiez	—
	ils/elles	peuvent	pouvaient	pourront	purent	puissent	
prendre (*to take*)	je	prends	prenais	prendrai	pris	prenne	
prenant	tu	prends	prenais	prendras	pris	prennes	prends
pris	il/elle	prend	prenait	prendra	prit	prenne	
	nous	prenons	prenions	prendrons	prîmes	prenions	prenons
	vous	prenez	preniez	prendrez	prîtes	preniez	prenez
	ils/elles	prennent	prenaient	prendront	prirent	prennent	
recevoir (*to receive*)	je	reçois	recevais	recevrai	reçus	reçoive	
recevant	tu	reçois	recevais	recevras	reçus	reçoives	reçois
reçu	il/elle	reçoit	recevait	recevra	reçut	reçoive	
	nous	recevons	recevions	recevrons	reçûmes	recevions	recevons
	vous	recevez	receviez	recevrez	reçûtes	receviez	recevez
	ils/elles	reçoivent	recevaient	recevront	reçurent	reçoivent	

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
rire (*to laugh*)	je	ris	riais	rirai	ris	rie	
riant	tu	ris	riais	riras	ris	ries	ris
ri	il/elle	rit	riait	rira	rit	rie	
	nous	rions	riions	rirons	rîmes	riions	rions
	vous	riez	riiez	rirez	rîtes	riiez	riez
	ils/elles	rient	riaient	riront	rirent	rient	
savoir (*to know;*	je	sais	savais	saurai	sus	sache	
to know how)	tu	sais	savais	sauras	sus	saches	sache
sachant	il/elle	sait	savait	saura	sut	sache	
su	nous	savons	savions	saurons	sûmes	sachions	sachons
	vous	savez	saviez	saurez	sûtes	sachiez	sachez
	ils/elles	savent	savaient	sauront	surent	sachent	
sentir (*to feel;*	je	sens	sentais	sentirai	sentis	sente	
to smell)	tu	sens	sentais	sentiras	sentis	sentes	sens
sentant	il/elle	sent	sentait	sentira	sentit	sente	
senti	nous	sentons	sentions	sentirons	sentîmes	sentions	sentons
	vous	sentez	sentiez	sentirez	sentîtes	sentiez	sentez
	ils/elles	sentent	sentaient	sentiront	sentirent	sentent	
sortir (*to leave*)	je	sors	sortais	sortirai	sortis	sorte	
sortant	tu	sors	sortais	sortiras	sortis	sortes	sors
sorti	il/elle	sort	sortait	sortira	sortit	sorte	
	nous	sortons	sortions	sortirons	sortîmes	sortions	sortons
	vous	sortez	sortiez	sortirez	sortîtes	sortiez	sortez
	ils/elles	sortent	sortaient	sortiront	sortirent	sortent	
souffrir (*to suffer;*	je	souffre	souffrais	souffrirai	souffris	souffre	
to bear)	tu	souffres	souffrais	souffriras	souffris	souffres	souffre
souffrant	il/elle	souffre	souffrait	souffrira	souffrit	souffre	
souffert	nous	souffrons	souffrions	souffrirons	souffrîmes	souffrions	souffrons
	vous	souffrez	souffriez	souffrirez	souffrîtes	souffriez	souffrez
	ils/elles	souffrent	souffraient	souffriront	souffrirent	souffrent	
tenir (*to hold*)	je	tiens	tenais	tiendrai	tins	tienne	
tenant	tu	tiens	tenais	tiendras	tins	tiennes	tiens
tenu	il/elle	tient	tenait	tiendra	tint	tienne	
	nous	tenons	tenions	tiendrons	tînmes	tenions	tenons
	vous	tenez	teniez	tiendrez	tîntes	teniez	tenez
	ils/elles	tiennent	tenaient	tiendront	tinrent	tiennent	

Conjugation of Irregular Verbs

Infinitive Present participle Past participle	Person	Present Indicative	Imperfect Indicative	Future Indicative	Past Indicative	Present Subjunctive	Imperative
vendre (*to sell*)	je	vends	vendais	vendrai	vendis	vende	
vendant	tu	vends	vendais	vendras	vendis	vendes	vends
vendu	il/elle	vend	vendait	vendra	vendit	vende	
	nous	vendons	vendions	vendrons	vendîmes	vendions	vendons
	vous	vendez	vendiez	vendrez	vendîtes	vendiez	vendez
	ils/elles	vendent	vendaient	vendront	vendirent	vendent	
venir (*to come*)	je	viens	venais	viendrai	vins	vienne	
venant	tu	viens	venais	viendras	vins	viennes	viens
venu	il/elle	vient	venait	viendra	vint	vienne	
	nous	venons	venions	viendrons	vînmes	venions	venons
	vous	venez	veniez	viendrez	vîntes	veniez	venez
	ils/elles	viennent	venaient	viendront	vinrent	viennent	
vêtir (*to dress*)	je	vêts	vêtais	vêtirai	vêtis	vête	
vêtant	tu	vêts	vêtais	vêtiras	vêtis	vêtes	vêts
vêtu	il/elle	vêt	vêtait	vêtira	vêtit	vête	
	nous	vêtons	vêtions	vêtirons	vêtîmes	vêtions	vêtons
	vous	vêtez	vêtiez	vêtirez	vêtîtes	vêtiez	vêtez
	ils/elles	vêtent	vêtaient	vêtiront	vêtirent	vêtent	
vivre (*to be alive, to live*)	je	vis	vivais	vivrai	vécus	vive	
	tu	vis	vivais	vivras	vécus	vives	vis
vivant	il/elle	vit	vivait	vivra	vécut	vive	
vécu	nous	vivons	vivions	vivrons	vécûmes	vivions	vivons
	vous	vivez	viviez	vivrez	vécûtes	viviez	vivez
	ils/elles	vivent	vivaient	vivront	vécurent	vivent	
voir (*to see*)	je	vois	voyais	verrai	vis	voie	
voyant	tu	vois	voyais	verras	vis	voies	vois
vu	il/elle	voit	voyait	verra	vit	voie	
	nous	voyons	voyions	verrons	vîmes	voyions	voyons
	vous	voyez	voyiez	verrez	vîtes	voyiez	voyez
	ils/elles	voient	voyaient	verront	virent	voient	
vouloir (*to want; to wish*)	je	veux	voulais	voudrai	voulus	veuille	
	tu	veux	voulais	voudras	voulus	veuilles	veuille
voulant	il/elle	veut	voulait	voudra	voulut	veuille	
voulu	nous	voulons	voulions	voudrons	voulûmes	voulions	veuillons
	vous	voulez	vouliez	voudrez	voulûtes	vouliez	veuillez
	ils/elles	veulent	voulaient	voudront	voulurent	veuillent	

German

Strong and Irregular Verbs

In German, verbs that form the past stem by changing the vowel of the infinitive stem are known as strong verbs. Both the principal parts and the endings of these verbs must be known in order to derive all their conjugational forms. For most such verbs, the principal parts consist of the infinitive, the third person singular of the past tense, and the past participle. For some of these verbs, the third person singular of the present tense (which also provides the stem for the second person singular) is needed; for a few others, the general subjunctive stem is needed.

In German, most strong verbs follow one of the patterns in the table. Two regular verbs for each class are given. Some common verbs that conform to the classes (except for minor irregularities) are shown. Unclassified strong verbs and the irregular weak verbs are also given.

The examples in the table are given in the following order: **A** is the infinitive; **B** is the third person singular, present tense, indicative; **C** is the third person singular, past tense, indicative; and **D** is the past participle.

Conjugation of Classified Strong Verbs

	Class I	II	III	IV	V	VI	VII
	Stem Vowel		nd nn				
A	ei	ie	i ng i mm	e	e	a	a
B	ei	ie	i nk i	(a)i (b)ie	(a)ie (b)i	ä	
C	(a)ie (b)i	o	a a	a	a	u	ie
D	ie i	o	u o	o	e	a	a
A	bleiben (*to stay*)	bieten (*to bid; to offer*)	finden (*to find*)	helfen (*to help*)	lesen (*to read*)	wachsen (*to grow*)	lassen (*to leave*)
B	bleibt	bietet	findet	hilft	liest	wächst	läßt
C	blieb	bot	fand	half	las	wuchs	ließ
D	geblieben	geboten	gefunden	geholfen	gelesen	gewachsen	gelassen
A	greifen (*to grasp*)	gießen (*to pour*)	beginnen (*to begin; to start*)	stehlen (*to steal*)	messen (*to measure*)	fahren (*to go; to travel; to drive*)	raten (*to guess; to advise*)
B	greift	gießt	beginnt	stiehlt	mißt	fährt	rät
C	griff	goß	begann	stahl	maß	fuhr	riet
D	gegriffen	gegossen	begonnen	gestohlen	gemessen	gefahren	geraten
A				nehmen (*to take*)		schaffen (*to manage*)	
B				nimmt		schafft	
C				nahm		schuf	
D				genommen		geschaffen	

Conjugation of Unclassified Strong Verbs

A	kommen (*to come*)	gehen (*to go*)	stehen (*to stand*)	tun (*to do*)	NOTE: **Class I** verb stems ending in a vowel change the vowel of the past tense and of the past participle to *i* before -en.
B	kommt	geht	steht	tut	
C	kam	ging	stand	tat	
D	gekommen	gegangen	gestanden	getan	

Conjugation of Irregular Strong Verbs

	Class I	II	III	IV	V	VI	VII
	Stem Vowel		nd nn				
A	ei	ie	i ng i mm	e	e	a	a
B	ei	ie	i nk i	(a)i (b)ie	(a)ie (b)i	ä	
C	(a)ie (b)i	o	a a	a	a	u	ie
D	ie i	o	u o	o	e	a	a
A	schneiden (*to cut; to divide; to separate; to slice; to chop*)	ziehen (*to pull*)	erlöschen (*to go out; to die*)	treten (*to kick*)	geben (*to give*)	heißen (*to be called*)	laufen (*to run*)
B	schneidet	zieht	erlischt	tritt	gibt	heißt	läuft
C	schnitt	zog	erlosch	trat	gab	hieß	lief
D	geschnitten	gezogen	erloschen	getreten	gegeben	geheißen	gelaufen
A	leiden (*to suffer*)	heben (*to lift*)	saufen (*to drink*)	essen (*to eat*)	sitzen (*to sit*)	stoßen (*to push*)	fangen (*to catch*)
B	leidet	hebt	säuft	ißt	sitzt	stößt	fängt
C	litt	hob	soff	aß	saß	stieß	fing
D	gelitten	gehoben	gesoffen	gegessen	gesessen	gestoßen	gefangen
A		schmelzen (*to melt*)	betrügen (*to cheat*)		liegen (*to lie—on a bed*)	hauen (*to hit; to carve*)	hängen (*to hang*)
B		schmilzt	betrügt		liegt	haut	hängt
C		schmolz	betrog		lag	hieb	hing
D		geschmolzen	betrogen		gelegen	gehauen	gehangen
A		lügen (*to lie—tell an untruth*)			bitten (*to ask*)		rufen (*to call*)
B		lügt			bittet		ruft
C		log			bat		rief
D		gelogen			gebeten		gerufen

Conjugation of Irregular Weak Verbs

A	bringen (*to bring*)	denken (*to think*)	kennen (*to know*)	senden (*to send*)	NOTE: **Class IV** verbs, helfen (*to help*), sterben (*to die*), verderben (*to ruin; to spoil*), and werfen (*to throw*), have the umlauted vowel *ü* in the general subjunctive. The unclassified verb stehen (*to stand*) has *ü* for *ä*. *general subjunctive stem
B	bringt	denkt	kennt	sendet	
C	brachte	dachte	kannte	sandte	
D	gebracht	gedacht	gekannt	gesandt	
*	brächt-	dächt-	kennt-	sendet-	
			also: brennen (*to burn*) nennen (*to call*) rennen (*to run*)	*also:* wenden (*to turn*)	

Conjugation of Irregular Verbs

Some Latin verbs have irregular principal parts; some lack one or two of the principal parts altogether; some lack some forms that are found in the conjugations of other verbs. Aside from its inherent importance, the forms of the irregular verb "to be" must be mastered because they are used to build the perfect tense system.

fero, ferre, tuli, latum (*to bear*; *to carry*)						
Indicatve (Active)	**Present**	**Imperfect**	**Future**	**Perfect**	**Pluperfect**	**Future Perfect**
	fero	ferebam	feram	tuli	tuleram	tulero
	fers	ferebas	feres	tulisti	tuleras	tuleris
	fert	ferebat	feret	tulit	tulerat	tulerit
	ferimus	ferebamus	feremus	tulimus	tuleramus	tulerimus
	fertis	ferebatis	feretis	tulistis	tuleratis	tuleritis
	ferunt	ferebant	ferent	tulerunt	tulerant	tulerint
Subjunctive (Active)	feram	ferrem	——	tulerim	tulissem	——
	feras	ferres	——	tuleris	tulisses	——
	ferat	ferret	——	tulerit	tulisset	——
	feramus	ferremus	——	tulerimus	tulissemus	——
	feratis	ferretis	——	tuleritis	tulissetis	——
	ferant	ferrent	——	tulerint	tulissent	——
Indicative (Passive)	feror	ferebar	ferar	latus sum	latus eram	latus ero
	ferris	ferebaris	fereris	latus es	latus eras	latus eris
	fertur	ferebatur	feretur	latus est	latus erat	latus erit
	ferimur	ferebamur	feremur	lati sumus	lati eramus	lati erimus
	ferimini	ferebamini	feremini	lati estis	lati eratis	lati eritis
	feruntur	ferebantur	ferentur	lati sunt	lati erant	lati erunt
Subjunctive (Passive)	ferar	ferrer	——	latus sim	latus essem	——
	feraris	ferreris	——	latus sis	latus esses	——
	feratur	ferretur	——	latus sit	latus esset	——
	feramur	ferremur	——	lati simus	lati essemus	——
	feramini	ferremini	——	lati sitis	lati essetis	——
	ferantur	ferrentur	——	lati sint	lati essent	——
Active		**Participle**			**Infinitive**	
Present		ferens			ferre	
Future		laturus			laturus esse	
Perfect		——			tulisse	
Passive		**Participle**			**Infinitive**	
Present		——			ferri	
Future		——			latum iri	
Perfect		latus			latus esse	

Conjugation of Irregular Verbs

sum, esse, fui, -, (*to be*)

Indicative (Active)	**Present**	**Imperfect**	**Future**	**Perfect**	**Pluperfect**	**Future Perfect**
	sum	eram	ero	fui	fueram	fuero
	es	eras	eris	fuisti	fueras	fueris
	est	erat	erit	fuit	fuerat	fuerit
	sumus	eramus	erimus	fuimus	fueramus	fuerimus
	estis	eratis	eritis	fuistis	fueratis	fueritis
	sunt	erant	erunt	fuerunt	fuerant	fuerint
Subjunctive (Active)	sim	essem	——	fuerim	fuissem	——
	sis	esses	——	fueris	fuisses	——
	sit	esset	——	fuerit	fuisset	——
	simus	essemus	——	fuerimus	fuissemus	——
	sitis	essetis	——	fueritis	fuissetis	——
	sint	essent	——	fuerint	fuissent	——
Active		**Participle**			**Infinitive**	
Present		——			esse	
Future		futurus			futurus esse/ fore	
Perfect		——			——	

eo, irte, ii, itum (*to go*)

* The *v* of the perfect, pluperfect, and future perfect tenses may be omitted.

Indicatve (Active)	**Present**	**Imperfect**	**Future**	**Perfect***	**Pluperfect***	**Future Perfect***
	eo	ibam	ibo	ivi	iveram	ivero
	is	ibas	ibis	ivisti	iveras	iveris
	it	ibat	ibit	ivit	iverat	iverit
	imus	ibamus	ibimus	ivimus	iveramus	iverimus
	itis	ibatis	ibitis	ivistis	iveratis	iveritis
	eunt	ibant	ibunt	iverunt	iverant	iverint
Subjunctive (Active)	eam	irem	——	iverim	ivissem	——
	eas	ires	——	iveris	ivisset	——
	eat	iret	——	iverit	ivisset	——
	eamus	iremus	——	iverimus	ivissemus	——
	eatis	iretis	——	iveritis	ivissetis	——
	eant	irent	——	iverint	ivissent	——
Active		**Participle**			**Infinitive**	
Present		iens, euntis			ire	
Future		iturus			iturus esse	
Perfect		——			ivisse	

Conjugation of Irregular Verbs

fio, fieri, factus sum (*to be made; to become*)						
Indicatve (Active)	**Present**	**Imperfect**	**Future**	**Perfect**	**Pluperfect**	**Future Perfect**
	fio	fiebam	fiam	factus sum	factus eram	factus ero
	fis	fiebas	fies	factus es	factus eras	factus eris
	fit	fiebat	fiet	factus est	factus erat	factus erit
	fimus	fiebamus	fiemus	facti sumus	facti eramus	facti erimus
	fitis	fiebatis	fietis	facti estis	facti eratis	facti eritis
	fiunt	fiebant	fient	facti sunt	facti erant	facti erunt
Subjunctive (Active)	fiam	fierem	——	factus sim	factus essem	——
	fias	fieres	——	factus sis	factus esses	——
	fiat	fieret	——	factus sit	factus esset	——
	fiamus	fieremus	——	facti simus	facti essemus	——
	fiatis	fieritis	——	facti sitis	facti essetis	——
	fiant	fierent	——	facti sint	facti essent	——
Active		**Participle**			**Infinitive**	
Present		——			fieri	
Future		——			factum iri	
Perfect		factus			factus esse	

malo, malle, malui, -, (*to prefer*)						
Indicative (Active)	**Present**	**Imperfect**	**Future**	**Perfect**	**Pluperfect**	**Future Perfect**
	malo	malebam	malam	malui	malueram	maluero
	mavis	malebas	males	maluisti	malueras	malueris
	mavult	malebat	malet	maluit	maluerat	maluerit
	malumus	malebamus	malemus	maluimus	malueramus	maluerimus
	mavultis	malebatis	maletis	maluistis	malueratis	malueritis
	malunt	malebant	malent	maluerunt	maluerant	maluerint
Subjunctive (Active)	malim	mallem	——	maluerim	maluissem	——
	malis	malles	——	malueris	maluisses	——
	malit	mallet	——	maluerit	maluisset	——
	malimus	mallemus	——	maluerimus	maluissemus	——
	malitis	mallemus	——	maluerimus	maluissetis	——
	malint	mallent	——	maluerint	maluissent	——
		Participle			**Infinitive**	
Present		——			malle	
Future		——			——	
Perfect		——			maluisse	

Conjugation of Irregular Verbs

nolo, nolle, nolui, -, (*to be unwilling; to not want; to not wish*)

	Present	Imperfect	Future	Perfect	Pluperfect	Future Perfect
Indicatve (Active)	nolo	nolebam	nolam	nolui	nolueram	noluero
	nonvis	nolebas	noles	noluisti	nolueras	nolueris
	nonvult	nolebat	nolet	noluit	noluerat	noluerit
	nolumus	nolebamus	nolemus	noluimus	nolueramus	noluerimus
	nonvultis	nolebatis	noletis	noluistis	nolueratis	nolueritis
	nolunt	nolebant	nolent	nolerunt	noluerant	noluerint
Subjunctive (Active)	nolim	nollem	——	noluerim	noluissem	——
	nolis	nolles	——	nolueris	noluisses	——
	nolit	nollet	——	noluerit	noluisset	——
	nolimus	nollemus	——	noluerimus	noluissemus	——
	nolitis	nolletis	——	nolueritis	noluissetis	——
	nolint	nollent	——	noluissent	noluissent	——
		Participle			**Infinitive**	
Present		nolens			nolle	
Future		——			——	
Perfect		——			noluisse	

possum, posse, potui, -, (*to be able to*)

	Present	Imperfect	Future	Perfect	Pluperfect	Future Perfect
Indicatve (Active)	possum	poteram	potero	potui	potueram	potuero
	potes	poteras	poteris	potuisti	potueras	potueris
	potest	poterat	poterit	potuit	potuerat	potuerit
	possumus	poteramus	poterimus	potuimus	potueramus	potuerimus
	potestis	poteratis	poteritis	potuistis	potueratis	potueritis
	possunt	poterant	poterunt	potuerunt	potuerant	potuerint
Subjunctive (Active)	possim	possem	——	potuerim	potuissem	——
	possis	posses	——	potueris	potuisses	——
	possit	posset	——	potuerit	potuisset	——
	possimus	possemus	——	potuerimus	potuissemus	——
	possitis	possetis	——	potuerimus	potuissemus	——
	possint	possent	——	potuerint	potuissent	——
		Participle			**Infinitive**	
Present		——			posse	
Future		——			——	
Perfect		——			potuisse	

Latin

Conjugation of Irregular Verbs

volo, velle, volui, -, (*to be willing; to want; to wish*)						
Indicatve (Active)	**Present**	**Imperfect**	**Future**	**Perfect**	**Pluperfect**	**Future Perfect**
	volo	volebam	volam	volui	volueram	voluero
	vis	volebas	voles	voluisti	volueras	volueris
	vult	volebat	volet	voluit	voluerat	voluerit
	volumus	volebamus	volemus	voluimus	volueramus	voluerimus
	vultis	volebatis	voletis	voluistis	volueratis	volueritis
	volunt	volebant	volent	volerunt	voluerant	voluerint
Subjunctive (Active)	velim	vellem	——	voluerim	voluissem	——
	velis	velles	——	volueris	voluisses	——
	velit	vellet	——	voluerit	voluisset	——
	velimus	vellemus	——	voluerimus	voluissemus	——
	velitis	velletis	——	volueritis	voluissemus	——
	velint	vellent	——	voluerint	voluissent	——
		Participle			**Infinitive**	
Present		volens			velle	
Future		——			——	
Perfect		——			voluisse	

Numbers

	Cardinal Numbers	Ordinal Numbers	Roman Numerals
1	unus, una, unum	primus, -a, -um	I
2	duo, duae, duo	secundus	II
3	tres, tria	tertius	III
4	quattuor	quartus	IIII or IV
5	quinque	quintus	V
6	sex	sextus	VI
7	septem	septimus	VII
8	octo	octavus	VIII
9	novem	nonus	VIIII or IX
10	decem	decimus	X

	Cardinal Numbers	Ordinal Numbers	Roman Numerals
11	undecim	undecimus	XI
12	duodecim	duodecimus	XII
13	tredecim	tertius decimus	XIII
14	quattuordecim	quartus decimus	XIIII or XIV
15	quindecim	quintus decimus	XV
16	sedecim	sextus decimus	XVI
17	septendecim	septimus decimus	XVII
18	duodeviginti	duodevicensimus	XVIII
19	undeviginti	undevicensimus	XVIIII or XIX
20	viginti	vicensimus	XX

Nouns

In Latin sentences, each noun has number (singular or plural), gender (masculine, feminine, or neuter), and case (grammatical role). Gender is a fixed quality, while number and case change depending on how the noun is being used.

Case. A declension is a pattern of case endings. There are five declensions; each noun may belong to only one. The dictionary entry for a Latin noun gives you first the nominative case, then an abbreviation for the genitive case, and then the gender.

Declensions

	1st	2nd			3rd			4th	5th
Singular:									
Nominative	carta (*charter*)	dominus (*lord, master*)	puer (*boy*)	vinum (*wine*)	rex (*king*)	civis (*citizen*)	ius (*right; justice*)	redditus (*rent*)	res (*object; thing; matter*)
Vocative	carta	domine	puer	vinum	rex	civis	ius	redditus	res
Accusative	cartam	dominum	puerum	vinum	regem	civem	ius	redditum	rem
Genitive	carte	domini	pueri	vini	regis	civis	iuris	redditus	rei
Dative	carte	domino	puero	vino	regi	civi	iuri	redditui	rei
Ablative	carta	domino	puero	vino	rege	cive	iure	redditu	re
Plural:									
Nominative	carte	domini	pueri	vina	reges	cives	iura	redditus	res
Vocative	carte	domini	pueri	vina	reges	cives	iura	redditus	res
Accusative	cartas	dominos	pueros	vina	reges	cives	iura	redditus	res
Genitive	cartarum	dominorum	puerorum	vinorum	regum	civium	iurum	reddituum	rerum
Dative	cartis	dominis	pueris	vinis	regibus	civibus	iuribus	redditibus	rebus
Ablative	cartis	dominis	pueris	vinis	regibus	civibus	iuribus	redditibus	rebus

Case

Case	Major grammatical role
Nominative	Subject of a verb
Genitive	Possession
Dative	Indirect object
Accusative	Direct object / Object of a preposition
Ablative	Adverbial use / Object of a preposition
Vocative	Direct address

Spanish

Conjugation of Stem-Changing Verbs

First class. Certain verbs of the first and second conjugations with stem vowels ***e*** or ***o*** change ***e*** to ***ie***, ***o*** to ***ue*** when stressed. Note that **jugar** (*to play*) is conjugated as if it were a first class stem-changing verb with stem vowel ***o***.

EXAMPLES:

pensar *(to think)*	volver *(to return)*	**Present participle**			
		pensando	volviendo		
Present indicative		**Present subjunctive**		**Imperative**	
pienso	vuelvo	piense	vuelva		
piensas	vuelves	pienses	vuelvas	piensa	vuelve
piensa	vuelve	piense	vuelva	piense	vuelva
pensamos	volvemos	pensemos	volvamos	pensemos	volvamos
pensáis	volvéis	penséis	volváis	pensad	volved
piensan	vuelven	piensen	vuelvan	piensen	vuelvan

Second class. Certain verbs of the third conjugation with stem vowels of ***e*** or ***o*** change ***e*** to ***ie***, ***o*** to ***ue*** when stressed. Changes are the same as those listed above with the addition of ***e*** changing to ***i***, and ***o*** to ***u***, when the following syllable contains a stressed ***a***, ***ie***, or ***io***.

EXAMPLES:

(ie) sentir *(to feel)*	(ue) dormir *(to sleep)*	**Present participle**			
		sintiendo	durmiendo		
Present indicative		**Present subjunctive**		**Preterite**	
siento	duermo	sienta	duerma	sentí	dormí
sientes	duermes	sientas	duermas	sientas	dormiste
siente	duerme	sienta	duerma	sintió	durmió
sentimos	dormimos	sintamos	durmamos	sentimos	dormimos
sentís	dormís	sintáis	durmáis	sentisteis	dormisteis
sienten	duermen	sientan	duerman	sintieron	durmieron
Imperfect subjunctive		**Imperative**			
sintiera	durmiera				
sintieras	durmieras	siente	duerme		
sintiera	durmiera	sienta	duerma		
sintiéramos	durmiéramos	sintamos	durmamos		
sintierais	durmierais	sentid	dormid		
sintieran	durmieran	sientan	duerman		

Third class. Certain verbs of the third conjugation with stem vowel ***e*** change ***e*** to ***i*** in all forms affected in the first and second classes of stem-changing verbs.

EXAMPLES:

(i) pedir *(to ask for)*		**Present participle**		
		pidiendo		
Present Indicative	**Present subjunctive**	**Preterite**	**Imperfect subjunctive**	**Imperative**
pido	pida	pedí	pidiera	
pides	pidas	pediste	pidieras	pide
pide	pida	pidió	pidiera	pida
pedimos	pidamos	pedimos	pidiéramos	pidamos
pedís	pidáis	pedisteis	pidierais	pedid
piden	pidan	pidieron	pidieran	pidan

Spelling Change Verbs

This class of verbs undergoes changes in spelling during conjugation, with the effect of preserving the sound of the consonant found in the infinitive.

The chart below has examples of changes in the verbs **buscar** (to search), **jugar** (to play), **averiguar** (to find out), **vencer** (to conquer), **conocer** (to know), **leer** (to read), **escoger** (to choose), and **seguir** (to follow).

Verb ending	Spelling change	Examples
in verbs ending in **car**	*c* changes to *qu* before an *e*	buscar → busqué
in verbs ending in **gar**	insert *u* before an *e*	jugar → jugué
in verbs ending in **guar**	put *dieresis* over *u* before an *e*	averiguar → averigüé
in verbs ending in **cer** or **cir** preceded by a consonant	replace the *c* by *z* before an *a* or *o*	vencer → venzo
in verbs ending in **cer** or **cir** preceded by a vowel	insert *z* before *c* when followed by an *a* or *o*	conocer → conozco
in verbs ending in **er** or **ir**	change the *i* of the endings *ie* and *io* to *y*	leer → leyó
in verbs ending in **ger** or **gir**	replace the *g* with *j* before an *a* or *o*	escoger → escojo
in verbs ending in **guir** when the *u* is silent	drop the *u* before an *a* or *o*	seguir → sigo

Irregular Verbs

Irregular verbs are the most difficult verbs to learn. Yet, perhaps unfortunately, they describe actions and states of being that are most frequently needed in speaking and writing Spanish. The verb forms given in the chart below and on the following pages take thirty-eight irregular verbs through nine tenses. As you will see, the verb endings for certain tenses are the same as those you will encounter in conjugating regular verbs.

Conjugation of Irregular Verbs

Infinitive	Indicative				Conditional	Subjunctive			Imperative
	Present	Imperfect	Preterite	Future	Present	Present	Imperfect (1st form)	(2nd form)	
abrir	abro	abría	abrí	abriré	abriría	abra	abriera	abriese	
(*to open*)	abres	abrías	abriste	abrirás	abrirías	abras	abrieras	abrieses	abre
	abres	abría	abrió	abrirá	abriría	abra	abrira	abriese	abre
	abrimos	abríamos	abrimos	abriremos	abriríamos	abramos	abriéramos	abriésemos	abramos
	abrís	abríais	abristeis	abriréis	abriríais	abráis	abrierais	abrieseis	abrid
	abren	abrían	abrieron	abrirán	abrirían	abran	abrieran	abriesen	abran
andar	ando	andaba	anduve	andaré	andaría	ande	anduviera	anduviese	
(*to go;*	andas	andabas	anduviste	andarás	andarías	andes	anduvieras	anduvieses	anda
to walk)	anda	andaba	anduvo	andará	andaría	ande	anduviera	anduviese	ande
	andamos	andábamos	anduvimos	andaremos	andaríamos	andemos	anduviéramos	anduviésemos	andemos
	andáis	andabais	anduvisteis	andaréis	andaríais	andéis	anduvierais	anduvieseis	andad
	andan	andaban	anduvieron	andarán	andarían	anden	anduvieran	anduviesen	anden

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive	Indicative				Conditional	Subjunctive			Imperative
	Present	Imperfect	Preterite	Future	Present	Present	Imperfect (1st form)	(2nd form)	
buscar	busco	buscaba	busqué	buscaré	buscaría	busque	buscara	buscase	
(*to search*)	buscas	buscabas	buscaste	buscarás	buscarías	busques	buscaras	buscases	busca
	busca	buscabas	buscó	buscará	buscaría	busque	buscara	buscase	busque
	buscamos	buscábamos	buscamos	buscaremos	buscaríamos	busquemos	buscáramos	buscásemos	busquemos
	buscáis	buscabais	buscasteis	buscaréis	buscaríais	busquéis	buscarais	buscaseis	buscad
	buscan	buscaban	buscaron	buscarán	buscarían	busquen	buscaran	buscasen	busquen
caer	caigo	caía	caí	caeré	caería	caiga	cayera	cayese	
(*to fall*)	caes	caías	caíste	caerás	caerías	caigas	cayeras	cayeses	cae
	cae	caía	cayó	caerá	caería	caiga	cayera	cayese	caiga
	caemos	caíamos	caímos	caeremos	caeríamos	caigamos	cayéramos	cayésemos	caigamos
	caéis	caíais	caísteis	caeréis	caeríais	caigáis	cayerais	cayeseis	caed
	caen	caían	cayeron	caerán	caerían	caigan	cayeran	cayesen	caigan
cerrar	cierro	cerraba	cerré	cerraré	cerraría	cierre	cerrara	cerrase	cierra
(*to close*)	cierras	cerrabas	cerraste	cerrarás	cerrarías	cierres	cerraras	cerrases	cierre
	cierra	cerraba	cerró	cerrará	cerraría	cierre	cerraras	cerrase	cerremos
	cerramos	cerrábamos	cerramos	cerraremos	cerraríamos	cerremos	cerráramos	cerrásemos	cerrad
	cerráis	cerrabais	cerrasteis	cerraréis	cerraríais	cerréis	cerrarais	cerraseis	cierren
	cierran	cerraban	cerraron	cerrarán	cerrarían	cierren	cerraran	cerrasen	
conocer	conozco	conocia	conocí	conoceré	conocería	conozca	conociera	conociese	
(*to know*)	conoces	conocías	conociste	conocerás	conocerías	conozcas	conocieras	conocieses	conoce
	conoce	conocía	conoció	conocerá	conocería	conozca	conociera	conociese	conozca
	conocemos	conocíamos	conocimos	conoceremos	conoceríamos	conozcamos	conociéramos	conociésemos	conozcamos
	conocéis	conocíais	conociesteis	conoceréis	conoceríais	conozcáis	conocierais	conocieseis	conoced
	conocen	conocían	conocieron	conocerán	conocerían	conozcan	conocieran	conociesen	conozcan
dar	doy	daba	di	daré	daría	dé	diera	diese	
(*to give*)	das	dabas	diste	darás	darías	des	dieras	dieses	da
	da	daba	dio	dará	daría	dé	diera	diese	dé
	damos	dábamos	dimos	daremos	daríamos	demos	diéramos	diésemos	demos
	dais	dabais	disteis	daréis	daríais	deis	dierais	dieseis	dad
	dan	daban	dieron	darán	darían	den	dieran	diesen	den
decir	digo	decía	dije	diré	diría	diga	dijera	dijese	
(*to say;*	dices	decías	dijiste	dirás	dirías	digas	dijeras	dijeses	di
tell)	dice	decía	dijo	dirá	diría	diga	dijera	dijese	diga
	decimos	decíamos	dijimos	diremos	diríamos	digamos	dijéramos	dijésemos	digamos
	decís	decíais	dijisteis	diréis	diríais	digáis	dijerais	dijeseis	decid
	dicen	decían	dijeron	dirán	dirían	digan	dijeran	dijesen	digan

Conjugation of Irregular Verbs

Infinitive	Indicative				Conditional	Subjunctive			Imperative
	Present	Imperfect	Preterite	Future	Present	Present	Imperfect (1st form)	(2nd form)	
empezar	empiezo	empezaba	empecé	empezaré	empezaría	empiece	empezara	empezase	
(*to start*)	empiezas	empezabas	empezaste	empezarás	empezarías	empieces	empezaras	empezases	empieza
	empieza	empezaba	empezó	empezará	empezaría	empiece	empezara	empezase	empiece
	empezamos	empezábamos	empezamos	empezaremos	empezaríamos	empecemos	empezáramos	empezásemos	empecemos
	empezáis	empezabais	empezasteis	empezaréis	empezaríais	empecéis	empezarais	empezaseis	empezad
	empiezan	empezaban	empezaron	empezarán	empezarían	empiecen	empezaran	empezasen	empiecen
encontrar	encuentro	encontraba	encontré	encontraré	encontraría	encuentre	encontrara	encontrase	
(*to find*)	encuentras	encontrabas	encontraste	encontrarás	encontrarías	encuentres	encontraras	encontrases	encuentra
	encuentra	encontraba	encontró	encontrará	encontraría	encuentre	encontrara	encontrase	encuentre
	encontramos	encontrábamos	encontramos	encontraremos	encontraríamos	encontremos	encontráramos	encontrásemos	encontremos
	encontráis	encontrabais	encontrasteis	encontraréis	encontraríais	encontréis	encontrarais	encontraseis	encontrad
	encuentran	encontraban	encontraron	encontrarán	encontrarían	encuentren	encontraran	encontrasen	encuentren
escoger	escojo	escogia	escogí	escogeré	escogería	escoja	escogiera	escogiese	
(*to choose*)	escoges	escogías	escogiste	escogerás	escogerías	escojas	escogieras	escogieses	escoge
	escoge	escogía	escogió	escogerá	escogería	escoja	escogiera	escogiese	escoja
	escogemos	escogíamos	escogimos	escogeremos	escogeríamos	escojamos	escogiéramos	escogiésemos	escojamos
	escogéis	escogíais	escogisteis	escogeréis	escogeríais	escojáis	escogierais	escogieseis	escoged
	escogen	escogían	escogieron	escogerán	escogerían	escojan	escogieran	escogiesen	escojan
estar	estoy	estaba	estuve	estaré	estaría	esté	estuviera	estuviese	
(*to be*)	estás	estabas	estuviste	estarás	estarías	estés	estuvieras	estuvieses	está
	está	estaba	estuvo	estará	estaría	esté	estuviera	estuviese	esté
	estamos	estábamos	estuvimos	estaremos	estaríamos	estemos	estuviéramos	estuviésemos	estemos
	estáis	estabais	estuvisteis	estaréis	estaríais	estéis	estuvierais	estuvieseis	estad
	están	estaban	estuvieron	estarán	estarían	estén	estuvieran	estuviesen	estén
haber	he	había	hube	habré	habría	haya	hubiera	hubiese	
(*to have*)	has	habías	hubiste	habrás	habrías	hayas	hubieras	hubieses	he
	ha	había	hubo	habrá	habría	haya	hubiera	hubiese	haya
	hemos	habíamos	hubimos	habremos	habríamos	hayamos	hubiéramos	hubiésemos	hayamos
	habéis	habíais	hubisteis	habréis	habríais	hayáis	hubierais	hubieseis	habed
	han	habían	hubieron	habrán	habrían	hayan	hubieran	hubiesen	hayan
hacer	hago	hacía	hice	haré	haría	haga	hiciera	hiciese	
(*to make;*	haces	hacías	hiciste	harás	harías	hagas	hicieras	hicieses	haz
to do)	hace	hacía	hizo	hará	haría	haga	hiciera	hiciese	haga
	hacemos	hacíamos	hicimos	haremos	haríamos	hagamos	hiciéramos	hiciésemos	hagamos
	hacéis	hacíais	hicisteis	haréis	haríais	hagáis	hicierais	hicieseis	haced
	hacen	hacían	hicieron	harán	harían	hagan	hicieran	hiciesen	hagan

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive	Indicative				Conditional	Subjunctive			Imperative
	Present	Imperfect	Preterite	Future	Present	Present	Imperfect (1st form)	(2nd form)	
ir	voy	iba	fui	iré	iría	vaya	fuera	fuese	
(*to go*)	vas	ibas	fuiste	irás	irías	vayas	fueras	fueses	ve
	va	iba	fue	irá	iría	vaya	fuera	fuese	vaya
	vamos	íbamos	fuimos	iremos	iríamos	vayamos	fuéramos	fuésemos	vamos
	vais	ibais	fuisteis	iréis	iríais	vayáis	fuerais	fueseis	id
	van	iban	fueron	irán	irían	vayan	fueran	fuesen	vayan
jugar	juego	jugaba	jugué	jugaré	jugaría	juegue	jugara	jugase	
(*to play*)	juegas	jugabas	jugaste	jugarás	jugarías	juegues	jugaras	jugases	juega
	juega	jugaba	jugó	jugará	jugaría	juegue	jugara	jugase	juegue
	jugamos	jugábamos	jugamos	jugaremos	jugaríamos	juguemos	jugáramos	jugásemos	juguemos
	jugáis	jugabais	jugasteis	jugaréis	jugaríais	juguéis	jugarais	jugaseis	jugad
	juegan	jugaban	jugaron	jugará	jugarían	jueguen	jugaran	jugasen	jueguen
lavar	lavo	lavaba	lavé	lavaré	lavaría	lave	lavara	lavase	
(*to wash*)	lavas	lavabas	lavaste	lavarás	lavarías	laves	lavaras	lavases	lava
	lava	lavaba	lavó	lavará	lavaría	lave	lavara	lavase	lave
	lavamos	lavábamos	lavamos	lavaremos	lavaríamos	lavemos	laváramos	lavásemos	lavemos
	laváis	lavabais	lavasteis	lavaréis	lavaríais	lavéis	lavarais	lavaseis	lavad
	lavan	lavaban	lavaron	lavarán	lavarían	laven	lavaran	lavasen	laven
leer	leo	leía	leí	leeré	leería	lea	leyera	leyese	
(*to read*)	lees	leías	leíste	leerás	leerías	leas	leyeras	leyeses	lee
	lee	leía	leyó	leerá	leería	lea	leyera	leyese	lea
	leemos	leíamos	leímos	leeremos	leeríamos	leamos	leyéramos	leyésemos	leamos
	leéis	leíais	leísteis	leeréis	leeríais	leáis	leyerais	leyeseis	leed
	leen	leían	leyeron	leerán	leerían	lean	leyeran	leyesen	lean
llegar	llego	llegaba	llegué	llegaré	llegaría	llegue	llegara	llegase	
(*to arrive*)	llegas	llegabas	llegaste	llegarás	llegarías	llegues	llegaras	llegases	llega
	llega	llegaba	llegó	llegará	llegaría	llegue	llegara	llegase	llegue
	llegamos	llegábamos	llegamos	llegaremos	llegaríamos	lleguemos	llegáramos	llegásemos	lleguemos
	llegáis	llegabais	llegasteis	llegaréis	llegaríais	lleguéis	llegarais	llegaseis	llegad
	llegan	llegaban	llegaron	llegerá	llegarían	lleguen	llegaran	llegasen	lleguen
mirar	miro	miraba	miré	miraré	miraría	mire	mirara	mirase	
(*to look at*)	miras	mirabas	miraste	mirarás	mirarías	mires	miraras	mirases	mira
	mira	miraba	miró	mirará	miraría	mire	mirara	mirase	mire
	miramos	mirábamos	miramos	miraremos	miraríamos	miremos	miráramos	mirásemos	miremos
	miráis	mirabais	mirasteis	miraréis	miraríais	miréis	mirarais	miraseis	mirad
	miran	miraban	miraron	mirarán	mirarían	miren	miraran	mirasen	miren

Conjugation of Irregular Verbs									
Infinitive	**Indicative**				**Conditional**	**Subjunctive**			**Imperative**
	Present	**Imperfect**	**Preterite**	**Future**	**Present**	**Present**	**Imperfect (1st form)**	**(2nd form)**	
oir (*to hear*)	oigo oyes oye oímos oís oyen	oía oías oía oíamos oiais oian	oí oíste oyó oímos oísteis oyeron	oiré oirás oirá oiremos oiréis oirán	oiría oirías oiría oiríamos oiríais oirian	oiga oigas oiga oigamos oigáis oigan	oyera oyeras oyera oyéramos oyerais oyeran	oyese oyeses oyese oyésemos oyeseis oyesen	oye oiga oigamos oíd oigan
olvidar (*to forget*)	olvido olvidas olvida olvidamos olvidáis olvidan	olvidaba olvidabas olvidaba olvidábamos olvidabais olvidaban	olvidé olvidaste olvidó olvidamos olvidasteis olvidaron	olvidaré olvidarás olvidará olvidaremos olvidaréis olvidará	olvidaría olvidarías olvidaría olvidaríamos olvidaríais olvidarían	olvide olvides olvide olvidemos olvidéis olviden	olvidara olvidaras olvidara olvidáramos olvidarais olvidaran	olvidase olvidases olvidase olvidásemos olvidaseis olvidasen	olvida olvide olvidemos olvidad olviden
pagar (*to pay*)	pago pagas paga pagamos pagáis pagan	pagaba pagabas pagaba pagábamos pagabais pagaban	pagué pagaste pagó pagamos pagasteis pagaron	pagaré pagarás pagará pagaremos pagaréis pagarán	pagaría pagarías pagaría pagaríamos pagaríais pagarían	pague pagues pague paguemos paguéis paguen	pagara pagaras pagara pagáramos pagarais pagaran	pagase pagases pagase pagásemos pagaseis pagasen	paga pague paguemos pagad paguen
perder (*to lose*)	pierdo pierdes pierde perdemos perdéis pierden	perdía perdías perdía perdíamos perdíais perdían	perdí perdiste perdió perdimos perdisteis perdieron	perderé perderás perderá perderemos perderéis perderán	perdería perderías perdería perderíamos perderíais perderían	pierda pierdas pierda perdamos perdáis pierdan	perdiera perdieras perdiera perdiéramos perdierais perdieran	perdiese perdieses perdiese perdiésemos perdieseis perdiesen	pierde pierda perdamos perded pierdan
poder (*to be able*)	puedo puedes puede podemos podéis pueden	podía podías podía podíamos podíais podían	pude pudiste pudo pudimos pudisteis podieron	podré podrás podrá podremos podréis podrán	podría podrías podría podríamos podríais podrían	pueda puedas pueda podamos podáis puedan	pudiera pudieras pudiera pudiéramos pudierais pudieran	pudiese pudieses pudiese pudiésemos pudieseis pudiesen	puede pueda podamos poded puedan
poner (*to put*)	pongo pones pone ponemos ponéis ponen	ponía ponías ponía poníamos poníais ponían	puse pusiste puso pusimos pusisteis pusieron	pondré pondrás pondrá pondremos pondréis pondrán	pondría pondrías pondria pondríamos pondríais pondrían	ponga pongas ponga pongamos pongáis pongan	pusiera pusieras pusiera pusiéramos pusierais pusieran	pusiese pusieses pusiese pusiésemos pusieseis pusiesen	pon ponga pongamos poned pongan

Irregular Verbs

Conjugation of Irregular Verbs

Infinitive	Indicative				Conditional	Subjunctive			Imperative
	Present	Imperfect	Preterite	Future	Present	Present	Imperfect (1st form)	(2nd form)	
quedar	quedo	quedaba	quedé	quedaré	quedaría	quede	quedara	quedase	
(*to stay*)	quedas	quedabas	quedaste	quedarás	quedarías	quedes	quedaras	quedases	queda
	queda	quedaba	quedó	quedará	quedaría	quede	quedara	quedase	quede
	quedamos	quedábamos	quedamos	quedaremos	quedaríamos	quedemos	quedáramos	quedásemos	quedemos
	quedáis	quedabais	quedasteis	quedaréis	quedaríais	quedéis	quedarais	quedaseis	quedad
	quedan	quedaban	quedaron	quedarán	quedarían	queden	quedaran	quedasen	queden
querer	quiero	quería	quise	querré	querría	quiera	quisiera	quisiese	
(*to want;*	quieres	querías	quisiste	querrás	querrías	quieras	quisieras	quisieses	quiere
love)	quiere	quería	quiso	querrá	quirría	quiera	quisiera	quisiese	quiera
	queremos	queríamos	quisimos	querremos	querríamos	queramos	quisiéramos	quisiésemos	quieramos
	queréis	quieríais	quisisteis	querréis	querríais	queráis	quisierais	quisieseis	quiered
	quieren	querían	quisieron	querrán	querrían	quieran	quisieran	quisiesen	quieran
saber	sé	sabía	supe	sabré	sabría	sepa	supiera	supiese	
(*to know*)	sabes	sabías	supiste	sabrás	sabrías	sepas	supieras	supieses	sabe
	sabe	sabía	supo	sabrá	sabría	sepa	supiera	supiese	sepa
	sabemos	sabíamos	supimos	sabremos	sabríamos	sepamos	supiéramos	supiésemos	sepamos
	sabéis	sabíais	supisteis	sabréis	sabríais	sepáis	supierais	supieseis	sabed
	saben	sabían	supieron	sabrán	sabrían	sepan	supieran	supiesen	sepan
sacar	saco	sacaba	saqué	sacaré	sacaría	saque	sacara	sacase	
(*to take*	sacas	sacabas	sacaste	sacarás	sacarías	saques	sacaras	sacases	saca
out)	saca	sacaba	sacó	sacará	sacaría	saque	sacara	sacase	saque
	sacamos	sacábamos	sacamos	sacaremos	sacaríamos	saquemos	sacáramos	sacásemos	saquemos
	sacáis	sacabais	sacasteis	sacaréis	sacaríais	saquéis	sacarais	sacaseis	sacad
	sacan	sacaban	sacaron	sacarán	sacarían	saquen	sacaran	sacasen	saquen
seguir	sigo	seguía	seguí	seguiré	seguiría	siga	siguiera	siguiese	
(*to follow*)	sigues	seguías	seguiste	seguirás	seguirías	sigas	siguieras	siguieses	——
	sigue	seguía	siguió	seguirá	seguiría	siga	siguiera	siguiese	——
	seguimos	seguíamos	seguimos	seguiremos	seguiríamos	sigamos	siguiéramos	siguiésemos	——
	seguís	seguíais	seguisteis	seguiréis	seguiríais	sigáis	siguierais	siguieseis	——
	siguen	seguían	siguieron	seguirán	seguirían	sigan	siguieran	siguiesen	——
salir	salgo	salía	salí	saldré	saldría	salga	saliera	saliese	
(*to go*	sales	salías	saliste	saldrás	saldrías	salgas	salieras	salieses	sal
out)	sale	salía	salió	saldrá	saldría	salga	saliera	saliese	salga
	salimos	salíamos	salimos	saldremos	saldríamos	salgamos	saliéramos	saliésemos	salgamos
	salís	salíais	salisteis	saldréis	saldríais	salgáis	salierais	salieseis	salid
	salen	salían	salieron	saldrán	saldrían	salgan	salieran	saliesen	salgan

Conjugation of Irregular Verbs

Infinitive	Indicative				Conditional	Subjunctive			Imperative
	Present	Imperfect	Preterite	Future	Present	Present	Imperfect (1st form)	(2nd form)	
ser	soy	era	fui	seré	sería	sea	fuera	fuese	
(*to be*)	eres	eras	fuiste	serás	serías	seas	fueras	fueses	sé
	es	era	fue	será	sería	sea	fuera	fuese	sea
	somos	éramos	fuimos	seremos	seríamos	seamos	fuéramos	fuésemos	seamos
	sois	erais	fuisteis	seréis	seríais	seáis	fuerais	fueseis	sed
	son	eran	fueron	serán	serían	sean	fueran	fuesen	sean
tener	tengo	tenía	tuve	tendré	tendría	tenga	tuviera	tuviese	
(*to have*)	tienes	tenías	tuviste	tendrás	tendrías	tengas	tuvieras	tuvieses	ten
	tiene	tenía	tuvo	tendrá	tendría	tenga	tuviera	tuviese	tenga
	tenemos	teníamos	tuvimos	tendremos	tendríamos	tengamos	tuviéramos	tuviésemos	tengamos
	tenéis	teníais	tuvisteis	tendréis	tendríais	tengáis	tuvierais	tuvieseis	tened
	tienen	tenían	tuvieron	tendrán	tendrían	tengan	tuvieran	tuviesen	tengan
traer	traigo	traía	traje	traeré	traería	traiga	trajera	trajese	
(*to bring*)	traes	traías	trajiste	traerás	traerías	traigas	trajeras	trajeses	trae
	trae	traía	trajo	traerá	traería	traiga	trajera	trajese	traiga
	traemos	traíamos	trajimos	traeremos	traeríamos	traigamos	trajéramos	trajésemos	traigamos
	traéis	traíais	trajisteis	traeréis	traeríais	traigáis	trajerais	trajeseis	traed
	traen	traían	trajeron	traerán	traerían	traigan	trajeran	trajesen	traigan
valer	valgo	valía	valí	valdré	valdría	valga	valiera	valiese	
(*to be*	vales	valías	valiste	valdrás	valdrías	valgas	valieras	valieses	val
worth)	vale	valía	valió	valdrá	valdría	valga	valiera	valiese	valga
	valemos	valíamos	valimos	valdremos	valdríamos	valgamos	valiéramos	valiésemos	valgamos
	valéis	valíais	valisteis	valdréis	valdríais	valgáis	valierais	valieseis	valed
	valen	valían	valieron	valdrán	valdrían	valgan	valieran	valiesen	valgan
venir	vengo	venía	vine	vendré	vendría	venga	viniera	viniese	
(*to come*)	vienes	venías	viniste	vendrás	vendrías	vengas	vinieras	vinieses	ven
	viene	venía	vino	vendrá	vendría	venga	viniera	viniese	venga
	venimos	veníamos	vinimos	vendremos	vendríamos	vengamos	veniéramos	viniésemos	vengamos
	venís	veníais	vinisteis	vendréis	vendríais	vengáis	vinierais	vinieseis	venid
	vienen	venían	vinieron	vendrán	vendrían	vengan	vinieran	viniesen	vengan
ver	veo	veía	vi	veré	vería	vea	viera	viese	
(*to see*)	ves	veías	viste	verás	verías	veas	vieras	vieses	ve
	ve	veía	vio	verá	vería	vea	viera	viese	vea
	vemos	veíamos	vimos	veremos	veríamos	veamos	viéramos	viésemos	veamos
	veis	veíais	visteis	veréis	veríais	veáis	vierais	vieseis	ved
	ven	veían	vieron	verán	verían	vean	vieran	viesen	vean

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PREFACE

Although vast amounts of material exist on the Internet and elsewhere, much of it is unsubstantiated, unreliable, and/or unverifiable. This volume has been created expressly to serve as a primary filter for students looking for topics and credible information for their report writing.

This book and its accompanying web portal contain articles on ninety-five separate research topics, which in many cases will include sufficient material for an entire paper. Each article also includes several Hot Topics, suggesting additional ideas and directions you might like to pursue.

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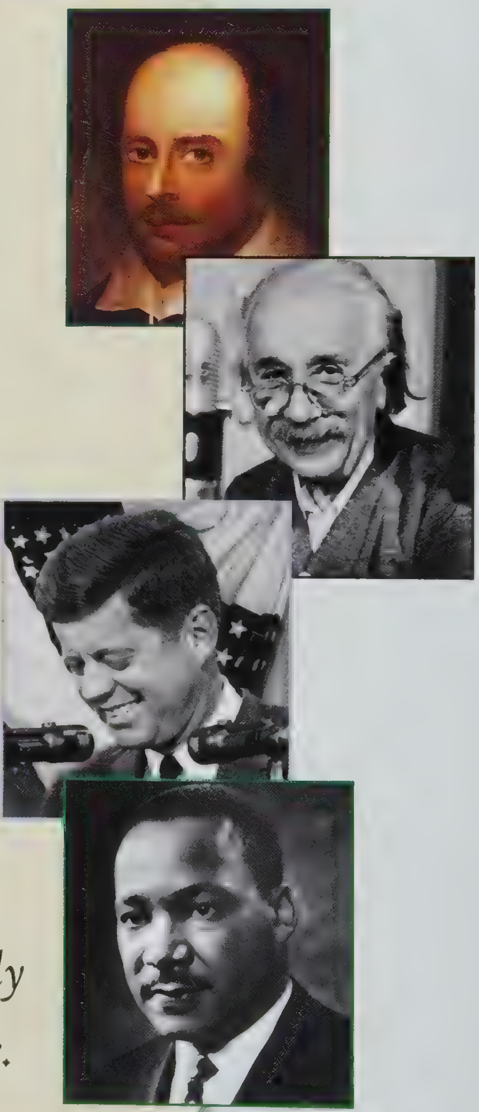

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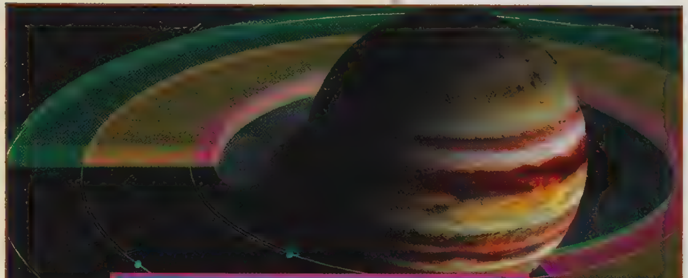

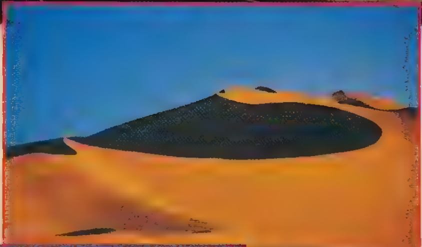

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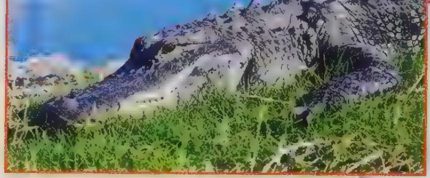

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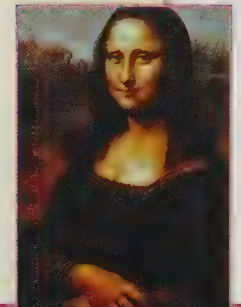

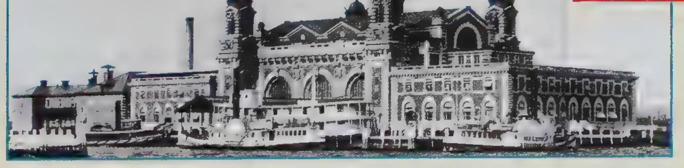

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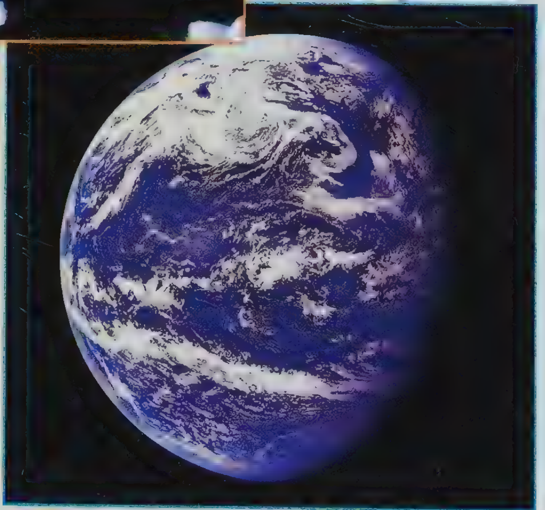

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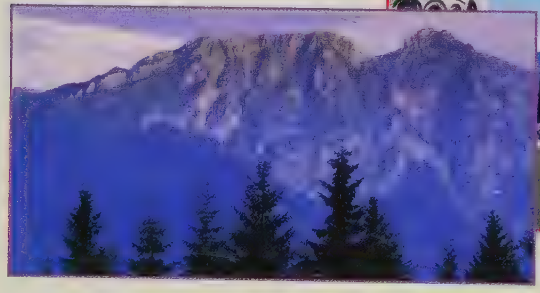

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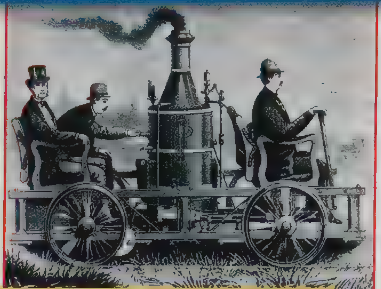

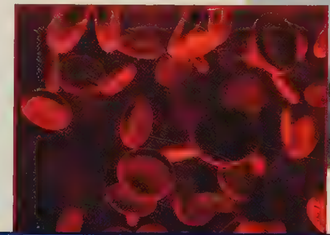

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Abraham Lincoln

The sixteenth president of the United States, Abraham Lincoln (1809–1865), is known best for his presidential efforts during the most devastating era in American history, the American Civil War (1861–1865), ending slavery in the United States, and being the first president of the United States to be assassinated.

HOT topics

- Greatest Speeches
- Emancipation Proclamation
- Critical Involvements in War Effort
- Highlights of Lincoln's Administration
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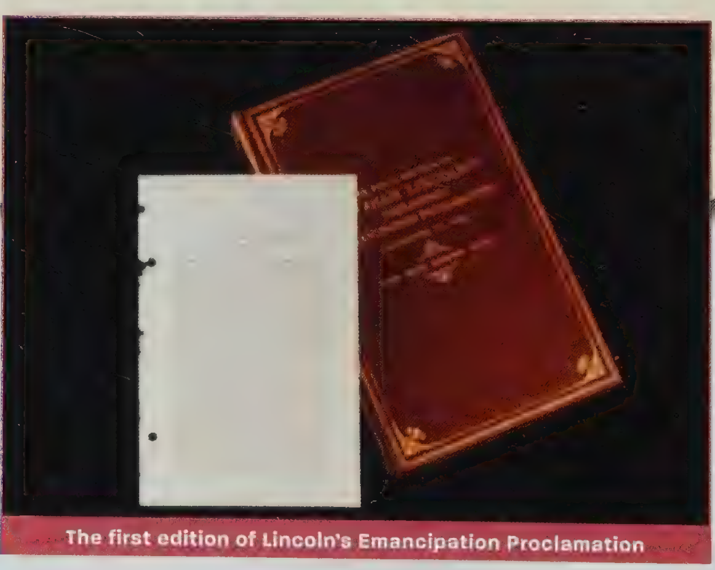
HOT topics

Greatest Speeches. Lincoln's Gettysburg Address, given during the dedication ceremonies at a cemetery on the Gettysburg battlefield on November 19, 1863, is well-remembered and often quoted. Also of historical significance is Lincoln's Second Inaugural Address. Both speeches are etched in stone on the Lincoln Memorial in Washington, D.C.

Emancipation Proclamation. On January 1, 1863, President Lincoln issued the Emancipation Proclamation, declaring freedom for slaves within the Confederacy. The proclamation also



The first edition of Lincoln's Emancipation Proclamation

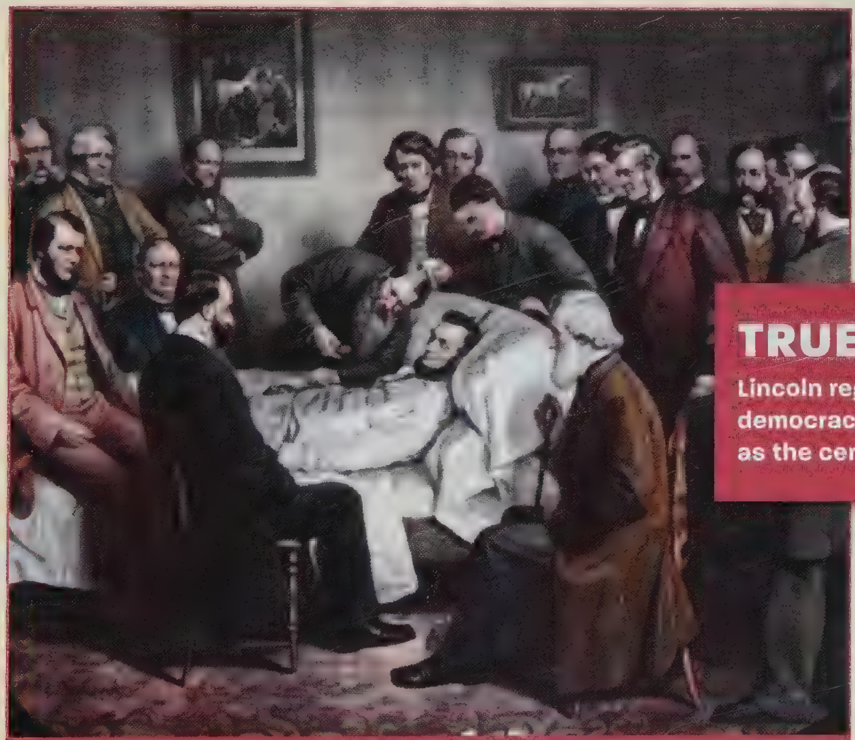
HOT topics

allowed blacks to serve in the Union Army and Navy, which greatly affected the outcome of the war. Although the Emancipation Proclamation did not free all slaves, it led to the Thirteenth Amendment to the Constitution, which became law in 1865 and officially ended slavery in all of the United States.

Critical Involvements in War Effort. Some historians believe Lincoln was the chief architect of military strategy for the Union. He chose which officers would lead specific military campaigns. He blocked the Southern ports, called for 75,000 men for the army, and ordered spending of federal funds. He also suspended *habeas corpus*, thus allowing Southern sympathizers to be detained or imprisoned. He strengthened relations with border states as well as foreign nations (notably during the "Trent Affair").

Highlights of Lincoln's Administration. Besides conducting the war effort and issuing the Emancipation Proclamation, Lincoln's administration also approved the Homestead Act of 1862, which encouraged farm ownership; signed the Pacific Railroad Act in 1862, starting the transcontinental railroad; signed the Land Grant Act, which set aside land for state colleges; and approved the Nation Bank Act of 1863, which established the current American system of national banks.

Assassination. Abraham Lincoln was the first president of the United States to be assassinated. On April 14, 1865, less than a week after the war ended, John Wilkes Booth shot President Lincoln in the head while Lincoln was attending a performance at Ford's Theatre in Washington. Booth escaped but was killed later by federal troops in an attempt to capture him. Several people were tried as conspirators with Booth in Lincoln's assassination and in a plot to kill other government officials.



Lincoln on his deathbed

TRUE or FALSE?

Lincoln regarded the fate of world democracy, not abolition of slavery, as the central issue of the Civil War.

THE BASICS

Abraham Lincoln (1809–1865) was one of the truly great men of all time. He led the United States during the American Civil War (1861–1865), which was the greatest crisis in U.S. history. Lincoln helped end slavery in the nation and helped keep the American Union from splitting apart during the war. Lincoln thus believed that he proved to the world that democracy can be a lasting form of government. Lincoln's Gettysburg Address, Second Inaugural Address, and many of his other speeches and writings are classic statements of democratic beliefs and goals. In conducting a bitter war, Lincoln never became bitter himself. He showed a nobility of character that has worldwide appeal. Lincoln, a Republican, was the first member of his party to become president. He was assassinated near the end of the Civil War and was succeeded by Vice President Andrew Johnson. Lincoln was the first U.S. president to be assassinated.

Portrait of Abraham Lincoln

The American people knew little about Lincoln when he became president. Little in his past experience indicated that he could successfully deal with the deep differences between Northerners and Southerners over slavery. Lincoln received less than 40 percent of the popular vote in winning the presidential election of 1860. But by 1865, he had become in the eyes of the world equal in importance to George Washington. Through the years, many people have regarded Lincoln as the greatest person in United States history.

During the Civil War, Lincoln's first task was to win the war. He had to view nearly all other matters in relation to the war. It was "the progress of our arms," he once said, "upon which all else depends." But Lincoln was a peace-loving man who had earlier described military glory as "that attractive rainbow, that rises in showers of blood—that serpent's eye that charms to destroy." The Civil War was by far the bloodiest war in U.S. history. In the Battle of Gettysburg, for example, the more than 45,000 total casualties (people killed, wounded, captured, or missing) exceeded the number of casualties in all previous American wars put together.

Lincoln became a remarkable war leader. Some historians believe he was the chief architect of the Union's victorious military strategy. This strategy called for Union armies to advance against the enemy on all fronts at the same time. Lincoln also insisted that the objective of the Union armies should be the destruction of opposing forces, not the conquest of territory. Lincoln changed generals several times because he could not find one who would fight the war agressively. When he finally found such a general, Ulysses S. Grant, Lincoln stood firmly behind him.

Lincoln's second great task was to keep up Northern morale throughout the horrible war in which many relatives in the North and South fought against one another and hundreds of thousands died. He understood that the Union's resources vastly exceeded those of the Confederacy, and that the Union would eventually triumph if it remained dedicated to victory. For this reason, Lincoln used his great writing and speechmaking abilities to spur on his people.

If the Union had been destroyed, the United States could have become two, or possibly more, nations. These nations separately could not have become as prosperous and important as the United States is today. By preserving the Union, Lincoln influenced the course of world history. By ending slavery, he helped assure the moral strength of the United States. His own life story, too, has been important. He rose from humble origins to the nation's highest office. Millions of people regard Lincoln's career as proof that democracy offers all people the best hope of a full and free life.

Life in the United States during Lincoln's administration revolved around the war. But almost miraculously, the nation also laid out a blueprint for modern America during the war years. Economic development played an important role in Lincoln's vision of America's future, in which all people would have the right to rise in life. National banking legislation provided for paper money as we know it today—and for federal controls to assure sound banking and credit. U.S. tariffs on European manufactured goods helped limit foreign competition and encouraged the growth of American industry. The administration encouraged labor unions. The government's homestead laws gave free land to settlers. Immigration was encouraged, as was the settlement of the West. Land was also granted for colleges that later became great state universities and for the construction of the nation's first transcontinental railroad. In addition, the nation's first income tax was levied to provide funds for the war.

Soldiers and civilians alike sang "The Battle Hymn of the Republic" or "Dixie." Winslow Homer's painting *Prisoners from the Front* brought him his first fame. Patriotic literature of the time included John Greenleaf Whittier's poem "Barbara Frietchie" and Edward Everett Hale's story "The Man Without a Country." Lincoln and numerous other Americans chuckled at the humorous writings of Artemus Ward and admired the patriotic prints of Currier and Ives.

Early Life

Family background. Soon after Lincoln was nominated for the presidency, he wrote an autobiography. It began: "Abraham Lincoln was born February 12, 1809, then in Hardin, now in the more recently formed county of Larue, Kentucky. His father, Thomas, and grandfather, Abraham, were born in Rockingham County, Virginia, whither their ancestors had come from Berks County, Pennsylvania. His lineage has been traced no farther back than this."

Statue of Lincoln at the Lincoln Memorial, Washington, D.C.

Since Lincoln's time, his ancestry has been traced to a weaver named Samuel Lincoln, who emigrated from Hingham, England, to Hingham, Massachusetts, in 1637. Samuel Lincoln founded the Lincoln family in America. The families of several of his children played important parts in Massachusetts history.

Descendants of Mordecai Lincoln, a son of Samuel, moved to New Jersey, Pennsylvania, and Virginia. One was a great-great-grandson named Abraham. This Abraham Lincoln was the grandfather of the future president. He owned a farm in the Shenandoah Valley of Virginia during the Revolutionary War. In 1782, he and his wife and five small children traveled to the wilderness of Kentucky. An Indian killed him there in 1786.

One of his sons, Thomas Lincoln, became the father of the future president. In later years, the president said his father was "a wandering laboring boy, and grew up literally without education." Thomas Lincoln worked as a frontier farm hand during most of his youth. But he learned enough skill at woodworking to earn a living as a carpenter. In 1806, when he was twenty-eight years old, he married Nancy Hanks. Nancy came from what her son described as an "undistinguished" Virginia family of humble, ordinary people. Historians know only that she was the daughter of a Lucy Hanks.

Thomas and Nancy Lincoln lived in Elizabethtown, Kentucky, for the first eighteen months of their marriage. Their first child, Sarah, was born there in 1807. The next year, Thomas Lincoln bought a farm on the South Fork of the Nolin River, about 5 miles (8 kilometers) south of Elizabethtown. Abraham Lincoln was born on this farm.

Boyhood. The Lincolns lived for two years on the farm where Abraham was born. Then they moved to a farm on Knob Creek, 10 miles (16 kilometers) away. When Sarah and Abraham could be spared from their chores, they went to a log schoolhouse. There the children learned reading, writing, and arithmetic.

Many people believe that because Lincoln began his life in a log cabin, he was born in poverty. But many people then lived in log cabins. The Lincolns were as comfortable as most of their neighbors, and Abraham and Sarah were well fed and well clothed for the times. A third child, Thomas, died in infancy.

Thomas Lincoln had trouble over property rights throughout his years in Kentucky. In 1816, he decided to move to Indiana, where people could buy land directly from the government. Besides, Thomas Lincoln did not believe in slavery, and Indiana had no slavery.

The Lincolns loaded their possessions into a wagon. They traveled northward to the Ohio River and were ferried across. Then they traveled through the thick forests to Spencer County, in southwestern Indiana. There, Thomas Lincoln began the task of changing 160 acres (65 hectares) of forest land into a farm.

The Lincolns found life harder in Indiana than in Kentucky. They arrived early in winter and needed shelter at once. Thomas and his son built a three-sided structure made of logs, called a "half-faced camp." A fire on the fourth side burned night and day. Soon after finishing this shelter, the boy and his father began to build a log cabin. The family moved into it in February 1817.

Bears and other wild animals roamed the forests of this remote region. Trees had to be cut and fields cleared so that a crop could be planted. Although Abraham was only eight years old, he was large for his age and had enough strength to swing an ax. For as long as he lived in Indiana, he was seldom without his ax. He later called it "that most useful instrument."

Lincoln's childhood home, with a rail fence that he built.

Slowly, life became happier on the farm. But in October 1818, Nancy Lincoln died of what the pioneers called "milk sickness." This illness was probably caused by poison in the milk of cows that had eaten snakeroot. Thomas buried his wife among the trees on a hill near the cabin.

Life on the farm became dull and cheerless after the death of Nancy Lincoln. Sarah, now twelve, kept house as well as she could for more than a year. Then Thomas Lincoln returned to Kentucky for a visit. While there, on December 2, 1819, he married Sarah Bush Johnston, a widow. He had known her before her first marriage. The new Mrs. Lincoln brought along her three children, aged twelve, eight, and five. Her arrival at the cabin in Indiana ended the long months of loneliness.

Education. Abraham Lincoln grew from a boy of seven to a man of twenty-one on the wild Indiana frontier. His education can best be described in his own words:

"There were some schools, so called; but no qualification was ever required of a teacher, beyond 'readin, writin, and cipherin,' to the Rule of Three. If a straggler supposed to understand latin, happened to sojourn in the neighborhood, he was looked upon as a wizzard. There was absolutely nothing to excite

Lincoln as a young boy. The woman may be his mother, Nancy Hanks Lincoln.

ambition for education. Of course when I came of age I did not know much. Still somehow, I could read, write, and cipher to the Rule of Three; but that was all."

Lincoln's formal schooling totaled less than a year. Books and paper were scarce on the frontier. Like other boys and girls of his time, Lincoln made his own arithmetic textbook. Several of its pages still exist. Abraham often worked his arithmetic problems on boards, then shaved the boards clean with a drawknife, and used them again and again. He would walk a great distance for a book. The few he could borrow were good ones. They included *Robinson Crusoe*, *Pilgrim's Progress*, and Aesop's fables. Lincoln also borrowed a history of the United States and a schoolbook or two.

In 1823, when Abraham was fourteen, his parents joined the Pigeon Creek Baptist Church. There was bitter rivalry among Baptists, Methodists, Presbyterians, and other denominations. This may help explain why Lincoln never joined any church and why he never attended church regularly. Yet he became a man of deep religious feelings. He came to know the Bible thoroughly. Biblical references and quotations enriched his later writings and speeches. As president, he kept a Bible on his desk and often opened it for comfort and guidance.

Another book also impressed young Lincoln deeply—a biography of George Washington. He told about it years later in a speech before the New Jersey Senate:

"May I be pardoned if, on this occasion, I mention that away back in my childhood, the earliest days of my being able to read, I got hold of a small book, such a one as few of the younger members have ever seen, Weems's *Life of Washington*. I remember all the accounts there given of the battle fields and struggles for the liberties of the country...and you all know, for you have all been boys, how these early impressions last longer than any others. I recollect thinking then, boy even though I was, that there must have been something more than common that those men struggled for."

Youth on the frontier. Abraham reached his full height of 6 feet 4 inches (193 centimeters) long before he was twenty. He was thin and awkward, big-boned and strong. The young man developed great strength in his chest and legs, and especially in his arms. He had a homely face and dark skin. His hair was black and coarse and stood on end.

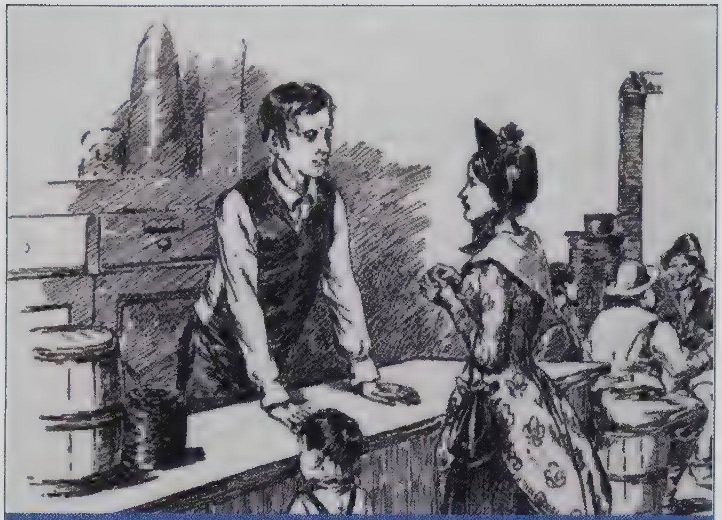
Even as a boy, Lincoln showed ability as a speaker. He often amused himself and others by imitating some preacher or politician who had spoken in the area. People liked to gather at the general store in the crossroads village of Gentryville. Lincoln's gift for telling stories made him a favorite with the people there. In spite of his youth, he was well known in his neighborhood.

A boy of Lincoln's size and strength had no trouble finding hard work. People always needed great piles of cut wood for cooking and for warmth. He could split logs for fence rails. He could plow fields, cut and husk corn, and thresh wheat with a flail. Lincoln worked for a neighbor when his father could spare him.

The Ohio River, 15 miles (24 kilometers) away, attracted Lincoln strongly. The first money he earned was for rowing passengers to a steamboat in midstream. In 1828, he helped take a flatboat loaded with farm produce to New Orleans. The trip gave him his first view of the world beyond his own community. That same year, his sister died in childbirth.

In 1830, Thomas Lincoln decided to move again. The years in Indiana had not been successful. The dreaded milk sickness was again striking down settlers. Relatives in Illinois sent word of deep, rich, black soil on the treeless prairies. The Lincolns and several other families started west. They reached their destination two weeks later, and settled 10 miles (16 kilometers) west of Decatur, on the north bank of the Sangamon River.

Lincoln was now twenty-one and free to strike out for himself. But he remained with his father



A depiction of Lincoln clerking at Offutt's store

one more year. He helped plant the first crop, and he split rails for a cabin and fences. He worked for neighboring settlers during the winter. In the spring of 1831, a trader named Denton Offutt hired Lincoln and two other young men to take a flatboat to New Orleans. This trip gave Offutt a good impression of his lanky boat hand. He hired Lincoln as a clerk in his new store in the village of New Salem, Illinois, 20 miles (32 kilometers) northwest of Springfield. While Lincoln was away, his parents moved to Coles County, where they lived for the rest of their lives.

New Salem Years

Life on his own began for Lincoln when he settled in New Salem. He lived there almost six years, from July 1831 until the spring of 1837. The village consisted of log cabins clustered around a mill, a barrel-maker's shop, a wool-carding machine, and a few general stores.

The villagers helped Lincoln in many ways. The older women mended his clothes and often gave him meals. Jack Kelso, the village philosopher, introduced him to the writings of the English dramatist William Shakespeare and the Scottish poet Robert Burns. These works, and the Bible, became his favorite reading.

Lincoln arrived in New Salem, as he said, "a piece of floating driftwood." He earned little and slept in a room at the rear of Offutt's store. Within a few months the business failed. Lincoln would have been out of a job if the Black Hawk War had not begun in 1832.

The Black Hawk War. By late 1831, the federal government had moved most of the Sauk and Fox Indians from Illinois to Iowa. In the spring of 1832, Chief Black Hawk led a band of several hundred Indians back across the Mississippi River to try to regain their lands near Rock Island. The governor called out the militia, and Lincoln volunteered for service.

Lincoln's company consisted of men from the New Salem area. The men promptly elected him captain. This was only nine months after he had settled in the village. Even after he had been nominated for president, Lincoln said this honor "gave me more pleasure than any I have had since." It provided the first significant indication of his gift for leadership. Lincoln's comrades liked his friendliness, his honesty, and his skill at storytelling. They also admired his great strength and his sportsmanship in wrestling matches and other contests.

Lincoln's term of service ended after thirty days, but he reenlisted, this time as a private. A month later, he enlisted again. He served a total of ninety days, but saw no fighting. He later described his militia experiences as "bloody struggles with the musquetoes" and "charges upon the wild onions."

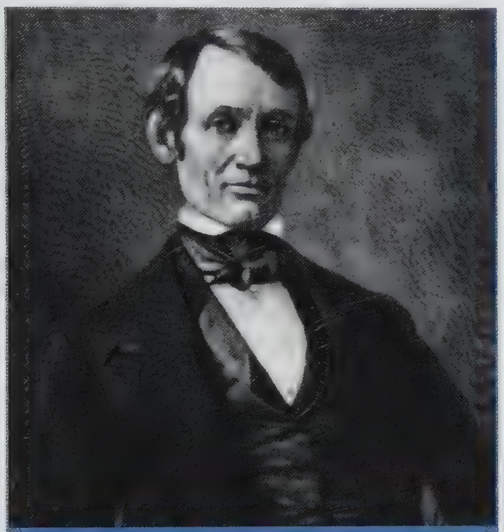
Search for a career. Before his military service, many of Lincoln's friends had encouraged him to become a candidate for the state legislature. Spurred by their faith, he announced his candidacy in March 1832. The Black Hawk War prevented him from making much of a campaign. He arrived home in July, only two weeks before the election. Lincoln was defeated in the election, but the people in his own precinct gave him 277 of their 300 votes.

Lincoln faced the problem of making a living. He thought of studying law, but decided he could not succeed without a better education. Just then, he had a chance to buy a New Salem store on credit, in partnership with William F. Berry. Lincoln later recalled that the partnership "did nothing but get deeper and deeper in debt." The store failed after a few months.

In May 1833, Lincoln was appointed postmaster of New Salem. Soon afterward, the county surveyor offered to make him a deputy. Lincoln knew nothing about surveying, but he prepared for the work by hard study. Odd jobs and fees from his two public offices earned him a living.

Berry died in 1835, leaving Lincoln liable for the debts of the partnership, about $1,100. It took Lincoln several years to pay these debts, but he finally did it. His integrity helped him earn the nickname "Honest Abe."

In New Salem, Lincoln grew close to a girl named Ann Rutledge. When she died in the summer of 1835, he grieved deeply. Scholars have debated their relationship for years. Although Ann was engaged to another man, it seems likely that she and Lincoln did fall in love. But, within 18 months, Lincoln recovered from his loss and proposed marriage to a Kentucky girl, Mary Owens. He met her while she was visiting her sister in New Salem. Their affection for each other was not especially deep, however, and Owens rejected him.



Lincoln as a lawyer, before becoming president

Success in politics. In 1834, Lincoln again ran for the legislature. He had become better known by this time and won election as a Whig. He served four successive two-year terms in the lower house of the Illinois General Assembly. During his first term, he met a young Democratic legislator, Stephen A. Douglas.

Lincoln quickly came to the front in the legislature. He was witty and ready in debate. His skill in party management enabled him to become the Whig floor leader at the beginning of his second term. He took leading parts in the establishment of the Bank of Illinois and in the adoption of a plan for a system of railroads and canals. This plan broke down after the Panic of 1837. Lincoln also led a successful campaign for moving the state capital from Vandalia to Springfield.

While in the legislature, Lincoln made his first public statement on slavery. In 1837, the legislature passed, by an overwhelming majority, resolutions condemning abolition societies. These societies urged freedom for slaves. Lincoln and another legislator, Dan Stone, filed a protest. They admitted that Congress had no power to interfere with slavery in the states where it existed. They believed "the promulgation of abolition doctrines tend rather to increase than abate its evils." Their protest arose from the legislature's failure to call slavery an evil practice. Lincoln and Stone declared that "the institution of slavery is founded on both injustice and bad policy." Slavery had become a much greater issue twenty-three years later, when Lincoln was nominated for president. He said then that his protest in the Illinois legislature still expressed his position on slavery.

Lincoln the Lawyer

"I go for all sharing the privileges of the government who assist in bearing its burdens."
—Letter to the editor of the Sangamon Journal, *New Salem, Illinois, June 13, 1836*

Study. In 1834, during Lincoln's second campaign for the legislature, John T. Stuart had urged him to study law. Stuart was an attorney in Springfield and a leading member of the legislature. Lincoln overcame his doubts about his education. He borrowed law books from Stuart and studied them. He sometimes walked 20 miles (32 kilometers) from New Salem to Springfield for books. Henry E. Dummer, Stuart's law partner, recalled:

"Sometimes he walked, but generally rode. He was the most uncouth looking young man I ever saw. He seemed to have but little to say; seemed to feel timid, with a tinge of sadness visible in the countenance, but when he did talk all this disappeared for the time and he demonstrated that he was both strong and acute. He surprised us more and more at every visit."

On September 9, 1836, Lincoln received his license to practice law, although his name was not entered on the roll of attorneys until March 1, 1837. The population of New Salem had dropped by that time, and Lincoln decided to move to the new state capital. Carrying all he owned in his saddlebags, he rode into Springfield on April 15, 1837. There he became the junior partner in the law firm of Stuart and Lincoln.

In Lincoln's time, there were few law schools. Most lawyers simply "read law" in the office of an attorney. Years later, in giving advice to a law student, Lincoln explained his method of study:

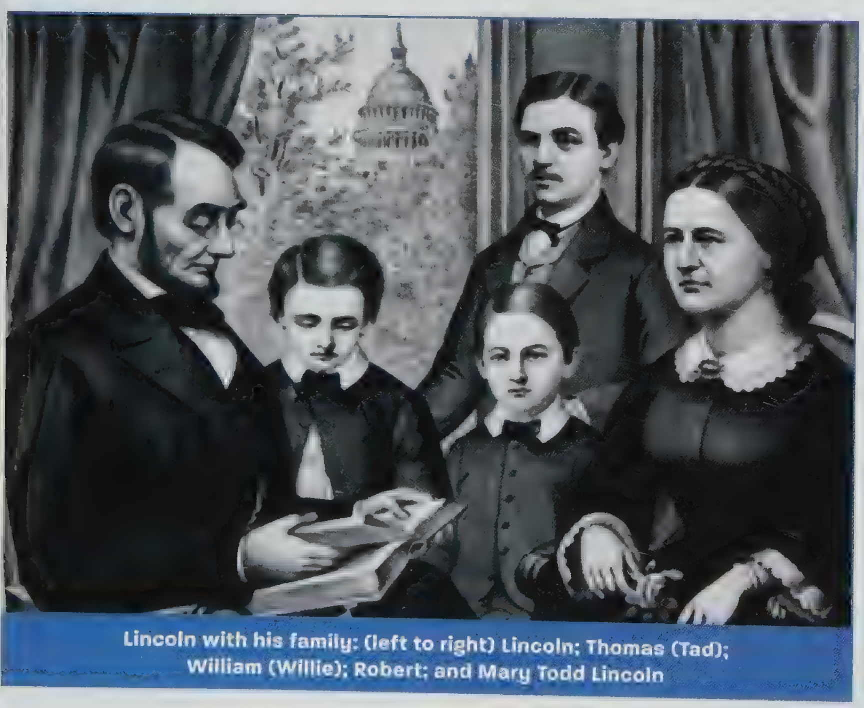
"If you are resolutely determined to make a lawyer of yourself, the thing is more than half done already.

> "If destruction be our lot we must ourselves be its author and finisher. As a nation of free men we must live through all time, or die by suicide. . . Towering genius disdains a beaten path. It seeks regions hitherto unexplored."
>
> *—Speech in Springfield, Illinois, January 27, 1838*

It is but a small matter whether you read with anybody or not. I did not read with anyone. Get the books, and read and study them till you understand them in their principal features; and that is the main thing. It is of no consequence to be in a large town while you are reading. I read at New Salem, which never had three hundred people living in it. The books, and your capacity for understanding them, are just the same in all places. . .Always bear in mind that your own resolution to succeed is more important than any other one thing."

Early practice. Lincoln's partnership with Stuart lasted until the spring of 1841. Then he became the junior partner of Stephen T. Logan, one of the greatest lawyers who ever practiced in Illinois. This partnership ended in the fall of 1844.

Lincoln then asked William H. Herndon to become his partner. Herndon, nine years younger than Lincoln, had just received his license to practice law. Lincoln called him "Billy," but Herndon always called his partner "Mr. Lincoln." The two men never formally dissolved their law firm. More than sixteen years later, Lincoln visited his old office on his last day in Springfield before leaving for Washington to be inaugurated as president. He noticed the firm's signboard at the foot of the steps and said: "Let it hang there undisturbed. Give our clients to understand that the election of a president makes no change in the firm of Lincoln and Herndon."



Lincoln with his family: (left to right) Lincoln; Thomas (Tad); William (Willie); Robert; and Mary Todd Lincoln

The practice of law in Illinois was not specialized in Lincoln's time. He tried his first case in the circuit court of Sangamon County. He practiced in the Illinois federal courts within two years after his admission to the bar. A year later, he tried the first of many cases in the state supreme court. But all the while, he also handled cases before justices of the peace. He also gave advice and opinions on many matters for small fees.

Lincoln's family. Soon after Lincoln moved to Springfield, he met Mary Todd (December 13, 1818–July 16, 1882), a woman from Kentucky who lived there with a married sister. They had a stormy courtship and at one time broke their engagement. They were married on November 4, 1842, when Lincoln was thirty-three and his bride was twenty-three.

Mary Todd Lincoln was high-strung and socially ambitious. Lincoln tended to be moody and absent-minded. Their contrasting personalities sometimes caused friction. But they had a loving marriage that lasted until Abraham's death.

Lincoln and his bride first lived in a Springfield boardinghouse, where they paid $4 a week. Eighteen months after his marriage, Lincoln bought the plain but comfortable frame house in which the family lived until he became president. By the time he bought the house, his first son, Robert Todd, was nine months old. The Lincolns' second son, Edward Baker, was born in 1846, but died four years later. William Wallace, born in 1850, died in the White House at the age of eleven. Their fourth son, Thomas, usually called Tad, became ill and died in 1871 at age eighteen.

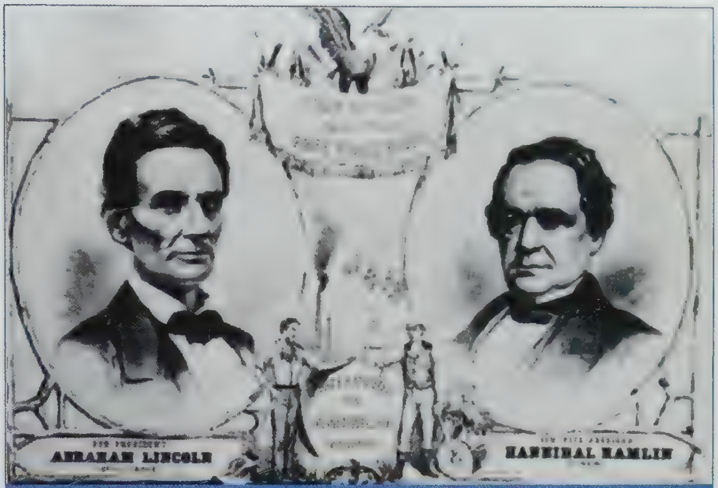
The family lived comfortably. Lincoln became a highly successful lawyer and politician, and was not the poverty-stricken failure sometimes portrayed in legend. He often cared for his own horse and milked the family cow, but so did most of his neighbors. The Lincoln family usually employed a servant to help with the housework.

Riding the circuit. The state of Illinois was, and still is, divided into circuits for judicial purposes. Each circuit consisted of several counties where court was held in turn. The judge and many lawyers traveled from county to county. They tried such cases as came their way during each term.

Lincoln "traveled the circuit" for six months each year. He loved this kind of life. The small inns where the lawyers stayed had few comforts, but they offered many opportunities for meeting people.

Lively talk and storytelling appealed to Lincoln. He also liked the long rides across the prairies. Lincoln's circuit at its largest included fifteen counties and covered about 8,000 square miles (21,000 square kilometers).

Lincoln developed traits as a lawyer that made him well known throughout Illinois. He could argue a case strongly. He sometimes persuaded clients to settle their differences out of court, which meant a smaller fee, or no fee at all, for him. In court, Lincoln could present a case so that twelve jurors, often poorly educated, could not fail to understand it. He could also argue a complicated case before a well-informed judge. He prepared his cases thoroughly and was unfailingly honest.



An 1860 campaign poster

National Politics

Search for advancement. After four terms in the Illinois legislature, Lincoln wanted an office with greater prestige. He had served the Whig Party well, and election to Congress became his goal.

In 1840, Lincoln made a speaking tour of the state for William Henry Harrison, the Whig candidate for president. He campaigned on the issue of a sound central banking system, speaking out in favor of rechartering the Bank of the United States. Lincoln believed his service had earned him the nomination for Congress from his district. In 1843, and again in 1844, the nomination went to other candidates.

Disappointed, but not bitter, Lincoln worked for the election of Henry Clay, the Whig presidential candidate in 1844. During this campaign, Lincoln focused on the need for a tariff, which would raise the cost of imports and aid American industrial growth. Two years later, Lincoln received his reward and won the Whig nomination for the U.S. House of Representatives. His opponent in the election was Peter Cartwright, a well-known Methodist circuit rider. The Whigs firmly controlled Lincoln's district, and he got 6,340 of the 11,418 votes cast.

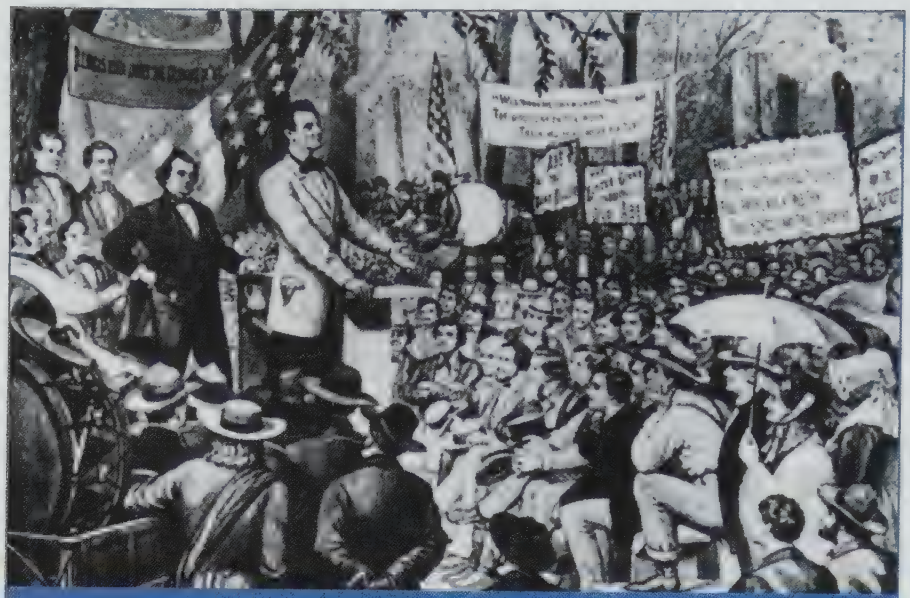
Congressman. Lincoln took his seat in Congress on December 6, 1847. By that time, the United States had won the Mexican War, although a peace treaty between Mexico and the United States had not yet been signed. Lincoln joined his fellow Whigs in blaming President James K. Polk for the war. He said of Polk, "The blood of this war, like the blood of Abel, is crying to heaven against him." He warned that "military glory" was an "attractive rainbow" that could result in "showers of blood." But he would not abandon U.S. troops on the battlefield and voted to supply them with equipment.

Lincoln failed to make the reputation he had hoped for in Congress. He gave notice that he intended to introduce a bill to free the slaves in the District of Columbia, but he never did. He emphasized his position on slavery by supporting the Wilmot Proviso, which would have banned slavery in any territory acquired from Mexico.

Throughout his term, Lincoln supported the Whig policy of having the federal government pay for internal improvements. He made several speeches in support of this policy, and once reproved President Polk for vetoing funds to make rivers and harbors more navigable and thus increase commerce. Lincoln worked for the nomination and election of Zachary Taylor, the Whig candidate for president in 1848.

Return to law. Lincoln's term ended on March 4, 1849. He tried unsuccessfully to get an appointment as Commissioner of the General Land Office. The administration offered to appoint him secretary, then governor, of Oregon Territory. Lincoln refused both offers.

Lincoln returned to Springfield. He practiced law more earnestly than ever before. He continued to travel the circuit, but appeared more often in the higher courts. He also handled more important cases. Corporations and big businesses were becoming increasingly important in Illinois and neighboring states. Lincoln represented them frequently in lawsuits, and he soon prospered. The



A depiction of a Lincoln-Douglas debate

"Moral principle is a looser bond than pecuniary interest."
—Speech, October 1856

"With public sentiment, nothing can fail; without it, nothing can succeed."
—Speech in Ottawa, Illinois, July 31, 1858

largest fee he ever received, $5,000, was for his successful defense of the Illinois Central Railroad in an important tax case. After 1849, Lincoln's reputation grew steadily. In the 1850s, he was known as one of the leading lawyers of Illinois.

Reentry into politics. A sudden change in national policy toward slavery brought Lincoln back into the center of political activity in Illinois. The Missouri Compromise of 1820 had prohibited slavery in new territories north of an east-west line that was an extension of Missouri's southern boundary. Early in 1854, Senator Stephen A. Douglas of Illinois introduced a bill to organize the territories of Kansas and Nebraska. As approved by Congress, this Kansas-Nebraska Act repealed the Missouri Compromise. It provided that the settlers of new territories should decide for themselves whether they wanted slavery.

Lincoln and many others had believed that slavery had been permanently limited and would in time die. Lincoln believed that the new policy gave new life to slavery, and it outraged him.

Lincoln revered the Founding Fathers. He believed they had written a promise of freedom and equality into the Declaration of Independence. He once said: "I have never had a feeling politically which did not spring from the sentiments embodied in the Declaration of Independence." During his early years in politics, Lincoln had looked up to Henry Clay as an ideal politician. But he looked to Thomas Jefferson for his democratic political principles and to Alexander Hamilton for his economic principles.

Lincoln always opposed slavery, but he never became an abolitionist. He believed that the bonds holding the nation together would be strained if Americans made a rapid break with the past. He also knew that abolitionists could never aim for national office. Lincoln granted that slavery should have the protection that the Constitution gave it. But he wanted the people to realize that slavery was evil and that it should be put on "the road to ultimate extinction."

Douglas refused to admit that slavery was wrong. He said he did not care whether the people of new territories voted for or against slavery. Lincoln believed that the nation stood for freedom and equality. He felt it must not be indifferent to the unjust treatment of any person. To ignore moral values, he said, "deprives our republican example of its just influence in the world." It enabled the enemies of free institutions "to taunt us as hypocrites." Lincoln resolved to do what he could to reverse the Kansas-Nebraska Act.

A turning point in Lincoln's life came with the rise of the slavery controversy. Fighting against what he termed the "cancer" of bondage, he rose to the presidency and directed a bloody civil war that would put an end to what he saw as an evil institution. But first, many political battles had to be fought.

Lincoln entered the congressional election campaign of 1854 to help a candidate who opposed the Kansas-Nebraska Act. But when Senator Douglas returned to Illinois to justify the new law, Lincoln opposed him wherever he could. At Springfield, Peoria, and Chicago, Lincoln delivered such powerful speeches that he became known as the leader of the Illinois forces opposing the Kansas-Nebraska Act. He was again elected to the Illinois legislature, but resigned in order to run for the United States Senate.

At that time, the legislature elected senators. On the first ballot, Lincoln received forty-five votes, which was five short of a majority. On each succeeding ballot, his vote dwindled. Finally, to keep a Douglas supporter from being elected, Lincoln persuaded his followers to vote for Lyman Trumbull, who had started with only five votes. Trumbull was elected.

The Whig Party began falling apart during the 1850s, largely because party members in various parts of the country could not agree on a solution to the slavery problem. In 1856, Lincoln joined the antislavery Republican Party, then only two years old. During the presidential election campaign that year, he made more than a hundred speeches in behalf of John C. Fremont, the Republican candidate. Fremont lost the election to Democrat James Buchanan. But Lincoln had strengthened his own position in the party through his unselfish work.

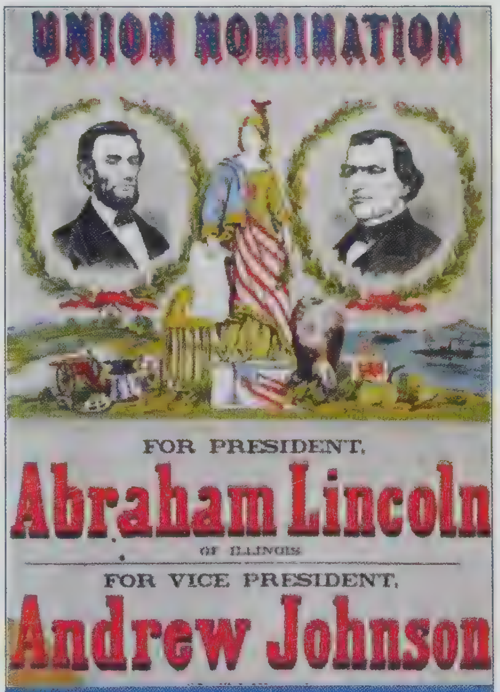
The debates with Douglas. In 1858, Lincoln was nominated to run against Douglas for the U.S. Senate. He accepted the honor with a speech that aroused much controversy. Many people thought his remarks stirred up conflict between the North and South. Lincoln said:

"A house divided against itself cannot stand. I believe this government cannot endure, permanently half slave and half free. I do not expect the Union to be dissolved—I do not expect the house to fall—but I do expect it will cease to be divided. It will become all one thing, or all the other. Either the opponents of slavery will arrest the further spread of it, and place it where the public mind shall rest in the belief that it is in the course of ultimate extinction; or its advocates will push it forward till it shall become alike lawful in all the States—old as well as new, North as well as South."

After a few speeches, Lincoln challenged Douglas to a series of debates. Douglas accepted and named seven towns for the meetings. The first debate was held at Ottawa, Illinois, on August 21, 1858. The last was at Alton, Illinois, on October 15. Each candidate spoke for an hour and a half. Large crowds attended each debate except the one at Jonesboro, in the southernmost part of the state. Newspapers from around the country reported the debates, and the two men drew national attention.

The debates centered on the extension of slavery into free territory. Douglas defended the policy of the Kansas-Nebraska Act. He called this policy *popular sovereignty*. His opponents ridiculed it as *squatter sovereignty* and warned that it would make slavery national and permanent. Lincoln argued that the Supreme Court of the United States, in the *Dred Scott* decision, had opened the way for slavery to enter all the territories. In the debate held at Freeport, Illinois, Douglas denied this argument. He contended that the people of any territory could keep slavery out of that territory simply by refusing to pass local laws protecting it. This position became known as the *Freeport Doctrine*. Lincoln insisted that there was a fundamental difference between Douglas and himself. Douglas ignored the moral question of slavery, but Lincoln regarded slavery "as a moral, social, and political evil." Douglas, in turn, tried to depict Lincoln as a dangerous radical who favored racial equality.

In addition to the debates, both men spoke almost daily to rallies of their own. Each traveled throughout the state. Before the exhausting campaign ended, Douglas' deep bass voice had become so husky that it was hard to understand him. Lincoln's high, penetrating voice still reached the limits of a large audience.



Campaign poster for Lincoln and Andrew Johnson

In the election, Lincoln candidates for the legislature received more votes than their opponents. But the state was divided into districts in such a way that Douglas candidates won a majority of the seats. As a result, Douglas was reelected by a vote of fifty-four to forty-six.

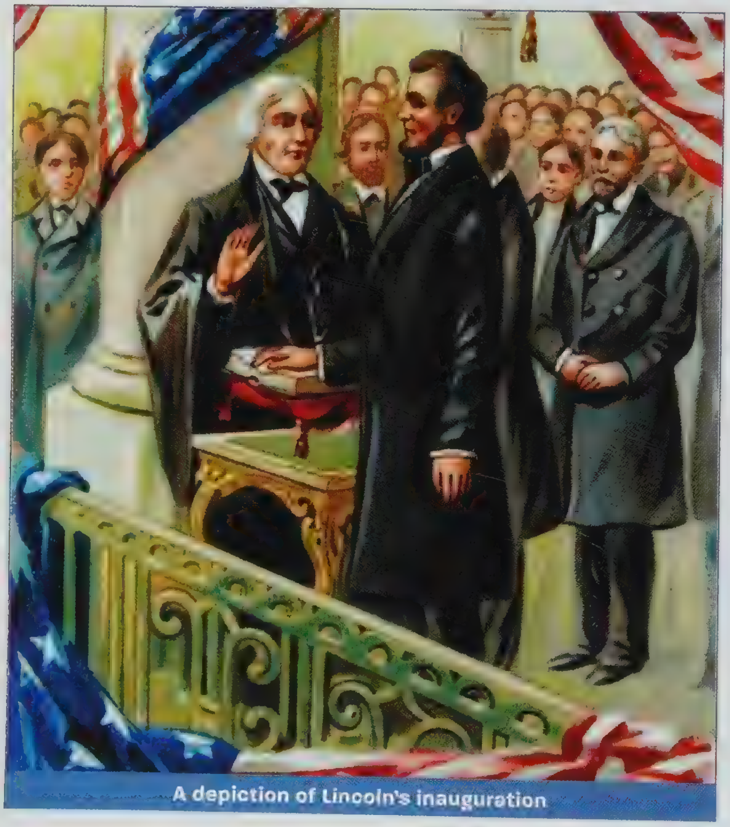
"What constitutes the bulwark of our own liberty and independence? It is not our frowning battlements, our bristling sea coasts, our army and our navy. . . Our reliance is in the love of liberty which God has planted in us. Our defense is in the spirit which prized liberty as the heritage of all men, in all lands everywhere. Destroy this spirit and you have planted the seeds of despotism at your own doors. Familiarize yourselves with the chains of bondage and you prepare your own limbs to wear them. Accustomed to trample on the rights of others, you have lost the genius of your own independence and become the fit subjects of the first cunning tyrant who rises among you."

—Speech at Edwardsville, Illinois, September 11, 1858

"...he who would be no slave must consent to have no slave. Those who deny freedom to others deserve it not for themselves, and under a just God, cannot long retain it."
—Letter to H. L. Pierce and others, April 6, 1859

The debates made Lincoln a national figure. Early in 1860, he delivered an address at Cooper Union in New York City. The speech ended with the famous plea: "Let us have faith that right makes might, and in that faith let us to the end dare to do our duty as we understand it." This address and others delivered later in New England made a strong impression on many influential eastern Republicans.

Election of 1860. The Republican National Convention met in Chicago on May 16, 1860. Lincoln was by no means unknown to the delegates. The week before, at the Illinois state Republican convention, his supporters had nicknamed him "the Railsplitter." This nickname, recalling the days when Lincoln had split rails for fences, helped make him even better known to the delegates. But other party leaders had larger followings. Senator William H. Seward of New York had the strongest support, but he also had many enemies. Senator Salmon P. Chase of Ohio lacked the united support of even his own state. Lincoln had never held a prominent national office and had no bitter enemies. He held moderate views on the slavery question. His humble background could be counted on to arouse great enthusiasm among the voters.



A depiction of Lincoln's inauguration

On the first ballot, Seward received 173½ votes, Lincoln 102, and Chase 49. Lincoln gained the support of Pennsylvania and Indiana on the second ballot and received 181 votes to 184½ for Seward. During the third ballot, Lincoln continued to gain strength. Before the result was announced, Ohio switched four votes from Chase to Lincoln. This gave Lincoln more than the 233 votes needed to win the nomination. The delegates nominated Senator Hannibal Hamlin of Maine for vice president.

Like other presidential candidates of his period, Lincoln felt it was undignified to campaign actively. He stayed quietly in Springfield during the election campaign. His followers more than made up for his inactivity. The opposing Democratic Party broke into two factions, which helped Lincoln immensely. Senator Douglas, the nation's leading Democrat, had angered the proslavery wing of his party. Northern Democrats nominated him for president. The Southern faction of the Democratic Party chose Vice President John C. Breckinridge. A fourth party, calling itself the Constitutional Union Party, nominated former Senator John Bell of Tennessee.

Lincoln won election easily, receiving 180 electoral votes to 72 for Breckinridge, 39 for Bell, and 12 for Douglas. But more Americans voted against Lincoln than for him. The people gave him 1,865,908 votes, compared to a combined total of 2,819,122 for his opponents. All Lincoln's electoral votes, and nearly all his popular votes, came from the North. Never before had a president been elected by one region alone.

Lincoln's Administration (1861–1865)

The South secedes. During the months before Lincoln's inauguration, many Southern leaders threatened to withdraw their states from the Union if Lincoln should win the election. On December 20, 1860, South Carolina passed an Ordinance of Secession that declared the Union dissolved as far as that state was concerned. By the time Lincoln became president, six other Southern states had withdrawn from the Union. Four more states followed later. The seceded states organized themselves into the Confederate States of America.

First inauguration. Lincoln said farewell to his Springfield neighbors on February 11, 1861. He parted with these words: "Here I have lived a quarter of a century, and have passed from a young to an old man. Here my children have been born, and one is buried. I now leave, not knowing when, or whether ever, I may return, with a task before me greater than that which rested upon Washington. Without the assistance of that Divine Being who ever attended him, I cannot succeed. With that assistance I cannot fail. Trusting in Him, who can go with me, and remain with you and be everywhere for good, let us confidently hope that all will yet be well. To His care commending you, as I hope in your prayers you will commend me, I bid you an affectionate farewell."

The long train trip to Washington, D.C., had been carefully planned to include stops at most large Eastern cities. This allowed many thousands of people to see the man who would be their next president. Lincoln had grown a beard that gave him a more dignified appearance. While in Philadelphia, Lincoln heard a report of an assassination plot in Baltimore, the only Southern stop along his route. His advisers persuaded him to cut short his trip. Lincoln continued in secret to Washington, arriving early on the morning of February 23.

On March 4, 1861, Lincoln took the oath of office and became the sixteenth president of the United States. In his inaugural address, Lincoln denied that he had any intention of interfering with slavery in states where the Constitution protected it. He urged the preservation of the Union. Lincoln warned that he would use the full power of the nation to "hold, occupy, and possess" the "property and places" belonging to the federal government. By "property and places," he meant forts, arsenals, and custom houses. Lincoln's closing passage had great beauty and literary power. He appealed to "the mystic chords of memory, stretching from every battlefield and patriot grave to every living heart and hearthstone all over this broad land."

Lincoln announced his Cabinet the day after his inauguration. Four members—William H. Seward, Salmon P. Chase, Simon Cameron, and Edward Bates—had been his rivals for the presidential nomination. The Cabinet members represented many shades of opinion within the Republican Party and included a Democrat for balance. On the whole, they were an exceptionally able group.

Fort Sumter and war. As the Southern states seceded, they seized most of the federal forts within their boundaries. Lincoln had to decide whether the remaining forts should be strengthened and whether to try to retake the forts already in Southern hands.

An interior view of Fort Sumter after its evacuation by Major Anderson

Fort Sumter, in Charleston Harbor, became a symbol of an indivisible Union. Major Robert Anderson commanded the Union garrison there. If Lincoln withdrew the troops, a storm of protest would rise in the North. If he reinforced Fort Sumter, the South would consider it an act of war.

As a compromise, Lincoln decided to send only provisions to Anderson, whose supplies were running low. He informed South Carolina of his intention. Leaders of the state regarded the relief expedition as a hostile act and demanded Anderson's surrender. Anderson refused, and on April 12, General Pierre G. T. Beauregard ordered Confederate artillery to fire on the fort. Anderson surrendered the next day. The attack on Fort Sumter marked the start of the Civil War.

Lincoln met the crisis with energetic action. He called out the militia to suppress the "insurrection." He proclaimed a blockade of Southern ports and expanded the army beyond the limit set by law. However, Southern sympathizers living in the North objected to and obstructed the war effort. As a result, Lincoln suspended the privilege of *habeas corpus* in areas where these Southern sympathizers were active. The action meant that antiwar Northerners could be arrested and held without formal charges. In addition, Lincoln ordered the spending of federal funds without waiting for congressional appropriations.

Lincoln believed all these actions to be within the war powers granted the president by the Constitution. He justified his acts when Congress met for the first time in his administration in July 1861. The message Lincoln delivered to Congress ranks as one of his greatest state papers. Chief Justice Roger B. Taney had attacked Lincoln bitterly for suspending habeas corpus. In his message, Lincoln posed a question that even today is difficult to answer: "Are all the laws but one to go unexecuted, and the government itself go to pieces lest

"Work, work, work, is the main thing."
—Letter to John M. Brockman, September 25, 1860

"Ballots are the rightful and peaceful successors to bullets."
—Message to Congress, July 1861

"Fair play is a jewel."
—Letter to Simon Cameron, August 10, 1861

Ancient Egypt

Ancient Egypt was an advanced culture that originated in the Nile River Valley of northeastern Africa about 5,000 years ago. This exceptional civilization lasted over 2,000 years. Ancient Egyptians created a 365-day calendar, basic forms of math, a national government, hieroglyphics, and papyrus. They are best-known for building pyramids.

HOT topics

- **Afterlife/Mummification Process/Rituals**
- **Egyptian Gods and Goddesses**
- **Egyptian Pyramids**
- **Hieroglyphics**
- **Rosetta Stone**

HOT topics

Afterlife/Mummification Process/Rituals.

Ancient Egyptians mummified their dead in the belief that the body had to be preserved for the afterlife. This process required that the bodies be embalmed and dried. Then, after certain organs were removed, the body was treated with resins used to seal out moisture. After that it was wrapped in linen bandages. The body would then be placed inside a coffin or a number of coffins, each inside of the other. Finally, the mummies were hidden inside secret chambers in large tombs, along with gold treasures, precious objects, and their internal organs, which were placed in canopic jars.

DEFINITIONS

dynasty: a series of rulers who belong to the same family; a line of kings or princes

hieroglyphic: a picture, character, or symbol standing for a word, idea, or sound; hieroglyph. The ancient Egyptians used hieroglyphics instead of an alphabet like ours.

papyrus: a tall water plant from which the ancient Egyptians, Greeks, and Romans made a kind of paper to write on. It belongs to the sedge family. Papyrus has stems three to ten feet high, and though it is comparatively rare today, it is still found in Egypt, Ethiopia, Syria, and Sicily. Papyrus is the bulrush of the Bible.

Pharaoh or pharaoh: the title given to the kings of ancient Egypt.

the Pyramids: any one of the huge, massive stone pyramids, serving as royal tombs, built by the ancient Egyptians.

Sphinx: a huge stone statue with a man's head and a lion's body, near Cairo, Egypt.

HOT topics

Egyptian Gods and Goddesses. Many gods and goddesses were worshiped by the ancient Egyptians. For example, the sun god Re and the goddess Rennutet were worshiped to ensure good harvests. The goddess Isis was worshiped as a devoted mother and wife, Osiris governed vegetation and the dead, and Horus was the lord of heaven. In each city and town, the people worshiped their own special deity as well as the major ones.

Egyptian Pyramids. Ancient Egyptians built pyramids as tombs for their rulers, who were mummified and then hidden inside secret chambers within the pyramids. The pyramids were, and still are, architectural marvels. Temples were usually attached to the pyramids and used for funeral ceremonies. There are ten pyramids at Giza including the Great Pyramid, the largest, which was built for King Khufu. The Great Sphinx, a huge statue of a sphinx, also stands amongst these pyramids.

Hieroglyphics. Hieroglyphics are picture symbols used to represent ideas and sounds. Ancient Egyptians, as well as some other ancient cultures, used hieroglyphics as a form of writing. After the Egyptians replaced hieroglyphics with a simpler alphabet, eventually the ability to read hieroglyphics was lost, that is, until the discovery and deciphering of the Rosetta stone.

Rosetta Stone. In 1799, a stone tablet was discovered near Rosetta, Egypt. This Rosetta stone was inscribed in three scripts—Egyptian hieroglyphic, Egyptian demotic, and Greek. Up until this time, the ability to read hieroglyphics had been lost. But, in 1822, by using the Egyptian demotic and the Greek inscriptions, Jean François Champollion was able to decipher the entire text, which turned out to be a decree honoring King Ptolemy V. The ability to decipher hieroglyphics has given us the ability to learn ancient Egyptian history.

Sarcophagus

The Great Sphinx

TRUE or FALSE?
Ancient Egyptians did not use money.

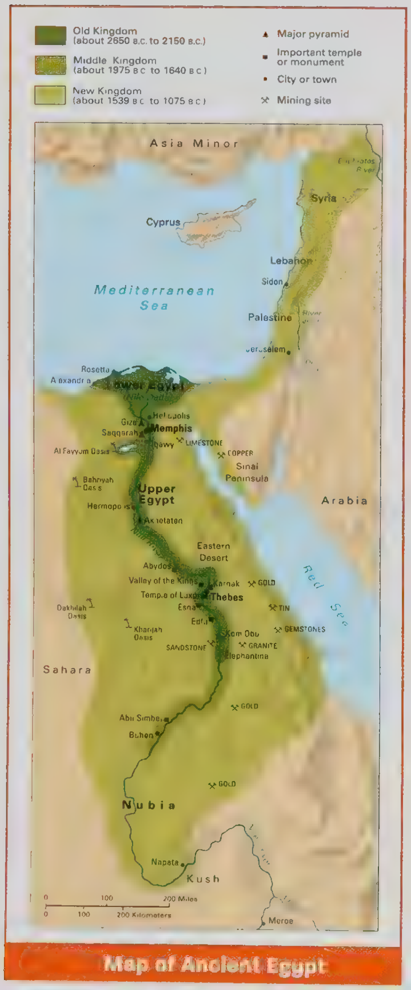
THE BASICS

Ancient Egypt was the birthplace of one of the world's first civilizations. This advanced culture arose about 5,000 years ago in the Nile River Valley in northeastern Africa. It thrived for over 2,000 years and so became one of the longest lasting civilizations in history.

Hieroglyphics

The mighty Nile River was the lifeblood of ancient Egypt. Every year, it overflowed and deposited a strip of rich, black soil along each bank. The fertile soil enabled farmers to raise a huge supply of food. The ancient Egyptians called their country *Kemet*, meaning *Black Land*, after the dark soil. The Nile also provided water for irrigation and was Egypt's main transportation route. For all these reasons, the ancient Greek historian Herodotus called Egypt "the gift of the Nile."

The ancient Egyptians made outstanding contributions to the development of civilization. They created the world's first national government, basic forms of arithmetic, and a 365-day calendar. They invented a form of picture writing called *hieroglyphics*. They also invented *papyrus*, a paper-like writing material made from the stems of papyrus plants. The Egyptians developed one of the first religions to emphasize life after death. They built great cities in which many skilled architects, doctors, engineers, painters, and sculptors worked.


Old Kingdom (about 2650 B.C. to 2150 B.C.)
Middle Kingdom (about 1975 B.C. to 1640 B.C.)
New Kingdom (about 1539 B.C. to 1075 B.C.)
▲ Major pyramid
■ Important temple or monument
• City or town
⚒ Mining site
Asia Minor
Euphrates River
Syria
Cyprus
Lebanon
Sidon
Mediterranean Sea
Palestine
Jerusalem
Rosetta
Alexandria
Lower Egypt
Heliopolis
Giza
Memphis
Saqqarah
LIMESTONE
Al Fayyum Oasis
COPPER
Sinai Peninsula
Bahriyah Oasis
Upper Egypt
Hermopolis
Arabia
Akhetaten
Eastern Desert
Abydos
Valley of the Kings
Karnak
GOLD
Red Sea
Temple of Luxor
Thebes
Dakhilah Oasis
Esna
TIN
Edfu
Kharijah Oasis
Kom Ombo
GEMSTONES
SANDSTONE
GRANITE
Elephantine
Sahara
GOLD
Abu Simbel
Buhen
GOLD
Nubia
Napata
Kush
0 100 200 Miles
0 100 200 Kilometers
Meroe
Map of Ancient Egypt

The best-known achievements of the ancient Egyptians, however, are the pyramids they built as tombs for their rulers. The most famous pyramids stand at Giza. These gigantic stone structures—marvels of architectural and engineering skills—have been preserved by the dry climate for about 4,500 years. They serve as spectacular reminders of the glory of ancient Egypt.

The Egyptian World

The land. Ancient Egypt was a long, narrow country through which the Nile River flowed. Deserts bordered the country on the east, south, and west. The Mediterranean Sea lay to the north. The Nile River flowed north out of central Africa through the Egyptian desert to the Mediterranean. The Egyptians called the desert *Deshret*, meaning *Red Land*. The Nile's course through Egypt was about 600 miles (1,000 kilometers). The river split into several channels north of what is now Cairo, forming the Nile Delta. Rolling desert land lay west of the Nile Valley, and mountains rose to the east.

The Nile River flooded its banks each year. The flooding started in June, with the rainy season in central Africa. The rains raised the level of the river as the Nile flowed northward. The floodwaters usually went down in September, leaving a strip of fertile land about 6 miles (10 kilometers) wide on each riverbank. Farmers then plowed and seeded the rich soil. The Egyptians also depended on the Nile as their chief transportation route. Memphis and Thebes—two capitals of ancient Egypt—and many other cities developed along the river because of its importance to farming and transportation.

The people. Most people of ancient Egypt lived in the Nile River Valley. Scholars believe the valley had from about one million to four million people, and possibly more, at various times during ancient Egypt's history. The rest of the population lived in the delta and in oases west of the river, in the Libyan Desert.

The ancient Egyptians had dark skin and dark hair. They spoke a language that was related both to the Semitic languages of southwestern Asia and to certain languages of northern Africa. The Egyptian language was written in hieroglyphics, a system of picture symbols that stood for ideas and sounds. The Egyptians began to use this system about 3000 B.C. It consisted of more than 700 picture symbols. The Egyptians used hieroglyphics to inscribe monuments and temples and to record official texts. For everyday use, they developed simpler hieroglyphic forms called *hieratic* and *demotic.*

During the New Kingdom (about 1539–1075 B.C.), ancient Egypt had three main social classes—upper, middle, and lower. The upper class consisted of the royal family, rich landowners, government officials, high-ranking priests and army officers, and doctors. The middle class was made up chiefly of merchants, manufacturers, and craft workers. The lower class, the largest class by far, consisted of unskilled laborers. Most of them worked on farms. Prisoners captured in foreign wars became slaves and formed a separate class.

Ancient Egypt's class system was not rigid. People in the lower or middle class, as well as slaves, could move to a higher position. They improved their status mainly through marriage or success in their jobs. Even slaves had rights. They could own personal items, get married, and inherit land. They could also be given their freedom.

Life of the People

Family life. The father headed the family in ancient Egypt. Upon his death, his oldest son became the head. Women had almost as many rights as men. They could own and inherit property, buy and sell goods, and make a will. A wife could obtain a divorce. Few other ancient civilizations gave women all these rights.

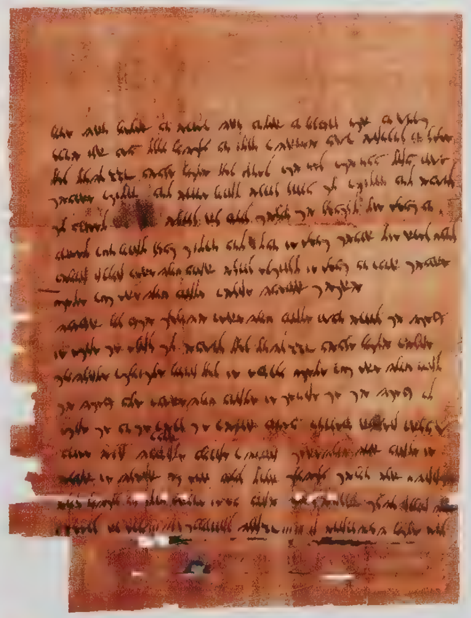
Children played with dolls, tops, and stuffed leather balls. They had board games with moves determined by the throw of dice. They also had several kinds of pets, including cats, dogs, monkeys, baboons, and birds.

Kings commonly had several wives at once. In many cases, a king's chief wife belonged to the royal family.

Education. Only a small percentage of boys and girls went to school in ancient Egypt, and most of them came from upper-class families. These students attended schools for scribes. Scribes made written records for government offices, temples, and other institutions. They also read and wrote letters for the large numbers of Egyptians who could not read and write.

The king's palace, government departments, and temples operated the scribal schools. All the schools prepared the students to become scribes or to follow other careers. The main subjects were reading, literature, geography, mathematics, and writing. The students learned writing by copying literature, letters, and business accounts. They used papyrus, the world's first paper-like material, and wrote with brushes made of reeds whose ends were softened and shaped. The Egyptians made ink by mixing water and soot, a black powder formed in the burning of wood or other substances.

Most Egyptian boys followed their fathers' occupations and were taught by their fathers. Some boys thus learned a trade, but the majority became farmers. Many parents placed their sons with master craftsmen, who taught carpentry, pottery making, or other skills. Boys who wanted to become doctors probably went to work with a doctor after finishing their basic schooling. Most girls were trained for the roles of wife and mother. Their mothers



Papyrus document dated July 3 (449 B.C.)

taught them cooking, sewing, and other skills.

Ancient Egypt had many libraries. A famous library in Alexandria had about 500,000 papyrus scrolls, which dealt with astronomy, geography, and many other subjects. Alexandria also had an outstanding museum.

Food, clothing, and shelter. Bread was the chief food in the diet of most ancient Egyptians, and beer was the favorite beverage. The bread was made from wheat, and the beer from barley. Many Egyptians also enjoyed a variety of vegetables and fruits, fish, milk, cheese, butter, and meat from ducks and geese. Wealthy Egyptians regularly ate beef, antelope and gazelle meat, and fancy cakes and other baked goods. They drank grape, date, and palm wine. The people ate with their fingers.

The Egyptians generally dressed in white linen garments. Women wore robes or tight dresses with shoulder straps. Men wore skirts or robes. The Egyptians often wore colored, shoulder-length headdresses. Rich Egyptians wore wigs, partly for protection against the sun. Wealthy Egyptians also wore sandals made of papyrus or palm leaves. The common people usually went barefoot. Young children rarely wore any clothes.

Sandals were painted on this mummy.

The ancient Egyptians liked to use cosmetics and wear jewelry. Women wore red lip powder, dyed their hair, and painted their fingernails. They outlined their eyes and colored their eyebrows with gray, black, or green paint. Men also outlined their eyes and often wore as much makeup as women. Both sexes used perfume and wore necklaces, rings, and bracelets. Combs, mirrors, and razors were common grooming aids.

The Egyptians built their houses with bricks of dried mud. They used trunks of palm trees to support the flat roofs. Many city houses were narrow buildings with three or more floors. Most poor Egyptians lived in one- to three-room huts. The typical middle-class Egyptian lived in a one- or two-story house with several rooms. Many rich Egyptians had houses with as many as seventy rooms. Some of these homes were country estates with orchards, pools, and large gardens. Egyptian houses had small windows placed high in the walls to help keep out the sun. The people spread wet mats on the floors and hung them on the walls and porches to help cool the air inside their houses. On hot nights, they often slept on the roof, where it was cooler.

Ancient Egyptian furniture included wooden stools, chairs, beds, and chests. People used pottery to store, cook, and serve food. They cooked food in clay ovens or over fires and used charcoal and wood for fuel. Oil lamps provided lighting. The lamps had flax wicks and burned oil in shallow bowls or hollowed-out stones.

Recreation. The ancient Egyptians enjoyed numerous leisure activities. They fished and swam in the Nile River. Sailing on the Nile was a popular family activity. Adventurous Egyptians hunted crocodiles, lions, hippopotamuses, and wild cattle with bows and arrows or spears. Many Egyptians liked to watch wrestling matches. At home, the Egyptians played *senet,* a board game similar to backgammon.

Religion

Gods and goddesses.

The ancient Egyptians believed that various *deities* (gods and goddesses) influenced every aspect of nature and every human activity. They therefore worshiped many *deities.* The main god was the sun god Re. The Egyptians relied on Re and the goddess Rennutet for good harvests. The most important goddess was Isis. She represented the devoted mother and wife. Her husband and brother, Osiris, ruled over vegetation and the dead. Horus, son of Isis and Osiris, was god of the sky. He was called the lord of heaven and was often pictured with the head of a falcon.

In each city and town, the people worshiped their own special deity as well as the major ones. The people of Thebes worshiped Amun. Amun was later identified with Re and called Amun-Re. In time, Amun-Re became the chief deity. Other local deities and their main centers of worship included Ptah, the creator god of Memphis; Thoth, the god of wisdom and writing in Hermopolis; and Khnum, the creator god of Elephantine. Many deities were pictured with human bodies and the heads of animals. Such a head suggested a real or imagined quality of the animal and made identification of the deity easy.

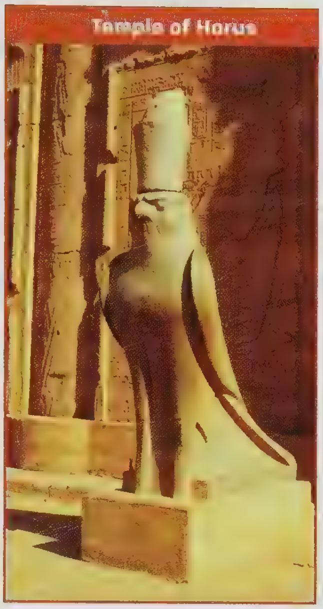
Most ancient Egyptians prayed at home because the temples did not offer regular services for people. Each temple was either regarded as the home of a certain deity or dedicated to a dead king. A temple built in honor of Amun-Re at Karnak was the country's largest temple. It had more than 130 columns that rose about 80 feet (24 meters). Brilliantly colored paintings decorated the columns and walls in the temple's Great Hall, which still ranks as the largest columned hall ever built.

The priests' main job was to serve the deity or king, who was represented by a statue in the temple. The king reigning at the time was considered the chief priest of Egypt. Each day, he or other local priests washed and dressed the statue and brought it food. Priests also offered prayers requested by individuals.

The afterlife. The ancient Egyptians believed that they could enjoy life after death. This belief in an *afterlife* sometimes led to much preparation for death and burial. It resulted, for example, in the construction of the pyramids and other great tombs for kings and queens. Other Egyptians had smaller tombs.

The Egyptians believed that the bodies of the dead had to be preserved for the next life, and so they *mummified* (embalmed and dried) corpses to prevent them from decaying. After a body was mummified, it was wrapped in layers of linen strips and placed in a coffin. The mummy was then put in a tomb. Some Egyptians mummified pets, including cats and monkeys. A number of Egyptian mummies have lasted to the present day.

Temple of Horus



The Egyptians filled their tombs with items for use in the afterlife. These items included clothing, wigs, food, cosmetics, and jewelry. The tombs of rich Egyptians also had statues representing servants who would care for them in the next world. Scenes of daily life were painted on walls inside the tombs. The Egyptians believed that certain prayers said by priests would bring the scenes, as well as the dead, to life.

Many Egyptians bought texts containing prayers, hymns, spells, and other information to guide souls through the afterlife, protect them from evil, and provide for their needs. Egyptians had passages from such texts carved or written on walls inside their tombs or had a copy of a text placed in their tombs. Collections of these texts are known as the Book of the Dead.

Work of the People

Most of the workers in the fertile Nile Valley were farm laborers. Great harvests year after year helped make Egypt rich. Many other people made their living in manufacturing, mining, transportation, or trade.

The Egyptians did not use money. Instead, they traded goods or services directly for other goods or services. Under

Stone jars, luxuries thought to be needed in the afterlife

this *barter* system, workers were often paid in wheat and barley. They used any extra quantities they got to trade for needed goods.

Agriculture. Most farm laborers worked on the large estates of the royal family, the temples, or other wealthy landowners. They received small amounts of crops as pay, partly because landowners had to turn over a large percentage of all farm production in taxes. Some farmers were able to rent fields from rich landowners.

Ancient Egypt was a hot country in which almost no rain fell. But farmers grew crops most of the year by irrigating their land. They built canals that carried water from the Nile to their fields. Farmers used wooden plows pulled by oxen to prepare the fields for planting.

Wheat and barley were the main crops of ancient Egypt. Other crops included lettuce, beans, onions, figs, dates, grapes, melons, and cucumbers. Parts of the date and grape crops were crushed to make wine. Many farmers grew flax, which was used to make linen. The Egyptians raised dairy and beef cattle, goats, ducks, sheep, geese, and donkeys. Some people kept bees for honey.

Manufacturing and mining. Craftsmen who operated small shops made most of the manufactured goods in ancient Egypt. The production of linen clothing and linen textiles ranked among the chief industries. Other important products included pottery, bricks, tools, glass, weapons, furniture, jewelry, and perfume. The Egyptians also made many products from plants, including rope, baskets, mats, and sheets of writing material.

Ancient Egypt had rich supplies of minerals. Miners produced large quantities of limestone, sandstone, and granite for the construction of pyramids and monuments. They also mined copper, gold, and such semiprecious gems as turquoise, amethyst, and malachite. Much of Egypt's gold came from the hills east of the Nile and from Nubia, a country south of Egypt.

Trade and transportation. Ancient Egyptian traders sailed to lands bordering the Aegean, Mediterranean, and Red seas. They acquired silver, horses, and cedar logs from Syria, Lebanon, and other areas of southwestern Asia. They got ivory, leopard skins, gold, cattle, and spices from Nubia. For these goods, the Egyptians bartered gold, other minerals, wheat, barley, papyrus sheets, and wine.

Transportation within ancient Egypt was chiefly by boats and barges on the Nile River. The earliest Egyptian boats were made of papyrus reeds. Moved by poles at first, they later were powered by rowers with oars. By about 3200 B.C., the Egyptians had invented sails and begun to rely on the wind for power. About 3000 B.C., they started to use wooden planks to build ships.

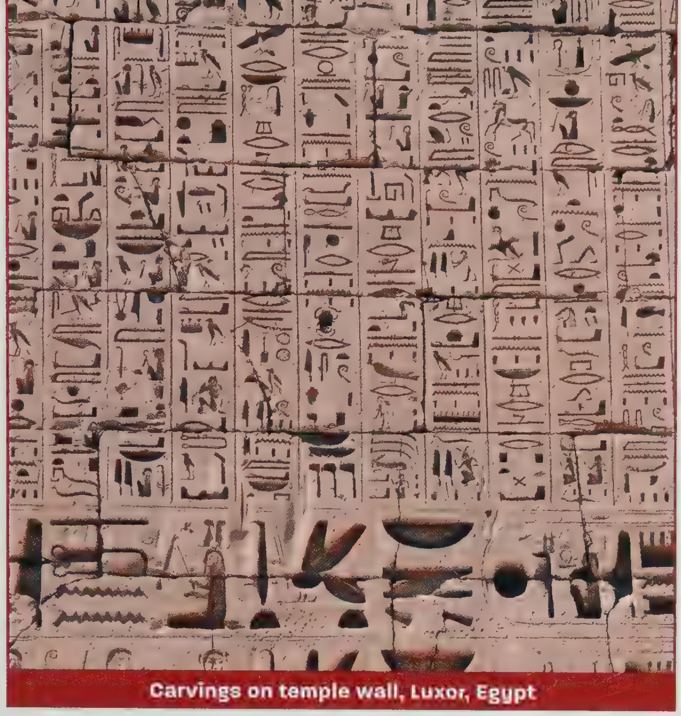
During ancient Egypt's early history, most people walked or rode donkeys when they traveled by land. Wealthy Egyptians were carried on special chairs. During the 1600s B.C., the Egyptians began to ride in horse-drawn chariots.

Crafts and professions. The royal family and the temples of ancient Egypt employed many skilled architects, engineers, carpenters, artists, and sculptors. They also hired bakers, butchers, teachers, scribes, accountants, musicians, butlers, and shoemakers. The Egyptians' belief that their bodies had to be preserved for the afterlife made embalming a highly skilled profession. Many Egyptians served in the army and navy. Others worked on cargo ships or fishing boats.

Arts and Sciences

Architecture. Ancient Egypt's pyramids are the oldest and largest stone structures in the world. The ruins of about ninety pyramids still stand along the Nile. Three huge pyramids at Giza rank as one of the Seven Wonders of the Ancient World, a list of sights compiled by ancient travelers. The first Egyptian pyramids were built about 4,500 years ago. The largest one, the Great Pyramid at Giza, stands about 450 feet (140 meters) high. Its base covers about 13 acres (5 hectares). This pyramid was built with more than two million limestone blocks, each weighing an average of 2½ tons (2.3 metric tons).

Eighteenth Dynasty jewelry chest



Carvings on temple wall, Luxor, Egypt

The ancient Egyptians also built temples of limestone. They designed parts of the temples to resemble plants. For example, some temples had columns carved to look like palm trees or papyrus reeds. The temples had three main sections—a small shrine, a large hall with many columns, and an open courtyard.

Painting and sculpture. Many of ancient Egypt's finest paintings and other works of art were produced for tombs and temples. Artists covered the walls of tombs with bright, imaginative scenes of daily life and pictorial guides to the afterlife. The tomb paintings were not simply decorations. They reflected the Egyptians' belief that the scenes could come to life in the next world. The tomb owners therefore had themselves pictured not only as young and attractive but also in highly pleasant settings that they wished to enjoy in the afterlife.

Ancient Egyptian sculptors decorated temples with carvings showing festivals, military victories, and other important events. Sculptors also carved large stone sphinxes. These statues were supposed to represent Egyptian kings or gods and were used to guard temples and tombs. The Great Sphinx, for example, is believed to represent either King Khafre or the god Re-Harakhte. This magnificent statue has a human head and the body of a lion. It is 240 feet (73 meters) long and about 66 feet (20 meters) high. The Great Sphinx, which is near the Great Pyramid at Giza, was carved about 4,500 years ago. Sculptors also created small figures from wood, ivory, alabaster, bronze, gold, and turquoise. Favorite subjects for small sculptures included cats, which the Egyptians considered sacred and valued for protecting their grain supplies from mice.

Music and literature. The ancient Egyptians enjoyed music and singing. They used harps, lutes, and other string instruments to accompany their singing. Egyptian love songs were poetic and passionate.

Writers created many stories that featured imaginary characters, settings, or events and were clearly meant to entertain. Other writings included essays on good living called "Instructions."

Sciences. The ancient Egyptians made observations in the fields of astronomy and geography that helped them develop a calendar of 365 days a year. The calendar was based on the annual flooding of the Nile. The flooding began soon after the star Sirius reappeared on the eastern horizon after months of being hidden. This reappearance occurred about June 20 each year. The calendar enabled the Egyptians to date much of their history. Dated materials from ancient Egypt have helped scholars date events in other parts of the ancient world.

The ancient Egyptians could measure areas, volumes, distances, lengths, and weights. They used geometry to determine farm borders. Mathematics was based on a system of counting by tens, but the system had no zeros.

Ancient Egyptian doctors were the first physicians to study the human body scientifically. They studied human anatomy and knew that the pulse was in some way connected with the heart. They could set broken bones, care for wounds, and treat many illnesses. Some doctors specialized in a particular field of medicine, such as eye defects or stomach disorders.

Government

Kings ruled ancient Egypt throughout most of its history. Sometime between 1554 and 1304 B.C., the people began to call the king *pharaoh*. The word *pharaoh* comes from words that meant *great house* in Egyptian. The Egyptians believed that each of their kings was the god Horus in human form. This belief helped strengthen the authority of the kings.

The position of king was inherited. It passed to the eldest son of the king's chief wife. Many Egyptian kings had several other wives, called *lesser wives*, at the same time. Some chief wives gave birth to daughters but no sons, and several of those daughters claimed the right to the throne. At least four women became rulers.

Officials called *viziers* helped the king govern ancient Egypt. By the 1400s B.C., the king appointed two of them. One vizier administered the Nile Delta area, and the other one managed the region to the south. The viziers acted as mayors, tax collectors, and judges, and some even controlled temple treasuries. Other high officials included a treasurer and army commander. The government collected taxes from farmers in the form of crops. Skilled workers paid taxes in the goods or services they produced. The treasuries of kings and temples were thus actually warehouses consisting largely of crops and various manufactured goods. The government also levied a *corvee* (tax paid in the form of labor) to obtain troops and government workers.

For purposes of local government, ancient Egypt was divided into forty-two provinces called *nomes*. The king appointed an official known as a *nomarch* to govern each province. There were courts in each nome and a high court in the capital. Viziers judged most cases. Kings decided cases involving crimes punishable by death.

In its early days, ancient Egypt had a small army of foot soldiers equipped with spears. During the 1500s B.C., Egypt built up a large army. The army included archers who were trained to shoot accurately while riding in fast-moving, horse-drawn chariots. Egypt had a large navy of long ships. These ships were powered chiefly by oarsmen, though most vessels also had sails.

Pyramid of Khafre, Giza, Egypt

History

Beginnings. The earliest known communities in ancient Egypt were villages established over 5,000 years ago. In time, the villages became part of two kingdoms. One of these kingdoms controlled the villages that lay on the Nile Delta, and the other controlled the villages south of the delta. The delta area was known as Lower Egypt. The southern region was called Upper Egypt.

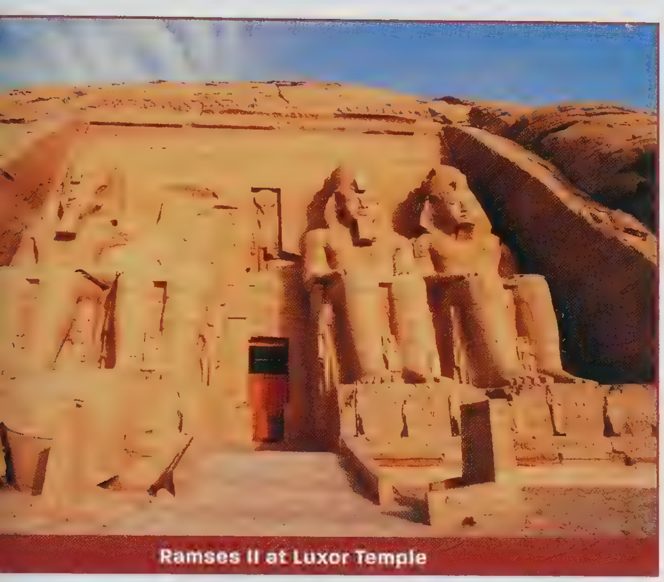
Egyptian civilization began about 3100 B.C. According to tradition, King Menes of Upper Egypt conquered Lower Egypt around that time. He then united the country and formed the world's first national government. Menes founded Memphis as his capital near the site of present-day Cairo. He also established the first of thirty Egyptian *dynasties* (series of rulers in the same family).

The early dynastic period of ancient Egyptian history covered Dynasties I and II, which ruled for about 300 years. During this period, the kings built a temple to Ptah, the chief god of Memphis, and erected several palaces near the temple. The Egyptians also developed irrigation systems, invented ox-drawn plows, and started to develop a bureaucracy during the first two dynasties.

The Old Kingdom, which lasted from about 2650 to 2150 B.C., included Dynasties III through VIII. By the time Dynasty III began, Egypt had a strong central government. The Old Kingdom became known for the construction of Egypt's giant pyramids. This period is sometimes called the Pyramid Age.

The first known Egyptian pyramid was built for King Zoser at Saqqarah about 2650 B.C. The tomb rises about 200 feet (60 meters) in six giant steps and is called the Step Pyramid. During Dynasty IV, workers built the Great Pyramid and other pyramids at Giza. The Great Pyramid was built for King Khufu. Huge pyramids were built nearby for his son, King Khafre, and for King Menkaure. Farm laborers worked on the pyramids when floodwaters of the Nile covered their fields.

By Dynasty VI, the king's authority had begun to weaken as high priests and government officials fought for power. Dynasties VII through XI had weak rulers. Dynasties IX and X and the first half of Dynasty XI ruled during what is known as the



Ramses II at Luxor Temple

first intermediate period. The capital was finally moved to Thebes.

The Middle Kingdom was the period that began in the last half of Dynasty XI and continued through Dynasty XIII or XIV. The Middle Kingdom began in the early 1900s B.C., when Mentuhotep II unified Egypt. Amenemhet I, a vizier in southern Egypt, began Dynasty XII around 1938 B.C., when he seized the throne and moved the capital to Itjtawy, near Memphis. Amenemhet and his strong successors, including Senusret I, Senusret III, and Amenemhet III, helped restore Egypt's wealth and power. During Dynasty XII, Egypt conquered Nubia and promoted trade with Palestine and Syria in southwestern Asia. The arts flourished under this dynasty.

Weak kings ruled Egypt toward the end of Dynasty XII. Settlers from Asia gradually invaded the Nile Delta, using horse-drawn chariots, improved bows, and other tools of war unknown to the native Egyptians. The invaders, called the Hyksos, seized control of Egypt about 1630 B.C., and their leaders founded Dynasty XV. The Middle Kingdom ended in the mid-1600s, but the Hyksos continued to dominate Egypt until 1520 B.C.

The New Kingdom was a period of nearly 500 years in which ancient Egypt became the world's strongest power. The period began around 1539 B.C., with Dynasty XVIII. During this dynasty, native Egyptians drove the Hyksos forces out of Egypt, and Thebes regained its importance. Amun, a god worshiped mainly in Thebes, was increasingly identified with the god Re and called Amun-Re.

At the beginning of Dynasty XVIII, Egypt developed a permanent army that used horse-drawn chariots and other advanced military techniques introduced during the Hyksos period. The dynasty's early rulers led military forces into southwestern Asia. Thutmose I apparently reached the Euphrates River. Queen Hatshepsut, his daughter, also led armies in battle. Egypt developed a great empire and reached the height of its power during the 1400s B.C., under King Thutmose III. He led military campaigns into Asia almost yearly for twenty years and brought the eastern coast of the Mediterranean Sea into the Egyptian empire. Thutmose also reestablished Egyptian control over Kush and surrounding Nubia, which were valuable sources of slaves, copper, gold, ivory, and ebony. As a result of these victories, Egypt became the strongest and wealthiest nation in the world.

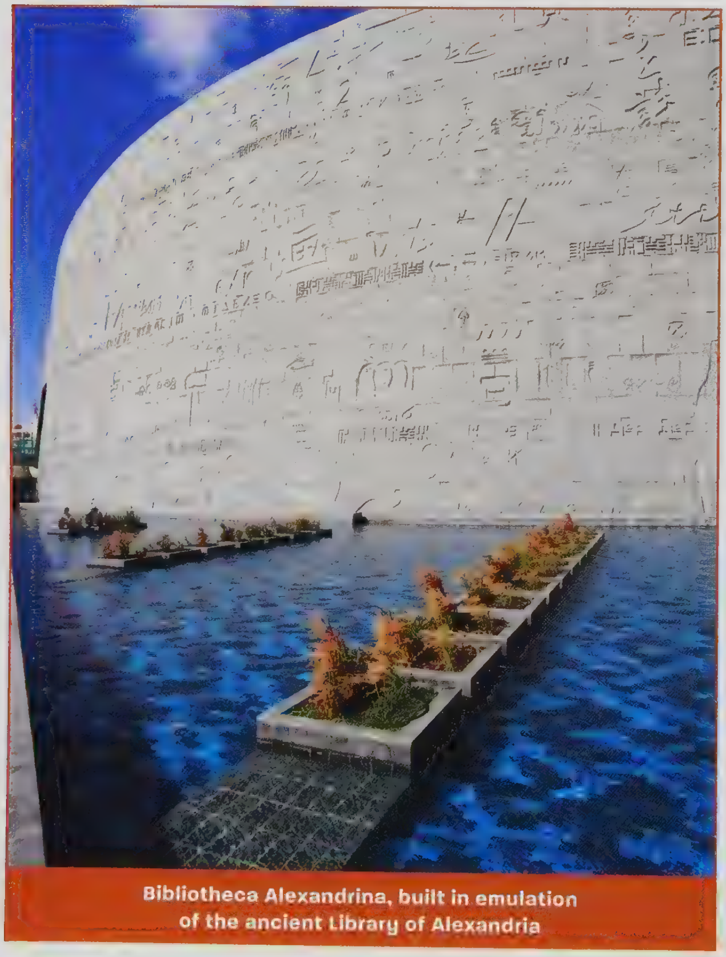
The course of Egyptian history changed unexpectedly after Amenhotep IV came to the throne around 1353 B.C. He devoted himself to a sun god called the Aten, represented as the disk of the sun. Amenhotep changed his own name to Akhenaten and declared that the Aten had replaced Amun and all other gods except Re. He believed that Re was part of the sunlight that came from the Aten. The king also moved the capital to a new city, Akhetaten, about 175 miles (280 kilometers) north of Thebes. Ruins of the city lie near what is now Tell el Amarna. Akhenaten's religious reforms, known as the Amarna Revolution, led to an outpouring of art and sculpture that glorified the Aten. But the changes angered many Egyptians.

Akhenaten's immediate successors ended the unrest. King Tutankhaten removed *-aten* from his name and became Tutankhamun. He restored the old state religion, allowing the worship of the old deities as well as the Aten. Horemheb, the last Dynasty XVIII king, completely rejected Akhenaten's religious beliefs. Dynasty XIX kings built temples to many gods throughout Egypt. Two of the kings, Seti I and his son, Ramses II, regained Asian territories lost after the reign of Thutmose III.

Ancient Egypt began to decline during Dynasty XX. Bitter struggles for power by priests and nobles broke the country into small states. Egypt lost its territories abroad, and its weakness attracted a series of invaders.

The periods of foreign control. Ancient Egypt's decline accelerated rapidly after about 1075 B.C., when Dynasty XX ended. During the next 750 years, ten dynasties ruled Egypt. Most of them were formed by Nubian, Assyrian, and Persian rulers. In 332 B.C., the Macedonian conqueror Alexander the Great added Egypt to his empire. In 331, Alexander founded the city of Alexandria in the delta.

The Ptolemies. Alexander died in 323 B.C., and his generals divided his empire. Ptolemy, one of the generals, gained control of Egypt. About 305 B.C., he took the title of king and founded a dynasty known as the Ptolemaic dynasty. The dynasty's rulers spread Greek culture in Egypt. They also built temples to Egyptian gods, developed Egypt's natural resources, and increased foreign trade. Alexandria became Egypt's capital, and its magnificent library and museum helped make the city one of the greatest cultural centers of ancient times.



Bibliotheca Alexandrina, built in emulation of the ancient Library of Alexandria

Roman rule. Queen Cleopatra VII of the Ptolemies probably married Mark Antony, a co-ruler of Rome, in 37 B.C. Antony and his rival co-ruler, Octavian, each wanted to rule the vast Roman lands by himself. In 32 B.C., Antony combined his and Cleopatra's military forces to fight forces led by Octavian. The navy of Antony and Cleopatra lost the vital Battle of Actium to Octavian's fleet in 31 B.C. The couple committed suicide in 30 B.C., and Octavian then made Egypt a Roman province. Rome's control of Egypt gradually weakened after A.D. 395, when the Roman Empire split into eastern and western parts. By A.D. 642, Arab Muslims had conquered Egypt.

Learning About Ancient Egypt

The study of ancient Egypt is called *Egyptology*, and experts in the field are *Egyptologists*. Much of their knowledge comes from studying the architecture and other arts of ancient Egypt. Ruins of magnificent temples stand at Abydos, Kom Ombo, Edfu, Esna, Luxor, and Karnak. Excavations of pharaohs' tombs, such as those in a burial ground called the Valley of the Kings, near Luxor, have yielded superb paintings. Tutankhamun's tomb was filled with stunning examples of the ancient Egyptians' skill in woodworking and metalworking.

Information about ancient Egypt also comes from written records made by the Egyptians themselves and by such ancient Greek writers as Herodotus and Strabo. The Egyptians used hieroglyphics until sometime after they came under Roman rule. The ability of anyone to read Egyptian hieroglyphics was then quickly lost.

For over 1,000 years, scholars tried but failed to decipher the writing system of ancient Egypt. Then, in 1799, a rock slab with ancient Greek and Egyptian writing was found outside Rosetta, a city near Alexandria. A French scholar named Jean François Champollion began to compare the Greek and Egyptian words on the so-called Rosetta stone. By 1822, he had deciphered the hieroglyphics. Dictionaries developed since then have helped scholars translate the writings on many monuments and in temples and tombs.

SPHINX is an imaginary creature of ancient myths. The Egyptians, Greeks, and peoples of the Near East all had stories about such creatures. According to various tales, the sphinx had the body of a lion and the head of a human, falcon, or ram. Some sphinxes also had wings and a serpent tail.

The term *sphinx* is a Greek word that originally referred to an imaginary evil monster. The ancient Greeks used the term to describe the huge stone statues of lions with human heads that they saw during their visits to Egypt.

Egyptian sphinxes. Most Egyptian sphinxes had the head of a man and the body, feet, and tail of a lion. Others had heads of rams or falcons. Egyptians often made statues of sphinxes to honor a king or queen. The sculptors modeled the face of such a sphinx after the honored person. Egyptian art frequently showed kings as lions conquering their enemies, and sphinxes became symbols of royal protection. Statues of sphinxes often lined avenues leading to temples, such as those near the great temple at Karnak. Other sphinxes represented the god Horus, the sky god and sun god who was thought to be a protector of the king.

The largest, oldest, and most famous sphinx statue lies in the desert near Giza, Egypt. It is called the *Great Sphinx*. The monument stretches 240 feet (73 meters) long and stands about 66 feet (20 meters) high. The width of its face measures 13 feet 8 inches (4.17 meters). Egyptians built the Great Sphinx about 4,500 years ago. They carved its head and body directly out of a giant rock in a limestone formation and cut stone blocks to form the paws and legs. This limestone formation had supplied much of the stone used to build several great pyramids.

The Great Sphinx wears a royal headdress and lies near the pyramid of King Khafre. Historians believe that the sphinx's face is a portrait of Khafre, who probably had the monument built.

Sand has often buried the Great Sphinx up to its neck. King Thutmose IV of Egypt (who ruled about 1400 to 1390 B.C.) cleared the sand away, supposedly after dreaming that the god Horus asked him to do so. During modern times, workers removed the sand in 1818, 1886, 1926, and 1938.

Through the years, desert sand, wind, rain, and sun have worn away part of the stone of the Great Sphinx. A broken section of the head indicates that it may have been used as a target for gun practice at various times. In the 1970s, scientists began efforts to preserve the crumbling stone of the Great Sphinx by treating it with special chemicals.

HIEROGLYPHICS

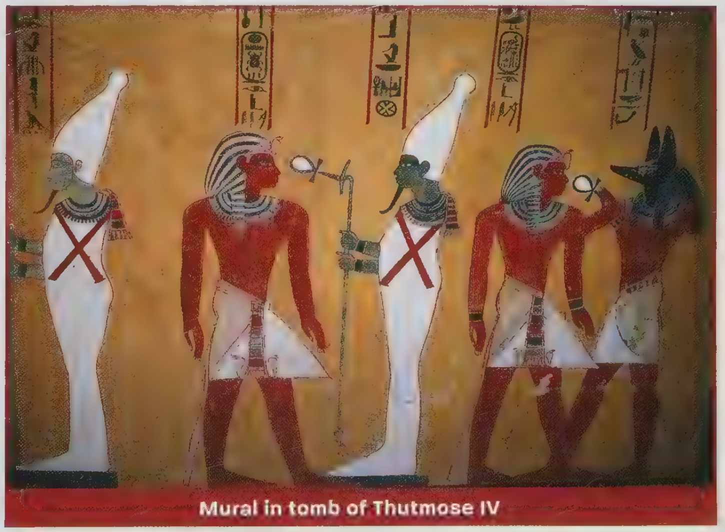
HIEROGLYPHICS is a form of writing in which picture symbols represent ideas and sounds. The word *hieroglyphics* comes from two Greek words that mean *sacred carving*. Hieroglyphics usually refers to the writing of ancient Egypt. However, forms of picture writing were used in other ancient cultures as well, notably by the Hittites, who lived in the region that is now Turkey, and by the Maya and Aztec Indians of Central America.

The ancient Egyptians used hieroglyphic writing for more than 3,000 years. They used hieroglyphic writing primarily for religious inscriptions on temples and stone monuments and to record the words and deeds of royalty. In fact, the Egyptians called their writing *the words of God*. The inscriptions were written or carved by highly trained men who were called *scribes*.

After the A.D. 300s, the Egyptians replaced their other forms of writing with a simpler alphabet that was borrowed mainly from Greek. Knowledge of the meaning of hieroglyphic symbols was soon lost and remained a mystery until the early 1800s, when scholars deciphered the writing.

Development of hieroglyphic writing. The ancient Egyptians borrowed the idea of hieroglyphic writing from Mesopotamia about 3000 B.C. Egyptian hieroglyphics generally included about 800 symbols. However, by about 300 B.C., there were more than 6,000 symbols. The symbols have the elegant, stiff quality typical of ancient Egyptian art.

The earliest *hieroglyphs* (symbols) contained many pictorial characters known as *pictographs* or *ideograms*. These characters were literal representations of ideas. For example, Egyptians who wished to express the idea of a woman drew a picture of a woman.



Mural in tomb of Thutmose IV

The earliest writings of the Egyptians also included phonetic hieroglyphs, also called *phonograms*. Such hieroglyphs, like the characters of modern alphabets, represented the sounds of the language. Some represented only one sound. Others represented combinations of two or three sounds that formed syllables. But the phonetic symbols represented only the sounds of consonants. The Egyptians did not write the vowels. Thus, scholars remain unsure of how the ancient Egyptian language was pronounced.

Egyptian hieroglyphs included *determinatives*. Determinatives indicated the class of object to which the preceding hieroglyphs belonged. An example of a determinative might be the symbol for water placed after the name of a specific lake. Such symbols helped explain and emphasize the meaning of other hieroglyphs.

Some hieroglyphic texts are read from right to left and others from left to right, depending on the direction the hieroglyphs face. Scribes also wrote in columns, which were read from top to bottom. Hieroglyphs often served as decoration. Sometimes, the symbols were painted with brilliant colors or covered with gold.

As writing became more common, the need developed for a material that was easier than stone to write on, store, and transport. For this purpose, the Egyptians invented *papyrus*, a paper-like material made from a reed plant. Scribes wrote on papyrus with brushes made of reeds. The tips of the reeds were softened and shaped into a point. Soot mixed with water served as ink.

The Egyptians used this simplified *cursive* (connected) script called hieratic writing for writing quickly on papyrus. Hieratic writing resembled hieroglyphic writing in much the same way that modern longhand resembles printing. Scribes used hieratic script for both religious and nonreligious purposes.

Hieroglyphic columns from Temple of Amun

Later, about 700 B.C., a script called *demotic* became popular. It was simpler and could be written faster than hieratic writing. Scribes also used the script for correspondence and record-keeping. Demotic writing was widely used for about 1,000 years.

Deciphering hieroglyphic writing. The Egyptians eventually stopped using their own writing systems, replacing them with an essentially Greek alphabet. The phonetic values and other uses of the hieroglyphs were forgotten. People came to believe that the hieroglyphs really represented a secret and magical code used by Egyptian priests.

In 1799, a French officer in the army of Napoleon I discovered a stone tablet near the mouth of the Nile River near Rosetta, Egypt. The tablet, named the Rosetta stone, carried an inscription in three scripts—Egyptian hieroglyphic, Egyptian demotic, and Greek. By reading the Greek portion of the stone, scholars learned that the text consisted of a decree, issued in 196 B.C., honoring King Ptolemy V.

Scholars attempted to translate the Egyptian script using methods that were similar to modern cryptography. In 1814, Thomas Young, an English physician and scholar, discovered that some hieroglyphics were phonetic signs. In addition, scholars learned that hieroglyphs enclosed in an oval ring, called a *cartouche*, represented names of individuals.

In 1822, a French scholar named Jean François Champollion achieved a breakthrough in deciphering the hieroglyphs of the Rosetta stone. By studying the position and repetition of proper names in the Greek script, he picked out the same names in the Egyptian scripts. In addition, Champollion's knowledge of Coptic, the last stage of the ancient Egyptian language written with mostly Greek letters, helped him recognize many ancient Egyptian words in the hieroglyphic part of the text. Eventually, he deciphered the entire text.

Today, grammars and dictionaries enable scholars to read Egyptian hieroglyphic writing easily. Knowledge of ancient Egyptian history would be nearly impossible to obtain without the ability to read hieroglyphics.

Other hieroglyphic writing. In Central America, the earliest examples of Maya hieroglyphs date from about A.D. 250. Maya hieroglyphs consisted of a combination of ideograms that represented entire ideas or words and phonetic hieroglyphs that represented syllables. The Maya carved hieroglyphs on buildings and on large stone monuments called *stelae*. These carvings frequently recorded important historical events in the lives of Maya rulers. In addition, the Maya painted hieroglyphs on pottery and wrote on paper made from fig tree bark. The Maya are the only ancient American people known to have developed a writing system that could express all the words in their language.

Aztec hieroglyphs consisted of pictographs, which also had a phonetic value. The Aztec combined the symbols of several objects to form the sound or name of an object or abstract idea not represented by a pictograph. These symbols resemble *rebus* writing.

The Hittites developed a hieroglyphic writing system about 1500 B.C. Some Hittite symbols represented words. Others represented phonetic syllables.

PYRAMIDS are large structures with square bases and four smooth, triangular-shaped sides that come to a point at the top. Several ancient peoples used pyramids as tombs or temples. The most famous pyramids are those built about 4,500 years ago as tombs for Egyptian kings. These Egyptian pyramids are among the Seven Wonders of the Ancient World.

Egyptian pyramids. The ruins of thirty-five major pyramids still stand near the Nile River in Egypt. Each was built to protect the body of an Egyptian king. The Egyptians thought that a person's body had to be preserved and protected so the soul could live forever. The Egyptians *mummified* (embalmed and dried) their dead and hid the mummies in large tombs. From about 2700 to 1700 B.C., the bodies of Egyptian kings were buried inside or beneath a pyramid in a secret chamber that was filled with treasures of gold and precious objects.

Many scholars believe that the pyramid shape has a religious meaning to the Egyptians. The sloping sides may have reminded the Egyptians of the slanting rays of the sun, by which the soul of the king could climb to the sky and join the gods.

Funeral ceremonies were performed in temples that were attached to the pyramids. Most pyramids had two temples that were connected by a long stone passageway. Sometimes a smaller pyramid for the body of the queen stood next to the king's pyramid. Egypt has at least forty smaller pyramids that were used for queens or as memorial monuments for kings. The king's relatives and officials were buried in smaller rectangular tombs called *mastabas*. These buildings had sloping sides and flat roofs.

The first pyramids. Imhotep, a great architect and statesman, built the first known pyramid for King Zoser about 2650 B.C. Zoser's tomb rose in a series of giant steps, or terraces, and is called the *Step Pyramid*. This pyramid still stands at the site of the ancient city of Memphis, at Saqqarah, near Cairo.

Mayan hieroglyphs, near Yucatan, Mexico

The first smooth-sided pyramid was built about 2600 B.C. It still stands at Medum. It began as a stepped pyramid, and then the steps were filled in with casing stones to give the building smooth, sloping sides. Other pyramids built during a period of Egyptian history called the Old Kingdom (about 2650–2150 B.C.) can be seen at Abusir and Dahshur. During the Middle Kingdom (about 1975–1640 B.C.), pyramids were built at Hawara, Illahun, Lisht, and Dahshur—near what is now Cairo. The remains of these pyramids are still impressive.

The pyramids of Giza (Al Jizah) stand on the west bank of the Nile River outside Cairo. There are ten pyramids at Giza, including three of the largest and best preserved of all Egyptian pyramids. They were built for kings about 2600 to 2500 B.C. The largest was built for King Khufu (called Cheops by the Greeks). The second was built for King Khafre (Chephren), and the third for King Menkaure (Mycerinus). A huge statue of a sphinx, called the Great Sphinx, was probably built for Khafre. It stands near his pyramid.

The pyramid of Khufu, called the *Great Pyramid,* contains more than 2 million stone blocks that average 2½ tons (2.3 metric tons) each. It was originally 481 feet (147 meters) tall, but some of its upper stones are gone now and it stands about 450 feet (140 meters) high. Its base covers about 13 acres (5 hectares).

A study of the Great Pyramid shows how these gigantic structures were built. The ancient Egyptians had no machinery or iron tools. They cut big limestone blocks with copper chisels and saws. Most of the stones came from quarries nearby. But some came from across the Nile River, and others came by boat from distant quarries. Gangs of men dragged the blocks to the pyramid site and pushed the first layer of stones into place. Then they built long ramps of earth and brick, and dragged the stones up the ramps to form the next layer. As they finished each layer, they raised and lengthened the ramps. Finally, they covered the pyramid with an outer coating of white casing stones. They laid these outer stones so exactly that from a distance the pyramid appeared to have been cut out of a single white stone. Most of the casing stones are gone now, but a few are still in place at the bottom of the Great Pyramid.

The Step Pyramid

The burial chamber is inside the Great Pyramid. A corridor leads from an entrance on the north side to several rooms within the pyramid. One of the rooms is called the *Queen's Chamber,* although the queen is not buried there. The room was planned as the king's burial chamber. But Khufu changed the plan and built another burial chamber, called the *King's Chamber.* The *Grand Gallery,* a corridor 153 feet (47 meters) long and 28 feet (8.5 meters) high, leads to Khufu's chamber. It is considered a marvel of ancient architecture.

No one knows how long it took to build the Great Pyramid. The ancient Greek historian Herodotus said that the work went on in four-month shifts, with 100,000 workers in each shift. Scholars now doubt that account and believe that about 100,000 men worked on the pyramids for three or four months each year. Farm laborers built the pyramids. They worked on the tombs during periods when floodwaters of the Nile covered the fields and made farming impossible.

Thieves broke into most of the pyramids, stole the gold, and sometimes destroyed the bodies. Later Egyptian kings stopped using pyramids, and built secret tombs in cliffs. But some kings of the Kushite kingdom in Nubia, south of Egypt, built pyramids long after they were no longer used in Egypt.

American pyramids. Indians of Central and South America also built pyramids. They built stepped pyramids that had flat tops. They used the flat tops as platforms for their temples.

The Moche Indians of Peru built large brick pyramids. *The Temple of the Sun,* near what is now Trujillo, on Peru's northern coast, has a terraced brick pyramid on top of a stepped platform. The ancient Maya of Central America built pyramid-shaped mounds of earth with temples on top.

Indians of central Mexico also built big stepped pyramids. For example, Indians constructed the great pyramids of the Sun and Moon that still stand at Teotihuacan, near Mexico City. The Toltec Indians built a stepped pyramid at Cholula that is one of the largest structures in the world. The Spanish conquerors destroyed most pyramids of the later Aztec Empire in Mexico. These pyramids were built in steps or terraces like the other American pyramids and had temples on top. Two of the greatest were at Tenochtitlan (now Mexico City). Mound-building Indians of North America built some pyramid-shaped mounds, but they were not true pyramids.

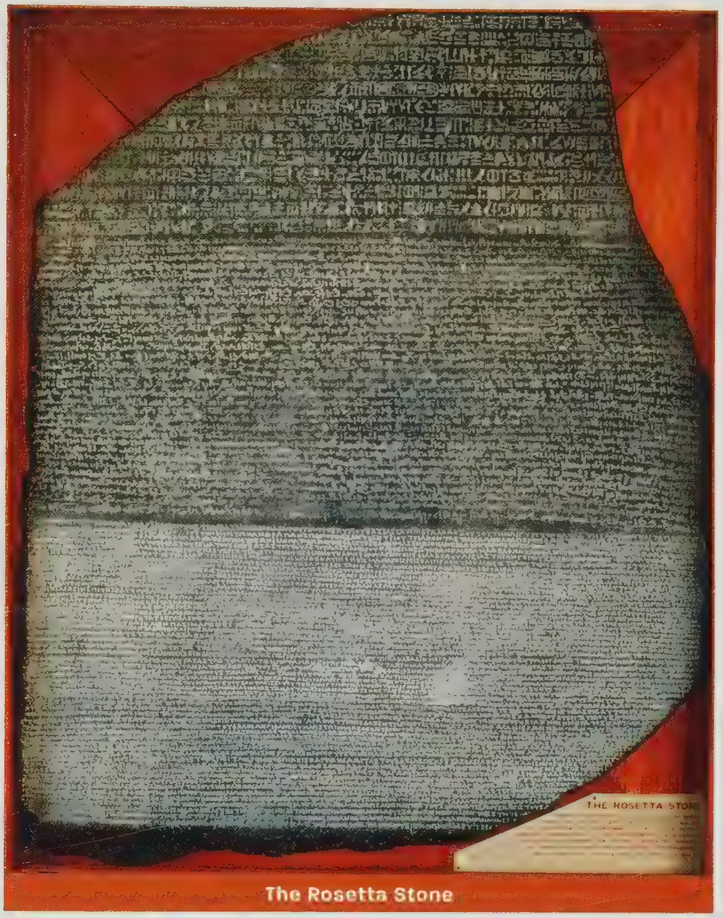
THE ROSETTA STONE gave the world the key to the long-forgotten language of ancient Egypt. A French officer of Napoleon's engineering corps discovered it in 1799. He found the stone half buried in the mud near Rosetta, a city near Alexandria, Egypt. The Rosetta stone was later taken to England, where it is still preserved in the British Museum.

On the stone is carved a decree by Egyptian priests to commemorate the crowning of Ptolemy V Epiphanes, king of Egypt from 205 to 180 B.C. The first inscription is in ancient Egyptian hieroglyphics. The second is in demotic, the popular language of Egypt at that time. At the bottom of the stone the same message is written again in Greek.

The stone is made of a dark gray granitelike rock with a pinkish tone and a pink streak at the top. It is about 11 inches (28 centimeters) thick, 3 feet 9 inches (114 centimeters) high, and 2 feet 4½ inches (72 centimeters) across. Part of the top and a section of the right side are missing.

The language of ancient Egypt had been a riddle to scholars for hundreds of years. A French scholar named Jean François Champollion used the Rosetta stone to solve the riddle. Using the Greek text as a guide, he studied the position and repetition of proper names in the Greek text and was able to pick out the same names in the Egyptian text. This enabled him to learn the sounds of many of the Egyptian hieroglyphic characters.

Champollion had a thorough knowledge of Coptic, the last stage of the Egyptian language that was written mainly with Greek letters. This knowledge enabled him to recognize the meanings of many Egyptian words in the upper part of the inscription. After much work, Champollion could read the entire text. In 1822, he published a pamphlet, *Lettre à M. Dacier*, containing the results of his work. This pamphlet enabled scholars to read the literature of ancient Egypt.



The Rosetta Stone

MLA Citation

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DATA

Important Dates in Egypt's History

c. 3100 B.C.	Egyptian civilization began with the union of Lower and Upper Egypt.
2686–2181 B.C.	The Old Kingdom was a period known for the construction of great pyramids.
1938 B.C.	King Amenemhet founded Dynasty XII, which greatly increased Egypt's power.
c. 1630 B.C.	Hyksos rulers formed a dynasty that ruled Egypt for about 100 years.
1479–1425 B.C.	The Egyptian empire reached its height during the reign of King Thutmose III.
1353 B.C.	Akhenaten became king of Egypt and introduced major religious reforms.
c. 1075 B.C.	Dynasty XX ended, and Egypt began to decline rapidly as a strong nation.
332 B.C.	Alexander the Great added Egypt to his empire.
31 B.C.	A Roman fleet crushed an Egyptian force in the Battle of Actium, leading to Rome's takeover of Egypt in 30 B.C.
A.D. 642	Muslims from Arabia seized Alexandria and completed their conquest of Egypt.

ADDITIONAL RESOURCES

Books to Read

Level I

Fletcher, Joann. *Exploring the Life, Myth, and Art of Ancient Egypt*. Rosen Publishing, 2010.
Macdonald, Fiona. *Egyptians*. Sea-to-Sea Publications, 2011.
Platt, Richard. *The Egyptians*. Sea-to-Sea Publications, 2011.
Smith, Miranda. *Ancient Egypt*. Kingfisher, 2010.

Level II

Ancient Egypt and the Near East. Marshall Cavendish Reference, 2011.
Bierbrier, Morris L. *Historical Dictionary of Ancient Egypt*. 2nd ed. Scarecrow, 2008.
Wendrich, Willeke. *Egyptian Archaeology*. 5th ed. Wiley-Blackwell, 2010.
Wilkinson, Toby A. H. *Lives of the Ancient Egyptians*. Thames & Hudson, 2007.
Wilkinson, Toby A. H. *The Rise and Fall of Ancient Egypt*. Random House, 2010.

Web Sites

A Multimedia Guide to Art of the Ancient World

http://www.artic.edu/cleo/

An interactive journey through the collection of ancient art at The Art Institute of Chicago. This Web site takes visitors on an expedition to the ancient cultures of Egypt, Greece, and Rome.

Cleopatra: Lost and Found

http://americanart.si.edu/exhibitions/online/lewis/

An on-line exhibit from the Smithsonian Institute.

Egyptian Artifacts Exhibit

http://www.memphis.edu/egypt/artifact.html

Selected artifacts from the University of Memphis Institute of Egyptian Art and Archaeology, with text and photos.

The Institute of Egyptian Art and Archaeology

http://www.memphis.edu/egypt/

Information about art and relics from Ancient Egypt from the University of Memphis Institute of Egyptian Art and Archaeology.

Theban Mapping Project

http://www.thebanmappingproject.com/

Discover each tomb in the Valley of the Kings in this interactive atlas. Investigate a database of information about each tomb and view a compilation of nearly 2,000 images.

China

China is a large country in eastern Asia. It is the world's largest country in population with over 1.3 billion people—about 20 percent of all the people in the world.

HOT topics

- Family Life in China
- Religion
- Higher Education
- Hong Kong

HOT topics

Family Life in China. Family life has always been extremely important in Chinese culture. For thousands of years, the Chinese people practiced loyalty to family, obedience to the father, and reverence for ancestors. Traditionally, Chinese families valued sons far more than daughters. A husband could divorce his wife if she failed to give birth to sons. In some cases, daughters were killed at birth to save resources for the sons. Today, social policy in China stresses that families should value girls and boys equally. Relationships within Chinese families have become less formal, and parents no longer expect their children to show unquestioning obedience.

Religion. Religion is tolerated, but restricted, by the Communist government of China. However, it played an important part in traditional Chinese life, Confucianism, Taoism, and Buddhism were major religions throughout most of China's history. The religious beliefs of many Chinese people included elements of all three. Confucianism is based on the ideas of Confucius, a Chinese philosopher born about 550 B.C. More a moral code than a religion, it stresses the importance of ethical standards and of a well-ordered society. Taoism, also native to China, was influenced by folk religion. It teaches that a person should live in harmony with nature. Buddhism reached China from India before A.D. 100. It taught strict moral standards and the ideas of rebirth and life after death.

HOT topics

Higher Education. Nationwide examinations in China determine who may advance to higher education and at what kind of school. Students study intensely for the tests. Those who do best on the tests enter a university. Some wealthier students who do not qualify may pay to attend private universities. Others who pass the examination with lower scores may enter a technical college or vocational university. These schools train students for jobs in business and industry. Many students complete two- or three-year programs in such fields as agriculture, industrial arts, and nursing.

Graduation ceremony at Tsinghua University, July 2007

Hong Kong. An island with a population of nearly seven million, Hong Kong is a special administrative region of China. It lies on China's southern coast, near the mouth of the Zhu Jiang (Pearl River). Hong Kong is a major port of Asia. It is also a center of trade, finance, and tourism. Hong Kong was part of China from ancient times until the 1800s. Through treaty agreements with China, the United Kingdom gained control of Hong Kong Island in 1842. In 1898 China agreed to lease Hong Kong to the United Kingdom for 99 years. At the end of the lease in 1997, China demanded the return of the island, and power was transferred back to the Chinese. Today Hong Kong is a vital node in the global economy.

TRUE or FALSE?

The Chinese name for China, *Zhongguo*, means *High Country*. It comes from the ancient Chinese belief that their country sat upon the highest mountains in the world.

An aerial view of Hong Kong harbor at dusk

THE BASICS

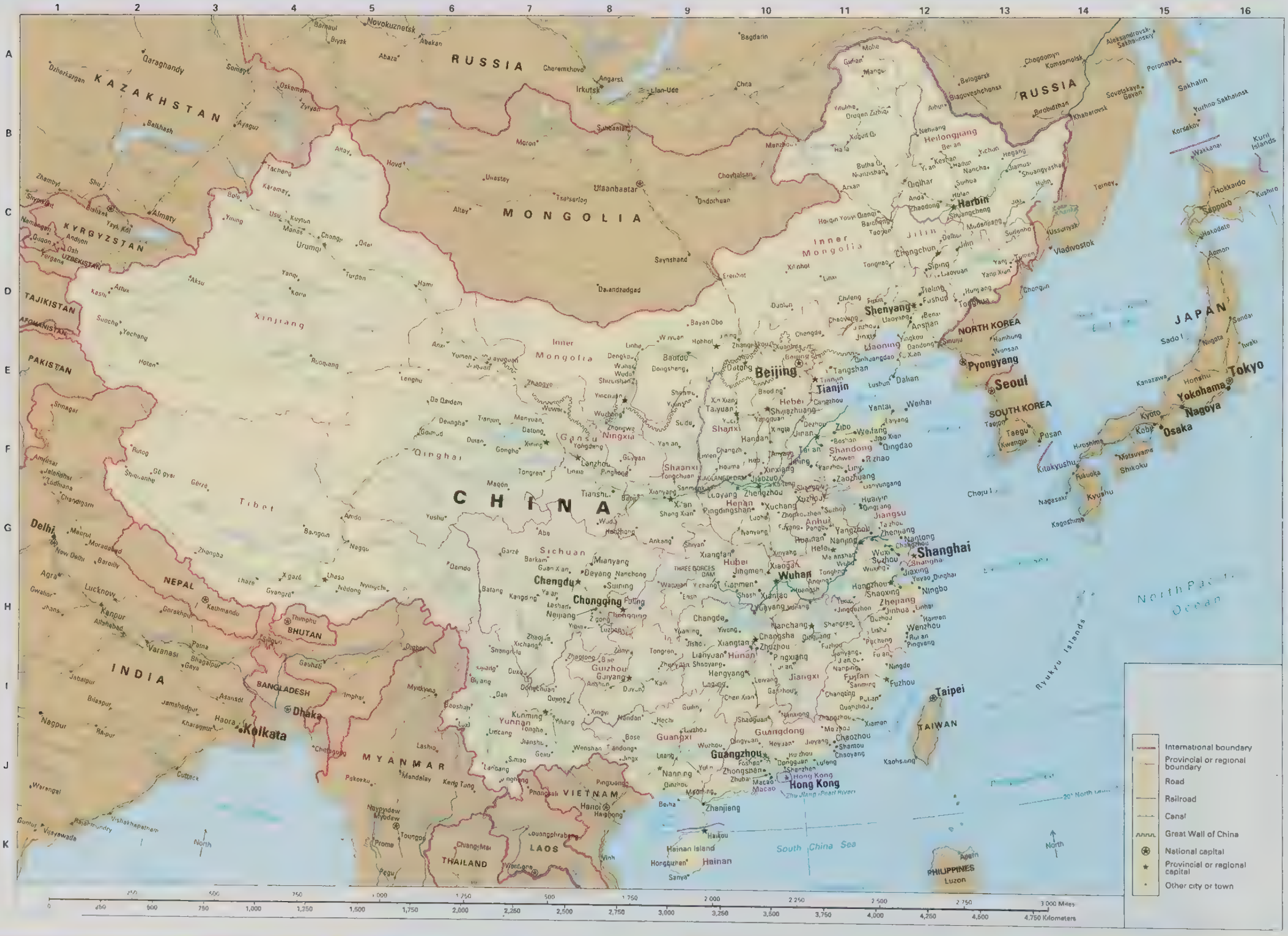
China is a large country in eastern Asia. It is the world's largest country in population with over 1.3 billion people—about 20 percent of all the people in the world. China is the third largest country in area. Only Russia and Canada have more territory. China is also one of the world's oldest countries, with a rich history that stretches over thousands of years.

The Chinese call their country *Zhongguo,* which means *Middle Country.* This name probably came from the ancient Chinese belief that their country was the geographical center of the world and the most cultured civilization. The English name *China* probably came from *Qin,* the name of an early Chinese *dynasty* (series of rulers from the same family).

More than 90 percent of China's people live in the eastern half of China, which has most of China's major cities and nearly all the land suitable for farming. Western China, by contrast, has far fewer people and resources. It is home to many of the country's minority groups.

Agriculture has always been the chief economic activity in China. Most of the people live in rural villages, and over half of all workers are farmers. However, China has some of the world's largest cities. They include Shanghai and Beijing (formerly Peking), the nation's capital.

China is one of the world's oldest living civilizations. Its written history goes back about 3,500 years. The Chinese were the first to develop the compass, paper, porcelain, and silk cloth. They undertook huge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VIETNAM
NORTH KOREA
SOUTH KOREA
JAPAN
TAIWAN
PHILIPPINES
CHINA
Xinjiang
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Yinchuan
Guiyang
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South China Sea
North Pacific Ocean
Ryukyu Islands
Kuril Islands
Sakhalin
Hokkaido
Honshu
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International boundary
Provincial or regional boundary
Road
Railroad
Canal
Great Wall of China
National capital
Provincial or regional capital
Other city or town

Map of China

construction projects, such as the Great Wall. Over the centuries, Japan, Korea, Vietnam, and other Asian lands borrowed from Chinese art, language, literature, religion, and technology.

In early times, the country that is now China was divided into small states, which were sometimes allied and sometimes at war. In 221 B.C., the Qin dynasty conquered the states and created a strong central government, forming the first united Chinese empire. Such empires continued to rule for more than 2,000 years. Chinese empires expanded the country's territory, built great cities, and sponsored magnificent works of literature and art. Nomadic groups from the north sometimes conquered all or part of the country. But the invaders generally adopted more from Chinese civilization than the Chinese adopted from them.

In the 1800s, China's last empire, the Qing, began to weaken. In 1911, revolutionaries overthrew the Qing, and the next year, China became a republic. But the *Kuomintang* (Nationalist Party), which ruled the republic, never established effective government over all of China. In 1949, the Chinese Communist Party defeated the Nationalists and set up China's present government. The Communists called the country *Zhonghua Renmin Gongheguo* (People's Republic of China). The Nationalists fled to the island of Taiwan, where they reestablished their government. The People's Republic claims that Taiwan should be part of its territory. This article discusses only the People's Republic of China.

The Communists made many major changes in China. They placed all important industries under state ownership and direction. The government also took control of most trade and finance. In the late 1900s, the Communists began to loosen their grip on the nation's economy and to allow more free enterprise. China has one of the world's largest economies, and many of its people prosper. But the majority of Chinese still live modestly.

Government

China's government is dominated by the Chinese Communist Party, the military, and a branch of government known as the State Council. The Communist Party is the most powerful group. All people who hold middle- or lower-level positions in the party or the government are called *cadres* or *ganbu.*

The Communist Party. China remains a one-party state. A number of minor political parties exist, but they have no power. Millions of Chinese belong to the Communist Party, but members make up only about 5 percent of the total population. In 2002, the party began allowing business owners to join. Admitting business owners was a major change because Communists traditionally had considered the owners of businesses and other means of production to be enemies of the working class. Although the party still supports Communist ideals, it recognized that business owners play an important role in modern China.

The Communist Party has four main administrative bodies: the National Party Congress, the Central Committee, the *Politburo* (Political Bureau), and the Secretariat. The National Party Congress has nearly 3,000 representatives, selected by party members throughout the nation. The Central Committee consists of leading party members and is elected by the National Party Congress. The Central Committee has about 200 voting members and 150 alternates. The Politburo has about 25 members, who are top party leaders elected by the Central Committee. The Standing Committee is a smaller group within the Politburo and is made up of some of the most important members of the Communist Party. In addition, several powerful leaders belong to the Secretariat, which is chosen by the Central Committee.

Deng Xiaoping

The Communist Party's constitution states that the National Party Congress and the Central Committee are the most important bodies, but they have little real power. In general, they automatically approve party policies and guidelines set by the Politburo and its Standing Committee. The Secretariat is responsible for carrying out the day-to-day activities of the party.

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National government. China's Constitution establishes the National People's Congress as

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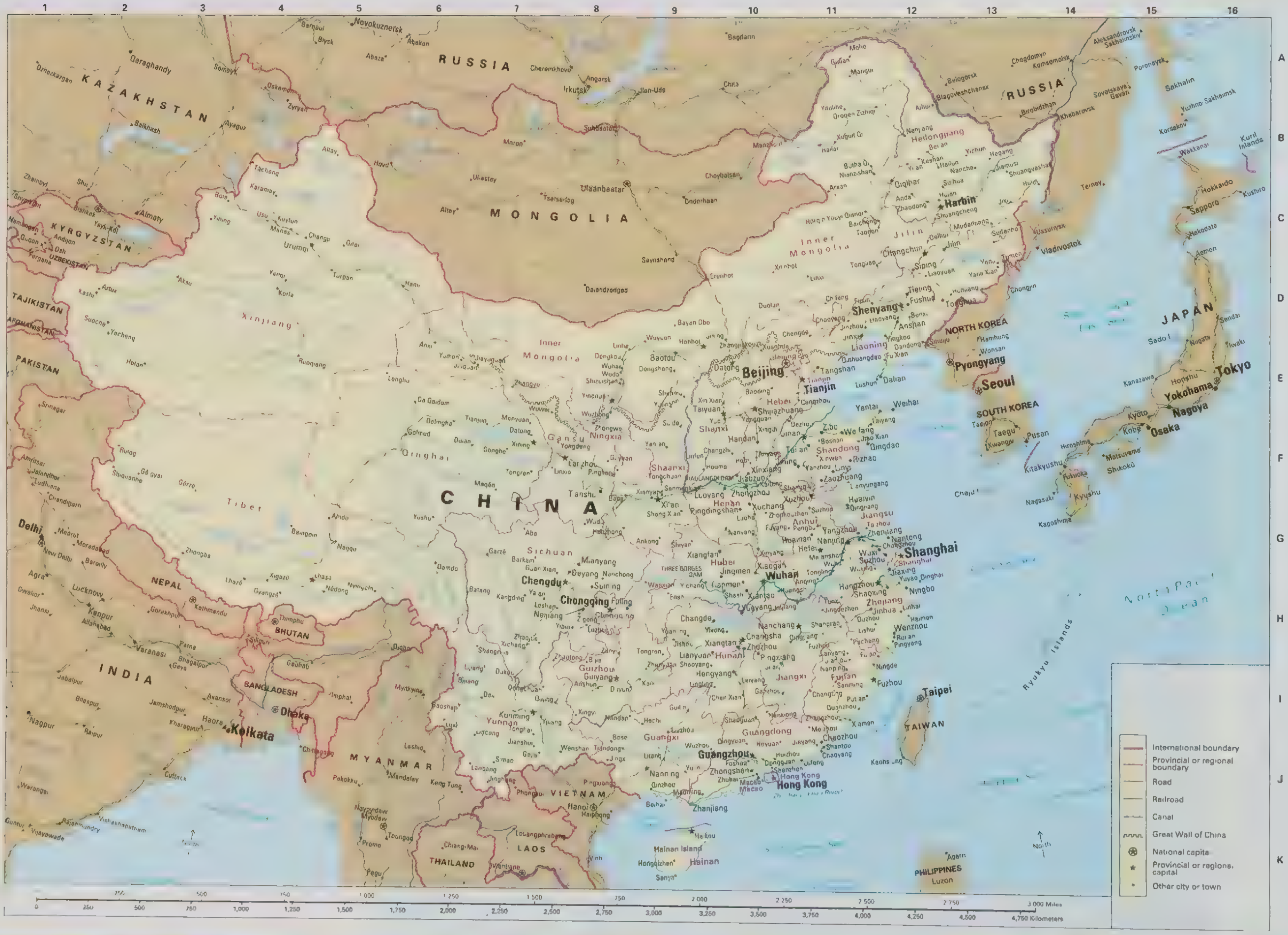
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MYANMAR
LAOS
THAILAND
VIETNAM
NORTH KOREA
SOUTH KOREA
JAPAN
TAIWAN
PHILIPPINES
CHINA
Xinjiang
Tibet
Qinghai
Inner Mongolia
Gansu
Sichuan
Yunnan
Guizhou
Guangxi
Guangdong
Hunan
Hubei
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Tianjin
Shanghai
Chongqing
Chengdu
Wuhan
Guangzhou
Hong Kong
Harbin
Shenyang
Taipei
Seoul
Pyongyang
Tokyo
Ulaanbaatar
Delhi
Kolkata
Dhaka
Hanoi
International boundary
Provincial or regional boundary
Road
Railroad
Canal
Great Wall of China
National capital
Provincial or regional capital
Other city or town
North
3,000 Miles
4,750 Kilometers

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National government. China's Constitution establishes the National People's Congress as the highest government authority. Members of the National People's Congress are elected by local and regional people's congresses and by the armed forces. The members of the National People's Congress serve five-year terms. The chief function of the congress is legislative. Its powers include adopting laws, approving the national budget, and appointing government officials. A standing committee of about 150 members handles the work of the congress when it is not in session.

The State Council serves as the executive branch. It carries on the day-to-day affairs of the government. The council is led by the premier, China's head of government. The premier is chosen by the National People's Congress, upon nomination by the president. The president's duties are largely ceremonial. The premier is assisted by several vice premiers, state councilors, and a number of ministers and heads of special commissions. The ministers are in charge of government departments, such as the ministries of defense, education, and finance.

Political divisions. China has 33 major political divisions—22 provinces, 5 *autonomous* (self-governing) regions, 4 nationally governed municipalities, and 2 special administrative regions. The autonomous regions—Guangxi, Inner Mongolia, Ningxia, Tibet, and Xinjiang—have many people who belong to China's minority ethnic groups. Although the regions are called autonomous, they are actually governed much like the rest of the nation. The nationally governed municipalities—Beijing, Chongqing, Shanghai, and Tianjin—are large metropolitan areas that are administered by the national government. Each of these municipalities consists of an urban center and a rural area. The special administrative regions are Hong Kong and Macao, which were controlled for many years by the United Kingdom and Portugal, respectively. Hong Kong and Macao have their own executive, legislative, and judicial bodies. China handles their defense and foreign policy.

The People's Liberation Army

China has three levels of local government. The 33 major political units are divided into more than 300 prefectures and more than 650 major cities. Counties, other cities, and districts of major cities make up the next level. The counties are subdivided into thousands of townships and towns. Each political unit has a people's congress with a standing committee, and an executive body patterned after the State Council.

Courts in China do not function as a completely independent branch of government as they do in many Western nations. Instead, the courts base their decisions largely on the policies of the Communist Party.

The highest court in China is the Supreme People's Court. It interprets the national laws and supervises the local people's courts. It also makes the final judgment on cases that have been appealed from lower courts. The Supreme People's Procuratorate hears cases that involve violations by government officials and sees that the national Constitution and the regulations of the National People's Congress are observed.

The armed forces of China are jointly commanded by the Central Military Commission of the Communist Party and the Central Military Commission of the government. China has an army, navy, and air force, which together make up the People's Liberation Army (PLA). The PLA has about $1\frac{1}{2}$ million male and female regular members. About $1\frac{1}{2}$ million men and women serve in China's *militia* (citizens' army). There are also about 800,000 army reserves. Men and women between 18 and 22 years of age may be drafted for military service.

The armed forces hold enormous political power in the People's Republic of China. Military officers make up a large percentage of the members on the Communist Party's Central Committee. In addition to its military duties, the People's Liberation Army helps carry out party policies and programs.

People

Population. About a fifth of the world's people live in China. Shanghai is China's largest city and one of the world's largest as well. Beijing, the country's capital, is the second largest city. About 100 Chinese cities each have more than a million people. However, most of the country's people live in rural villages and small towns in eastern China. The western half of China has less than 10 percent of the population.

China's government sees the country's population as both a great resource and a great burden. China's huge workforce could make its economy the most powerful in the world. But unless China limits its population growth, far more children would be born than China could adequately feed, house, educate, or employ under present conditions. By law, to limit population increases, men may not marry until they are 22 years old, and women until they are 20. People are encouraged to postpone marriage until they are in their late 20s and to have no more than two children.

Nationalities. About 92 percent of the people belong to the Han nationality. The rest of the population consists of over 50 minority groups, including Kazakhs, Mongols, Tibetans, Uygurs, and Zhuang. The different nationality groups are distinguished chiefly by language and culture.

Most of China's minority peoples live in the border regions and western China. A few groups, such as the Mongols in the north and the Kazakhs in the northwest, have a long tradition of herding sheep, goats, and other livestock. Some are still nomads, moving from place to place during the year to feed their herds on fresh pastures. The Uygurs raise livestock and grow crops on oases in the deserts of northwestern China. The Tibetan people practice simple forms of agriculture and herding in China's southwestern highlands. Many Koreans are farmers near the border with North Korea.

A number of minority groups inhabit the far southern parts of China. Some of these groups, such as the Zhuang, live much like their neighbors, the Han. Other minority groups are related to the peoples of Laos, Myanmar, Thailand, or Tibet. Many of these people, who live in less developed mountain areas, retain their traditional language and way of life.

Languages. Chinese, the native language of the Han, is actually a group of closely related languages. Early in the 1900s, China's government made northern Chinese, which was spoken in Beijing, the official language. This version of Chinese is often called Mandarin in English, but the Chinese call it *Putonghua* (common language). Most people in northern China speak Putonghua, and it is the language of instruction in almost all schools. Other varieties of Chinese include Northern Min (spoken in Fujian province), Southern Min (spoken mainly in Guangdong and Hainan), Wu (spoken in Shanghai, Jiangsu, and Zhejiang), and Yue or Cantonese (spoken in Guangdong and Guangxi). Each language has several local dialects.

Chinese characters

Although each version of Chinese has its own pronunciation, all Chinese is written in a similar way. The Chinese writing system uses *characters* instead of an alphabet. Each character is a symbol that represents a word or part of a word.

The minority peoples of China speak many languages, including Korean, Mongolian, Uygur, and Zhuang. Many groups use their own language in their schools and publications. Many members of China's minority groups learn Chinese as a second language. A few minority groups speak Chinese as their primary language.

Way of Life

Family life has always been extremely important in Chinese culture. For thousands of years, the Chinese people practiced loyalty to family, obedience to the father, and reverence for ancestors. Chinese philosophy and religion emphasized these values. In the mid-1900s, the Communists tried to replace loyalty to the family with loyalty to the work group. Other social and economic changes in the late 1900s further disrupted traditional family values.

Relationships within Chinese families have become less formal, and parents no longer expect their children to show unquestioning obedience. In the past, parents arranged marriages for their children and chose whom the children would marry. Many young people today choose their own marriage partners, although usually with the consent of their parents. Parents still help arrange some marriages in rural areas.

Chinese families traditionally valued sons far more than daughters. A husband could divorce his wife if she failed to give birth to sons. In some cases, daughters were killed at birth to save resources for the sons. Today, social policy in China stresses that families should value girls and boys equally. The Communist government strongly supports the idea that women should contribute to the family income and participate in social and political activities. Women do many kinds of work outside the home. Many young husbands share in the shopping, housecleaning, cooking, and caring for the children. However, equality between the sexes is more widely accepted in the cities than in the countryside.

Rural life. Traditionally, most Chinese lived in small villages. Many families owned their land, though it was often not large enough to support them. Many other families owned no land but worked as tenants or laborers for landowners and rich farmers.

After the Communists took control of China, they organized a collective ownership system, in which large groups of peasants owned land, tools, work animals, and workshops in common. The highest level of the collective system was the *commune,* which administered the economic activity for groups of about 5,000 families. Smaller units called *production brigades* were further divided into *production teams,* which were the equivalent of a small village. These units planned and performed most day-to-day farm work. In some cases, each family owned its house and a plot on which it could grow vegetables and raise chickens or hogs for its own use. If a family grew a surplus of crops, it could sell the surplus in a local market.

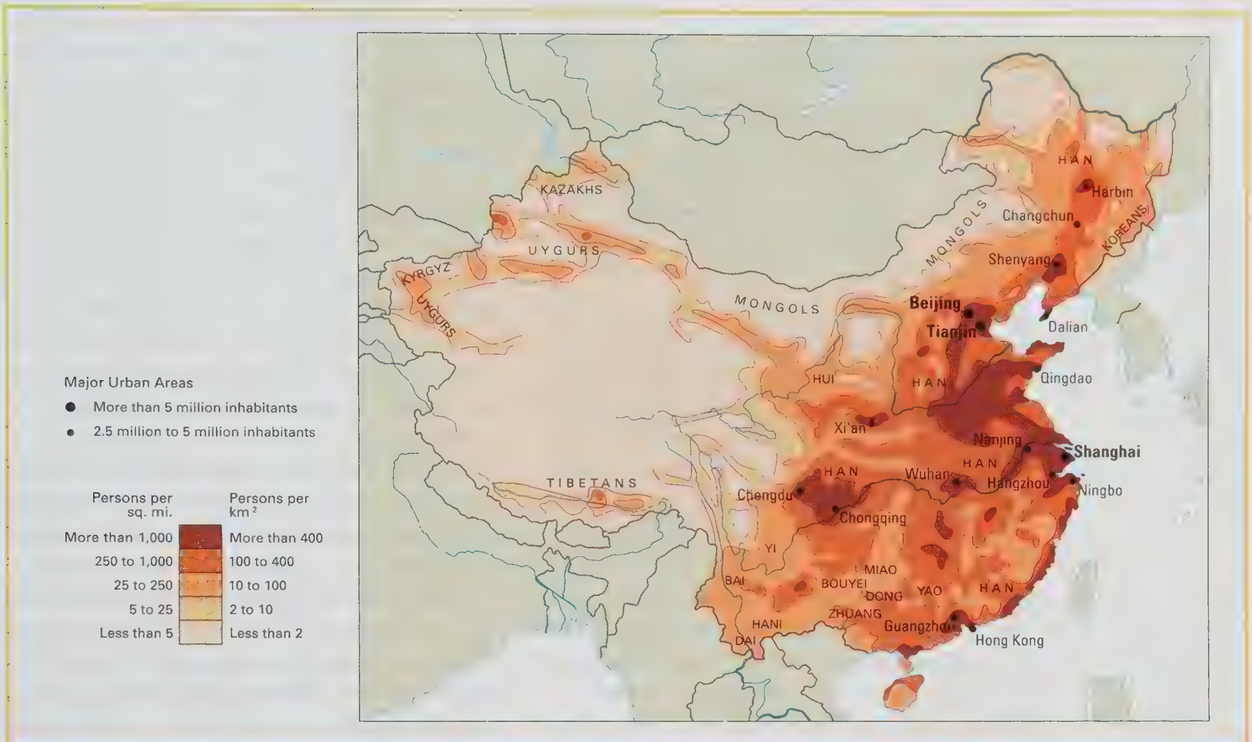
In 1979, the government began a new system to gradually abolish communes, brigades, and teams. Now, cooperative groups known as *collectives* make contracts with individual families. The contract tells how much land a family can work, what crops and livestock the family will raise, and how much it will sell to the government at a set price. After fulfilling its contract, the farm family may use the remainder of its production as it wishes. Most families use some for food and sell the rest on the open market.

Some rural families sign contracts as *specialized households.* These households may specialize in raising only one commodity, such as chickens or silk. Or they may provide farm machinery, repairs, or handicrafts on the free market instead of doing full-time farm work. After paying an agreed amount to the government, the specialized household keeps any profit. Some households operate businesses or small factories and hire employees. Some of them have become relatively wealthy.

The standard of living in rural China today is much higher than it was before the Communists came to power. The average income in rural areas is still low, but most families have enough food and clothing and also own a bicycle or motor scooter, a radio or television set, and a sewing machine. Some rural families own a refrigerator or a washing machine. Most rural families live in five- or six-room houses. Older houses are made of wood or mud bricks and have a tile or thatched roof. Newer houses are made of clay bricks or stone and have a tile roof. Some villages have apartment buildings. Except in remote areas, most houses and apartments have electric power.

An older wooden house in Zhaoxing village

Rural people work many hours a day, especially at planting and harvesting time. They also attend meetings and night classes, where they learn to read and write or how to use scientific farming methods. Even so, the people have time for recreation. Many villages have a small library and a recreation center


Major Urban Areas
● More than 5 million inhabitants
• 2.5 million to 5 million inhabitants
Persons per sq. mi.
Persons per km²
More than 1,000 — More than 400
250 to 1,000 — 100 to 400
25 to 250 — 10 to 100
5 to 25 — 2 to 10
Less than 5 — Less than 2
KAZAKHS
UYGURS
KYRGYZ
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Beijing
Tianjin
Dalian
Qingdao
Xi'an
Nanjing
Shanghai
Wuhan
Hangzhou
Ningbo
Chengdu
Chongqing
Guangzhou
Hong Kong

Where the people of China live

with a television or a film projector, and sometimes a computer. Villages may also have sports facilities, music groups, or theater groups.

City life. China's cities are crowded, and housing is in great demand. Many city residents live in older neighborhoods where the houses resemble those in the countryside. Many other city dwellers live in large new apartment complexes. City governments construct some apartment buildings, and large businesses build others to house their workers. Most families pay rent for their apartments, but some have the opportunity to buy them.

Most city neighborhoods or apartment complexes have an elected residents' committee, also called a neighborhood committee. The committee supervises various neighborhood facilities and programs, such as daycare centers, evening classes, and after-school activities for children. When fights, petty crimes, or acts of juvenile delinquency occur in the neighborhoods, committee members talk with the people involved and try to help them solve the problem. These neighborhood organizations seek to keep crime from becoming a serious problem despite the overcrowding in China's cities.

In general, people in China's cities have a higher standard of living than people in the countryside. Their wages are low compared with those of workers in Western industrial countries. But most households have at least two wage earners, and rents and the cost of food are low. Most city people can afford a bicycle or motor scooter, a television set, and some household appliances. Some are able to buy a computer or a car. City people also have more cultural advantages than do rural people. They can attend a greater variety of classes and meetings, and cities typically have more theaters, museums, and other cultural activities.

Food. Grains are the main foods in China. Rice is the favorite grain among people in the south. In the north, people prefer wheat, which they make into dumplings and noodles. Corn, millet, and sorghum are also eaten. Vegetables, especially cabbages and *tofu* (soybean curd), rank second in the Chinese diet. Pork and poultry are favorite meats. People in China also like eggs, fish, fruits, and shellfish. Fast food, such as hamburgers and french fries, has become popular in China's cities.

Breakfast foods in China include rice porridge, stuffed pocket bread, or deep-fried pastries that taste like doughnuts. Favorite lunchtime foods include egg rolls and dumplings stuffed with meat or shrimp. A typical main meal includes vegetables with bits of meat or seafood, soup, and rice or noodles. Chopsticks and soup spoons serve as the utensils at Chinese meals.

Tea is the traditional favorite Chinese beverage. But soft drinks and beer have also become popular beverages. Ice cream is a favorite treat in China's cities.

Fancy Chinese cooking varies from region to region. *Beijing* (formerly *Peking*) *duck* is a northern specialty. It consists of slices of crisp roast duck eaten with thin rolled pancakes and a sweet sauce made from soybean paste. Foods from the coastal areas include fish, crab, and shrimp. The spiciest foods come from Sichuan and Hunan. Chinese cooks vary the texture of dishes by adding crunchy bamboo shoots and *water chestnuts* (thickened stems of an aquatic plant). The Chinese occasionally eat things rarely used as food elsewhere, such as tiger lily buds, sea animals called *sea cucumbers*, and snake meat. Shark's fin soup is an expensive delicacy.

Recreation. The Chinese enjoy many recreational activities that are popular throughout the world. Watching television, listening to radio, reading, going to the movies or opera, and shopping are common. *Karaoke* clubs, where guests sing along to recorded music, have become popular. Badminton, basketball, soccer, and table tennis are favorite sports. Chinese often invite guests over for meals, but going to restaurants is also popular. Many Chinese urban youth use computers at Internet cafés.

A Taoist monk at Xian monastery, Shaanxi

Traditional Chinese martial arts, such as *t'ai chi ch'uan* (also written *taijiquan*), are popular. Ballroom dancing parties take place both indoors and outdoors. In parks, the Chinese play *xiang qi* (a Chinese version of chess) and Chinese card games. Another favorite game is *mah-jongg*, which is played with engraved tiles.

Clothing. Most Chinese wear clothing similar to that of Europeans and North Americans. In urban areas, fashionable designs are popular, especially among younger people. Members of certain religious and ethnic groups may wear special costumes and headgear. In rural areas, people often make their own clothes.

Health care in China combines traditional Chinese medicine and modern Western medicine. Traditional medicine is based on the use of herbs, attention to diet, and ancient treatments, such as *acupuncture*. In acupuncture, thin needles are inserted into the body at certain points to relieve pain or treat disease. From Western medicine, the Chinese have adopted many drugs and surgical methods.

Hospitals and clinics in China may be either publicly or privately owned. Hospitals in large cities provide access to advanced medical technologies. In rural areas, some villages have medical workers or rural doctors, although some of them do not have much medical training. Village health care providers can treat simple cases and prescribe drugs. For more advanced care, rural Chinese may have to travel a great distance to reach a township health center or a county hospital.

Beginning in the late 1970s, China's government greatly reduced health care funding. Now, patients are expected to pay more of their health care expenses. Most urban residents live near good facilities and can afford to pay for their care. But many rural Chinese no longer have access to affordable health care.

Religion is tolerated, but restricted, by the Communist government of China. However, it played an important part in traditional Chinese life. Confucianism, Taoism (also spelled Daoism), and Buddhism were major religions throughout most of China's history. The religious beliefs of many Chinese people included elements of all three.

Confucianism is based on the ideas of Confucius, a Chinese philosopher born about 550 B.C. More a moral code than a religion, it stresses the importance of ethical standards and of a well-ordered society. In the ideal Confucian society, parents have the right to rule their children, men to rule women, and the educated to rule the common people. Confucianism strongly emphasizes deep respect for one's ancestors and for the past.

Taoism, also native to China, teaches that a person should live in harmony with nature. Taoism began during the 300s B.C. and is based largely on the book *Tao Te Ching (The Classic of the Way and the Virtue)*. Taoism came to include many elements of Chinese folk religion and so became a religion with many protective gods.

Buddhism reached China from India before A.D. 100 and became well established throughout the country during the 300s. Under the influence of Taoism, Chinese varieties of Buddhism developed. They taught strict moral standards and the ideas of rebirth and life after death. The Chinese Buddhists worshiped many gods and appealed to them for help.

China's Communist government regarded religion as part of China's past that would die out. It expected scientific and Marxist thought to replace religion and for many years persecuted religious believers. The Communists destroyed some Taoist and Buddhist temples and other religious buildings, and it turned others into museums, schools, and meeting halls. Beginning in the 1970s, the government adopted a more tolerant attitude toward religion. The government now allows the open practice of religion and the publication of religious works. It restored and reopened some religious buildings. But the government still tries to control religious organizations.

Christian missionaries worked in China for many years before the Communists came to power. The Communists expelled foreign missionaries and closed most Christian churches. In the late 1900s, the government permitted many Christian churches to reopen, but it suppressed Christian movements that organized outside of government control.

Muslims make up a small percentage of the Chinese population. They live mainly in northwestern China.

Chinese schoolchildren

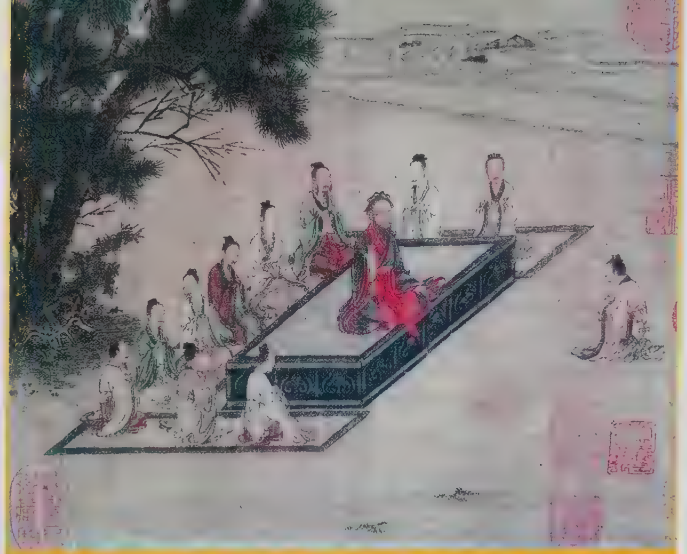
A spiritual movement called Falun Gong appeared in China in the early 1990s and grew rapidly. Falun Gong teaches techniques of meditation through exercises as a means of improving physical health and spiritual purity. The movement claims to have millions of followers in China. Members of Falun Gong staged a 10,000-person demonstration outside central government buildings in Beijing in 1999. This demonstration led the Chinese government to suppress Falun Gong and to issue an arrest warrant for its founder.

Education. The Chinese have always prized education and respected scholars. The Confucians believed that people could perfect themselves through study. They made no sharp distinction between academic education and moral education. For many years, candidates for government jobs had to pass an examination based on the Confucian works.

The Communists regard education as a key to reaching their goals. They have conducted literacy programs in rural areas in an effort to teach all Chinese to read and write. In the 1950s, they began a language reform program to help reduce illiteracy. The program included simplifying more than 2,000 of the most basic Chinese characters by reducing the number of strokes in each character. Such changes helped make Chinese easier to write. Today, most Chinese 15 years of age or older can read and write.

Moral education is important in China. However, the Chinese teach morality as defined in a Communist sense. They say students should be both politically committed to Communist ideas and technically skilled. Courses in China combine the teaching of academic facts and political values.

In the mid-1980s, China's government began trying to get children to attend school for at least nine years. Students who show outstanding ability on nationwide examinations go to *key schools*,



A silk painting showing Confucius lecturing students

which have the best faculties and facilities. Key schools offer education at the elementary, secondary, and college levels. Some private schools exist, although their fees are high.

Elementary and secondary schools. Children in China enter elementary school at the age of 6 or 7. Nearly all of the country's children attend elementary school, which lasts for 5 or 6 years. Elementary school courses include art, Chinese, English, geography, history, mathematics, music, science, physical education, and political education.

After completing elementary school, students may enter secondary schools called *middle schools*. Junior middle school lasts three years, and senior middle school continues for another two or three years. Middle school courses include many subjects studied in elementary school plus biology, chemistry, physics, and foreign languages. Vocational and technical middle schools offer training in agriculture, industrial technology, and other work-related subjects. Almost all elementary school graduates enter middle school, but far fewer continue on to the senior level.

Higher education. Nationwide examinations determine who may advance to higher education and at what kind of school. Students study intensely for the tests. Those who do best on the tests enter a university. Some wealthier students who do not qualify may pay to attend private universities. The chief university subjects include economics, education, engineering, literature, medicine, and science.

Others who pass the examination with lower scores may enter a technical college or vocational university. These schools train students for jobs in business and industry. Many students complete two- or three-year programs in such fields as agriculture, industrial arts, and nursing.

China has about 2,000 institutions of higher learning, including both universities and schools for adult education. The number of students who desire a university education exceeds the number of openings available. But in addition to technical and vocational schools, adult students can continue their education at "workers' colleges" run by factories. These schools offer short-term courses for employees. Other adult education includes part-time study, radio, television, and correspondence courses, and distance learning programs conducted over the Internet.

The Arts

The Chinese have one of the longest and greatest artistic traditions in the world. Chinese pottery and jade from the 4000s B.C. showed a great deal of technical skill and artistic refinement. During the Shang (1766–1045 B.C.) and Zhou (1045–256 B.C.) dynasties, Chinese metalworkers excelled in bronzework. Late in the Zhou dynasty, the Chinese produced remarkable textiles and lacquered items. Most early Chinese art reflected the power and mystery of nature. But around 200 B.C., Chinese art and literature began to focus on mythical and historical figures, and human situations and values.

By the Song dynasty (A.D. 960–1279), nature had again become a prominent theme in Chinese art and literature. Chinese artists emphasized the balance between two principal forces of nature, called *yin* and *yang*. Landscape painters, for example, aimed for harmony, rhythm, and balance in their compositions. Chinese writers, musicians, and architects also tried to capture these qualities in their work.

In the late 1800s and early 1900s, European and American culture began to influence Chinese life, including the arts. But after the Chinese Communists gained control of the country in 1949, they required art and literature to express Communist values and ideals. Since the late 1970s, the government has relaxed its demands, allowing the revival of Chinese traditions in the arts and permitting experimentation in non-Chinese styles.

Literature. The earliest Chinese literature was inscribed on pieces of bone or turtle shell called *oracle bones* from about 1500 to 1045 B.C., during the

Shang dynasty. These inscriptions recorded the administrative duties, dreams, and future concerns of the royal families. A period of great philosophical activity in the latter part of the Zhou dynasty produced the Confucian, Taoist, and other classic writings. These classics, written in a highly refined script known as *wenyan* (patterned words), became models of Chinese literature.

For most of Chinese history, poetry was considered the highest form of literary achievement. It was esteemed not only on its own, but also as an important part of dramas, stories, and novels. Poetry was even inscribed on paintings.

Painting. Chinese potters painted sophisticated designs on their vessels as early as the 4000s B.C. Painting on silk began during the Shang dynasty. Painting on paper began after the Chinese invented paper in the 200s B.C. Early paintings showed people, animals, spirits, or abstract designs. But landscapes became the chief subject of Chinese painting by the A.D. 900s. During the Song dynasty (960–1279), many artists painted landscapes showing towering mountains and vast expanses of water. These artworks expressed a harmony between nature and the human spirit.

Chinese *calligraphy* (fine handwriting) has long been closely linked with the arts of poetry and painting. The use of a brush for writing became common during the Han dynasty (206 B.C.–A.D. 220). The Chinese traditionally considered calligraphy as the highest form of art.

In the A.D. 1000s, painters began to combine landscapes and other subjects with written inscriptions that added to the overall design. These inscriptions typically described the artist's feelings about the scene or the circumstances under which the painting was created. Owners of such paintings would often add inscriptions to the work, recording their own reactions.

In traditional Chinese painting, artists use the same kind of brush for painting as for calligraphy. It consists of a wooden or bamboo handle with bristles of animal hair arranged to form an extremely fine point. The artist can paint many kinds of lines by adjusting the angle of the brush and the pressure on it. Chinese artists paint chiefly with black ink made of pine soot and glue. They sometimes use plant or mineral pigments to add color to their paintings. Chinese painters have created many works on paper or silk scrolls, which can be rolled up for storage and safekeeping. Other paintings have been done on plaster walls or on flat pieces of silk or paper.

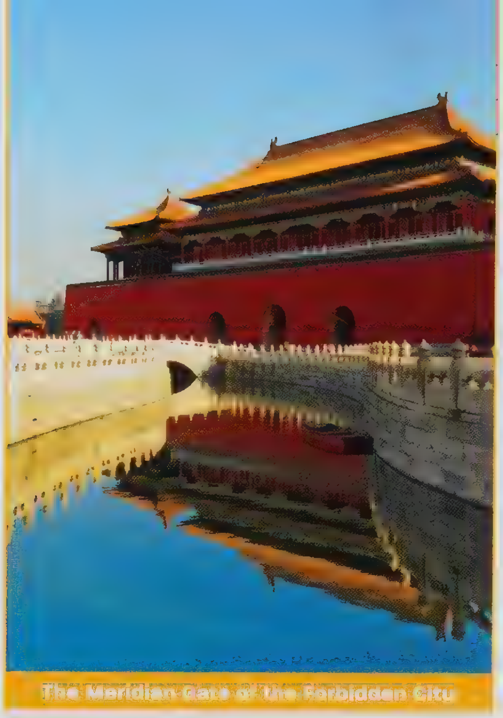
Bronze and jade. The earliest Chinese bronzes were highly decorated vessels created for use in rituals during the Shang and Zhou dynasties. Bronze workers cast the vessels in *piece molds,* clay molds made of separate pieces of baked clay, which had to be destroyed to free the vessels. Art collectors have prized early Chinese bronzes for centuries, valuing them both for their exquisite design and their antiquity.

The Chinese have always held jade in high regard. Their language has many words built on *yu,* the word for jade. These words often convey notions of beauty, virtue, long life, and purity. The Chinese used various forms of jade in ceremonies, buried it with the dead, displayed it in homes and palaces, and wore it for both decoration and protection. Jade *amulets* (charms) supposedly provided protection from evil spirits and preserved the owner's "life force." The Chinese esteemed jade chimes for their clear and uplifting sound.

Sculpture and pottery. Most Chinese sculptures are associated with ritual and religion. They often stand as tomb guardians above ground or as burial attendants in or near graves. Since 1974, thousands of earthenware figures of people, horses, and chariots have been discovered near Xi'an in burial pits near the tomb of Shi Huangdi, the first emperor of the Qin dynasty. These figures, the earliest known life-sized Chinese sculptures, date from the late 200s B.C.

Buddhism reached China from India toward the end of the Han period (206 B.C. to A.D. 220), and sculptors began to turn their skills to the service of this new religion. The Buddhists built temples in or near cities. In rural areas, they hollowed out cliffsides to form chapels. Sculptors decorated the chapels with figures of Buddha and his attendants. Some sculptures were carved from local stone. Others were molded of clay, fired, painted, and glazed. Still other sculptures were cast of bronze and coated with gold.

The Chinese have made pottery since prehistoric times. They began to use the potter's wheel before 3000 B.C. and produced glazed pottery as early as the 1300s B.C. The Chinese developed the world's first porcelain in the A.D. 100s, during the Han dynasty. They admired porcelain, and wrote many essays and poems about it. Among the most esteemed types of porcelain were "official ware"



The Meridian Gate of the Forbidden City

(*guanyao*), produced for the emperor's household; bluish-white *qingbai* ware; and crackled *ge* ware.

Other kinds of porcelains were decorated with folk symbols and the bright colors of folk and religious art. Some featured beautifully painted flowers, landscapes, historical scenes, and calligraphic inscriptions. Porcelain dishware and vases produced during the Tang (618–907), Song (960–1279), and Ming dynasties (1368–1644) and the early part of the Qing dynasty (1644–1912) are among the greatest treasures of Chinese art.

Architecture. Traditionally, most of the public buildings in China were constructed of wood on a stone or earthen platform. The most outstanding feature of Chinese architecture was a large tile roof with extending edges that curved gracefully upward, a development dating from the Tang dynasty. Wooden columns were connected to the ceiling beams by wooden brackets that branched out to support these roofs. Walls did not carry weight but merely provided privacy. Most buildings had only one story, but the Chinese also built many-storied towers called *pagodas*. Today, Chinese architects seldom construct buildings in the traditional styles, and new buildings in Chinese cities look much like those in any modern city.

China's traditional landscape gardens were designed primarily to serve as Taoist retreats. They combine artistic design, complex symbolism, and careful arrangement. Their structure reflects the interplay between the principles of yin and yang. For example, garden designers place angular buildings among rounded natural features and rock formations next to water. Light areas alternate with dark, and empty spaces with solids.

Music was extremely important in traditional Chinese society, both for ritual and recreational purposes. The *Record of Rites*, an ancient Confucian text, states that music is the best way to achieve harmony in the universe. The occasions for music in China ranged from grand public ceremonies and festivals to marriages, funerals, and simple social gatherings.

Traditional Chinese music sounds much different from the music of Europe and America because it uses a different scale. The scales most commonly used in Western music have eight tones, but most Chinese scales have five tones. Melody is the most important element in Chinese music. Instruments and voices follow the same melodic line instead of following different lines that harmonize.

Chinese instruments include the *qin*, a seven-stringed instrument, and the *sheng*, a mouth organ made of 17 bamboo pipes. The Chinese also have a lute-like instrument called the *pipa* and two kinds of flutes, the *xiao* and the *di*. Today, Chinese musicians also play non-Chinese instruments and perform the music of many American and European composers, jazz artists, and rock stars.

Theater. There are several types of Chinese drama: classic *zaju* (variety performance) of the Yuan dynasty (1279–1368); southern drama (*xiwen* and *chuanqi*); various regional styles, such as *Kunqu*, from the region of Suzhou; and a Qing dynasty hybrid form known as *jingxi* or Beijing opera. Although each of these traditional dramatic forms has its own special features, all combine spoken language, music, acting, and sometimes mime, dance, or acrobatics.

Chinese drama often uses poetry to convey dramatic mood. Characters that appear frequently in traditional Chinese drama include scholars, military men (both good and bad), heroic women, and silly comic characters. Since the late 1800s, forms of drama from Europe and the United States have influenced Chinese plays.

The Land

China is the world's third largest country in area. Only Russia and Canada are larger. China's land is as varied as it is vast. It ranges from subarctic regions in the north to tropical lowlands in the south and from fertile plains in the east to deserts in the west.

Northeastern China was once called Manchuria, but today it is called the Northeast (*Dongbei* in Chinese). Xinjiang covers the far northwest, and Tibet (or *Xizang*) covers the far southwest. Inner Mongolia lies in the north. The eastern half of China, south of the Northeast and Inner Mongolia, is sometimes called *China proper*. It has always had most of China's people.

China can be divided into eight major land regions. They are (1) Tibetan Highlands, (2) Xinjiang-Mongolian Uplands, (3) Inner Mongolian Border Uplands, (4) Eastern Highlands, (5) Eastern Lowlands, (6) Central Uplands, (7) Sichuan Basin, and (8) Southern Uplands.

Much of China is so densely populated that little wildlife remains. But rugged mountain forests at the eastern edge of the Tibetan Highlands shelter pandas, golden monkeys, takins, and other rare animals. A few elephants and gibbons inhabit the tropical Southern Uplands region. A few Amur, or Siberian, tigers live in remote forests of the Northeast.

The Tibetan Highlands lie in southwestern China. The region consists of a vast plateau bordered by towering mountains—the Himalaya on the south, the Karakoram Range on the west, and the Kunlun on the north. The world's highest mountain, Mount Everest, rises 29,035 feet (8,850 meters) above sea level in the Himalaya in southern Tibet. Two of the world's longest rivers, the Huang He and the Yangtze, begin in the highlands and flow eastward across China to the sea. The Yangtze River is called the Chang Jiang in China.

The endangered giant panda can be found in Wolong Valley, China.

Tibet suffers from both drought and extreme cold. Most of the region is a wasteland of rock, gravel, snow, and ice. A few areas provide limited grazing for hardy yaks—woolly oxen that furnish food, clothing, and transportation for the Tibetans. Crops grow only in a few lower-lying areas, largely in the east.

The Xinjiang-Mongolian Uplands occupy the vast dry stretches of northwestern China. The region has plentiful mineral resources. However, it is thinly populated because of its remoteness and harsh climate.

The eastern part of the region consists of two deserts, the Mu Us and part of the Gobi. The western part of the region is divided into two areas by the Tian Shan range, which has peaks over 20,000 feet (6,096 meters) above sea level. South of the mountains lies one of the world's driest deserts, the Taklimakan. The Turpan Depression, an oasis near the northern edge of the Taklimakan, is the lowest point in China. It lies 505 feet (154 meters) below sea level. To the north of the Tian Shan, the Junggar Basin stretches northward to the Altai Mountains along the Mongolian border.

The Inner Mongolian Border Uplands lie between the Gobi and the Eastern Lowlands. The Greater Hinggan Range forms the northern part of the region. The terrain there is rugged, and little agriculture is practiced. The southern part of the region is a plateau thickly covered with *loess*, a fertile, yellowish soil deposited by the wind. Loess consists of tiny mineral particles and is easily worn away. The Huang He and its tributaries have carved out hills and steep-sided valleys in the soft soil. The name *Huang He* means *Yellow River* and comes from the large amounts of silt carried by the river.

The Eastern Highlands consist of the Shandong Peninsula and the eastern part of the Northeast. The Shandong Peninsula is a hilly region with excellent harbors and rich deposits of coal. The hills of the eastern part of the Northeast have some of China's best forests. The highest hills are the Changbai Mountains (Long White Mountains) along the border with North Korea. To the north, the Amur River forms the border with Russia. Just south of the Amur is the Lesser Hinggan Range.

Traditional houses by the Grand Canal

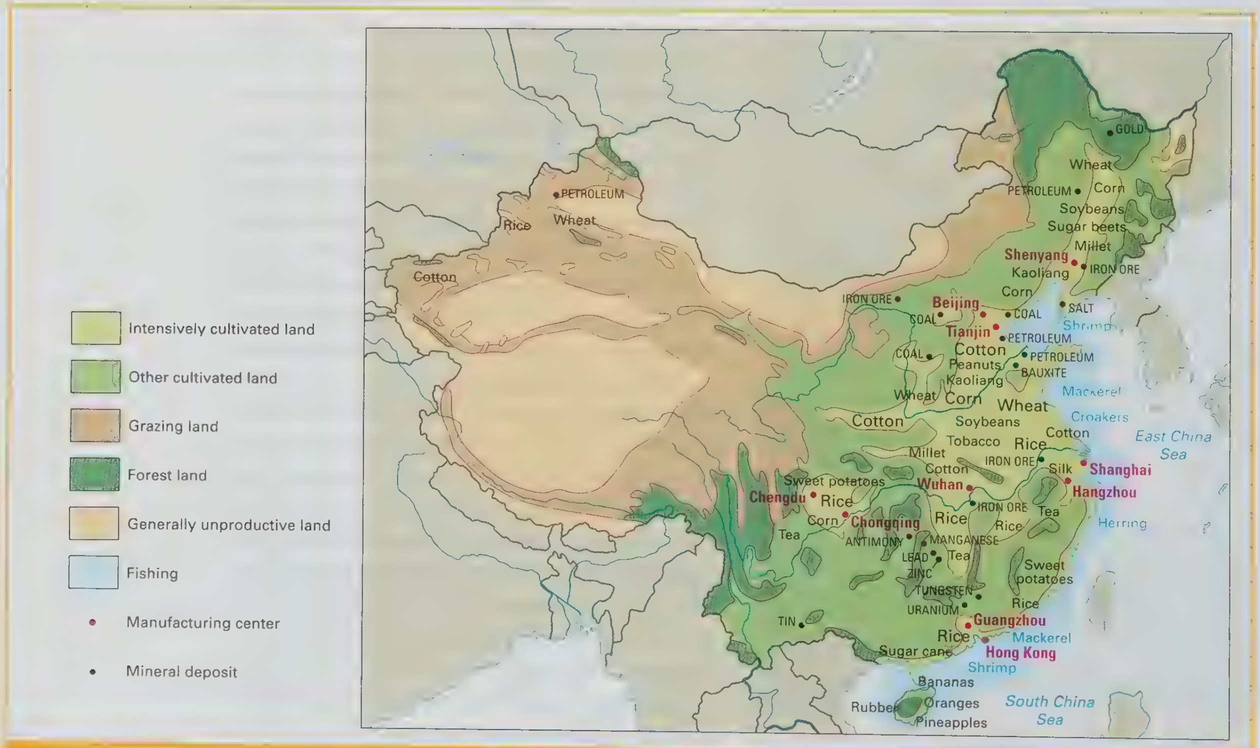
The Eastern Lowlands lie between the Inner Mongolian Border Uplands and the Eastern Highlands and extend south to the Southern Uplands. From north to south, the region consists of the Northeastern Plain, the North China Plain, and the valley of the Yangtze River. The Eastern Lowlands have China's best farmland and many of the largest cities.

The Northeastern Plain has fertile soils and large deposits of coal and iron ore. Most of the Northeast's people live on the southern part of the plain near the Liao River.

Farther south, the wide, flat North China Plain lies in the valley of the Huang He. Wheat is the main crop in this highly productive agricultural area. Major flooding formerly occurred in the valley, which earned the river the nickname *China's Sorrow*. Today, dams, dikes, and reservoirs, and the use of river water for irrigation, control most floods. The Grand Canal, which is the world's longest artificially created waterway, extends more than 1,000 miles (1,600 kilometers) across the North China Plain.

The lower Yangtze Valley has the best combination of level land, fertile soil, and rainfall anywhere in China. In the so-called Fertile Triangle between Nanjing, Shanghai, and Hangzhou, the population density is extremely high. The Yangtze River and its many tributaries have long formed the most important water route for trade within China.

The Central Uplands are an area of hills and mountains between the Eastern Lowlands and the Tibetan Highlands. The Qin Ling, a mountain range, is the chief physical feature of the region. Peaks in the range rise more than 12,000 feet (3,658 meters) above sea level. The Qin Ling crosses the region from east to west. It forms a natural barricade against seasonal winds that carry rain from the south and dust or cold air from the north. To the north of the mountains are dry wheat-growing areas. To the south lie warm, humid areas where rice is the major crop.


Intensively cultivated land
Other cultivated land
Grazing land
Forest land
Generally unproductive land
Fishing
Manufacturing center
Mineral deposit
GOLD
Wheat
PETROLEUM
Corn
Soybeans
Sugar beets
Millet
Shenyang
Kaoliang
IRON ORE
Corn
PETROLEUM
Rice
Wheat
Cotton
IRON ORE
Beijing
COAL
COAL
SALT
Shrimp
Tianjin
PETROLEUM
Cotton
COAL
PETROLEUM
Peanuts
BAUXITE
Kaoliang
Mackerel
Wheat
Corn
Wheat
Croakers
Cotton
Soybeans
Cotton
East China Sea
Tobacco
Rice
Millet
IRON ORE
Cotton
Silk
Shanghai
Sweet potatoes
Wuhan
Hangzhou
Chengdu
Rice
IRON ORE
Tea
Corn
Chongqing
Rice
Rice
Herring
Tea
ANTIMONY
MANGANESE
LEAD
Tea
ZINC
Sweet potatoes
TUNGSTEN
URANIUM
Rice
TIN
Guangzhou
Rice
Mackerel
Sugar cane
Hong Kong
Shrimp
Bananas
Rubber
Oranges
Pineapples
South China Sea

Major uses of land in China

The Sichuan Basin, a large basin surrounded by high mountains, lies southwest of the Central Uplands. Its mild climate and long growing season make it one of China's main agricultural regions. Most crops are grown on *terraced fields*—level strips of land cut out of the hillsides. The name *Sichuan* means *Four Rivers* and refers to the four streams that flow into the Yangtze River in the region. The rivers have carved out deep gorges in the region's red sandstone, which makes land travel difficult. Small ships can travel on the Yangtze into central Hubei, but only boats and other small craft can navigate farther west into Sichuan and the river's swift-flowing tributaries.

The Southern Uplands cover southeastern China, including the island of Hainan. The Southern Uplands are a region of green hills and mountains. The deltas of the Xi Jiang (West River) and Min rivers are almost the only flat areas in the region. The Xi Jiang and its tributaries form the main transportation route for southern China. Deep, rich soils and a tropical climate help make the delta area an extremely productive agricultural region.

Much of the Southern Uplands is so hilly and mountainous that little land can be cultivated, even by terracing. The central part of the region, near the city of Guilin, is one of the most scenic areas in China. It has many isolated limestone hills that rise 100 to 600 feet (30 to 182 meters) almost straight up.

Climate

China has a wide range of climates because it is such a large country and has such a variety of natural features. The most severe climatic conditions occur in the Taklimakan and Gobi deserts. Daytime temperatures in these deserts may exceed 100°F (38°C) in summer, but nighttime lows may fall to -30°F (-34°C) in winter. Tibet and Heilongjiang province in northeastern China have long, bitterly cold winters. In contrast, coastal areas of southeastern China have a tropical climate.

Seasonal winds called *monsoons* greatly affect China's climate. In winter, monsoons carry cold, dry air from central Asia across China toward the sea. These high winds often create dust storms in the north. From late spring to early fall, the monsoons blow from the opposite direction and spread warm, moist air inland from the sea. Because of the monsoons, more rain falls in summer than in winter throughout China. Most parts of the country receive more than 80 percent of their rainfall between May and October.

Summers tend to be hot and humid in southeastern China and in the southern parts of the Northeast. In fact, summer temperatures average about 80°F (27°C) throughout much of China. However, northern China has longer and much colder winters than the south has. In January, daily low temperatures average about -13°F (-25°C) in Heilongjiang and about 20°F (-7°C) throughout much of the eastern third of the country. However, the coastal areas of the Southern uplands are much warmer. Mountains shield southern China and the Yangtze Valley west of Wuhan from the winter winds. The Sichuan Basin is especially well protected, and frost occurs only a few days each winter.

The amount of precipitation varies greatly from region to region in China. The deserts of Xinjiang and Inner Mongolia receive less than 4 inches (10 centimeters) of rain yearly. More than 40 inches (100 centimeters) of rain falls each year in many parts of southeastern China. Some areas near the southeastern coast receive up to 80 inches (200 centimeters) annually. In northern China, the amount of precipitation varies widely from year to year. However, most areas in northern China receive less than 40 inches (100 centimeters) yearly. For example, annual precipitation averages about 25 inches (63 centimeters) in Beijing and 28 inches (70 centimeters) in Shenyang. Snowfalls occur only in the north. But even there, they are infrequent and usually light.

Economy

China has one of the world's largest economies in terms of its *gross domestic product* (GDP), the value of all goods and services it produces in a year. But in terms of *per capita* (per person) GDP, China ranks low. More than half of the world's countries have a higher per capita GDP than China. Because of this, many economists still consider China a developing country.

The national government exercises much control over China's economy. It controls the most important industrial plants and operates most of the nation's banks, most long-distance transportation, and foreign trade. It also sets the prices of certain key goods and services.

China's government makes national economic plans that cover five-year periods. These plans determine how the government will work to improve different areas of the economy. Through the 2000s, the Communist government achieved an impressive record of economic growth.

In the early 1980s, the Chinese government began a series of economic reforms that led to less government control over some business activities. Since then, the number of privately owned and operated businesses has increased dramatically. Many experts believe the increased ownership of business has contributed significantly to China's economic growth. However, it has also led to increased unemployment and labor migration.

In the 1990s, the government began spending huge amounts of money to improve China's *infrastructure* (roads, bridges, dams, and other public works). These projects include the Three Gorges Dam, canals to bring water from the south to the north, and subway systems in major cities.

Manufacturing and mining contribute more to China's GDP than any other category of economic activity. Shanghai is one of the world's leading manufacturing centers. Its industrial output far exceeds that of any other place in China. Beijing and Tianjin are also important industrial centers. Others include Shenyang and Harbin in the Northeast; Guangzhou, Hangzhou, Shenzhen, and Wuhan in southeastern China; and Chengdu and Chongqing in the Sichuan Basin.

After the Communists came to power, they began to rebuild China's factories in an effort to make the nation an industrial power. They concentrated on the development of heavy industries, such as the production of metals and machinery. Today, China produces more steel than any other country. The machine-building industry provides metalworking tools and other machines for new factories. Other major manufactured products include aircraft, automobiles, cement and other building materials, fertilizer and other chemicals, military equipment, ships, and trucks.

The largest consumer goods industries include the clothing and textile industry, the food-processing industry, and electronics. As the standard of living in China improves, demand is growing for such consumer goods as automobiles, refrigerators, and television sets. As a result, the Chinese are increasing their production of consumer items.

Hydroelectric power station at Three Gorges Dam

To help continue the country's industrial expansion, China's leaders made contracts with foreign companies to modernize factories and to build new ones. The government is also improving and expanding scientific and technical education in China and sending students abroad for training. However, waste and inefficiency in industry remain problems.

China is one of the world's largest producers and users of coal. Coal deposits lie in many parts of China, but the best fields are in the north. Coal-burning plants provide about 70 percent of China's electric energy. Hydroelectric plants and oil-burning plants supply most of the rest. The largest oil field in China is at Daqing in Heilongjiang. Other major Chinese oil fields include those at Dagang, near Tianjin; at Liaohe, in Liaoning; and at Shengli on the Shandong Peninsula.

China is a leading producer of iron ore. Most of the ore comes from large, low-grade deposits in the northeastern provinces. Mines in the central and southwestern parts of the country also yield iron ore.

China outranks all other countries in the production of aluminum, lead, magnesium, tungsten, and zinc. It is a leading producer of copper, gold, silver, and tin.

Service industries provide services rather than produce goods. They include trade; banking and finance; communication; education; health care; insurance; recreation; and transportation. Together, they contribute over two-fifths of China's GDP.

Tourism has become an important part of this group. Each year, tens of millions of tourists visit China from other countries, and tens of millions of Chinese travel within their country. More information on transportation and communication appears later in this section.

Agriculture contributes only about one-tenth of China's GDP, but it ranks as the country's largest employer. Nearly half of China's workers are farmers. In southeastern and east-central China, farmers grow cotton, rice, sweet potatoes, tea, and wheat. Much corn is grown in the northeast. China produces more apples, cabbages, carrots, cotton, pears, potatoes, rice, tobacco, tomatoes, and wheat than any other country. In addition, it is a leading producer of corn, melons, rubber, soybeans, sugar beets, sugarcane, and tea. Farmers in the far south grow tropical crops, such as bananas, oranges, and pineapples.

In rural areas, many families raise chickens and ducks. China has more domesticated ducks than any other country. Many farmers raise hogs for both meat and fertilizer. China has nearly half of all the hogs in the world. China also has many cattle, goats, horses, and sheep.

Only about 15 percent of China's land area can be cultivated. Thus, farmers have little cropland on which to grow food for themselves and the rest of the country's huge population. However, they manage to provide almost enough food for all the people. Southern China has a long growing season, so farmers there can grow two or more crops on the same land each year. However, droughts and floods often interrupt production. Chinese farmers do most of their work by hand with simple tools. They use irrigation and fertilizers and practice soil conservation.

During the 1950s, the Communists *collectivized* China's agriculture. They organized the peasants to farm the land cooperatively in units called *communes*. In the 1980s, emphasis on communes declined, and individual families farmed more of the land. The families must pay taxes on their land and must sell an agreed amount of farm products to the state at a fixed price. They may then sell their surplus crops at farm markets, sometimes to city dwellers.

Fishing industry. China has the world's largest fishing industry. The Chinese catch tens of millions of tons of fish, shellfish, and other seafood annually. Fish farming is an important industry in China. Fish farmers raise fish in ponds both for food and for use in fertilizer.

Forestry. China is a world leader in producing forest products. Its timber industry is concentrated in the southeastern and northeastern parts of the country. But China does not produce enough timber to meet its own needs and must import many forest products.

International trade is vital to China's economic development. Foreign investments help China increase its imports and exports. In 1999, China signed a landmark trade agreement with the United States that lowered many barriers to trade. In 2001, China joined the World Trade Organization, a group that promotes international trade.

China imports machinery and other technology needed to modernize the economy. Other leading imports include chemicals, metals, petroleum and petroleum products, and plastics. China's main exports include clothing, furniture, food, electronic devices, textiles, and toys. Much of China's international trade passes through Hong Kong. China's chief trading partners include Germany, Japan, South Korea, and the United States.

A fish farmer with his harvest of catfish

Transportation. Many Chinese still rely on simple, traditional means for transportation over short distances. Rural people carry loads fastened to their back or hanging from poles carried across their shoulders. Carts and wagons are pulled either by people or by donkeys, horses, mules, or oxen. Bicycles, motor scooters, taxis, and buses are widely used for local travel.

Railroads make up the most important part of China's modern transportation system. Rail lines link the major cities and manufacturing centers. The railroads transport both freight and passenger traffic.

China has an extensive network of roads that reaches almost every town in the nation. Highway

Playing an on-line game in an Internet cafe

traffic in China consists mostly of trucks and buses. On rural roads, tractors are common. Many cars are owned by government agencies or taxi companies, but growing numbers of Chinese are buying them for personal use.

Ships transport freight and boats carry passengers and light loads on several Chinese rivers, especially the Yangtze. The Grand Canal, completed about A.D. 1327, extends more than 1,000 miles (1,600 kilometers) from Hangzhou in the south to Beijing in the north. However, silt deposits clog the canal, and only part of it still serves as a water route.

China's major ports include Dalian, Guangzhou, Hong Kong, Ningbo, Qingdao, Shanghai, Shenzhen, and Tianjin. The chief airports are at Beijing, Guangzhou, Hong Kong, and Shanghai, but many other cities have airports. Chinese and foreign airlines link China with cities around the world.

Communication in China comes under strict government control. During the early years of Communism, newspapers, radio, and television were devoted mostly to political propaganda. But in the 1980s, the government began allowing the media to provide general information and entertainment. Educational programs, concerts, plays, and films often appear on television.

Hundreds of daily newspapers and many weeklies are published in China. The government and the Chinese Communist Party publish or support most of these.

In 1978, many people began to express political opinions using posters with large writing called *big-character posters.* These posters were pasted on walls that came to be known as *democracy walls.* Many posters complained about China's political system. In 1979, the government forbade posters that criticized its policies. Now, posters typically give such information as tips on health and physical fitness.

Radios and televisions are widespread throughout China, although they are much more common in urban areas. Radio programs are still broadcast over loudspeakers in some rural areas. A village or other group of people sometimes buys a television set and places it in a common area for public use.

Since the 1990s, the telephone system that had been reserved for official purposes has been expanded to allow private use. Cellular telephones are popular, especially among city dwellers. People also use the state-run postal system for personal communication.

The Internet is popular in China, but the government tightly regulates its use. Yet some people in China use the Internet to express their political views, much as the big-character posters were used in the late 1970s.

History

The oldest written records of Chinese history date from about 1500 B.C., during the Shang dynasty. These records are in the form of *oracle bones*—notations scratched on thousands of turtle shells and animal bones. About 100 B.C., a Chinese historian named Sima Qian wrote the first major history of China. Through the centuries, the Chinese always kept detailed records of the events of their times.

Beginnings of Chinese civilization. People lived in what is now China long before the beginning of written history. Early human beings probably inhabited parts of eastern China more than 1 million years ago. Prehistoric human beings known as the *Homo erectus* lived between about 770,000 and 400,000 years ago in what is now northern China. By about 10,000 B.C., a number of cultures had developed in this area. From two of them—the Yangshao and the Longshan—a distinctly Chinese civilization gradually emerged.

The Yangshao culture reached the peak of its development about 3000 B.C. The culture, which extended from the central valley of the Huang He to the present-day province of Gansu, was based on millet farming. About the same time, the Longshan culture spread over much of what is now the eastern third of the country. The Longshan people lived in walled communities, cultivated rice, and raised cattle and sheep.

The first dynasties. The Xia culture, which some scholars consider China's first dynasty, arose during the 2100s B.C. For many years, experts doubted that the Xia really existed and thought it was only part of Chinese mythology. But archaeologists found evidence of its existence in what is now Henan province.

The Shang dynasty arose from the Longshan and Xia cultures about 1766 B.C. The Shang king-

Detail of some of the 6,000 statues in the Army of Terra-cotta Warriors, 2,000 years old, from the tomb of the first emperor of China

dom was centered in the eastern Huang He Valley. It became a highly developed society governed by a hereditary class of aristocrats. The dynasty's accomplishments included magnificent bronze vessels, horse-drawn war chariots, and a system of writing.

About 1045 B.C., the Zhou people overthrew the Shang from the west and established their own dynasty. The Zhou dynasty ruled China until 256 B.C. The dynasty directly controlled only the western part of their territory. To the east, the Zhou gave authority to certain followers. These followers became lords of semi-independent states. As time passed, these lords grew increasingly independent of the royal court and so weakened its power. In 771 B.C., invaders forced the Zhou to abandon their capital, near what is now Xi'an, and move eastward to Luoyang. Battles between the Zhou rulers and with non-Chinese invaders further weakened the dynasty.

About 500 B.C., the great philosopher Confucius proposed new moral standards to replace the magical practices of his time. During the later Zhou period, there were seven eastern states. The rulers of these states fought one another for the control of all China. In 221 B.C., the Qin state defeated the last of its rival states and established China's first empire controlled by a strong central government. The Qin believed in a philosophy called *Legalism*, and their victory resulted partly from following Legalistic ideas. Legalism emphasized the importance of authority, efficient administration, and strict laws. A combination of Legalistic administrative practices and Confucian moral values helped the Chinese empires endure for more than 2,000 years.

The Qin dynasty lasted only until 206 B.C., but it brought great changes that influenced all later empires in China. The first Qin emperor, Shi Huangdi, abolished the local states and set up a strong central government. His government standardized weights and measures, the currency, and the Chinese writing system. To keep out invaders, he ordered the construction of the Great Wall of China. Laborers built the wall by joining shorter walls constructed during the Zhou dynasty. The Great Wall, which was added to, rebuilt, and moved by later dynasties, extends from the Bo Gulf of the Yellow Sea to the the Jiayu Pass in what is now Gansu province in western China. It stretches about 5,500 miles (8,850 kilometers), including about 3,890 miles (6,260 kilometers) of handmade walls as well as such natural barriers as rivers and hills. Experts believe that the full extent of the Great Wall, including sections predating the Ming dynasty, measures about 13,170 miles (21,200 kilometers).

Shi Huangdi taxed the Chinese people heavily to support his military campaigns and his vast building projects. These taxes and the harsh enforcement of laws led to civil war soon after his death in 210 B.C. The Qin dynasty collapsed four years later.

The Han dynasty then gained control of China. It ruled from 206 B.C. to A.D. 220. During the Han period, Confucianism became the philosophical basis of government. Aristocrats held most important state offices. However, a person's qualifications began to play a role in the selection and placement of officials. Chinese influence spread into neighboring countries, and overland trade routes linked China with Europe for the first time.

In A.D. 9, a Han official named Wang Mang seized the throne and set up the Xin dynasty. However, the Han dynasty regained control of China by A.D. 25. Art, education, and science thrived. Writers produced histories and dictionaries. They also collected classics of literature from earlier times. During the late Han period, Buddhism was introduced into China from India.

Political struggles at the royal court and administrative dishonesty plagued the last century of Han rule. In addition, powerful regional officials began to mistreat the peasants. As a result, large-scale rebellion finally broke out, and the Han dynasty fell in 220.

Hurricanes and Tornadoes

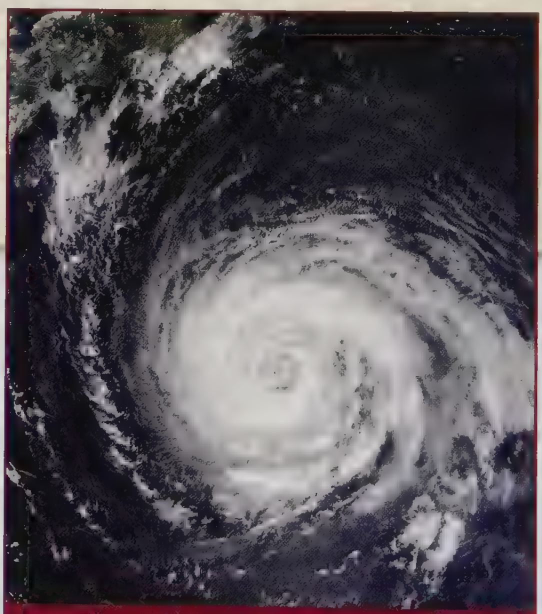
HOT topics

- Hurricane Katrina
- Tornado Chasers
- Anatomy of a Hurricane
- The Path of a Hurricane
- The Enhanced Fujita Scale and the Saffir-Simpson Hurricane Scale

A hurricane is a powerful, swirling storm that begins over a warm sea. When a hurricane hits land, it can cause great damage through fierce winds, torrential rain, flooding, and huge waves crashing ashore. A tornado is a spiral-shaped windstorm that usually forms over land and also can cause great damage from strong winds and flying debris.

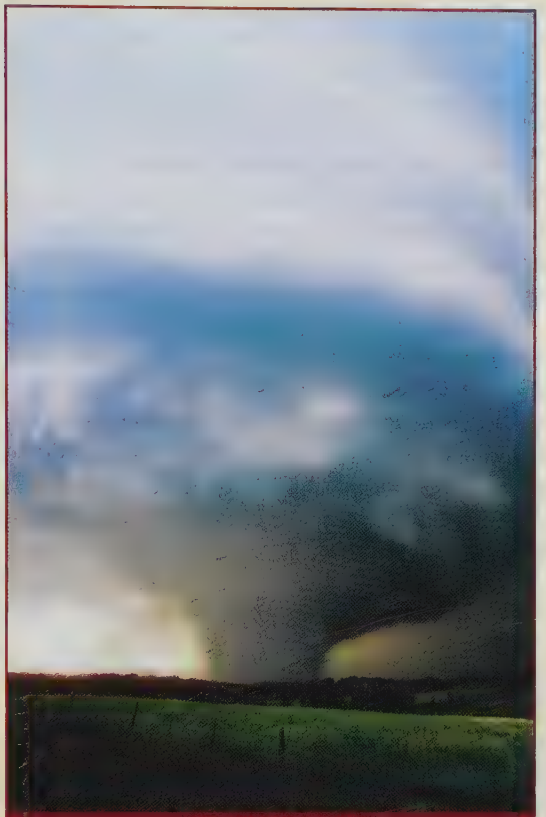
HOT topics

Hurricane Katrina. Hurricane Katrina was one of the most destructive storms ever to strike the United States. The storm killed about 1,800 people, caused tens of billions of dollars in damage, and left hundreds of thousands of people homeless. The government's handling of evacuations and relief efforts was heavily criticized. A panel set up by the U.S. House of Representatives placed much of the blame on a lack of leadership by federal officials.



Typhoon Fitow approaching Japan

HOT topics



A mile-wide tornado

Tornado Chasers. Tornadoes are difficult for scientists to study because they do not know ahead of time where they will occur and because they form in a small area, then vanish quickly. During the height of the tornado season, meteorologists and researchers form teams of "storm chasers" who drive in specially equipped vehicles to locate tornadoes and gather measurements and temperatures or film wind patterns.

Anatomy of a Hurricane. The winds of a hurricane swirl around a calm central zone called the *eye*. The eye usually measures 10 to 40 miles (16 to 64 kilometers) in diameter and is usually free of rain and large clouds. A band of tall clouds called the *eyewall* surrounds the eye. The strongest winds occur in and under the eyewall. Damaging winds may extend 250 miles (400 kilometers) from the eye. Heavy rains fall from the eyewall and from the bands of dense clouds.

The Path of a Hurricane. A long-lived hurricane may wander up to 4,000 miles (6,400 kilometers). Hurricanes in the Northern Hemisphere usually begin by traveling from east to west. As the storms approach the coast of North America or Asia, however, they shift to a more northerly direction. The path of an individual storm is difficult to predict, but most hurricanes turn gradually northwest, north, and finally northeast. In the Southern Hemisphere, the storms may travel westward, then southwest, south, and finally southeast.

The Enhanced Fujita Scale and the Saffir-Simpson Hurricane Scale. Tornadoes are rated on a scale based on the amount of damage that results. The categories range from EF0 (light damage, tree branches broken) to EF5 (incredible damage, houses lifted off foundation, car-sized objects thrown more than 300 feet). Hurricanes are rated on a scale based on wind speed and the height of the resulting storm surge—that is, how much the sea level rises above normal high tide. The categories range from 1 (weak, winds 74–95 mph) to 5 (devastating, winds 156+ mph).

TRUE or FALSE?

Hurricanes that occur in the Northwest Pacific Ocean are known as typhoons.

THE BASICS

A hurricane is a powerful, swirling storm that begins over a warm sea. Hurricanes then move westward and often toward the poles.

The winds of a hurricane swirl around a calm central zone called the *eye* surrounded by a band of tall, dark clouds called the *eyewall*. The eye is usually 10 to 40 miles (16 to 64 kilometers) in diameter and is free of rain and large clouds. In the eyewall, large changes in pressure create the hurricane's strongest winds. These winds can reach nearly 200 miles (320 kilometers) per hour. Damaging winds may extend 250 miles (400 kilometers) from the eye.

Storms known as *tropical cyclones* are referred to by different labels, depending on where they occur. They are called *hurricanes* when they happen over the North Atlantic Ocean, the Caribbean Sea, the Gulf of Mexico, or the Northeast Pacific Ocean. Such storms are known as *typhoons* if they occur in the Northwest Pacific Ocean, west of an imaginary line called the International Date Line. In the Southwest Pacific and Southeast Indian oceans, they are referred to as *severe tropical cyclones*. In the North Indian Ocean, they are known as *severe cyclonic storms*. In the Southwest Indian Ocean, such storms are referred to simply as tropical cyclones.

Hurricanes are most common during the summer and early fall. In the Atlantic and the Northeast Pacific, for example, August and September are the peak hurricane months. Typhoons occur throughout the year in the Northwest Pacific but are most frequent in summer. In the North Indian Ocean, tropical cyclones strike in May and November. In the South Indian Ocean, the South Pacific Ocean, and off the coast of Australia, the hurricane season runs from December to March. Approximately 90 hurricanes, typhoons, and tropical cyclones occur in a year throughout the world. In the rest of this article, the term *hurricane* refers to all such storms.

DEFINITIONS

hurricane *noun*. **a.** *Meteorology*. A wind having a velocity of 74 or more miles per hour (on the Beaufort scale, force 12). **b.** A tropical cyclone originating usually in the West Indies, often accompanied by violent thunderstorms. A storm with a violent wind and, usually, very heavy rain. *Figurative*. a sudden, violent outburst: *a hurricane of cheers*.

typhoon *noun*. A violent cyclone or hurricane occurring in the western Pacific, chiefly during the period from July to October. A violent storm or tempest occurring in Asia, especially in or near India. any violent storm.

cyclone *noun*. A storm or winds moving around and spiraling in toward a calm center of low pressure, which also moves. The winds of a cyclone move counterclockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. *Many errors in forecasting are caused by over attention to comparatively local conditions, without proper regard to the grand movement upon which the cyclones—the parents of rain and sunshine—are borne* (London Times). SYNONYM(S): typhoon, hurricane. Any very violent windstorm, such as a tornado. SYNONYM(S): whirlwind.

tornado *noun, plural* **-does** or **-dos**. **a.** An extremely violent and destructive whirlwind. A tornado extends down from a mass of dark clouds as a whirling funnel and moves over the land in a narrow path. **b.** Any extremely violent windstorm. A violent, whirling squall occurring during the summer on the west coast of Africa. *Figurative*. Any violent outburst: *a tornado of anger. In the fifteenth century a last tornado of nomadism arose in Western Turkestan* (H. G. Wells). *Obsolete*. A violent thunderstorm of the tropical Atlantic, with torrential rain.

How a Hurricane Forms

Hurricanes require a special set of conditions, including the ample heat and moisture that exists primarily over warm tropical oceans. For a hurricane to form, there must be a warm layer of water at the top of the sea. This warm seawater evaporates into the air. The moisture then *condenses* (changes into liquid), forming clouds. As the moisture condenses, it releases heat that warms the air, causing it to rise. The warm, rising air creates a region of relatively low *atmospheric pressure*. Atmospheric pressure is the weight of the air pressing down on a given area.

Air tends to move from areas of high atmospheric pressure to areas of low pressure, creating wind. In the Northern Hemisphere, the earth's rotation causes the wind to swirl into a low-pressure area in a counterclockwise direction. In the Southern Hemisphere, the winds rotate clockwise around a low. This effect of the rotating earth on wind flow is called the *Coriolis effect.* The Coriolis effect increases in intensity farther from the equator. To produce a hurricane, a low-pressure area must be more than 5 degrees of latitude north or south of the equator. Hurricanes seldom occur closer to the equator.

As the swirling winds increase in speed, more ocean water evaporates and then condenses. The moisture releases more heat, further warming the storm's core. The warm air rises faster, increasing surface wind speeds, and so on. This cycle, called a *positive feedback loop*, continues to strengthen the hurricane. When friction between the air and the water surface becomes great enough, the hurricane stops intensifying.

For a hurricane to develop, there must be little *wind shear*—that is, little difference in speed and direction between winds at upper and lower elevations. Uniform winds enable the warm inner core of the storm to stay intact. The storm would break up if the winds at higher elevations increased markedly in speed, changed direction, or both. The wind shear would disrupt the budding hurricane by tipping it over or by bringing dry air into the center of the storm.

The Life of a Hurricane

Meteorologists (scientists who study weather) divide the life of a hurricane into four stages: (1) tropical disturbance, (2) tropical depression, (3) tropical storm, and (4) hurricane.

A tropical disturbance is an area where rain clouds are building. The clouds form when moist air rises and becomes cooler. Cool air cannot hold as much water vapor as warm air can, and the excess water changes into tiny droplets of water that form clouds. The clouds in a tropical disturbance may rise to great heights, forming the towering thunderclouds that meteorologists call *cumulonimbus clouds.*

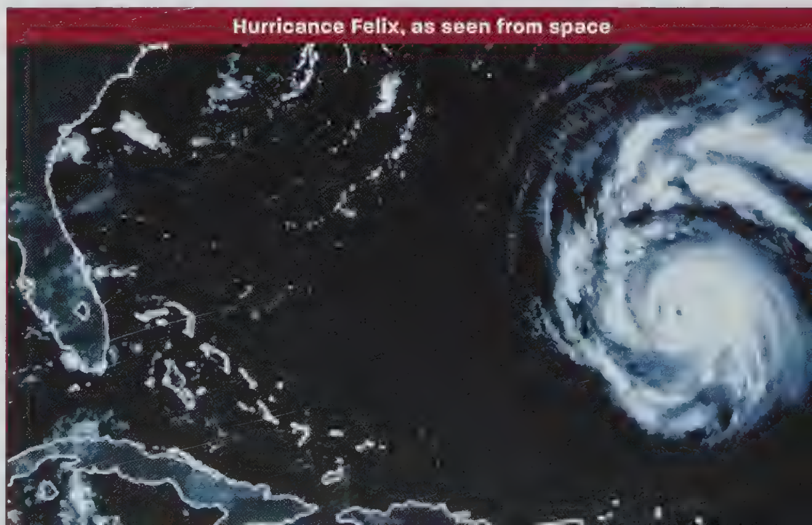
Cumulonimbus clouds usually produce heavy rains that end after an hour or two, and the weather clears rapidly. If conditions are right for a hurricane, however, there is so much heat energy and moisture in the atmosphere that new cumulonimbus clouds continually form from rising moist air.

A tropical depression is a low-pressure area surrounded by winds that have begun to blow in a circular pattern. A meteorologist considers a depression to exist when there is low pressure over a large enough area to be plotted on a weather map. On a map of surface pressure, such a depression appears as one or two circular *isobars* (lines of equal pressure) over a tropical ocean. The low pressure near the ocean surface draws in warm, moist air, which feeds more thunderclouds. The winds swirl slowly around the low-pressure area at first. As the pressure becomes even lower, the winds blow faster, and more ocean water evaporates.

Tropical storm. When the winds exceed 38 miles (61 kilometers) per hour, a tropical storm has developed. Viewed from above, the storm clouds now have a well-defined circular shape. The seas have become so rough that ships must steer clear of the area. The strong winds near the surface of the ocean draw more and more heat and water vapor from the sea. The increased warmth and moisture in the air feed the storm.

A tropical storm has a column of warm air near its center. The warmer this column becomes, the more the pressure at the surface falls. The falling pressure, in turn, creates more wind, which evaporates more ocean water and leads to even warmer air in the column.

Hurricane Felix, as seen from space



Each tropical storm receives a name. The names help meteorologists and disaster planners avoid confusion and quickly convey information about the behavior of a storm. The World Meteorological Organization (WMO), an agency of the United Nations, issues four alphabetical lists of names, one for the North Atlantic Ocean and the Caribbean Sea, and one each for the Eastern, Central, and Northwestern Pacific. The lists include both men's and women's names that are popular in countries affected by the storms.

Storms in the South Pacific and much of the Indian Ocean are named by the regional weather center located closest to the forming storm. These centers are found in Australia, India, Indonesia, New Zealand, and Papua New Guinea. The centers use a naming method similar to that of the WMO. Storms in the Southwestern Indian Ocean are named by the many African countries affected by such storms along with France. Each of the participating counties contributes at least one name to a given year's list.

Except in the Northwestern and Central Pacific, the first storm of the year gets a name beginning with *A*—such as Tropical Storm Alberto. If the storm intensifies into a hurricane, it becomes Hurricane Alberto. The second storm gets a name beginning with *B*, and so on through the alphabet. The lists do not use all the letters of the alphabet, however, since there are few names beginning with such letters as Q or U. For example, no Atlantic or Caribbean storms receive names beginning with Q, U, X, Y, or Z.

Because storms in the Northwestern Pacific occur throughout the year, the names run through the entire alphabet instead of starting over each year. The first typhoon of the year might be Typhoon Nona, for example. The Central Pacific usually has fewer than five named storms each year.

The system of naming storms has changed since 1950. Before that year, there was no formal system. Storms commonly received women's names and names of saints of both genders. From 1950 to 1952, storms were given names from the United States military alphabet—Able, Baker, Charlie, and so on. The WMO began to use only the names of women in 1953. In 1979, the WMO began to use men's names as well.

A palm tree is lashed by the strong winds of a hurricane.

Hurricane. A storm achieves hurricane status when its winds exceed 74 miles (119 kilometers) per hour. By the time a storm reaches hurricane intensity, it usually has a well-developed eye at its center. Surface pressure drops to its lowest in the eye.

In the eyewall, warm air spirals upward, creating the hurricane's strongest winds. Heavy rains fall from the eyewall and bands of dense clouds that swirl around the eyewall. These bands, called *rainbands*, can produce more than 2 inches (5 centimeters) of rain per hour. The hurricane draws large amounts of heat and moisture from the sea.

The Path of a Hurricane

Hurricanes last an average of 3 to 14 days. A long-lived storm may wander 3,000 to 4,000 miles (4,800 to 6,400 kilometers), typically moving over the sea at speeds of 10 to 20 miles (16 to 32 kilometers) per hour.

Hurricanes in the Northern Hemisphere usually begin by traveling from east to west. As the storms approach the coast of North America or Asia, however, they shift to a more northerly direction. Most hurricanes turn gradually northwest, north, and finally northeast. In the Southern Hemisphere, the storms may travel westward at first and then turn southwest, south, and finally southeast. The path of an individual hurricane is irregular and often difficult to predict.

All hurricanes eventually move toward higher latitudes where there is colder air, less moisture, and greater wind shears. These conditions cause the storm to weaken and die out. The end comes quickly if a hurricane moves over land, because it no longer receives heat energy and moisture from warm tropical water. Heavy rains may continue, however, even after the winds have diminished.

Hurricane Damage

Hurricane damage results from wind and water. Hurricane winds can uproot trees and tear the roofs off houses. The fierce winds also create danger from flying debris. Heavy rains may cause flooding and mudslides.

Hurricane-damaged beach house

The most dangerous effect of a hurricane, however, is a rapid rise in sea level called a *storm surge*. A storm surge is produced when winds drive ocean waters ashore. Storm surges are dangerous because many coastal areas are densely populated and lie only a few feet or meters above sea level. A 1970 cyclone in East Pakistan (now Bangladesh) produced a surge that killed about 266,000 people. A hurricane in Galveston, Texas, in 1900 produced a surge that killed about 6,000 people, the worst natural disaster in United States history.

Hurricane watchers rate the intensity of storms on a scale called the Saffir-Simpson scale, developed by American engineer Herbert S. Saffir and meteorologist Robert H. Simpson. The scale designates five levels of hurricanes, ranging from Category 1, described as weak, to Category 5, which can be devastating. Category 5 hurricanes have included Hurricane Camille, which hit the United States in 1969; Hurricane Gilbert, which raked the Caribbean Islands and Mexico in 1988; Hurricane Andrew, which struck the Bahamas, Florida, and Louisiana in 1992; and Hurricane Katrina, which caused widespread destruction in parts of Louisiana, Mississippi, and Alabama in 2005. In 2012, Hurricane Sandy, a Category 1, caused unprecedented damage and flooding from South Carolina to Maine.

Forecasting Hurricanes

Meteorologists use weather balloons, satellites, and radar to watch for areas of rapidly falling pressure that may become hurricanes. Specially equipped airplanes called *hurricane hunters* investigate budding storms.

If conditions are right for a hurricane, the National Weather Service issues a *hurricane watch*. A hurricane watch advises an area that there is a good possibility of a hurricane within 36 hours. If a hurricane watch is issued for your location, check the radio or television often for official bulletins. A *hurricane warning* means that an area is in danger of being struck by a hurricane in 24 hours or less. Keep your radio tuned to a news station after a hurricane warning. If local authorities recommend evacuation, move quickly to a safe area or a designated hurricane shelter.

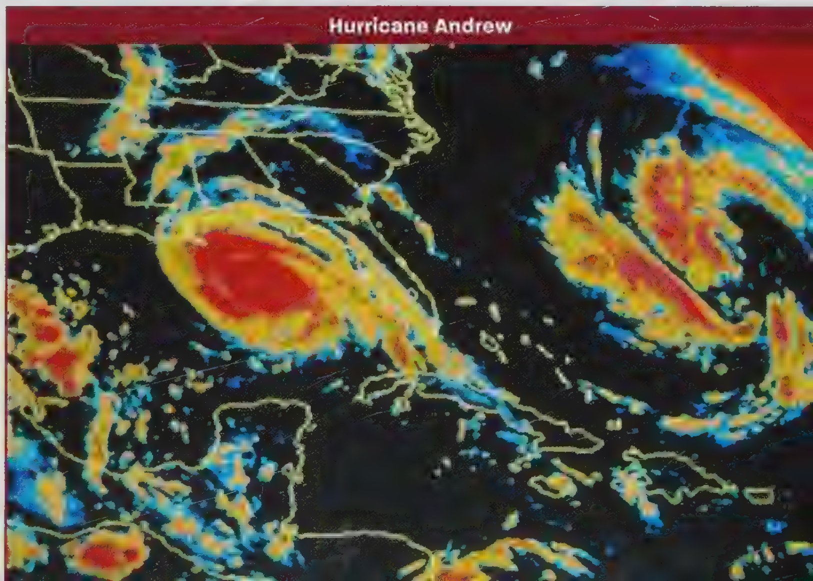
A typhoon is a violent cyclone that occurs in the northwest Pacific Ocean. Typhoons feature heavy rains and winds that maintain speeds equal to or greater than 74 miles (119 kilometers) per hour. Similar storms that occur in other parts of the world are called *tropical cyclones* or *hurricanes*. The word *typhoon* comes from the Chinese term *tai-fung*, meaning *great wind*.

Typhoons occur most frequently in the late summer. They form over warm seas between about 5 and 20 degrees of latitude from the equator. They tend to move west, northwest, and eventually northeast at speeds of 10 to 20 miles (16 to 32 kilometers) per hour. Inside a typhoon, strong winds blow in a counterclockwise direction around an area of low pressure at the storm's center, which is called the *eye*. The eye usually measures about 10 to 40 miles (16 to 64 kilometers) in diameter. The strongest winds blow inside the *eyewall*, a ring of clouds that surrounds the eye. These winds often reach speeds of more than 110 miles (180 kilometers) per hour.

The heavy rains and strong winds of a typhoon can cause great loss of life and billions of dollars in property damage. As a typhoon approaches lands, its winds produce a rush of seawater called a *storm surge* that can devastate coastal areas.

Cyclone is often used to mean a violent, swirling windstorm. To scientists, however, the term cyclone more commonly refers to the weather system in which this type of storm occurs. In this sense, a cyclone is any weather system except a tor-

Hurricane Andrew



nado in which the air pressure at the earth's surface is relatively low.

A cyclone—in the sense of a low-pressure system—acts as a "weather maker" in the middle latitudes. These are zones that extend from the Tropic of Cancer to the Arctic Circle and from the Tropic of Capricorn to the Antarctic Circle. A cyclone may be accompanied by strong winds and widespread areas of cloudiness and precipitation. A single cyclone can affect the weather over a third of a continent or more.

Viewed from above, the surface winds of a cyclone in the Northern Hemisphere blow counterclockwise and inward. In the Southern Hemisphere, the winds blow clockwise and inward. The winds bring together contrasting masses of air in the middle latitudes and in the high latitudes north of the Arctic Circle and south of the Antarctic Circle. Air masses may differ in temperature, humidity, or both. Where contrasting air masses meet, warm and cold fronts develop and spiral outward from the cyclone center. Warm air rises along the fronts, often producing cloudiness and rain or snow. Prevailing *westerlies* (winds from the west) steer a middle-latitude cyclone to the east and northeast.

A different kind of cyclone develops in or near the tropics, the regions between the equator and the Tropics of Cancer and Capricorn. A tropical cyclone has about one-third the diameter of a middle-latitude cyclone; it forms in warm, humid air over very warm ocean water; and it has no fronts. Prevailing winds steer it to the west.

The most intense tropical cyclone is a storm with extremely low air pressure at its center, surface winds blowing at speeds greater than 74 miles (119 kilometers) per hour, and heavy rains. Such a storm is called a *hurricane* when it occurs over the North Atlantic Ocean, the Caribbean Sea, the Gulf of Mexico, or the Northeast Pacific Ocean. It is known as a *typhoon* in the Northwest Pacific Ocean. Near Australia and in the Indian Ocean, it is referred to as a *tropical cyclone*.

A tornado at sunset

A tornado is the most violent of all storms. A powerful tornado can lift cars, cattle, and even mobile homes into the air. It can destroy almost everything in its path.

A tornado is a rapidly rotating column of air known as a *vortex* that has reached the ground. It is often associated with a *funnel cloud*, a funnel-shaped cloud that may appear near the ground in a thunderstorm. Tornado winds swirl at speeds that may exceed 300 miles (480 kilometers) per hour on rare occasions. Tornadoes are also sometimes called *twisters*.

Scientists use the word *cyclone* to refer to all spiral-shaped windstorms. Cyclones circulate in a counterclockwise direction in the Northern Hemisphere and in a clockwise direction in the Southern Hemisphere. Such storms come in many sizes. Among the largest and most intense are hurricanes and typhoons, which may reach 250 miles (400 kilometers) across. Most tornadoes are small, intense cyclones. On rare occasions, tornado winds whirl in the direction opposite that of a cyclone—for example, clockwise in the Northern Hemisphere.

A destructive tornado may reach 1 mile (1.6 kilometers) in diameter. It may travel at 60 miles (100 kilometers) per hour and blow for more than an hour. Fortunately, most tornadoes are smaller and weaker. Most move at less than 35 miles (55 kilometers) per hour and last only a few minutes.

Tornado damage is often localized. A tornado may demolish one house and leave a nearby house untouched. Most tornadoes create a path of devastation less than 1,600 feet (500 meters) wide.

The United States has more tornadoes than any other country. Most of these storms occur in a belt known as Tornado Alley. It stretches across the Midwestern, Plains, and Southern states, especially Texas, Oklahoma, Kansas, Nebraska, and Iowa. However, tornadoes also strike other parts of the world. Areas where tornadoes occur include much of Europe, Japan, parts of China, South Africa, and parts of Argentina and Brazil. Australia ranks second to the United States in number of twisters. Many tornadoes also strike Bangladesh and eastern India.

How a Tornado Forms

The most damaging tornadoes form in storms called *supercells*. A supercell is a large, powerful thunderstorm. It contains a rapidly rotating air mass called a *mesocyclone*. For a supercell to form, and perhaps spawn a tornado, several basic conditions must exist. There must be an adequate supply of moisture to feed the storm. There must be a layer of warm, moist air near the ground and a layer of cool air above. Finally, the winds at higher elevations must differ from those at lower levels in speed, direction, or both.

Moisture. The first requirement for most tornadoes is moisture. In Tornado Alley, air from the Gulf of Mexico provides the moisture to fuel a twister. The warm water of the Gulf evaporates into the air.

Tornadoes and other severe storms often form along a *dryline*. In North America, the dryline is a boundary separating warm, moist air from the Gulf of Mexico and hot, dry air from the west.

If the humidity is high and rain-cooled air enters the main *updraft* (upward flow of air), a *wall cloud* forms. A wall cloud is a low-hanging, dark cloud. Most funnel clouds develop from a wall cloud. If the humidity is low beneath the wall cloud, a tornado may form only a column of dust with no visible funnel cloud.

Doppler radar installation, Virginia

Air temperature. If there is warm, moist air at a lower altitude and cold, dry air at a higher altitude, the warmer air may become buoyant and rise rapidly. The air cools as it rises. The faster the warm air rises, the larger and more violent the storm and the more likely it will spawn a tornado. Storms may develop when warm air rides up over a shallow layer of cooler air. Storms may also form when moist air lifts over mountains, hills, or other high spots.

Often, a *front* powers an updraft of warm, moist air. A front is the boundary between two air masses of different densities resulting from a difference in temperature, humidity, or both. As the warm, less dense air rises, it begins to cool. The moisture it holds condenses into water droplets, forming a cloud. When the air rises high enough and becomes cold enough, its moisture turns into ice crystals. High in the atmosphere, often far above 35,000 feet (10,700 meters), the cloud stops rising. Upon reaching its maximum height, its top spreads out in the shape of an anvil. Anvil-shaped storm clouds often produce thunder, heavy rain, lightning, and hail. In the right conditions, a deadly tornado may form under the base of the cloud.

Winds. Another requirement for a supercell is that winds at higher elevations greatly differ from those at lower levels in speed, direction, or both. A difference in wind speed or direction is called *wind shear*. Wind shear makes the column of rising air begin to rotate. At first, the swirling air forms a broad, horizontal tube. As the storm develops, the tube tilts upright. It becomes the rotating column of air called a mesocyclone.

Most tornadoes occur in supercells. But some appear in a large group of storms called a *mesoscale convective system* (MCS). *Mesoscale* means *medium-sized*. *Convective* refers to *convection*, the turbulent upward and downward motions of air among the storms. An MCS is a cluster of thunderstorms that act as a system and often produce severe weather.

The Life of a Tornado

Tornadoes occur most often during the spring and early summer. Most happen in the late after-noon and early evening. The majority of tornadoes develop from severe thunderstorms. A hurricane, when it makes landfall, can also generate tornadoes.

The first sign of an approaching tornado may be light rain. Heavier rain follows and then rain mixed with hail. The hailstones may grow to the size of golf balls or even oranges. After the hail ends, a tornado may strike. In most tornadoes, a funnel-shaped cloud forms and descends from the wall cloud until it touches the ground. However, there might be a tornado even if the air is too dry for a visible funnel cloud to form. Sometimes, the first sign of a tornado is dust swirling just above the ground.

Some tornadoes contain smaller, short-lived, rotating columns of air called *suction spots* or *suction vortices*. The suction spots revolve around the center of the tornado and can inflict great damage to small areas. When these smaller vortices form, the overall vortex or rotating tornado cloud tends to be wide.

Tornadoes form over water as well as over land. Tornadoes over water, called *waterspouts*, carry large amounts of mist and spray. Waterspouts occur frequently in summer over the Florida Keys. Waterspouts also form elsewhere in the Gulf, along the Atlantic and Pacific coasts, over the Great Lakes, and even over the Great Salt Lake in Utah.

A few small tornadoes begin near the ground and build upward, instead of descending from the clouds. These storms are often called *landspouts* because they look like waterspouts over land.

Damage by Tornadoes

Most tornado damage results from the wind. Each time the wind speed doubles, the force of the wind increases four times. For example, the force of the wind at 220 miles (350 kilometers) per hour is four times as great as the force at 110 miles (175 kilometers) per hour. This tremendous strength may knock over buildings and trees. Other damage occurs when gusts of wind pick up objects and hurl them through the air.

The Fujita scale. Scientists estimate the wind speed of a tornado by the damage it inflicts. For years, they used a system called the Fujita scale. The Japanese-born weather scientist T. Theodore Fujita developed the scale in 1971. In the early 2000s, scientists developed a revised system called the Enhanced Fujita scale. On the Enhanced Fujita scale, EF0 is the weakest rating and EF5 is the strongest. An EF5 tornado can remove a house from its foundation.

Air pressure. Air rising from the ground in the vortex of a tornado creates an area of low air pressure near the ground. For this reason, some people open their windows if a tornado threatens. This precaution is meant to help equalize the indoor pressure with the air outside. The people fear that the air pressure outside the building might drop so suddenly that the structure would explode outward. Safety experts know, however, that air moves in and out of most buildings quickly. The air pressure remains nearly equal inside and out, even during a tornado. Open windows do not reduce the damage. Instead, they may increase the destruction if the wind hurls loose objects through the openings.

Forecasting Tornadoes

Meteorologists (scientists who study weather) hope to learn more about tornadoes to better forecast these destructive storms. They can predict with some accuracy 12 to 48 hours in advance if severe weather, and possibly a tornado, threatens an area. Forecasts are made using data from weather balloons, radar, and satellites.

In the United States, the National Weather Service issues a *tornado watch* when weather conditions are right for the formation of tornadoes. If a tornado watch is issued in your area, you should keep alert for threatening weather. Listen to the radio or television or check the Internet for more information.

If radar detects a mesocyclone or a pattern characteristic of a tornado, the National Weather Service issues a *tornado warning*. The characteristic radar pattern is called a *tornado vortex signature*. It indicates a region of strong rotation in a thunderstorm. The Weather Service also issues a tornado warning if someone actually sees a funnel cloud or tornado.

If a tornado warning is issued for your location, take cover immediately. The safest place is a basement or other underground shelter. If no underground shelter is available, an interior bathroom or closet is best.

Studying Tornadoes

Tornadoes are difficult to study. They form fast, vanish quickly, and occupy a small area. Another problem is that scientists do not know exactly what causes tornadoes. As a result, they find it difficult to reach the right place at the right time to gather data.

Meteorologists investigate tornadoes using a combination of field studies, computer modeling, and studies with devices called *vortex chambers*. From this research, scientists hope to learn how, when, and why tornadoes form. This knowledge will enable them to increase warning times. It will also help to reduce the number of false alarms and provide more accurate warnings.

Field studies take place outdoors. Scientists go into the field to study tornadoes up close. Many meteorologists form mobile teams of "storm chasers" to study tornadoes. The storm chasers travel in specially equipped automobiles, trucks, vans, and aircraft. They try to get as close as is safe to a tornado to study it.

Radar. Meteorologists use a special type of radar called *Doppler radar* to look for mesocyclones. Doppler radar enables storm chasers to map the wind's speed and direction, rather than merely track areas of precipitation. Doppler radar works because radar waves change frequency depending on whether the objects they bounce off, such as raindrops or dust particles, are advancing or receding. This change in frequency is called the *Doppler effect*. It can reveal the rotating pattern of a mesocyclone. As a result, Doppler radar can detect the development of a tornado before it descends from its parent thunderstorm and touches down.

With Doppler radar, meteorologists can also study the changes that take place in a thunderstorm before a tornado forms. Pairs of similar radars at different locations can allow meteorologists to estimate both the horizontal and vertical movements of the wind.

Some radar systems can scan a section of the sky rapidly. Rapid-scanning radar enables scientists to record and study even the fastest-forming twisters.

A few radar systems enable scientists to distinguish among airborne debris, drops of drizzle, hailstones, insects, and raindrops. These radars scan by transmitting polarized beams of radiation both vertically and horizontally. Most Doppler radars typically use only horizontally polarized beams. Beginning in 2011, the National Weather Service is updating its network of Doppler radars to include this capability.

Clean air contains few particles large enough for regular radar to detect. To map winds that carry little moisture or dust, meteorologists use a Doppler *lidar* system. A lidar is a system similar to radar but uses a signal with a much shorter wavelength. Even clean air has extremely small particles called *aerosols* floating in it. Lidar's shorter wavelength signal can detect these tiny particles. But lidar cannot penetrate far into clouds, rain, or hail.

A man loads a weather probe into his tornado-chasing van.

Researchers may record a tornado and the flying debris around it on film or video to help analyze wind patterns. They then compare the film with the radar images of the storm. Scientists may also survey the damage done by a tornado and compare it with the radar data.

The VORTEX projects. Beginning in 1994, scientists carried out two major research projects to study tornadoes. The projects were called the Verification of the Origins of Rotation in Tornadoes Experiment (VORTEX). The first project, VORTEX1, took place in 1994 and 1995. Scientists traveled in both ground-based vehicles and aircraft equipped with customized weather instruments. The project produced insight into tornado formation. It also promoted advancements in Doppler radar.

VORTEX2 followed in 2009 and 2010. More than 100 scientists traveling in dozens of vehicles participated. Unlike many earlier experiments, VORTEX2 did not have a home base. Instead, the storm chasers roamed from place to place. They used weather forecasts and their own instruments to position themselves in areas where tornadoes would likely form.

The VORTEX2 team used a vast array of scientific equipment. Radar, lidar, video, and film recorded any storms. Mobile radars scanned supercells likely to produce tornadoes. Instruments called *disdrometers* measured the size of raindrops or hail in storms. Weather balloons launched in various locations recorded environmental conditions. VORTEX2 also used remotely controlled aircraft called *unmanned aerial vehicles* (UAVs) to fly near storms and gather meteorological data.

The VORTEX2 team also released nearly 40 instrument packages, called Sticknets and Tornado Pods, where the team hoped a tornado would pass. The instruments provided information about conditions inside the funnel cloud, such as air pressure and humidity.

Computer modeling. Meteorologists make extensive use of *computer models* to study tornadoes. A computer model is a set of mathematical equations processed by computers. Scientists can easily vary the conditions in the models to test their theories. This use of computer modeling is known as *simulation*. For example, scientists can simulate supercells, storms, and smaller-scale vortexes. Most models are so complex that they require fast computers and a large amount of data.

Analysis of the models helps scientists to understand the processes by which tornadoes form and change. Computer modeling can also help predict the weather events that might occur under certain conditions and the likelihood that an event will actually occur. Thus, forecasters can determine the probability of a tornado occurring in a certain area during a certain period.

Vortex chambers. Meteorologists simulate some aspects of tornadoes in the lab using special chambers of rotating air currents. These devices are called *vortex chambers*. By varying the strength of an exhaust fan above the chamber and the angle at which air enters at the bottom, a scientist can re-create different types of tornado structures. In one common structure, air sinks outside the tornado and rises at the center. In another common structure, air sinks at the center of the tornado and rises outside the center.

A mathematical technique called a *large-eddy simulation* (LES) modeling can show what happens when a rotating column of air comes into contact with the ground. Such models also explain aspects of tornado structure, such as how wind speeds vary with altitude. Based on LES models, scientists have developed new theories about tornado formation. For example, one theory proposes that tornadoes may develop when air near the ground is blocked from entering the vortex. Another factor may be the roughness of the surface, such as the amount and heights of buildings, grass, or trees in an area.

MLA Citation

"Hurricanes and Tornadoes." *The Southwestern Advantage Topic Source*. Nashville: Southwestern. 2019.

ADDITIONAL RESOURCES

Books to Read

Emanuel, Kerry A. *Divine Wind: The History and Science of Hurricanes*. Oxford, 2005.
Fradin, Judith B. and Dennis B. *Hurricanes*. National Geographic Society, 2007.
Longshore, David. *Encyclopedia of Hurricanes, Typhoons, and Cyclones*. Rev. ed. Checkmark, 2008.
Treaster, Joseph B. *Hurricane Force: In the Path of America's Deadliest Storms*. Kingfisher Books, 2007.

Web Sites

Atlantic Oceanographic & Meteorological Laboratory

http://www.aoml.noaa.gov/hrd/tcfaq/tcfaqHED.html

Site on hurricanes, typhoons, and tropical cyclones.

Hurricane Hunters

http://www.hurricanehunters.com/

Site of the Air Force Reserve Hurricane Hunters—the only Department of Defense organization still flying into tropical storms and hurricanes.

National Hurricane Center

http://www.nhc.noaa.gov/

A government site that predicts and reports on hurricanes.

Tropical Storms Worldwide

http://www.solar.ifa.hawaii.edu/Tropical/tropical.html

A University of Hawaii Web site reporting on current tropical storms worldwide.

DATA

The Saffir-Simpson Hurricane Scale

Hurricane category	Wind speed (Mph)	Wind speed (Kph)	Storm surge (Feet)	Storm surge (Meters)	Effects
Category 1 (weak)	74–95	119–153	4–5	1.2–1.5	Minimal damage to trees, shrubbery, and mobile homes.
Category 2 (moderate)	96–110	154–177	6–8	1.8–2.4	Considerable damage to trees, mobile homes, and piers; some damage to roofs.
Category 3 (strong)	111–130	178–209	9–12	2.7–3.7	Trees blown down or stripped of leaves; mobile homes destroyed; some damage to other buildings.
Category 4 (very strong)	131–155	210–250	13–18	4.0–5.5	Extensive damage to windows, doors, and roofs, especially near shore; possible flooding.
Category 5 (devastating)	156+	251+	19+	5.8+	Small buildings overturned or blown away; severe structural damage to other buildings.

The Enhanced Fujita Scale

Scale	Damage	Wind speed (Mph)	Wind speed (Kph)
EF0	Light: Tree branches broken, damage to chimneys and large signs.	65–85	105–137
EF1	Moderate: Trees snapped, surface of roofs peeled off, windows broken.	86–110	138–177
EF2	Considerable: Large trees uprooted, roofs torn off frame houses, mobile homes demolished.	111–135	178–217
EF3	Severe: Roof and some walls torn off well-constructed houses, cars overturned.	136–165	218–266
EF4	Devastating: Well-constructed houses leveled, cars and large objects thrown.	166–200	267–322
EF5	Incredible: Strong frame houses lifted off foundation and destroyed, car-sized objects thrown more than 300 feet (90 meters).	Over 200	Over 322

DATA

Some Famous Hurricanes, Typhoons, and Cyclones

1900	A hurricane and storm surge killed about 6,000 people in the Galveston, Texas, area.
1928	About 1,800 people died in a hurricane and floods in the Lake Okeechobee area of Florida. The storm also killed 300 people in Puerto Rico.
1938	A hurricane tore through Long Island and New England, killing about 600 and causing $400 million in damage.
1944	A typhoon in the Philippine Sea sank three destroyers and wrecked more than 100 aircraft of the U.S. Pacific fleet; 778 lives were lost.
1963	Hurricane Flora killed about 5,000 people in Haiti, more than 1,700 in Cuba, and more than 400 in the Dominican Republic.
1969	Hurricane Camille killed more than 250 people in seven states from Louisiana to Virginia. It caused about $1 billion in damage.
1970	The storm surge from a tropical cyclone drowned about 266,000 people in East Pakistan (now Bangladesh).
1972	Floods of Hurricane Agnes killed 122 people and caused $3 billion in damage from Florida to New York.
1974	Hurricane Fifi struck Honduras, killing about 8,000 people and causing $1 billion in damage.
1975	Floods well inland caused by Hurricane Agnes killed 117 people from Florida to New York.
1979	Hurricane David battered the Dominican Republic, killing 1,200 and causing $1 billion in damage. Hurricane Frederic struck Alabama and Mississippi, killing 8 and causing $1 billion in damage.
1980	Tropical Cyclone Hyacinthe looped around Reunion Island in the Indian Ocean, delivering 252 inches of rain in just 15 days.
1988	Hurricane Gilbert, the most powerful hurricane ever recorded in the Western Hemisphere, struck the West Indies and Mexico, causing about 300 deaths.
1992	Hurricane Andrew struck the Bahamas, Florida, and Louisiana, killing 65 and causing about $26 billion in damage.
1997	Typhoon Linda battered Vietnam's southern coast, killing more than 600 people.
1998	A cyclone devastated the western India province of Gujarat, killing more than 2,000 people.
1998	Storms and flooding from Hurricane Mitch caused billions of dollars worth of damage and killed more than 11,000 in Central America.
1999	About 9,600 people died in a cyclone and floods in the eastern India province of Orissa.
2004	Hurricanes Charley, Frances, Ivan, and Jeanne hit the U.S. Southeast in 2 months, causing over $42 billion in damage and killing more than 150 people. Jeanne killed more than 3,000 Haitians.
2005	Hurricane Katrina struck New Orleans and other parts of the U.S. Gulf Coast, killing more than 1,800 people and causing an estimated $100 billion of damage. Weeks later, Hurricane Rita caused an estimated $10 billion of damage in Texas and Louisiana.
2008	Cyclone Nargis struck the coast of Myanmar near Yangon, killing about 140,000.
2012	Hurricane Sandy produced widespread wind damage and unprecedented flooding from South Carolina to Maine. The storm moved inland, causing blizzard conditions in the higher elevations of the Applachian region.

BACKGROUND INFORMATION

The following articles were written during the year in which the events took place and reflect the style and thinking of that time.

Weather (1992)

The year 1992 began with the warmest winter on record in the contiguous United States, surpassing by nearly 1°F (0.5°C) the previous record set in the winter of 1953–1954. Above-normal temperatures prevailed in every major region of the nation, and five Midwestern states recorded their warmest winter since observations began. Only in northern Maine did winter temperatures average below normal.

Farther east, below-normal temperatures were the rule. Gander, Canada, experienced its coldest February on record. One of the worst winter storms in decades occurred in Canada's Atlantic Provinces from January 31 to February 3, 1992. Pressure in the offshore storm center dropped to 962 millibars (28.41 inches) of mercury. Wind gusts reached 96 miles per hour (mph) (155 kilometers per hour [kph]), and snowfall at Moncton, Canada, totaled 63 inches (160 centimeters).

In some areas of the northwestern United States, the winter was one of the driest on record, but northern Mexico and the southern fringe of the United States experienced heavy rainfall. Texas and New Mexico had their wettest recorded winter. On January 17 and 18, a vigorous storm along the northern Gulf of Mexico brought one of the 10 heaviest snowfalls of the 1900s—up to 8 inches (20 centimeters) to northern Texas and eastward to northern Georgia.

Heavy precipitation continued into much of the spring. In mid-May, a series of storms caused extensive flooding of the San Antonio and Guadalupe rivers. By the end of May, the city of San Antonio had accumulated more than 33 inches (84 centimeters) of rain, exceeding the city's normal yearly total.

Cool, wet summer. In contrast to the warm winter, the three midsummer months were the coolest since 1915 in the contiguous United States as a whole, and the third-coldest summer since recordkeeping began. The lowest temperatures were concentrated mainly in the east, however, with above-average temperatures prevailing in Nevada and the West Coast states. In the north-central states, the temperature for the season averaged up to 7°F (4°C) below normal.

For the contiguous United States as a whole, it was also the wettest summer since 1941. Precipitation was more than 50 percent greater than normal in the Southwest, Central Plains, and the Southeast.

Hurricane Andrew. The 1992 Atlantic hurricane season was slow to start. But on August 23, the year's first tropical storm, Andrew, suddenly intensified as it moved over the island of Eleuthera in the Bahamas with a 23-foot-high (7-meter-high) storm surge. Early the next morning, Andrew, now a hurricane, moved inland through a densely populated area south of downtown Miami. During the hurricane's four-hour trek across southern Florida, steady winds of an estimated 140 mph (225 kph) prevailed, with some gusts measured at 164 mph (264 kph). An estimated 72,000 homes, including mobile homes, were damaged or destroyed, and 200,000 people were left homeless. The estimated damage total of up to $20 billion made Andrew the costliest disaster in U.S. history.

After leaving Florida, the storm moved northwest across the Gulf of Mexico and made a second landfall in southwestern Louisiana during the late evening of August 25. Moving more slowly, the storm weakened quickly as it continued northward. After sending heavy rain and several tornadoes along its path, it finally died out in Pennsylvania.

Florida's well-executed evacuation plan helped keep the official death toll relatively low. Thirteen people were killed in Florida, and one person was killed in Louisiana. Perhaps as many as 39 people died either as a direct or indirect result of the storm.

Altogether, the Atlantic hurricane season produced only five storms. One of these, Danielle, lashed coastal areas with gales from Cape Hatteras, North Carolina, to eastern Pennsylvania on September 25 and 26.

Hurricane Iniki. Hawaiians on the island of Kauai had to cope with Hurricane Iniki, which roared across the Pacific on September 11 with 130-mph (209-kph) sustained winds. The storm left an estimated 8,000 of the island's 52,000 inhabitants homeless.

Collection of golf ball-size hailstones

Typhoons. In 1992, the western third of the tropical Pacific Ocean had a record number of typhoons. Guam weathered six in three months. The first one, named Omar, struck on August 28, with winds up to 150 mph (240 kph), causing major damage to more than 75 percent of the island's buildings. From October 9 to November 23, five more typhoons struck the island, but damage was less severe.

Hailstorms and tornadoes. On March 6, 1992, portions of Seminole County in central Florida were buried more than ankle deep in golf ball-size hailstones. Buildings and crops suffered $25 million in damage. On March 25, hail caused $60 million in damages to the nursery industry in the Orlando, Florida, area.

By December, there had been more confirmed tornado occurrences across the United States than in any previous year. In June, 399 tornadoes were sighted, a record for that month. From June 15 through 18, large hailstones, strong winds, and more than 200 tornadoes hit the area from the central and northern Plains eastward into the Appalachian Mountains.

November, usually a quiet month for tornado activity, brought 173 reports. Most of these occurred from November 20 to 24 in 12 states, including Mississippi, Georgia, North and South Carolina, Kentucky, and Tennessee. At least 25 people were killed.

Drought and floods. As 1992 began, California was experiencing its sixth winter in a row with below-normal rainfall. The snowpack in the Sierra Nevada, which feeds California's reservoirs as it melts throughout the year, was only 45 percent of its normal size.

Then, from February 5 to 21, a series of heavy rains caused flooding and mud slides in several communities northwest of Los Angeles. Above-normal levels of precipitation continued through most of March. Nevertheless, for the 1991–1992 water season—which began on October 1, 1991, and ended on September 30, 1992—the total precipitation was only 90 percent of normal, not enough to end the water shortage that had persisted since 1986.

In Oregon, Washington, Idaho, and Nevada, where much smaller amounts of precipitation fell, the drought became extreme. Hawaii experienced drought conditions as well.

Every country in southern Africa was affected by the worst growing-season drought of the 1900s. Crop production was estimated to be only 60 percent of the amount in 1991. In Zimbabwe, the hardest-hit area, rainfall was less than 20 percent of normal. Thousands of cattle deaths were reported.

In mid-September 1992, heavy flooding reportedly killed 2,000 people in northern Pakistan and 500 people in India. About 1,800 villages in the Indian state of Punjab were reportedly washed away by tidal surges that rushed down rivers. Every river bridge and half the crops in the state of Jammu and Kashmir were reported to have been destroyed.

Record-breaking December storm. The worst east-coast winter storm in decades, and probably the worst ever in December, developed and stalled over northeastern Virginia on December 11. High winds and heavy snow caused 640,000 power outages in Pennsylvania. The snow accumulated to more than 36 inches (91 centimeters) in western Pennsylvania and the mountains of southern New England. Coastal areas in New Jersey and the New York metropolitan area were extensively flooded.

Trying to explain the weather. The warm winter of 1991–1992 in the United States fueled the ongoing debate about global warming, the theory that increasing levels of carbon dioxide in the atmosphere are causing Earth's average surface temperature to rise. As cold weather became entrenched in the eastern half of the United States, however, meteorologists focused their interest on the June 1991 eruption of Mount Pinatubo in the Philippines. The eruption had sent dust and gases high into the atmosphere, forming a cloud that circled the globe. Scientists believed that the cloud may have caused a temporary lowering of Earth's average temperature of about 1.8°F (1°C). This would not account for the low summer temperatures of 1992 in the United States and Canada, however.

The most likely cause of 1992's unusual weather patterns was the reappearance of El Nino, a vast area of warm water in the tropical Pacific Ocean that appears at irregular intervals of 2 to 12 years. Meteorologists believed that the droughts in Hawaii and southern Africa and the heavy precipitation along the southern fringe of the United States were by-products of an El Nino that appeared in November 1991.

Weather (2005)

The 2005 hurricane season in the Atlantic produced more storms than had occurred in any other year since record keeping began in 1851. Twenty-seven named storms formed—including a record 14 hurricanes—and one storm, Katrina, produced one of the greatest natural disasters in the history of the United States.

Tornadoes. For the first time since record keeping began in 1950, no tornado-related deaths occurred during the height of the season (from April to June) despite more than 500 twisters. However, five separate tornado outbreaks led to 27 deaths in November 2005, including 23 deaths near Evansville, Indiana, on the 12th.

The Atlantic hurricane season of 2005 was unprecedented in how early it began, the high intensity of the storms in the Gulf of Mexico, and the damage that they caused in the United States. The first named tropical storm, Arlene, formed in the western Caribbean on June 9. Arlene crossed western Cuba before making landfall near Pensacola on June 11 with winds of 60 miles (97 kilometers) per hour.

The season's first hurricane, Dennis, formed in the eastern Caribbean on July 6. The storm swept ashore near Cienfuegos, Cuba, as the strongest Atlantic hurricane recorded so early in the year, a Category 4. (A Category 4 hurricane, according to the Saffir-Simpson hurricane classification system, generates winds of 131 to 155 miles [210 to 249 kilometers] per hour.) Dennis had weakened to a Category 3 storm, with winds of 111 to 130 miles (179 to 209 kilometers) per hour when it made landfall near Pensacola on July 10. Nevertheless, Dennis caused less damage than Category 3 Hurricane Ivan, which struck the same area in 2004.

As Dennis dissipated, Tropical Storm Emily formed in the eastern Atlantic in the second week of July. Emily became a Category 4 hurricane as it moved through the Caribbean, striking Jamaica. The storm made a second landfall in Mexico about 75 miles (121 kilometers) south of Brownsville, Texas, as a Category 3 hurricane on July 20. By the end of July, seven Atlantic storms had been named, a record for so early in the season.

Katrina, a storm destined to become one of the most destructive acts of nature on record in the United States, made landfall in Florida near North Miami Beach on August 25 as a Category 1 hurricane, with winds of 74 to 95 miles (119 to 153 kilometers) per hour. The storm moved southwest across Florida, where it was blamed for 11 deaths.

Homes were destroyed by the battering of hurricanes Ivan and Katrina, Dauphin Island, Alabama.

Over the eastern Gulf, Katrina intensified into the fourth strongest hurricane on record. The eye of the hurricane slammed into the southeastern Louisiana coast on August 29, 2005, with winds of 127 miles (204 kilometers) per hour. The storm center passed just east of New Orleans before moving into Mississippi. The *storm surge* (a sudden rise of ocean water that accompanies a hurricane at landfall) reached 20 to 30 feet (6 to 9 meters) along parts of the Mississippi coast, destroying all structures close to the beach.

In New Orleans, the combination of heavy rain and roiling waters in Lake Pontchartrain caused the levee system to fail at several points. Water inundated about 80 percent of the city, and thousands of stranded people had to be rescued by boat and helicopter in the days following the storm. Hurricane Katrina caused at least 1,200 deaths in Mississippi and Louisiana, as well as economic losses estimated at $100 billion to $200 billion.

Hurricane Rita, which formed in the eastern Atlantic, intensified as it swept by the Florida Keys on September 20 and 21. In the central Gulf of Mexico, the storm grew to be the third strongest hurricane on record. Rita made landfall near the Texas-Louisiana border on September 24 as a Category 3 storm, with peak winds of 120 miles (193 kilometers) per hour. The storm surge flooded much of the southwestern Louisiana coast.

In October, Hurricane Wilma became the most powerful storm ever recorded in the Atlantic, with winds of 175 miles (282 kilometers) per hour. At least 12 people were killed in Mexico and Cuba before Wilma weakened and struck the southwest coast of Florida near Naples as a Category 3 storm on October 24. The hurricane crossed Florida, causing more than 30 deaths and extensive damage.

In all, 27 named storms formed in the Atlantic in 2005, breaking the previous record of 21 storms set in 1933. After the U.S. National Hurricane Center exhausted its list of 21 names, meteorologists began using letters of the Greek alphabet—from Alpha to Zeta—to name the storms.

Global Warming

HOT topics

Preventive Measures

Carbon Sequestration

The Kyoto Protocol

Melting Arctic Ice

Global warming is the gradual increase in the temperature of the earth's atmosphere caused by the concentration of gases, especially carbon dioxide, that absorb heat from the sun. The human consumption of fossil fuels such as coal, oil, and gas has magnified this greenhouse effect, with consequences for the earth's climate.

HOT topics

Preventive Measures. Global warming has been accelerated by human activities that have increased the concentration of carbon dioxide in the earth's atmosphere. Quantities of carbon dioxide can be limited by replacing fossil fuels with energy sources that do not emit CO_2, such as wind and solar power, nuclear energy, and the geothermal generation of power from underground steam.

DEFINITIONS

carbon dioxide: a heavy, colorless, odorless gas, present in the atmosphere or formed when any fuel containing carbon is burned; carbonic-acid gas. The air that is breathed out of an animal's lungs contains carbon dioxide. Plants absorb it from the air and use it to make plant tissue. Carbon dioxide is used in soda water and in fire extinguishers.*Formula:* CO_2

fossil fuel: coal, oil, or natural gas.

global warming: the gradual warming of the earth's atmosphere caused by the concentration of gases, especially carbon dioxide, that absorb heat from the sun: Scientists were warning that continued release of carbon dioxide from burning large amounts of fossil fuels could cause a global warming (Rod Such).

greenhouse effect: the absorption and retention of the sun's radiation in the earth's atmosphere, resulting in an increase in the temperature of the earth's surface.

HOT topics

Carbon Sequestration. The amount of carbon dioxide released into the atmosphere can be limited by the development of ways to store (sequester) carbon and keep it from entering the air. Possible carbon storage methods include injecting industrial emissions of CO_2 into underground geologic formations, such as the cavities left by the extraction of coal and gas from the ground. Carbon dioxide could also be deposited into the earth's oceans, although the environmental consequences of carbon sequestration for ocean life remain unknown. Plants are the earth's own natural carbon containers. Forests and croplands are capable of storing great amounts of CO_2. However, the long-term reliability of this method requires keeping the earth's ecosystems (such as its rain forests) intact.

The Kyoto Protocol. The Kyoto Protocol calls for industrialized countries to restrict their emissions of CO_2. It also establishes systems by which countries can earn and trade emissions credits. By 2005, more than 100 countries, including Japan and Russia, had ratified the Kyoto agreement. The United States, however, refuses to support the protocol. According to measurements taken in 1990, of all the industrialized countries in the world, the United States was responsible for the greatest percentage of CO_2 emissions. U.S. President George W. Bush stated that the requirements of the Kyoto Protocol would be damaging to the U.S. economy while failing to impose tough enough environmental restrictions on developing nations.

Members of Japanese environmental group celebrating the Kyoto Protocol

Melting Arctic Ice. Often described as the barometer of global environmental health, the Arctic is experiencing dramatic changes in both its seasonal and year-round ice packs. For indigenous people, mushy ice and early thaws have made hunting on what used to be well-known ice highways more difficult and dangerous. Deteriorating ice is also threatening numerous animal species that rely on it for their survival. Polar ice melt is likely the cause of a startling drop in the average weight of polar bears, whose shortened hunting season may be contributing to an observed decline in birth and survival rates for the bears, as well as other species whose life cycles depend on the ice.

TRUE or FALSE?

Global warming is raising the temperatures of the world's oceans, which promotes the spread of diseases harmful to sea life.

THE BASICS

Global warming is an increase in the average temperature at Earth's surface. People often use the term *global warming* to refer specifically to the warming observed since the mid-1800s. Scientists estimate that Earth's average surface temperature rose by about 1.4°F (0.76°C) from the mid-1800s to the early 2000s. Researchers have also found that most of the temperature increase occurred from the mid-1900s to the 2000s.

Natural processes have caused Earth's climate to change in the distant past. But scientists have found strong evidence that human activities have caused most of the warming since the mid-1900s.

Scientists predict that Earth's average surface temperature will rise an additional 2.0 to 11.5°F (1.1 to 6.4°C) by 2100. They also predict that, if warming continues unchecked, it will damage human society and the environment. For example, global warming could melt enough of the ice on land near Earth's poles to raise sea level. It could lead to more widespread droughts. It could also risk extinction for many plant and animal species.

Researchers have developed a number of ways to limit global warming. But because the warming is a global problem, many strategies require the cooperation of a diversity of nations, each with its own interests. Nevertheless, many countries are taking action individually and by international agreement to limit future warming.

Causes of global warming. Global surface temperatures have risen chiefly because of a process called the *greenhouse effect.* In the greenhouse effect, certain gases in the atmosphere trap heat from the sun. They act much like the glass roof and walls of a greenhouse. The heat-trapping gases are called *greenhouse gases.* They include methane (CH_4), nitrous oxide (N_2O), and ozone (O_3). But the gas that has produced the most warming is carbon dioxide (CO_2).

Natural concentrations of greenhouse gases in the atmosphere help keep the planet warm enough to support life. Levels of greenhouse gases in the atmosphere have varied at different times. But they held relatively stable for several thousand years before industry began to grow rapidly in the 1800s.

Since the mid-1800s, however, modern industry has caused significant increases in *emissions* (releases) of greenhouse gases. The increase in CO_2 levels comes chiefly from the burning of *fossil fuels.* The fossil fuels are coal, oil, and natural gas. They contain carbon, and burning them creates carbon dioxide. CO_2 levels also rise due to the clearing of land. Trees and other green plants remove CO_2 from the air during *photosynthesis*—the process they use to produce their food. Thus, as land is cleared and forests are cut down, more CO_2 remains in the atmosphere.

Not all human activities contribute to global warming. Some things that people do actually cool Earth's surface. For example, many *aerosols* (suspensions of tiny particles) enter the atmosphere from automobile exhaust and factory smoke. The aerosols encourage the formation of clouds. Both aerosols and clouds reflect the sun's heat back into space. As a result, they exert a cooling influence on Earth's surface. But researchers estimate that overall, human activities have caused far more warming than cooling.

Scientists have also compared the influences of human activities on climate with the influences of natural processes. The only significant natural process was changes in the sun's energy output. The scientists found that human activities—mainly greenhouse gas emissions—produced more than 10 times the warming influence of changes in solar output.

Researchers have linked global warming to a number of potentially damaging effects on living things and their *ecosystems.* An ecosystem consists of a community of organisms along with its physical environment. Global warming is raising sea level. In addition, it is rapidly affecting the Arctic region. Global warming may also alter weather patterns and affect human health around the world. Scientists project that these effects will intensify and spread with further warming.

Effect on plant and animal life. Global warming affects many plants and animals by causing seasonal changes in temperature to occur at different times of the year. In many parts of the world, for example, warm spring weather has come earlier in the year. As a result, events that occur in the spring have begun earlier. Such events as flowering, egg laying, migration, and the growth of new leaves happen earlier. For example, lilac bushes have flowered earlier in the spring across much of the United States.

Butterflies have appeared earlier in the United Kingdom.

Rising temperatures have also forced many animals to move to cooler areas. These animals may move to higher latitudes—that is, toward Earth's poles. Or they may move to higher elevations. For example, white storks are nesting higher in the mountains in Poland. In Australia, the large bats called *flying foxes* have migrated toward cooler conditions in the south. Scientists are concerned about organisms that have difficulty spreading to new places. Many land animals and plants cannot easily relocate.

The faster global warming occurs, the more difficult it will become for many species to adapt. Scientists project that 20 to 30 percent of species would face a higher risk of extinction if the average temperature rose more than 2.7 to 4.5°F (1.5 to 2.5°C).

Rise in sea level. Average sea level rose about 7 inches (17 centimeters) over the 1900s. Global warming contributed to the rise. Part of the increase occurred because water expands as it warms. Rising temperatures also melt ice on land, which then flows into the oceans. Rising sea level has already contributed to coastal flooding, erosion, and the loss of wetlands.

Researchers project that further warming could cause sea level to rise another 7 to 23 inches (18 to 59 centimeters) by 2100. However, the projections do not include possible contributions from the melting of major ice sheets in Greenland and Antarctica. Increases in the rates at which the sheets are melting could lead to an additional rise in sea level.

Rapid change in the Arctic. Temperatures in the Arctic have increased about twice as fast as the global average. Due to the rapid warming, the area of the Arctic covered by sea ice in the summer has dropped since the late 1970s. The melting of sea ice has little effect on sea level because the ice already floats on the ocean. But the loss of sea ice threatens many Arctic species. The threatened species include polar bears, which hunt on the ice, and seals, which give birth on it.

Harm to ocean life. Warming seas have damaged marine ecosystems, particularly coral reefs. High ocean temperatures can cause *coral bleaching.* In this process, corals lose the colorful algae that live inside them and provide them with food. If temperatures remain too high, the corals turn white and die. Scientists project that only about 2 to 5°F (1 to 3°C) of further warming of the ocean surface could cause the death of many coral reefs. This fact is of particular concern because the reefs provide a habitat for huge numbers of ocean species.

Changing weather patterns. Researchers predict that global warming will lead to more extreme weather. Warming could increase the frequency of heavy rain and snowfall. It could also cause more frequent and intense heat waves, more floods, and more widespread droughts. These impacts could strain water supplies, damage crops, and harm ecosystems.

Threats to human health. More frequent and intense hot days and heat waves can contribute to heat-related death and illness. Scientists also project more deaths and diseases caused by storms, floods, droughts, and fires. On the other hand, higher temperatures can reduce cold-related deaths. However, researchers predict that the harmful effects of rising temperatures on human health will outweigh the benefits.

To limit global warming, emissions of greenhouse gases must slow to a rate that allows atmospheric levels to stabilize. But emissions are instead rising rapidly. In addition, because the climate system changes slowly, there is a delay between an increase in greenhouse gas levels and the full

Damsel fish swimming over bleached coral off the [illegible] Islands temperature rise from that increase. As a result, some warming will continue even if greenhouse gas levels stabilize. Scientists project that even if concentrations stay at their 2000 level, an additional warming of 1°F (0.6°C) would occur in this century.

Scientists have studied several ways to limit global warming. The most obvious method is to limit CO_2 emissions. Another method, called carbon *sequestration,* involves preventing CO_2 from entering the atmosphere or removing CO_2 already there. A third method, *geoengineering,* involves altering the environment to counteract warming.

Limiting CO_2 emissions chiefly involves burning less fossil fuels. This can be done by using alternative energy sources, using fossil fuels more efficiently, and practicing energy conservation.

Alternative energy sources that do not emit CO_2 include the wind, sunlight, nuclear energy, and Earth's internal heat. Devices known as *wind turbines* can convert wind energy to electric energy. *Solar cells* can convert sunlight to electric energy. *Geothermal* energy systems extract energy from steam or water heated deep underground. Many alternative sources of energy cost more to use than do fossil fuels. However, increased research and use is reducing the cost of alternative sources.

Increased fuel efficiency. CO_2 emissions could be reduced if vehicles used fuel more efficiently. Researchers are developing devices to replace fuel-burning engines or to make them more efficient. Fuel-saving cars known as *hybrids* have already entered the market. A hybrid combines the components of a battery-driven electric car with a small gasoline engine. Scientists are also studying the use of *biofuels* from crops or other plant material. Biofuels might replace gasoline and other fuels. Future cars may use *fuel cells,* devices that convert chemical energy to electric energy.

Hybrids use both electricity and gasoline.

Personal energy conservation saves energy, reducing the need to burn fossil fuels. For example, you can turn off and unplug computers, televisions, and other electronic devices when you are not using them. You can turn off lights when you are not in a room. You can replace traditional light bulbs with compact fluorescent bulbs, which use less energy. Outside the home, you can cut emissions by walking or biking instead of driving. If you must drive, you can carpool or combine multiple errands in one trip. Public transportation may also help reduce emissions in some areas.

Carbon sequestration requires a place to store unwanted CO_2. It could be stored underground, underwater, or in living plants.

Underground or underwater sequestration would involve injecting industrial emissions of CO2 into underground rock formations or into the ocean. Suitable rock formations include natural petroleum reservoirs from which most of the oil or gas has been removed. Pumping CO_2 into such a reservoir would have the added benefit of making it easier to remove the remaining oil or gas. But CO_2 injected in this way could escape due to an earthquake or other disruption. Some scientists think that CO_2 could also be stored in layers of *basalt,* a type of volcanic rock. The basalt could chemically convert the CO_2 gas into solid salts that would be unlikely to escape.

CO_2 readily dissolves in water, and the oceans naturally store much of the gas. Scientists are examining the possibility of sequestering carbon by pumping CO_2 directly into the deep sea. However, they must carefully study the effects on ocean life. For example, water and CO_2 combine to form carbonic acid, which would increase the acidity of the water.

Sequestration in living plants. Green plants absorb CO_2 from the atmosphere as they grow. They use the carbon for photosynthesis. Creating or enhancing ecosystems with abundant plant life, such as forests and even cropland, could therefore remove much carbon dioxide from the atmosphere. However, future generations would have to preserve the ecosystems to prevent the carbon from reentering the atmosphere as CO_2.

Geoengineering involves making large-scale changes to the environment to limit global warming. One approach would be to inject aerosols into the atmosphere to reflect sunlight and so cool the planet. Another scheme would involve adding iron to the oceans to promote the growth of *phytoplankton.* Phytoplankton are tiny marine organisms that capture CO_2 for use in photosynthesis. Another proposal is to put trillions of tiny screens in orbit around Earth to deflect some of the sun's rays.

Such proposals carry unknown risks and challenges. For example, injecting aerosols into the atmosphere could increase *acid rain*—rain and other precipitation polluted by certain acids. Acid rain can kill fish in lakes and streams. Adding large amounts of iron to the sea could harm marine environments. Putting sun screens into space would be expensive, and the screens would need maintenance.

Government Action on Global Warming

In an effort to limit global warming, many national and other governments have developed policies to reduce emissions of greenhouse gases. Most countries have agreed to the Kyoto Protocol, an international treaty aimed at cutting emissions.

The Kyoto Protocol requires *developed* (wealthy) countries to restrict their emissions of CO_2 and five other greenhouse gases. Different nations have different yearly emissions targets for the period from 2008 to 2012. As a whole, the developed countries must reduce their emissions to a yearly average of about 95 percent of their 1990 emissions. The protocol does not place restrictions on *developing* (less wealthy) countries.

Delegates from around the world adopted the Kyoto Protocol as a preliminary document in 1997 in Kyoto, Japan. To enter into force, the protocol had to be *ratified* (formally approved) by at least 55 countries. The countries that approved also had to account for at least 55 percent of the CO_2 emissions of all developed countries in 1990. Most nations eventually agreed to the protocol. It went into effect in 2005. However, the United States refused to ratify the treaty.

The Kyoto Protocol alone will do little to stop global warming. It limits emissions for only a short period. It limits emissions to levels too high to stop the increase in greenhouse gas concentrations. But the protocol establishes a basis for future measures. Delegates at an international climate conference in Bali in 2007 began negotiating a new agreement, for the period beyond 2012.

Logging may contribute to the greenhouse effect.

Types of policies. The Kyoto Protocol is a *cap-and-trade* system. It requires each developed country to *cap,* or limit, its emissions to a certain target. Nations that reduce their emissions below their targets can *trade,* or sell, *emissions credits* to other countries. Such credits can be used to offset emissions. Cap-and-trade policies can also be used to control emissions within one country or within a single industry.

A related approach enables groups and individuals to earn and sell credits called *carbon offsets.* Such a credit is similar to an emissions credit, but it is not earned by cutting one's own emissions. Instead, it is earned by undertaking other projects that help limit greenhouse gas levels overall. Carbon offsets often form part of cap-and-trade systems. Under the Kyoto Protocol, for example, developed countries can receive credit for investing in projects that reduce emissions in developing nations. Many companies now invest in emissions reduction projects to generate credits. They sell these credits to individuals and businesses that use them to offset emissions from such activities as air travel and power usage.

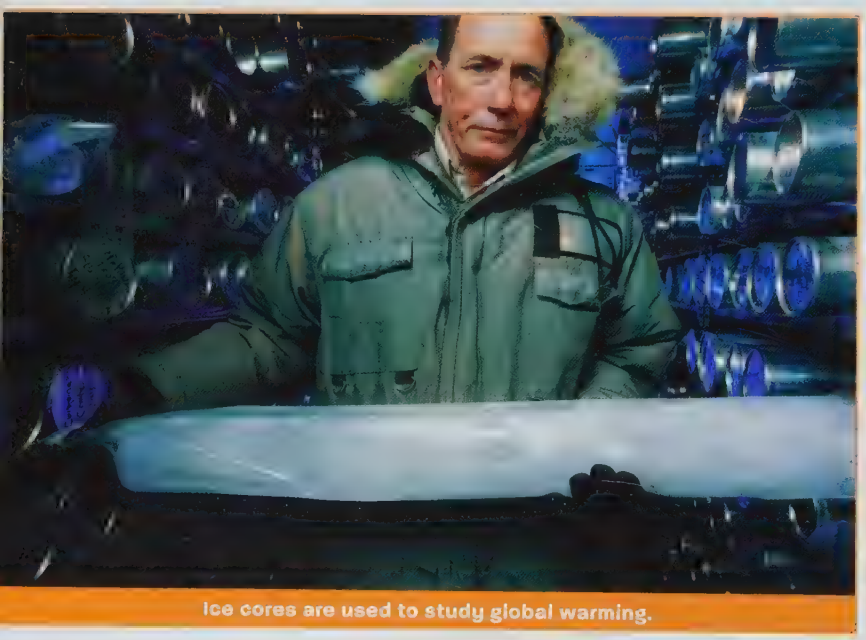
Another approach to limiting emissions is to impose a *carbon tax,* a tax on emissions of greenhouse gases. Such a tax can encourage companies to find ways to reduce emissions—ways that cost less than paying the tax.

Other measures. Some states, provinces, and countries have set goals that are more long-term than the Kyoto Protocol. For example, California has passed laws intended to reduce its total greenhouse gas emissions to 1990 levels by 2020. The state has proposed an 80-percent drop below 1990 levels by 2050. British Columbia has set a goal to cut emissions to 10 percent below 1990 levels, or 33 percent below 2007 levels, by 2020. The United Kingdom has proposed a 60-percent drop below 1990 levels for CO_2 emissions by 2050.

How Scientists Study Global Warming

Climatologists (scientists who study climate) use information from many sources to analyze global warming. The most reliable climate information comes from standardized measurements using weather instruments. But records based on reliable instruments date back only to the middle to late 1800s. Such records cannot show variations in climate that occurred long ago, before people began to measure with good instruments.

To look further back, climatologists analyze other types of evidence. Such evidence includes growth rings in trees, *cores* (cylindrical samples) of ice drilled from Antarctica and Greenland, and cores of sediment from ocean or lake beds. Tree growth rings, for example, can show when the weather was favorable for trees to grow rapidly. Ice cores hold tiny bubbles of air trapped when the ice formed. The bubbles can be analyzed to determine ancient greenhouse gas concentrations.


Ice cores are used to study global warming.

Evidence from these sources indicates that Earth is likely the warmest it has been in at least 1,300 years. It also shows that the temperature increase of the 1900s was the largest in more than 1,000 to 2,000 years. Data from ice cores also show that the levels of CO_2 and other greenhouse gases in the atmosphere are higher than they have been in at least the past 650,000 years.

Climatologists use computers to analyze past climate change and project future changes. First, scientists program a computer with a set of mathematical equations known as a *climate model.* The equations describe how various factors, such as the amount of CO_2 in the atmosphere, affect temperature. Next, the scientists enter data representing how those factors change over time. The computer then runs the model, describing how the climate conditions would vary. Such a representation is known as a *climate simulation.*

A complete understanding of global warming cannot be gained through simple observation. Its study relies on the scientific analysis of evidence and on climate simulations. This fact has enabled critics to dispute that warming has occurred. They also dispute that human activities caused the warming. To help establish a scientific *consensus* (general agreement), the United Nations established the Intergovernmental Panel on Climate Change (IPCC). The IPCC is a panel of scientists and governmental officials from more than 100 nations. It released several scientific reports during the 1990s and 2000s.

The IPCC conducted climate simulations as part of its studies. Its climatologists used different sets of simulations to model the warming of roughly the past 100 years. One set took into account both natural processes and human activities. Another set took into account only natural processes. The scientists then compared the temperatures produced by the simulations with temperatures recorded by thermometers. Only the set based on both natural processes and human activities corresponded closely to the actual temperature records. The results provided evidence that both natural processes and human activities have affected Earth's climate and that human activities are the main driver of the recent warming.

The IPCC also carried out simulations to predict the effects of natural and human influences on temperatures until 2100. The simulations showed that there can be no "quick fix" to global warming. Even if all emissions of greenhouse gases were to cease immediately, the temperature would continue to rise because of the greenhouse gases already in the atmosphere. However, the rate of emissions will determine the severity of global warming. The IPCC, along with former U. S. vice president Al Gore, won the 2007 Nobel Peace Prize for increasing public awareness of global warming.

THE KYOTO PROTOCOL

THE KYOTO PROTOCOL is an international agreement whose underlying purpose is to limit *global warming.* Global warming is an increase in the average temperature of Earth's surface since the late 1800s. The protocol requires many countries to limit their *emissions* (releases) of *greenhouse gases.* Such gases trap heat in the atmosphere, increasing Earth's surface temperature through a process known as the *greenhouse effect.* Human activities that increase greenhouse gas levels in the atmosphere include the clearing of land and the burning of *fossil fuels* (coal, oil, and natural gas). The Kyoto Protocol restricts emissions of carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and three other greenhouse gases. Most of the world's countries have agreed to the protocol. But a few nations, including the United States, have refused. The Kyoto Protocol entered into force in 2005.

The terms of the Kyoto Protocol distinguish between *developed* parties and *developing* parties. A developed party is a relatively wealthy country or group of countries. A developing party, in contrast, is one of the world's poorer nations. Under the protocol, each developed party must restrict its greenhouse gas emissions to a certain yearly target from 2008 to 2012. Parties that miss their targets under the Kyoto Protocol will be required to cut emissions by an even greater amount under the next such agreement. Each party's target is a certain percentage of its emission levels in 1990. As a whole, the developed parties must cut their emissions to an average of 5 percent below 1990 levels. Developing parties are not required to limit their emissions. The protocol places a heavier burden on developed countries because those countries can more easily pay the cost of cutting emissions.

Canadian Prime Minister Jean Chretien signs the Kyoto Protocol.

Developed countries must limit emissions within their own borders to meet their targets. But the protocol also allows countries to work with one another to earn and trade *emissions credits.* Developed parties can earn emissions credits by funding projects that reduce emissions in other nations. For instance, a developed party could help a developing country build a hydroelectric power plant instead of one that burns fossil fuels. In doing so, the developed party can earn emissions credits, which it could use to *offset* (make up for) emissions in excess of its target. Developed parties may also offset their emissions by enhancing *carbon sinks.* Carbon sinks are natural resources, such as forests, that remove greenhouse gases from the atmosphere.

Controversy. Some critics of the Kyoto Protocol doubt a connection between increased greenhouse gas emissions and global warming. To address concerns about the link between human activities and climate change, the United Nations (UN) created a committee called the Intergovernmental Panel on Climate Change (IPCC). The IPCC released several scientific reports during the 1990s and 2000s. The reports seek to establish a *consensus* (general agreement) among climate scientists from around the world. In its 2007 report, the IPCC cited a greater than 90-percent likelihood that most of the observed warming since the

The Larsen Ice Shelf is breaking apart.

mid-1900s is due to the human-caused increase in levels of greenhouse gases.

Some politicians have expressed concerns that the protocol could cause economic harm by discouraging the use of fossil fuels. The burning of such fuels is a major industrial activity in the United States and other developed countries. Other critics have claimed that the treaty does not require adequate participation from developing countries.

History. The Kyoto Protocol serves as an addition to the UN Framework Convention on Climate Change, a treaty adopted in 1992 to stabilize greenhouse gas concentrations. Delegates from more than 160 countries adopted the protocol at a 1997 meeting in Kyoto, Japan. To become legally enforceable, the protocol had to be *ratified* (formally approved) by at least 55 countries. In addition, the developed parties ratifying the protocol had to account for at least 55 percent of the total CO_2 emissions of all developed countries in 1990.

Delegates worked to complete the details of the agreement in a series of meetings. Most countries eventually agreed to the protocol, but some nations chose not to ratify it. These nations included the United States, whose 1990 emissions of CO_2 represented the highest share of the total for developed countries, 35 percent. In 1997, the U.S. Senate voted unanimously not to sign any agreement that did not include emissions reductions for developing countries or that would seriously harm the U.S. economy. In 1998, the administration of President Bill Clinton signed the Kyoto Protocol but did not submit it to the Senate for approval. In 2001, President George W. Bush rejected the Kyoto Protocol.

Rejection by the United States made it critical for other developed countries to ratify the protocol. Even if all the other developed countries except Russia ratified the protocol, the ratifying countries' 1990 emissions would fall short of the 55 percent required. In 2004, Russia's parliament voted to ratify the Kyoto Protocol, fulfilling the last requirement.

The protocol entered into force in 2005. In the same year, the European Union began working to meet its obligations by establishing its Greenhouse Gas Emission Trading Scheme, often called the EU ETS. The scheme sets limits on emissions by power plants and other large sources of greenhouse gases. It also enables facilities to buy and sell emissions allowances. The Kyoto Protocol also led to the creation of a global "carbon market" for the trading of greenhouse gas emissions. Developing countries began to participate in emissions reductions through hundreds of clean development projects.

In late 2005, delegates from the parties agreed to extend the Kyoto Protocol until the end of 2012. Parties to the protocol discussed extending the agreement again at the United Nations Climate Change Conference in Copenhagen, Denmark, in 2009, in Cancún, Mexico, in 2010, and in Durban, South Africa, in 2011. The discussions failed to produce an extension or an alternative to the protocol.

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ADDITIONAL RESOURCES

Books to Read

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Dutch, Steven I., ed. *Encyclopedia of Global Warming.* 3 vols. Salem Press, 2010.

Johansen, Bruce E. *The Encyclopedia of Global Warming Science and Technology.* 2 vols. Greenwood, 2009.

Simon, Seymour. *Global Warming.* Collins, 2010.

Web Sites

Global Warming

http://www.ncdc.noaa.gov/oa/climate/globalwarming.html

The U.S. National Oceanic and Atmospheric Administration (NOAA) presents answers to frequently asked questions about global warming.

Greenhouse Gas Online

http://www.ghgonline.org

A collection of articles on global warming from news media and peer-reviewed scientific journals.

U.S. EPA—Global Warming Site

http://www.epa.gov/climatechange

A U.S. Environmental Protection Agency site that provides information on current climate change issues as well as U.S. climate policy.

Global Warning: Early Warming Signs

http://www.climatehotmap.org

An interactive map of the world that illustrates the local consequences of global warming.

BACKGROUND INFORMATION

The following articles and features were written during the year in which the events took place and reflect the style and thinking of that time.

The Great Meltdown (2006)

Scientists warn that global warming is leading to the rapid and potentially catastrophic disappearance of ice sheets and glaciers worldwide.

—by Christina Johnson

The hard, cold fact about Earth's ice is that it is melting. In the Arctic, in the Antarctic, and on even the highest mountains, the ice cover is thinning, breaking up, and draining away. Ice in the Arctic Ocean is retreating so quickly that by 2100 the ocean could be ice-free in summer, opening the year-round seaway from the North Atlantic to the North Pacific oceans sought by early explorers to North America. From 1996 to 2006, the amount of Greenland's ice sliding into the North Atlantic each year jumped by about 250 percent, an amount equal to the volume of water in Lake Erie. The collapse of Antarctica's Larsen B ice shelf in 2002 dumped enough ice into the sea to give every person on Earth 1,800 bags of ice weighing 22 pounds (10 kilograms) each.

Most scientists agree that Earth's ice cover is eroding because of global warming—the increase of from 0.7 to 1.4 degrees Fahrenheit (0.4 to 0.8 degrees Celsius) in the planet's average surface temperature since the late 1800s. In fact, Earth is warmer now than at any time in the past 11,500 years, the National Academy of Sciences reported in September 2006. While cycles of cooling and warming as well as other natural processes affect Earth's climate, a widespread scientific consensus points to human activities, particularly emissions of heat-trapping greenhouse gases, as the main driving force behind the current warm-up. The most significant of these gases is carbon dioxide, produced by the burning of coal, gasoline, and wood.

Although the meltdown has—so far—hit hardest in the Arctic, its effects have been global and diverse. In 2005, the chairperson of the 150,000-member Inuit Circumpolar Conference said that the thinning and loss of Arctic sea ice (frozen seawater) was threatening the traditional lifestyles of indigenous (native) peoples. Also in 2005, scientists reported the first evidence that polar bears, which use sea ice as a platform for hunting seals, are drowning because of the growing distance between ice floes. Scientists also have linked a decline in sea ice to a 70 percent drop in the number of Adelie penguins living on one part of the Antarctic Peninsula. In the central Asian countries of Nepal and Bhutan, an international survey found at least forty-four mountain lakes filling so rapidly with meltwater from Himalayan glaciers that they could burst their banks within five years, causing potentially catastrophic floods in the populated valleys below.

Polar bears walking on melted ice

Earth is unlikely to become ice-free for at least a few centuries, even at current rates of warming. Nevertheless, the melting is seriously affecting human and animal populations as well as the environment. Numerous studies have predicted even more disturbing consequences for human health and safety as well as for ecosystems in the near future.

Ice in the Arctic. The Arctic, which is often described as the barometer of global environmental health, is experiencing dramatic changes in both its seasonal and perennial (year-round) ice packs. Seasonal ice, which grows and melts throughout the year, has been declining since at least 1978, the time when scientists began using satellites to monitor the ice. Historically, by March, all of the Arctic Ocean except for an area along the coasts of Norway and western Russia is locked in ice. By mid-September, only about 30 percent of the ice remains.

The amount of perennial ice surviving the summer melting season dropped by about 6.5 percent per decade from 1979 to 2001. After 2001, the retreat began to seem more like a rout. Between 2004 and 2005, perennial ice shrank by 14 percent to a record low of 2.05 million square miles (5.32 million square kilometers), according to a September 2006 report by scientists from NASA's Jet Propulsion Laboratory in Pasadena, California. The shrinkage represented an area equal to the size of Texas.

Just as disturbing, a 2006 study found that the amount of the Arctic Ocean covered by seasonal ice in March had dropped to its lowest average since 1978. The so-called winter ice maximum shrank by a record 6 percent—equivalent to an area roughly the size of New Mexico—from 2005 levels, the previous record. Moreover, in 2006 the winter ice shrank more than twice as fast as it did in the 1980s and 1990s.

The Arctic is experiencing "some of the most rapid and severe climate change on earth," according to the 2004 Arctic Climate Impact Assessment (ACIA). The ACIA emerged from a four-year study by hundreds of scientists working under a mandate from the United States and seven other Arctic nations, six organizations representing indigenous Arctic people, and 18 national academies of science. Signs of climate change documented by ACIA scientists included rising temperatures, shrinking and thinning sea ice, melting glaciers, and rising sea levels.

Melting's ripple effect. For indigenous people, mushy ice and unexpectedly early thaws have made hunting on what used to be well-known ice highways more difficult and dangerous. Some hunters, for example, have been forced to travel farther to find seals, walruses, and other Arctic prey. In addition, accidents on the ice are reportedly increasing, resulting in injuries and death, loss of valuable equipment, and expensive rescues, according to the National Snow and Ice Data Center (NSIDC) in Boulder, Colorado. The melting of the highways is also limiting the time available to truckers trying to deliver heating oil, gasoline, and other supplies to Far North communities. "The land is becoming a stranger to the Inuit," one Inuit official said in an interview.

Grey seal pups suffer because of the deteriorating sea ice.

Deteriorating sea ice is also threatening numerous animal species that depend on the ice for their survival. Seals give birth, raise their pups, and rest on the ice. In February 2006, Canadian scientists reported that about 1,500 grey seal pups were swept out to sea in a storm and drowned because a scarcity of sea ice forced females to give birth on the vulnerable beaches of a small island.

The increasingly earlier break-up of sea ice in the spring is likely the cause of a startling drop in the average weight of polar bears in western Hudson Bay. The bears feed almost exclusively in winter, when they hunt seals from the ice. Between 1980 and 2004, the average weight of adult female polar bears in that area fell from 650 to 507 pounds (295 to 230 kilograms). Because the female's ability to reproduce and care for cubs depends on her fat stores, a shortened hunting season may be contributing to an observed decline in birth and survival rates.

The dwindling of the ice has set off a scramble by the United States, Canada, Denmark, and other countries with borders in the Arctic to establish territorial claims in the Far North. Opening the Arctic, in addition to increasing ship traffic, would allow the development of what is reportedly about 25 percent of the world's remaining oil and gas reserves. Such development could further disrupt these troubled ecosystems.

Global changes. The effects of climate change in the Arctic may extend far beyond the frozen north. Although the Arctic Ocean is the smallest of the world's oceans, it plays a crucial role in regulating global climate. For example, the Arctic acts like a giant mirror, reflecting back into space up to 90 percent of incoming sunlight and, in the

Disappearing glaciers in the Himalayas could cause water shortages in the coming years.

process, helping to cool the planet. By contrast, the open ocean reflects only about 6 percent of the sunlight reaching it. In addition, thin ice, such as seasonal ice, reflects less solar energy than perennial ice. Using measurements made by U.S. submarines, scientists have found that, from the 1960s to the mid-1990s, the average thickness of Arctic ice in deep-water areas of the ocean dropped by about 40 percent, from 10 to 6 feet (3.1 to 1.8 meters). As the Arctic's ice cover disappears, more of the sun's heat is absorbed. The result is a "feedback loop" of rising temperatures that promote melting, which increases energy absorption and further raises temperatures.

Another possible consequence of Arctic melting involves the worldwide ocean current known as the thermohaline circulation (THC). Often described as a conveyor belt, the THC helps move solar energy from the tropics to polar regions and is partially responsible for the moderate winter temperatures in northwestern Europe, including the United Kingdom.

The THC exists because of regional differences in the density of seawater caused by temperature and salinity (saltiness). As seawater freezes, its salt separates from the growing ice crystals, creating briny (highly salty) pockets within the ice. Over time, the brine drips through the ice into the ocean, forming unusually dense layers of water. Briny water formed in the Arctic and other northern seas travels southward to the North Atlantic, where it joins and helps cool warm, salty water carried northward by the Gulf Stream. Several million cubic kilometers of the cold, briny water, which is denser than warm water, sinks toward the ocean bottom and begins moving south. After a 1,000-year journey through the South Atlantic, Indian, Pacific, and Southern (Antarctic) oceans, the water returns to the North Atlantic, where another circuit begins.

Many scientists have warned that rising surface temperatures and a loss of sea ice in the Arctic could disrupt the THC, causing dramatic changes in the global climate. For example, some computer models predict that if the THC were to slow or shut down, the Northern Hemisphere could become significantly cooler, while the Southern Hemisphere could become warmer. Fresh water flooding into the North Atlantic from melting glaciers in the Arctic and Greenland also could stall the THC by lowering the salinity of the water and forming a "cap" over the ocean.

The rising threat of higher sea levels. Perhaps the most discussed impact of a global ice meltdown is rising sea levels. The ACIA predicted that global sea levels could rise from 4 inches to 3 feet (10 to 90 centimeters) by 2100, with even faster increases as the century progresses. Currently, about half of the world's population lives within 125 miles (200 kilometers) of a coast, with many people residing much closer. A 1.5-foot (0.5-meter) increase in sea levels could cause shorelines in relatively flat areas to move inland by about 150 feet (50 meters). Higher sea levels would pollute underground freshwater aquifers and require the massive reengineering and reconstruction of coastal roads, power plants, and other basic infrastructure. Wetlands, marshes, and estuaries would be submerged, disrupting animal life on land and in the sea. Rising water would also result in major shifts in plant habitats, likely causing some species to become extinct.

In fact, sea levels have risen by about 4 to 9 inches (10 to 25 centimeters) since the early 1900s, according to the United Nations Environmental Programme (UNEP). Surprisingly, melting Arctic sea ice accounts for almost none of that total because the ice floats and, thus, occupies nearly the same volume as its meltwater would. About half of the increase has resulted from thermal expansion (the expansion and rise of the ocean caused directly by higher temperatures), UNEP scientists concluded. Meltwater from the world's two main ice sheets in Greenland and Antarctica and, to a much lesser degree, from glaciers outside polar regions accounts for the other half. An *ice sheet* is a dome-shaped glacier covering an area greater than 19,300 square miles (50,000 square kilometers).

Greenland's ice sheet, which covers the island's central plateau, ranks second only to Antarctica's ice sheets in volume. Two miles (3 kilometers) thick in places, it holds about 9 percent of the world's glacial ice and 10 percent of its freshwater, enough water to maintain the Mississippi River for at least 4,700 years, according to the U.S. Geological Survey. Surrounding the ice sheet are coastal mountains with glaciers that flow down the valleys to the sea. If Greenland's ice sheet were to melt completely, scientists have calculated, world oceans would rise by 20 feet (66 meters).

Vanishing glaciers. Glaciers outside Earth's polar regions are retreating because of both modern global warming and natural causes, particularly a gradual warm-up in Earth's temperature that followed a mini-ice age ending in the mid-1800s. A 2003 report by the conservation group WWF noted, "Since the early 1960s, mountain glaciers worldwide have experienced an estimated net loss of 960 cubic miles (4,000 cubic kilometers) of water—more than the annual discharge of the Orinoco, Congo, Yangtze, and Mississippi rivers combined; this loss was more than twice as fast in the 1990s than during the previous decades."

In Alaska, glaciers are thinning by an average of 6 feet (1.8 meters) a year, twice the rate in the mid-1990s. From 1850 to 2006, Glacier National Park lost 123 of its 150 glaciers. If current warming trends continue, all the park's glaciers—11,000-year-old remnants of the last Ice Age—will disappear sometime before 2050, according to the U.S. National Park Service. Glaciers in Asia's Himalaya-Hindu Kush region are retreating at the rate of about 30 to 50 feet (10 to 15 meters) annually, according to the WWF. Glaciers in the European Alps have lost from 10 to 20 percent of their volume since 1980, a number projected to rise to 80 percent by 2100. Because of the loss of the ice—and the reduction of the glaciers' weight on Earth's crust—Alpine peaks are rising. Mount Blanc in Switzerland, for example, is being lifted by about one-third inch (0.9 millimeter) per year.

Glaciers in equatorial regions are also draining away. In the Andes, glaciers are retreating at a rate exceeding anything seen in 5,000 years, according to a study released in June 2006. The Qori Kalis Glacier in the Peruvian Andes was retreating about 30 times faster in 2001 than it was in 1978. The Upsala Glacier in Argentina, the largest glacier in South America, is shrinking by 46 feet (14 meters) per year. Under current conditions, scientists predict, the Chacaltaya Glacier in Bolivia will disappear by about 2015. Once famous for the world's highest ski slope, Chacaltaya has lost 98 percent of its volume, most of it in the 1990s.

In Africa, about 80 percent of the ice on Mount Kilimanjaro, the continent's highest mountain, has disappeared since the early 1900s. Ice on Mount Kenya has shrunk by 40 percent since 1963. Glaciers in the mist-shrouded Ruwenzori Mountains of East Africa, known since ancient times as the Mountains of the Moon, have almost vanished.

Ilulissat Glacier, Greenland's fastest-moving glacier, is shrinking at an alarming rate.

The disappearance of Earth's glaciers has been described as a "time bomb." Tourists who visit the glaciers each year provide revenues crucial to the economies of numerous countries, especially in the developing world. Meltwater from glaciers powers hydroelectric dams. Peru, for example, depends on hydropower, mainly from glacial runoff, for a significant amount of its electric energy. Glacial meltwater also supports mining activities and is often essential for farming.

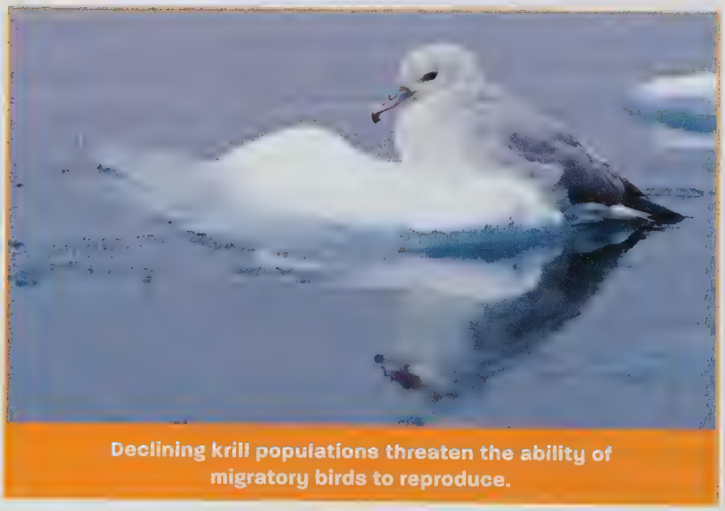
Most important, one-sixth of the world's population relies on glaciers and snow packs for its water supply. Glaciers in the Himalaya-Hindu Kush region, for example, help supply seven of Asia's most important rivers, including the Ganges, Yangtze, and Yellow. Municipal water systems in La Paz and El Alto, two of Bolivia's largest cities, depend on the Chacaltaya Glacier for their water. Glacier-fed rivers worldwide are also a crucial source of water for birds and other animals as well as for river and lake ecosystems. Computer models indicate that many of these rivers could all but disappear during the dry season.

Ice in the Antarctic. If all the glaciers outside Greenland melted, sea levels would rise only slightly. The melting of Antarctica's two enormous ice sheets would be another story. Antarctica is Earth's largest storehouse of ice, holding about 90 percent of all the planet's ice and 70 percent of its fresh water. Most of this ice lies in two massive ice sheets that can be as thick as 11,500 feet (3,500 meters). If both ice sheets melted, Earth's oceans would rise nearly 230 feet (70 meters).

Several important studies published in 2006 demonstrated that Antarctica is losing ice faster than it can be replaced by the continent's sparse snowfall. One study, by German and U.S. scientists analyzing gravity variations, showed that Antarctica's ice sheets decreased by 36 cubic miles (152 cubic kilometers) annually from 2002 to 2005. Most of that amount—equal to the total amount of water used in the United States for three months—came from West Antarctica, much of which lies below sea level. Air temperatures there have risen faster than any other place on Earth since the mid-1900s.

The second study was the first comprehensive survey of glaciers on the Antarctic Peninsula, a mountainous finger of West Antarctica. Since 1940, British and U.S. scientists found, 87 percent of the glaciers have retreated, by an average of 165 feet (50 meters) every year for the last five years. This retreat is faster than in earlier years.

The 2002 collapse of the northern section of the massive Larsen B ice shelf on the eastern side of the peninsula provided dramatic evidence of the changes occurring in the Antarctic. Over a 35-day period, 1,250 square miles (3,250 square kilometers) of the shelf shattered and separated from the continent, the largest single retreat of an ice shelf on the peninsula since the mid-1970s. At that time of its collapse, Larsen B, which was once 720 feet (220 meters) thick, had already lost 60 percent of its volume. In 2006, British and Belgian scientists determined that climate warming linked to human activity strengthened the westerly winds that blow across the northern part of the peninsula, allowing them to rise over mountains that normally block their flow.


Declining krill populations threaten the ability of migratory birds to reproduce.

Like melting sea ice, melting ice shelves have little effect on sea level. However, the ice shelves act as brakes on the ice flowing off the continent in glaciers. In fact, the four glaciers that had fed the northern section of Larsen B flowed up to eight times faster after the collapse. By contrast, the speed of nearby glaciers flowing toward the surviving section of the shelf remained unchanged. Moreover, the four glaciers that had been feeding Larsen B thinned by as much as 124 feet (38 meters) within six months.

Melting sea ice, aggravated by warming temperatures in the Southern Ocean, is affecting Antarctica's wildlife, particularly migratory birds. A 2006 study by French researchers found that nine species of sea birds that summer in Antarctica were migrating to the region an average of nine days later in 2004 than they were in the early 1950s. According to the scientists, one reason for the delay was a dramatic decline in populations of krill, tiny shrimplike animals that feed on algae that grow on the underside of the ice. A 2004 study by French researchers found that the number of krill, which form the base of the Antarctic food chain, had plunged by 80 percent since the 1970s. Shorter breeding seasons and less food threaten the birds' ability to reproduce.

Scientists know enough about the extent and effects of global warming to warn that actions should be taken to stabilize or reduce carbon-dioxide emissions. For this reason, the political and regulatory decisions made by governments and the choices made by individuals will continue to have a major influence on the future of climate change and Earth's icescape. In his 2006 book, *An Inconvenient Truth*, environmentalist and former U.S. Vice President Al Gore notes, "Our new technologies, combined with our numbers, have made us, collectively, a force of nature. And those with the most technology have the greatest moral obligation to use it wisely."

Global Warming (2006)

January 2006 began with a rare—for winter—tropical storm roaming the unusually warm waters of the Atlantic Ocean. Tropical Storm Zeta was the record-setting 27th storm of the Atlantic hurricane season, which had begun in June 2005.

The first month of 2006 was also the warmest January in the United States since record keeping began in 1895. Furthermore, the United States in 2006 had its second warmest summer since 1934.

The period from May through September 2006 was the warmest in central England since records were first kept there in the 1600s. During June and July 2006, sea ice in the Arctic shrank to its lowest values on record for those months.

Temperature trends reevaluated. In June, the National Research Council, a private research organization that advises the U.S. government, issued a report reevaluating a 2,000-year reconstruction of global surface temperatures that had been published in the 1990s by climatologist Michael Mann and colleagues. Mann's reconstruction was based on proxy data—that is, data from such sources as tree rings, ocean and lake sediments, ice cores, cave deposits, and historic drawings of glaciers. Scientists rely on proxy data to reconstruct long-term surface temperatures, because geographically widespread records of temperatures measured with instruments date back to only the mid-1800s.

Mann's reconstruction showed an unprecedented rapid rise in temperature since about 1970. His reconstruction also showed temperatures in the late 1990s reaching their highest values of the last 2,000 years.

The council concluded with high confidence that the proxy data demonstrates that the last few decades of the 1900s—averaged around the globe—have been warmer than any period since 1600. However, the report noted that much less confidence can be placed in temperature data before 1600.

Shrinking polar ice. Scientists from the National Snow and Ice Data Center at the University of Colorado in Boulder reported in October 2006 that the extent of sea ice in the Arctic reached its annual minimum in September at 2.2 million square miles (5.7 million square kilometers). This measurement was the second lowest value since records began in the 1970s. Only in 2005, when the minimum was 2.0 million square miles (5.2 million square kilometers), was there less Arctic sea ice. The researchers suggested that unusually cold weather in August probably prevented a new record from being established.

Hurricane Wilma in Cancun, Mexico, 2005

Polar sea ice is an important indicator of climate change, because it is sensitive to initial warming trends. As sea ice, which reflects much sunlight, melts, more of the darker ocean water is exposed. This darker surface absorbs more of the sun's heat energy, which further increases air temperatures near the water, ocean temperatures themselves, and the melting of sea ice.

Unexpected ocean cooling. Researchers at the National Oceanic and Atmospheric Administration's Pacific Marine Environmental Laboratory in Seattle reported in September 2006 that the average temperature of the top layer of Earth's oceans dropped slightly from 2003 to 2005. According to the scientists, the heat content of the upper 2,500 feet (750 meters) of ocean water decreased by 0.055 degrees Fahrenheit (0.031 degrees Celsius)—an amount that was equal to approximately 20 percent of the total heat gained by the world's oceans between 1955 and 2003.

The researchers noted that ocean water was still warmer in 2005 than it had been in the 1980s, but they had no explanation for the observed cooling, which was not predicted by any climate model. Because the ocean is a vast storehouse of energy that helps to drive the world's climate system,

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Preface

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The Graphics Used in This Book

To emphasize particular skills and strategies, we use several icons throughout this book. An icon in the margin will alert you that you should pay particular attention to the accompanying text. We use four icons:

Key Idea This icon points out a very important concept or fact that you should not pass over.

Strategy This icon calls your attention to a problem-solving strategy that you may want to try.

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AP English Language & Composition

Chapter

2

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How to Plan Your Time

IN THIS CHAPTER

Summary: Assess your own study patterns and preparation plans.

Key Idea

Key Ideas

- Explore three approaches.
- Choose a calendar that works for you.

Three Approaches to Prepare for the AP English Language and Composition Exam

No one knows your study habits, likes, and dislikes better than you. You are the only one who can decide which approach you want and/or need to adopt to prepare for the AP English Language and Composition exam. Look at the brief profiles below. These may help you to place yourself in a particular prep mode.

You are a full-year prep student (Approach A) if:

1. You like to plan for a vacation or the prom a year in advance.
2. You never think of missing a practice session, whether it's for your favorite sport, musical instrument, or activity.
3. You like detailed planning and everything in its place.
4. You feel you must be thoroughly prepared.
5. You hate surprises.
6. You are always early for appointments.

You are a one-semester prep student (Approach B) if:

1. You begin to plan for your vacation or the prom 4 to 5 months before the event.
2. You are willing to plan ahead to feel comfortable in stressful situations, but are okay with skipping some details.
3. You feel more comfortable when you know what to expect, but a surprise or two does not floor you.
4. You are always on time for appointments.

You are a 4- to 6-week prep student (Approach C) if:

1. You accept or find a date for the prom a week before the big day.
2. You work best under pressure and close deadlines.
3. You feel very confident with the skills and background you've learned in your AP English Language and Composition class.
4. You decided late in the year to take the exam.
5. You like surprises.
6. You feel okay if you arrive 10 to 15 minutes late for an appointment.

CALENDARS FOR EACH PLAN

Calendar for Approach A: Yearlong Preparation for the AP English Language and Composition Exam

Strategy Although its primary purpose is to prepare you for the AP English Language and Composition exam you will take in May, this book can enrich your study of language and composition, your analytical skills, and your writing skills.

SEPTEMBER–OCTOBER (Check off the activities as you complete them.)

_______ Determine into which student mode you would place yourself.
_______ Carefully read the Introduction and Chapter 1.
_______ Pay very close attention to the "Walk Through" the Diagnostic exam.
_______ Get on the Web and take a look at the AP website(s).
_______ Skim the Comprehensive Review section. (These areas will be part of your yearlong preparation.)
_______ Buy a highlighter.
_______ Flip through the entire book. Break the book in. Write in it. Highlight it.
_______ Get a clear picture of what your own school's AP English Language curriculum is.
_______ Review the Bibliography and establish a pattern of outside reading.
_______ Begin to use this book as a resource.

NOVEMBER (The first 10 weeks have elapsed.)

_______ Write the argumentative essay in the Diagnostic exam.
_______ Compare your essay with the sample student essays.
_______ Refer to Chapters 6 and 9 on the argumentative essay.
_______ Take five of our prompts and write solid opening paragraphs.

DECEMBER

_______ Maintain notes on literary works studied in and out of class.
_______ Refine analytical skills (see Chapters 5 and 8).
_______ Write one of the two analytical essays in the Diagnostic exam. (This will depend on the organization of your own curriculum.)
_______ Compare your essay with the sample student essays.

JANUARY (20 weeks have now elapsed.)

_______ Write the synthesis essay in the Diagnostic exam. (This will depend on your previous choice.)
_______ Compare your essay with the sample student essays.
_______ Refer to Chapters 7 and 10 on the synthesis essay.

FEBRUARY

_______ Take the multiple-choice section of the Diagnostic exam.
_______ Carefully go over the explanations of the answers to the questions.
_______ Score yourself honestly.
_______ Make a note of terms and concepts and types of questions that give you trouble.
_______ Review troublesome terms by checking the Glossary.

MARCH (30 weeks have now elapsed.)

_______ Form a study group.

_______ Choose a selection you have studied in class and create an essay question to go with it, or you can use one of our suggested prompts.

_______ Choose a passage from a current editorial and create an essay question to go with it, or you can choose one of our suggested prompts.

_______ Write one of the analytical essays.

_______ Write one of the synthesis essays.

_______ Compare essays and rate them with your study group. (Use our rubrics.)

APRIL

_______ Take Practice Exam 1 in the first week of April.

_______ Evaluate your strengths and weaknesses.

_______ Study appropriate chapters to correct weaknesses.

_______ Practice creating multiple-choice questions of different types with your study group.

_______ Develop and review worksheets for and with your study group.

MAY—First two weeks (THIS IS IT!)

_______ Highlight only those things in the Glossary about which you are still unsure. Ask your teacher for clarification. Study!

_______ Write at least three times a week under timed conditions.

_______ Take Practice Exam 2.

_______ Score yourself.

_______ Give yourself a pat on the back for how much you have learned and improved over the past nine months.

_______ Go to the movies. Call a friend.

_______ Get a good night's sleep. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST!

Calendar for Approach B:
Semester-Long Preparation for the AP English Language and Composition Exam

Strategy The following calendar assumes that you have completed one semester of language and composition and will use those skills you have been practicing to prepare you for the May exam. You still have plenty of time to supplement your course work by taking our study recommendations, maintaining literary notations, doing outside readings, and so forth. We divide the next 16 weeks into a workable program of preparation for you.

JANUARY–FEBRUARY (Check off the activities as you complete them.)

______ Carefully read the Introduction and Chapter 1.
______ Write the three essays on the Diagnostic exam.
______ Compare your essays with the sample student essays.
______ Complete the multiple-choice section of the Diagnostic exam.
______ Carefully go over the answers and explanations of the answers.
______ Take a close look at the Bibliography for suggestions regarding possible outside readings.

MARCH (10 weeks to go)

______ Form a study group.
______ Choose a favorite essay or excerpt from a book and create an essay question to go with it, or you can use one of our suggested prompts.
______ Choose a prose passage or essay and create an essay question to go with it, or you can choose one of our suggested prompts.
______ Write one of the analytical essays.
______ Write one of the synthesis essays.
______ Compare essays and rate them with your study group. (Use our rubrics.)

APRIL

______ Take Practice Exam 1 in the first week of April.
______ Evaluate your strengths and weaknesses.
______ Study appropriate chapters to correct weaknesses.
______ Practice creating multiple-choice questions of different types with your study group.
______ Develop and review worksheets for and with your study group.

MAY—First two weeks (THIS IS IT!)

______ Highlight only those things in the Glossary about which you are still unsure. Ask your teacher for clarification. Study!
______ Write at least three times a week under timed conditions.
______ Take Practice Exam 2.
______ Score yourself.
______ Give yourself a pat on the back for how much you have learned and improved over the past nine months.
______ Go to the movies. Call a friend.
______ Get a good night's sleep. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST!

Calendar for Approach C:
4- to 6-Week Preparation for the AP English Language and Composition Exam

Strategy At this point, we assume that you have been developing your argumentative, analytical, and writing skills in your English class for more than six months. You will, therefore, use this book primarily as a specific guide to the AP English Language and Composition exam. Remember, there is a solid review section in this book, to which you should refer.

Given the time constraints, now is not the time to try to expand your AP curriculum. Rather, it is the time to limit and refine what you already do know.

APRIL

_______ Skim the Introduction and Chapter 1.
_______ Carefully go over the "Rapid Review" sections of Chapters 5 through 10.
_______ Strengthen, clarify, and correct your weak areas after taking the Diagnostic exam.
_______ Write a minimum of three sample opening paragraphs for each of the three types of essays.
_______ Write a minimum of two timed essays for each type of essay on the exam.
_______ Complete Practice Exam 1.
_______ Score yourself and analyze your errors.
_______ Refer to the appropriate chapters to correct weaknesses.
_______ Refer to the Bibliography.
_______ If you feel unfamiliar with specific forms of discourse, refer to the list of suggested appropriate works.
_______ Develop a weekly study group to hear each other's essays and discuss writing.
_______ Skim and highlight the Glossary.

MAY—First two weeks (THIS IS IT!)

_______ Complete Practice Exam 2.
_______ Score yourself and analyze your errors.
_______ Refer to the appropriate chapters to correct weaknesses.
_______ Go to the movies. Call a friend.
_______ Get a good night's sleep. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST!

"One of the first steps to success on the AP exam is knowing your own study habits."
—Margaret R., AP Language teacher

SW
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Step 3

Develop Strategies for Success

CHAPTER 4 Section I of the Exam: How to Approach the Multiple-Choice Questions

CHAPTER 5 Section II of the Exam: How to Approach the Analysis Essay

CHAPTER 6 Section II of the Exam: How to Approach the Argumentative Essay

CHAPTER 7 Section II of the Exam: How to Approach the Synthesis Essay

SW
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Chapter 4

Section I of the Exam: How to Approach the Multiple-Choice Questions

IN THIS CHAPTER

Summary: Become comfortable with the multiple-choice section of the exam. If you know what to expect, you can prepare.

Key Idea

Key Ideas

- Prepare yourself for the multiple-choice section of the exam.
- Review the types of multiple-choice questions asked on the exam.
- Learn strategies for approaching the multiple-choice questions.
- Score yourself by checking the answer key and explanations for the multiple-choice section of the Diagnostic exam.

Introduction to the Multiple-Choice Section of the Exam

It has been our experience that the day after the exam finds students discussing the difficulties and uncertainties of Section I of the AP English Language and Composition exam.

We hope that by following along with us in this chapter, you will begin to feel a bit more familiar with the world of multiple-choice questions and, thereby, become a little more comfortable with the multiple-choice section of the exam.

What Is It About the Multiple-Choice Questions That Causes Such Anxiety?

Basically, a multiple-choice literature question is a flawed method of gauging understanding. Why? Because, by its very nature, a multiple-choice question forces you to play a cat-and-mouse game with the test maker, who demands that you concentrate on items that are incorrect before you can choose what is correct. We know, however, that complex literary works have a richness that allows for ambiguity. In the exam mode, you are expected to match someone else's reading of a work with your choice of answers. This is what often causes the student to feel that the multiple-choice section is unfair. And, perhaps, to a degree, it is. But, get with the program! It's a necessary evil. So, our advice to you is to accept the difficulties and limitations of Section I and to move on.

This said, it's wise to develop a strategy for success. Once again, practice is the key to this success.

"You know, when my teacher required us to make up multiple-choice questions that came from the AP prompts we wrote essays on, I really became more confident about how to answer these types of questions on the exam."
—Samantha T.,
AP student

You've answered all types of multiple-choice questions during your career as a student. The test-taking skills you have learned in your social studies, math, and science classes may also apply to this specific situation.

A word in defense of the test makers is in order here. The test is designed to allow you to shine, NOT to be humiliated. To that end, the people who design the multiple-choice questions take their job seriously and take pride in their product. You will not find "cutesy" questions, and they will not play games with you. What they will do is present several valid options in response to a challenging and appropriate question. These questions are designed to separate the knowledgeable, perceptive, and thoughtful reader from the superficial and impulsive one.

What Should I Expect in Section I?

For this first section of the AP English Language and Composition exam, you are allotted 1 hour to answer between 45 and 60 objective questions on four to five prose passages. The selections may come from works of fiction or nonfiction and be from different time periods, of different styles, and of different purposes. In other words, you will not find two essays by Thoreau in the multiple-choice section of the same test.

At least one of the readings will contain some type of citation, attribution, footnote, and so on. You will be expected to be able to determine how this citation, etc., is employed by the author to further his purpose. You will not be asked about specific formats such as MLA or APA.

These are not easy readings. They are representative of the college-level work you have been doing throughout the year. You will be expected to:

Key Idea ❯

- follow sophisticated **syntax**;
- respond to **diction**;
- be comfortable with upper-level vocabulary;
- be familiar with rhetorical terminology;
- make **inferences**;
- be sensitive to **irony** and **tone**;
- recognize components of organization and **style**;
- be familiar with modes of **discourse** and rhetorical strategies; and
- recognize how information contained in citations contributes to the author's purpose.

The good news is . . . the selection is self-contained. If it is about the Irish Potato Famine, you will not be at a disadvantage if you know nothing about Irish history. Frequently, there will be biblical references in a selection. This is especially true of works from an earlier time period. You are expected to be aware of basic **allusions** to biblical and mythological works often found in literary texts, but the passage will never require you to have any particular religious background.

Do not let the subject matter of a passage throw you. Strong analytical skills will work on any passage.

How Should I Begin to Work with Section I?

Take no more than a minute and thumb through the exam, looking for the following:

- The length of the selections
- The time periods or writing styles, if you can recognize them
- The number of questions asked
- A quick idea of the type of questions

This brief skimming of the test will put your mind into gear, because you will be aware of what is expected of you.

How Should I Proceed Through This Section of the Exam?

"Even though it's time-consuming, I find it invaluable to take class time to accurately simulate exam conditions."
—Cynthia N., AP teacher

Timing is important. Always maintain an awareness of the time. Wear a watch. (Some students like to put it directly in front of them on the desk.) Remember, this is not your first encounter with the multiple-choice section of the test. You've probably been practicing timed exams in class; in addition, this book provides you with three timed experiences. We're sure you will notice improvements as you progress through the timed practice activities.

Although the test naturally breaks into 15-minute sections, you may take less or more time on a particular passage, but know when to move on. The test DOES NOT become more difficult as it progresses; therefore, you will want to give yourself the opportunity to answer each set of questions.

Work at a pace of about one question per minute. Every question is worth the same number of points, so don't get bogged down on those that involve multiple tasks. Don't panic if a question is beyond you. Remember, it will probably be beyond a great number of the other students taking the exam. There has to be a bar that determines the 5's and 4's for this exam. Just do your best.

Reading the text carefully is a must. Begin at the beginning and work your way through.

Most people read just with their eyes. We want you to slow down and to read with your senses of sight, sound, and touch.

Strategy

- Underline, circle, and annotate the text.
- Read closely, paying attention to punctuation, syntax, diction, **pacing**, and organization.
- Read as if you were reading the passage aloud to an audience, emphasizing meaning and intent.
- As corny as it may seem, hear those words in your head.
- This technique may seem childish, but it works. Using your finger as a pointer, underscore the line as you are reading it aloud in your head. This forces you to slow down and to really notice the text. This will be helpful when you have to refer to the passage.
- Use all of the information given to you about the passage, such as title, author, date of publication, and footnotes.
- Be aware of organizational and rhetorical devices and techniques.

- Be aware of thematic lines and be sensitive to details that will obviously be material for multiple-choice questions.
- Quickly skim the questions, ignoring the choices. This will give you an idea as to what is expected of you as a reader of the given text.

Tip You can practice these techniques anytime. Take any work and read it aloud. Time yourself. A good rate is about 1½ minutes per page.

Types of Multiple-Choice Questions

Is the Structure the Same for All of the Multiple-Choice Questions?

No. There are several basic patterns that the AP test makers employ. These include:

The straightforward question.

- The passage is an example of ______________.
  C. a contrast/comparison essay
- The pronoun "it" refers to ______________.
  B. his gait

Draw a conclusion or interpret question. This question refers you to specific lines.

- Lines 52–57 serve to ______________.
  A. reinforce the author's thesis

All . . . except question. This question requires more time because it demands that you consider every possibility.

- The AP English Language and Composition exam is all of the following except:
  A. It is given in May of each year.
  B. It is open to high school seniors.
  C. It is published in the *New York Times.*
  D. It is used as a qualifier for college credit.
  E. It is a 3-hour test.

Make an inference or abstract a concept question. This question is not directly stated in the passage question.

- In "Letter from a Birmingham Jail," the reader can infer that the speaker is ______________.
  E. religious

The Roman numeral type question. This question is problematic and time consuming. You can be certain that each exam will have a couple of these questions within it. Skip it if it causes you problems and/or you are short on time.

- In the passage, "night" refers to ______________.
  I. the death of the young woman
  II. a pun on Sir William's title
  III. the end of the affair
    A. I only
    B. I and II
    C. I and III
    D. II and III
    E. I, II, and III

This is the type of question to skip if it causes you problems and/or you are short on time.

The footnote question. This is the question that requires you to abstract, interpret, or apply information contained in footnotes attached to passages.

- The purpose of the footnote is to ______________.
  A. cite a primary source
  B. verify the writer's assertions
  C. direct the reader to other sources
  D. cite a secondary source
  E. provide the writer's additional commentary

What Kinds of Questions Should I Expect on the Exam?

The multiple-choice questions center on **form** and content. Naturally, the test makers are assessing your understanding of the meaning of the selection as well as your ability to draw inferences and perceive implications based on the given work. They also want to know if you understand how an author develops his or her ideas.

Key Idea

The questions, therefore, will be factual, technical, analytical, and inferential. The brief chart below illustrates the types of key words/phrases in these four categories you can expect to encounter.

Note: Do not memorize these tables. Likewise, do not panic if a word or phrase is not familiar to you. You may or may not encounter any or all of these words or phrases on any given exam. You can, however, count on meeting up with many of these in our practice exams in this book.

FACTUAL	TECHNICAL	ANALYTICAL	INFERENTIAL
Words refer to	Sentence structure	Rhetorical strategy	Effect of diction
Allusions	Style	Shift in development	Tone
Antecedents	Grammatical purpose	Rhetorical stance	Inferences
Pronoun referents	Dominant technique	Style	Effect of description
	Imagery	**Metaphor**	Effect of last paragraph
	Point of view	Contrast	Effect on reader
	Organization of passage	Comparison	Narrator's attitude
		Cause/effect	Image suggests
	Narrative progress of passage	**Argument**	Effect of detail
		Description	Author implies
	Conflict	Narration	Author most concerned with
	Irony	Specific–general	**Symbol**
	Function of . . .	General–specific	
		How something is characterized	
		Imagery	
		Passage is primarily concerned with	
		Function of . . .	

Tip

A word about jargon: Jargon refers to words unique to a specific subject. A common language is important for communication, and there must be agreement on the basic meanings of terms. Although it is important to know the universal language of a subject, it is also important that you not limit the scope of your thinking to a brief definition. All of the terms used in the above chart are categorized only for easy reference. They also work in many other contexts.

Scoring the Multiple-Choice Section

How Does the Scoring of the Multiple-Choice Section Work?

The College Board has implemented a new scoring process for the multiple-choice section of the AP English Language and Composition exam. No longer are points deducted for incorrect responses, so there is no longer a penalty for guessing incorrectly. **Multiple-choice scores are based solely on the number of questions answered correctly.** Therefore, it is to your advantage to answer all of the multiple-choice questions. Your chances of guessing the correct answer improve if you skillfully apply the process of elimination to narrow the choices.

Strategies for Answering the Multiple-Choice Questions

As observed earlier, you've been answering multiple-choice questions most of your academic life, and you've probably figured out ways to deal with them. There may, however, be some points you have not considered that will be helpful for this particular exam.

General Guidelines

- **Work in order.** We like this approach for several reasons:
  — It's clear.
  — You will not lose your place on the scan sheet.
  — There may be a **logic** to working sequentially which will help you to answer previous questions. but, this is your call. If you are more comfortable moving around the exam, do so.
- **Write on the exam booklet.** Mark it up. Make it yours. Interact with the test.
- **Monitor your time.** Do not spend too much time on any one question.
- **Do not be misled by the length or appearance of a selection**. There is no correlation between this and the difficulty of the questions.
- **Work within the given context.** You may know other information about the subject of the text or a question. It's irrelevant.
- **Consider all the choices** in a given question. This will guard against your jumping to a false conclusion. It helps you to slow down and to look closely at each possibility. You may find that your first choice was not the best or most appropriate one.
- **Maintain an open mind** as you answer subsequent questions in a series. Sometimes a later question will contradict an answer to a previous one. Reconsider both. Likewise, even the phrasing of a question may point to an answer in a previous question.
- **Remember that all parts of an answer must be correct.**
- **When in doubt, go back to the text.**

"One of my biggest challenges in preparing for the exam was to learn not to jump to conclusions when I was doing the multiple-choice questions."
—Samantha S.,
AP student

Specific Techniques

Strategy

- **Process of Elimination**
  This is the primary technique when you do not have direct knowledge of the answer.
  1. Read the five choices.
  2. If no choice immediately strikes you as correct, you can:
     — eliminate any choices that obviously wrong;
     — eliminate those choices that are too narrow or too broad;
     — eliminate illogical choices;
     — eliminate choices that are synonymous;
     — eliminate choices that cancel each other out.
  3. If two answers are close,
     — find the one general enough to contain all aspects of the question
     OR
     — find the one limited enough to be the detail the question is seeking.

- **Substitution/Fill In the Blank**
  1. Rephrase the question, leaving a blank where the answer should go.
  2. Use each of the choices to fill in the blank until you find the one that is the best fit.

- **Using Context**
  1. Use this technique when the question directs you to specific lines, words, or phrases.
  2. Locate the given word, phrase, or sentence and read the sentence before and after the section of the text to which the question refers. Often this provides the information or clues you need to make your choice.

- **Anticipation**
  As you read the passage for the first time, mark any details and ideas that you would ask a question about. You may second-guess the test makers this way.

- **Intuition/The Educated Guess**
  You have a wealth of skills and knowledge in your language and composition subconscious. A question or a choice may trigger a "remembrance of things past." This can be the basis for your educated guess. Have the confidence to use an educated guess as a valid technique. Trust your own resources.

Strategy

A Survival Plan

If time is running out and you haven't finished the last selection,

1. Scan the remaining questions and look for:
   — the shortest questions; and/or
   — the questions that point you to a line.

These two types of questions are relatively easy to work with and to verify.

2. Look for specific detail/definition questions.
3. Look for self-contained questions.
   "The jail sentence was a bitter winter for his plan" is an example of
   C. an analogy.

You do not have to go to the passage to answer this question.

Some Thoughts About Guessing

Tip

You can't be hurt by making educated guesses based on a careful reading of the selection. However, understand that you need to come to this exam well prepared. You must have a foundation of knowledge and skills. You cannot guess through the entire exam and expect to do well.

Is There Anything Special I Should Know About Preparing for the Multiple-Choice Questions?

After you have finished with the Diagnostic exam, you will be familiar with the format and types of questions asked on the AP English Language and Composition exam. However, just practicing answering multiple-choice questions on specific works will not give you a complete understanding of this questioning process. We suggest the following to hone your multiple-choice skills with prose multiple-choice questions.

Strategy

- Choose a challenging passage from a full-length prose work or a self-contained essay, plus choose another that contains documentation/citations. (Take a close look at your science and social studies texts for examples.)
- Read the selection a couple of times and create several multiple-choice questions about specific sections of the selection.
  — Make certain the section is self-contained and complex.
  — Choose a speech, a philosophical passage, an essay, an editorial, a letter, a preface or epilogue, a significant passage from a chapter, or a news article.
- Refer to the chart given earlier in this chapter for suggested language and type.
- Administer your miniquiz to a classmate, study group, or class.
- Evaluate your results.
- Repeat this process through several different works during your preparation for the exam. The selections can certainly come from those you are studying in class.
- Create a variety of question types.

Here's What Should Happen as a Result of Your Using This Process

- Your expectation level for the selections in the actual test will be more realistic.
- You will become familiar with the language of multiple-choice questions.
- Your understanding of the process of choosing answers will be heightened.
- If you write unsatisfactory questions, your analytical skills will help you figure out "what went wrong."
- Your understanding of terminology will become more accurate.
- Bonus: If you continue to do this work throughout your preparation for the AP exam, you will have created a mental storehouse of literary and analytical information. So, when you are presented with an analytical or argumentative essay in Section II, you will have an extra resource at your disposal.

Tip ❯ You might want to utilize this process throughout the year with selections studied in and out of class and keep track of your progress. See the Bibliography at the back of this book.

The Time Is at Hand

It is now your turn to try the Diagnostic exam, Section I.

- Do this section in ONE sitting. Time yourself!
  — *Note:* If the 1 hour passes before you have a chance to finish all of the questions, stop where you are and score what you have done up to this point. Afterward, complete the remaining parts of the section, but do not count it as part of your score.
- When you have completed all of the multiple-choice questions in this diagnostic exam, carefully look at the explanations of the answers. Spend time here and assess which types of questions are giving you trouble. Use this book to learn from your mistakes.

ANSWER SHEET FOR DIAGNOSTIC MULTIPLE-CHOICE QUESTIONS

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
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55. ______
56. ______

My Diagnostic Exam Summary

______ I finished all the questions in the allotted 1-hour.
______ I did not finish all the questions in the allotted 1-hour.
______ (Qty.) answers were correct.
______ (Qty.) answers were incorrect.
______ (Qty.) answers were blank.

I have carefully reviewed the answer explanations, and I think I need to work on the following types of questions:

DIAGNOSTIC EXAM
ADVANCED PLACEMENT ENGLISH LANGUAGE AND COMPOSITION

Section I

Total Time—1 hour

Carefully read the following passages and answer the accompanying questions. Questions 1–12 are based on the following passage from "Samuel Johnson on Pope," which appeared in *The Lives of the English Poets* (1779–1781).

> The person of Pope is well known not to have been formed by the nicest model. He has compared himself to a spider and, by another, is described as protuberant behind and before. He is said to have been beautiful in his infancy, but he was of a constitution feeble and weak. As bodies of a tender frame are easily distorted, his deformity was probably in part the effect of his application. But his face was not displeasing, and his eyes were animated and vivid.
>
> By natural deformity, or accidental distortion, his vital functions were so much disordered, that his life was a "long disease."
>
> He sometimes condescended to be jocular with servants or inferiors; but by no merriment, either of others or his own, was he ever seen excited to laughter.
>
> Of his domestic character frugality was a part eminently remarkable. Having determined not to be dependent, he determined not to be in want, and, therefore, wisely and magnanimously rejected all temptations to expense unsuitable to his fortune.
>
> The great topic of his ridicule is poverty; the crimes with which he reproaches his antagonists are their debts and their want of a dinner. He seems to be of an opinion not very uncommon in the world, that to want money is to want everything.
>
> He professed to have learned his poetry from Dryden, whom he praised through his whole life with unvaried liberality; and perhaps his character may receive some illustration, if he be compared with his master.
>
> Integrity of understanding and nicety of discernment were not allotted in a less proportion to Dryden than to Pope. But Dryden never desired to apply all the judgment that he had. He wrote merely for the people. When he pleased others, he contented himself. He never attempted to mend what he must have known to be faulty. He wrote with little consideration and, once it had passed the press, ejected it from his mind.
>
> Pope was not content to satisfy; he desired to excel, and, therefore, always endeavored to do his best. Pope did not court the candor, but dared the judgment of his reader, and, expecting no indulgence from others, he showed none to himself. He examined lines and words with minute and punctilious observation, and he retouched every part with diligence, until he had nothing left to be forgiven.
>
> Poetry was not the sole praise of either; for both excelled likewise in prose. The style of Dryden is capricious and varied; that of Pope is cautious and uniform. Dryden observes the motions of his own mind; Pope constrains his mind to his own rules of composition. Dryden's page is a natural field, diversified by the exuberance of abundant vegetation. Pope's is a velvet lawn, shaven by the scythe, and leveled by the roller.
>
> If the flights of Dryden are higher, Pope continues longer on the wing. If of Dryden's fire the blaze is brighter, of Pope's the heat is more regular and constant. Dryden is read with frequent astonishment, and Pope with perpetual delight.

1. The passage is primarily a(n) ____________.
A. character sketch of Pope
B. discussion of poetic style
C. criticism of Dryden
D. model for future poets
E. opportunity for the writer to show off his own skills

2. The passage discusses a contrast among all of the following except:
A. prose and poetry
B. Pope and Dryden
C. body and mind
D. poverty and wealth
E. body and soul

3. "If the flights" (35) means ____________.
A. Pope's writing will outlast Dryden's
B. both Pope and Dryden are equal
C. Pope is not idealistic
D. Pope is more wordy
E. Pope is not as bright as Dryden

4. The character of Pope is developed by all of the following except:
A. examples
B. comparison
C. contrast
D. satire
E. description

5. According to the passage, Pope and Dryden are ____________.
A. rivals
B. equally intelligent
C. outdated
D. equally physically attractive
E. in debt

6. From the passage, the reader may infer that Pope ____________.
A. was extravagant
B. was a man of the people
C. was jealous of Dryden
D. had a desire to be popular
E. had a bitter, satirical nature

7. The tone of the passage is ____________.
A. informal and affectionate
B. formal and objective
C. condescending and paternalistic
D. laudatory and reverent
E. critical and negative

8. Lines 20–24 indicate that Dryden was what type of writer?
A. one who labored over his thoughts
B. one who wrote only for himself
C. one who wrote only for the critics
D. one who wrote to please Pope
E. one who did not revise

9. Using the context of lines 27–29, "punctilious" means ____________.
A. precise
B. timely
C. cursory
D. scholarly
E. philosophical

10. In the context of the passage, "until he had nothing left to be forgiven" (29) means ____________.
A. Pope outraged his readers
B. Pope suffered from writer's block
C. Pope exhausted his subject matter
D. Pope's prose was revised to perfection
E. Pope cared about the opinions of his readers

11. "Shaven" and "leveled" in line 34 indicate that Pope's style of writing was ____________.
A. natural
B. richly ornamented
C. highly controlled
D. mechanical
E. analytical

12. Based on a close reading of the final paragraph of the passage, the reader could infer that the author ____________.
A. looks on both writers equally
B. prefers the work of Pope
C. sees the two writers as inferior to his own writing style
D. indicates no preference
E. prefers the work of Dryden

Questions 13–23 are based on the following excerpt from Charlotte Perkins Gilman's "Politics and Warfare," which appears in *The Man-Made World: Our Androcentric Culture* (1911).

> 1 There are many today who hold that politics need not be at all connected with warfare; and others who hold that politics is warfare from start to finish.
>
> 2 The inextricable confusion of politics and warfare is part of the stumbling block in the minds of men. As they see it, a nation is primarily a fighting organization; and its principal business is offensive and defensive warfare; therefore the ultimatum with which they oppose the demand for political equality—"women cannot fight, therefore they cannot vote."
>
> 3 Fighting, when all is said, is to them the real business of life; not to be able to fight is to be quite out of the running; and ability to solve our growing mass of public problems; questions of health, of education, of morals, of economics; weighs naught against the ability to kill.
>
> 4 This naïve assumption of supreme value in a process never of the first importance; and increasingly injurious as society progresses, would be laughable if it were not for its evil effects. It acts and reacts upon us to our hurt. Positively, we see the ill effects already touched on; the evils not only of active war, but of the spirit and methods of war; idealized, inculcated, and practiced in other social processes. It tends to make each man-managed nation an actual or potential fighting organization, and to give us, instead of civilized peace, that "balance of power" which is like the counted time in the prize ring—only a rest between combats.
>
> 5 It leaves the weaker nations to be "conquered" and "annexed" just as they used to be; with "preferential tariffs" instead of tribute. It forces upon each the burden of armament; upon many the dreaded conscription; and continually lowers the world's resources in money and in life.
>
> 6 Similarly in politics, it adds to the legitimate expenses of governing the illegitimate expenses of fighting; and must needs have a "spoils system" by which to pay its mercenaries.
>
> 7 In carrying out the public policies the wheels of state are continually clogged by the "opposition"; always an opposition on one side or the other; and this slow wiggling uneven progress, through shorn victories and haggling concessions, is held to be the proper and only political method.
>
> 8 "Women do not understand politics," we are told; "Women do not care for politics"; "Women are unfitted for politics."
>
> 9 It is frankly inconceivable, from the androcentric viewpoint, that nations can live in peace together, and be friendly and serviceable as persons are. It is inconceivable also, that, in the management of a nation, honesty, efficiency, wisdom, experience and love could work out good results without any element of combat.
>
> 10 The "ultimate resort" is still to arms. "The will of the majority" is only respected on account of the guns of the majority. We have but a partial civilization, heavily modified to sex—the male sex.

13. The author's main purpose in the passage is to

________.

A. argue for women being drafted
B. criticize colonialism
C. present a pacifist philosophy
D. criticize the male-dominated society
E. protest tariffs

14. In paragraph 2, the author maintains that men support their position on equality for women based upon which of the following approaches?
A. begging the question
B. a syllogism using a faulty premise
C. an appeal to emotion
D. circular reasoning
E. an *ad hoc* argument

15. Using textual clues, one can conclude that "androcentric" most probably means ____________.
A. robot-centered
B. world-centered
C. female-centered
D. self-centered
E. male-centered

16. In addition to indicating a direct quotation, the author uses quotation marks to indicate ____________.
A. the jargon of politics and warfare
B. the coining of a phrase
C. a definition
D. the author's scholarship
E. that the author does not take responsibility for her words

17. In paragraph 4, "increasingly injurious as society progresses" is reinforced by all of the following except:
A. "ill effects already touched on" [paragraph 4]
B. "active war" [paragraph 4]
C. "weaker nations to be 'conquered' and 'annexed'" [paragraph 5]
D. "illegitimate expenses of fighting" [paragraph 6]
E. "Women do not understand politics" [paragraph 8]

18. According to the author, men view the primary purpose of government to be ____________.
A. educating the people
B. solving the "mass of public problems"
C. obtaining as much power as possible
D. economics
E. health

19. The argument shifts from a discussion of warfare to a discussion of politics in the first sentence of which of the following paragraphs?
A. paragraph 4
B. paragraph 5
C. paragraph 6
D. paragraph 7
E. paragraph 9

20. The tone of the passage is best described as ____________.
A. ambivalent
B. reverent
C. condescending
D. accusatory
E. indifferent

21. The style of the passage can best be described as ____________.
A. poetic and emotional
B. editorial and analytical
C. mocking and self-serving
D. preaching and moralistic
E. authoritative and pretentious

22. To present her argument, Gilman primarily uses which of the following rhetorical strategies?
A. process
B. definition
C. cause and effect
D. narration
E. description

23. "It," as used in paragraphs 4, 5, and 6, only refers to ____________.
A. "Fighting is to them the real business of life" [paragraph 3]
B. "evil effects" [paragraph 4]
C. "man-managed nation" [paragraph 4]
D. "preferential tariffs" [paragraph 5]
E. "spoils system" [paragraph 6]

Questions 24–33 are based on the following speech, "On the Death of Martin Luther King, Jr." by Robert F. Kennedy.

1 I have bad news for you, for all of our fellow citizens, and people who love peace all over the world, and that is that Martin Luther King was shot and killed tonight.

2 Martin Luther King dedicated his life to love and to justice for his fellow human beings, and he died because of that effort.

3 In this difficult day, in this difficult time for the United States, it is perhaps well to ask what kind of a nation we are and what direction we want to move in, for those of you who are black—considering the evidence there evidently is that there were white people who were responsible—you can be filled with bitterness, with hatred, and a desire for revenge. We can move in that direction as a country, in great polarization—black people amongst black, white people amongst white, filled with hatred toward one another.

4 Or we can make an effort, as Martin Luther King did, to understand and to comprehend, and to replace that violence, that stain of bloodshed that has spread across our land, with an effort to understand with compassion and love.

5 For those of you who are black and are tempted to be filled with hatred and distrust at the injustice of such an act, against all white people, I can only say that I feel in my own heart the same kind of feeling. I had a member of my family killed, [and] he was killed by a white man. But we have to make an effort in the United States, we have to make an effort to understand, to go beyond these rather difficult times.

6 My favorite poet was Aeschylus. He wrote: "In our sleep, pain which cannot forget falls drop by drop upon the heart until, in our own despair, against our will, comes wisdom through the awful grace of God."

7 What we need in the United States is not division; what we need in the United States is not hatred; what we need in the United States is not violence or lawlessness but love and wisdom, and compassion toward one another and a feeling of justice toward those who still suffer within our country, whether they be white or black.

8 So I shall ask you tonight to return home, to say a prayer for the family of Martin Luther King, that's true, but more importantly to say a prayer for our own country, which all of us love—a prayer for understanding and that compassion of which I spoke.

9 We can do well in this country. We will have difficult times. We've had difficult times in the past. We will have difficult times in the future. It is not the end of violence; it is not the end of lawlessness; it is not the end of disorder.

10 But the vast majority of white people and the vast majority of black people in this country want to live together, want to improve the quality of our life, and want justice for all human beings who abide in our land.

11 Let us dedicate ourselves to what the Greeks wrote so many years ago: to tame the savageness of man and to make gentle the life of this world.

12 Let us dedicate ourselves to that, and say a prayer for our country and for our people.

24. The primary purpose of RFK's speech is most probably to ______________.

A. inform the people of the event
B. praise the accomplishments of Martin Luther King, Jr.
C. offer condolences to King's family
D. call for calm and unity between blacks and whites
E. offer condolences to the black community at large

25. Which of the following paragraphs does not contain examples of parallel structure?
A. paragraph 3 beginning with "In this difficult . . ."
B. paragraph 6 beginning with "My favorite poet . . ."
C. paragraph 7 beginning with "What we need . . ."
D. paragraph 9 beginning with "We can do well . . ."
E. paragraph 10 beginning with "But the vast majority . . ."

26. Paragraph 5 contains an example of ____________.
A. understatement
B. figurative language
C. sarcasm
D. logical fallacy
E. analogous example

27. The tone of the speech can best be described as ____________.
A. elevated and conciliatory
B. angry and inflammatory
C. formal and detached
D. informal and emotional
E. accusatory and bitter

28. To keep his speech from leading to violence, RFK makes use of which of the following?
I. constantly repeating King's name and his desire for unity between races
II. an ethical appeal based on the power of religion
III. emphasizing a common bond to show the connection between himself and his audience
A. I
B. II
C. III
D. I and III
E. I, II, and III

29. All of the following paragraphs give support to the inference that RFK expected violence to follow the assassination except:
A. paragraph 3 beginning with "In the beginning . . ."
B. paragraph 4 beginning with "Or we can . . ."
C. paragraph 6 beginning with "My favorite . . ."
D. paragraph 7 beginning with "What we need . . ."
E. paragraph 9 beginning with "We can do well . . ."

30. RFK most probably chose to refer to the Greeks in paragraph 11 for all of the following reasons except:
A. to impress the audience with his scholarship
B. to concisely restate the theme of the speech
C. to provide a healing thought for the people to remember
D. to elevate the level of discourse
E. to reinforce the ideals of democracy with which the Greeks are associated

31. Paragraphs 7 and 8 are constructed around which of the following rhetorical strategies?
A. analysis
B. definition
C. narration
D. process
E. cause and effect

32. The quotation given in paragraph 6 can best be restated as ____________.
A. the process of healing is inevitable
B. time heals all wounds
C. sleep numbs those in pain
D. God is the source of humankind's grief
E. sleep is the only escape from pain

33. All of the following are effects of the repetition in paragraphs 11 and 12 except that it ____________.
A. links the speaker with the audience
B. refers to paragraph 2 and King's dedication
C. emphasizes dedication so that the audience will remember it
D. reinforces the tribute to Martin Luther King, Jr.
E. elevates the occasion to one which is worthy of honor

Questions 34–44 are based on the following letter.

Albert Einstein
Old Grove Road
Nassau Point
Peconic, New York
August 2, 1939

F. D. Roosevelt,
President of the United States,
White House
Washington, D.C.

Sir:

Some recent work by E. Fermi and L. Szilard, which has been communicated to me in manuscript, leads me to expect that the element uranium may be turned into a new and important source of energy in the immediate future. Certain aspects of the situation which has [*sic*] arisen seem to call for watchfulness and, if necessary, quick action on the part of the Administration. I believe therefore that it is my duty to bring to your attention the following facts and recommendations: 1

In the course of the last four months it has been made probable—through the work of Joliot in France as well as Fermi and Szilard in America—that it may become possible to set up a new nuclear chain reaction in a large mass of uranium, by which vast amounts of power and large quantities of new radium-like elements would be generated. Now it appears almost certain that this could be achieved in the immediate future. 2

This new phenomenon would also lead to the construction of bombs, and it is conceivable—though much less certain—that extremely powerful bombs of a new type may thus be constructed. A single bomb of this type, carried by boat and exploded in a port, might very well destroy the whole port together with some of the surrounding territory. However, such bombs might very well prove to be too heavy for transportation by air. 3

The United States has only very poor ores of uranium in moderate quantities. There is some good ore in Canada and the former Czechoslovakia, while the most important source of uranium is Belgian Congo. 4

In view of this situation you may think it desirable to have some permanent contact maintained between the Administration and the group of physicists working on chain reactions in America. One possible way of achieving this might be for you to entrust with this task a person who has your confidence and who could perhaps serve in an official capacity. His task might comprise the following: 5

a) to approach Government Departments, keep them informed of the further development and put forward recommendations for Government action;
b) giving particular attention to the problem of securing a supply of uranium ore for the United States;
c) to speed up the experimental work, which is at present being carried on within the limits of the budgets of University laboratories, by providing funds, if such funds be required, through his contacts with private persons who are willing to

make contributions for this cause, and perhaps also by obtaining the co-operation of industrial laboratories which have the necessary equipment.

I understand that Germany has actually stopped the sale of uranium from the Czechoslovakian mines which she has taken over. That she should have taken such early action might perhaps be understood on the ground that the son of the German Under-Secretary of State, von Weizacker, is attached to the Kaiser-Wilhelm-Institut in Berlin where some of the American work on uranium is now being repeated.

Yours very truly,
Albert Einstein

34. In both paragraphs 2 and 3, Einstein makes use of the dash ____________.

A. to emphasize the words set off
B. as an exception to the point immediately before it
C. to sound more scholarly and formal
D. as an informal aside to what was said previously
E. to summarize

35. The omission of a cordial opening and identification of the credentials of the writer imply all of the following except:

A. Einstein expects his name alone will identify him
B. Einstein assumes that the information he presents is compelling enough to command a response
C. Einstein believes himself too busy and important to waste time on pleasantries
D. As a scientist, Einstein was accustomed to having the facts speak for themselves
E. They've had previous contact

36. The purpose of the listing in paragraph 5 is to ____________.

A. secure Einstein's role as Roosevelt's "permanent contact"
B. suggest a plan of necessary action to ensure American security
C. increase research funding for further nuclear experimentation
D. end scientific research leading to the construction of nuclear bombs
E. send a letter of warning to Germany

37. Einstein's attitude can best be described as ____________.

A. confrontational
B. deferential
C. cautionary
D. complacent
E. antagonistic

38. Einstein's first paragraph suggests all of the following except:

A. FDR is not staying abreast of important scientific developments
B. Einstein is concerned about how the administration is handling the new developments in uranium research
C. Einstein is concerned that the administration may be unaware of important developments in the scientific community
D. Einstein is an authority in the use of uranium
E. FDR is familiar with the work of Fermi and Szilard

39. Which of the following best identifies Einstein's primary mode of discourse in his letter to FDR?

A. narration
B. process
C. analysis
D. persuasion
E. exposition

40. To illustrate the gravity of the situation, Einstein uses all of the following except:

A. "call for watchfulness" [paragraph 1]
B. "it is my duty" [paragraph 1]
C. "appears almost certain" [paragraph 2]
D. "in the immediate future" [paragraph 2]
E. "obtaining the co-operation" [paragraph 5]

41. Einstein understates the urgency of developing "chain reactions" in America ____________.

A. with the repetition of the words *might* and *may*
B. by excluding a fatalistic prediction
C. by mentioning "other countries repeating America's work"
D. with the phrase "though much less certain"
E. all of the above

42. To persuade Roosevelt to consider his recommendations, Einstein uses all of the following approaches except:

A. discussions with other members of the scientific community
B. appeals to fear
C. presentation of evidence
D. making predictions
E. offering a plan

43. In his letter, Einstein's own assumptions are all of the following except:

A. his interpretation of the manuscript is accessible
B. his reputation as a scientist lends weight to his opinion
C. his plan can be implemented quietly
D. his urgency concerning the situation is apparent
E. Germany recognizes the urgency of the situation

44. After a careful reading of the letter, which of the following inferences is *not* valid?

A. Einstein understood the urgency of addressing the nuclear problem.
B. Einstein assumed FDR would react to the letter.
C. Einstein viewed the private sector as a means of circumventing a possible governmental impasse.
D. The Germans could have possibly misunderstood the significance of this scientific discovery.
E. Einstein is suspicious of German espionage.

Questions 45–56 are based on the following passage entitled "Reading an Archive," by Allan Sekula, which appeared in *Blasted Allegories*, a collection of contemporary essays and short stories, published by MIT Press in 1987.

> . . . The widespread use of photographs as historical illustrations suggests that significant events are those which can be pictured, and thus history takes on the character of *spectacle*.[7] But this pictorial spectacle is a kind of rerun, since it depends on prior spectacles for its supposedly "raw" material.[8] Since the 1920's, the picture press, along with the apparatuses of a corporate public relations, publicity, advertising, and government propaganda, have contributed to a regularized flow of images: of disasters, wars, revolutions, new products, celebrities, political leaders, official ceremonies, public appearances, and so on. For a historian to use such pictures without remarking on these initial uses is naïve at best, and cynical at worst. What would it mean to construct a pictorial history of postwar coal mining in Cape Breton by using pictures from a company public relations archive without calling attention to the bias inherent in that source? What present interests might be served by such an oversight?
>
> The viewer of standard historical histories loses any ground in the present from which to make critical evaluations. In retrieving a loose succession of fragmentary glimpses of the past, the spectator is flung into a condition of imaginary temporal and geographical mobility. In this dislocated and disoriented state, the only coherence offered is that provided by the constantly shifting position of the camera, which provides the spectator with a kind of powerless omniscience. Thus, the spectator comes to identify with the technical apparatus, with the authoritative institution of photography. In the face of this authority, all other forms of telling and remembering begin to fade. But the machine establishes the truth, not by logical argument, but by providing an experience. This experience characteristically veers between nostalgia, horror, and an overriding sense of the exoticism of the past, its irretrievable otherness for the viewer in the present. Ultimately, then, when photographs are uncritically presented as historical documents, they are transformed into aesthetic objects. Accordingly, the

pretense to historical understanding remains, although that understanding has been replaced by aesthetic experience.[9]

But what of our second option? Suppose we abandoned all pretense to historical explanation, and treated these photographs as artworks of one sort or another? This book would then be an inventory of aesthetic achievement and/or an offering for disinterested aesthetic perusal. The reader may well have been prepared for these likelihoods by the simple fact that this book has been published by a press with a history of exclusive concern with the contemporary vanguard art of the United States and Western Europe (and, to a lesser extent, Canada). Further, as I've already suggested, in a more fundamental way, the very removal of these photographs from their initial contexts invites aestheticism.

I can imagine two ways of converting these photographs into "works of art," both a bit absurd, but neither without ample precedent in the current fever to assimilate photography into the discourse and market of the fine arts. The first path follows the traditional logic of romanticism, in its incessant search for aesthetic origins in a coherent and controlling authorial "voice." The second path might be labeled "post-romantic" and privileges the subjectivity of the collector, connoisseur, and viewer over that of any specific author. This latter mode of reception treats photographs as "found objects." Both strategies can be found in current photographic discourse; often they are intertwined in a single book, exhibition, or magazine or journal article. The former tends to predominate, largely because of the continuing need to validate photography as a fine art, which requires an incessant appeal to the myth of authorship in order to wrest photography away from its reputation as a servile and mechanical medium. Photography needs to be won and rewon repeatedly for the ideology of romanticism to take hold.[10]

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[7] See Guy DeBord, *La société du spectacle* (Paris: Editions Buchat-Chastel, 1967): unauthorized translation, *Society of the Spectacle* (Detroit: Black and Red, 1970: rev. ed. 1977).

[8] We might think here of the reliance, by the executive branch of the United States government, on "photo opportunities." For a discussion of an unrelated example, see Susan Sontag's dissection of Leni Reifenstahl's alibi that *Triumph of the Will* was merely an innocent documentary of the orchestrated-for-cinema 1934 Nuremberg Rally of the National Socialists. Sontag quotes Reifenstahl: "Everything is genuine. . . . It is *history—pure history*." Susan Sontag, "Fascinating Fascism," *New York Review of Books* 22, no. 1 (February 1975); reprinted in *Under the Sign of Saturn* (New York: Farrar, Straus, & Giroux, 1980), p. 82.

[9] Two recent books counter this prevailing tendency in "visual history" by directing attention to the power relationships behind the making of pictures: Craig Heron, Shea Hoffmitz, Wayne Roberts, and Robert Storey, *All that Our Hands Have Done: A Pictorial History of the Hamilton Workers* (Oakville, Ontario: Mosaic Press, 1981), and Sarah Graham-Brown, *Palestinians and Their Society, 1880–1946* (London: Quartet Books, 1980).

[10] In the first category are books that discover unsung commercial photographers: e.g., Mike Disfarmer, *Disfarmer: The Heber Springs Portraits*, text by Julia Scully (Danbury, N.H.: Addison House, 1976). In the second category are books that testify to the aesthetic sense of the collector: e.g., Sam Wagstaff, *A Book of Photographs from the Collection of Sam Wagstaff* (New York: Gray Press, 1978).

45. The first sentence (lines 1–3) does all of the following, except:

A. to indicate that material appears in this essay prior to this section
B. to indicate scholarly research
C. to indicate a cause/effect relationship
D. to state the thesis of the piece
E. to establish that the essay is based on the opinion of the author

46. The word *oversight* in line 12 refers to ______________.

A. "pictures from a company public relations archive" (10–11)
B. "without calling attention to the bias" (11)
C. "construct a pictorial history" (9–10)
D. "coal mining in Cape Breton" (10)
E. "present interests" (12)

47. An accurate reading of footnote 7 informs the reader that the author based his material on ____________.

A. *Society of the Spectacle*, rev. ed. 1977
B. *Society of the Spectacle*, 1970
C. *La société du spectacle*, 1967
D. The Black and Red, 1970
E. Buchat-Chastel, 1967

48. The author directly involves the reader using which of the following linguistic devices?

A. direct address
B. exhortation
C. metaphor
D. direct quotation
E. rhetorical question

49. "initial contexts" in line 35–36 refers to ____________.

A. "our second option" (28)
B. "historical explanation" (28–29)
C. "inventory of aesthetic achievement" (30)
D. "contemporary vanguard art" (33)
E. "disinterested aesthetic perusal" (31)

50. The main concern of the passage is contained in which of the following lines?

A. "Since the 1920's . . . and so on." (4–8)
B. "The viewer . . . critical evaluations." (13–14)
C. "In retrieving . . . geographical mobility." (14–16)
D. "I can imagine . . . of the fine arts." (37–39)
E. "The former . . . mechanical medium." (46–49)

51. The most probable implication of this passage is that ____________.

A. historians are cynical
B. historians are naïve
C. readers/viewers must be aware of the bias inherent in source material
D. viewers/readers are ill equipped to make critical evaluations
E. dealing with photographs demands a combination of the mechanical and the aesthetic

52. The purpose of footnote 9 is to ____________.

A. enhance the reputation of the writer
B. cite a primary source
C. direct the reader to opposing positions
D. compare differing cultures
E. provide a historical context

53. The tone of the passage can best be described as ____________.

A. argumentative and scholarly
B. romantic and artistic
C. philosophical and didactic
D. informative and sarcastic
E. informal and playful

54. According to the author, the power of photography as historical illustration is found in the ____________.

A. historian
B. spectator
C. picture press
D. image itself
E. camera

55. The last paragraph is primarily developed using which of the following rhetorical strategies?

A. cause and effect
B. comparison and contrast
C. definition
D. description
E. narration

56. The reader may infer from the footnotes that the author is a(n) ____________.

A. photographer himself
B. journalist reporting on photography
C. fan of Leni Reifenstahl
D. established authority in this field
E. art critic

Answers and Explanations

Explanations to the Samuel Johnson Essay

1. **A.** Although references to poetic style and to Dryden are contained in the passage, they are included to illuminate the character of Pope.
2. **E.** No references to body versus soul are in the passage. We do find references to both the prose and the poetry of Pope and Dryden. We are told of Pope's monetary concerns, and we can infer the contrast between Pope's broken body and healthy mind.
3. **A.** This is a fairly straightforward interpretation of a figurative line. The idea of "long on the wing" naturally leads the reader to think of endurance.
4. **D.** A careful reading of this passage allows you to locate each of the devices, except satire.
5. **B.** Lines 20–21 clearly state that the two men were equally gifted.
6. **E.** Lines 9 and 10 tell the reader that Pope's humor was condescending. Lines 14–15 allude to his use of ridicule, and the reader may infer that these characteristics were carried over into Pope's writing.
7. **B.** The author never interjects his own feelings, and the diction and syntax remain on a scholarly, elevated level.
8. **E.** Carefully read lines 23 and 24 and you will see a direct correlation between those lines and choice E.
9. **A.** This is strictly a vocabulary question. You should be able to use the context clues of "minute" and "diligent" to lead you to choose A.
10. **D.** If you go to lines 25–29, you will see that Pope demanded perfection of himself and his writing. This characteristic is further extended with the clause in line 29.
11. **C.** Both words indicate a practiced, continuous, and extreme control of the work at hand. Even the "velvet of the lawn" indicates a tightness, a smoothness, and a richness of form and content.
12. **B.** If it were a contest, Pope would be declared the winner by Johnson. A close reading of both the structure and the content of the paragraph leads the reader to Pope. When discussing Dryden and Pope, Pope has the last work. This allows Pope to linger in the reader's mind. "Frequent" with Dryden and "perpetual" with Pope is another indication of Samuel Johnson's preference.

Explanations to the Gilman Essay

13. **D.** Although Gilman touches upon each of the choices in the passage, A, B, C, and E are details used to support her argument that a man-managed nation is an imperfect culture.
14. **B.** The question requires the student to be familiar with methods of logical reasoning and logical fallacies. Gilman presents the syllogism men use to deny women the right to vote:
    Those who fight may vote.
    Women do not fight.
    Therefore, women may not vote.
    "Those" is understood to be men. The first premise is incorrect, as is the second premise. This being the case, the conclusion is invalid.
15. **E.** This question depends upon both vocabulary and careful reading. Paragraph 8 points to a philosophy that desires to exclude women from politics. Therefore, any political involvement must be male-centered.
16. **A.** Most readers expect quotations to be used to indicate a direct quotation or specific titles of works. However, there are other uses for these bits of punctuation. One is to set off specific words or phrases used by others in a given context. Here, Gilman is making direct reference to the words employed by society's male leadership.
17. **E.** Because the argument of the passage is to criticize the aggressive nature of politics in a male-managed society and to point out the results of combining politics and warfare, the question demands details that support the idea of aggression being detrimental to society. The only choice that does not reflect this idea is E.

18. C. Look carefully at the second paragraph to see the ranking Gilman sets up as the male-centered priorities. The only one ranked over the others is fighting and the ability to kill. Therefore, the only appropriate choice is C.

19. C. Syntactically, the phrase, "Similarly in politics" is an indicator that a comparison is being drawn between what came before and what comes after. No other phrase does this.

20. D. Because this is an argumentative selection, the author is expected to take a position on an issue. Because of this, the choices of "ambivalent" and "indifferent" are immediately eliminated. Keeping in mind the diction of the piece, you can see that "reverent" and "condescending" are also inappropriate.

21. B. Remember, all parts of your answer must be correct. The only choice that presents two correct descriptions of the style is B.

22. C. If you read the passage carefully, you cannot avoid the cause-and-effect sequencing throughout the excerpt. Look at paragraphs 2, 3, 4, 6, and the last. Remember that an author can use many different techniques in the same work, but only one will be predominant, and that strategy is what reinforces the author's purpose.

23. A. It is interesting to look at the singular use of this pronoun. In every instance, "it" refers to "fighting," while reinforcing the author's relentless focus on the essential problem.

Explanation of the Answers to the RFK Speech

24. D. Although RFK tells the audience of the death of Martin Luther King, Jr., and although he briefly asks the audience to pray for King's family, condolences are not the focus of the speech. No real listing or praise of King's accomplishments is given. Throughout his speech, Kennedy continually stresses the need for unity, wisdom, and compassion.

25. B. Every AP Comp student must be familiar with parallel structure and must be able to recognize it in writing. A close examination of paragraph 6 will reveal that no repetition of structure and/or diction is present.

26. E. RFK's comparison of his similar situation with regard to the assassination of his brother, John F. Kennedy, is the framework for this paragraph. No other choice is present.

27. A. Keeping in mind that both parts of the answer must be supported in the text, A is the only appropriate choice. All other choices are either unsupported or contradictory to the purpose of the speech.

28. E. This is a question that is helped by your close attention to the previous questions. In them you will see that all of the choices were referred to.

29. C. The paragraph is only concerned with the words of Aeschylus that are centered on the results of loss. The other paragraphs realistically acknowledge the violent history of this nation.

30. A. It is obvious that RFK does not want to separate himself from his audience. The hope is to take the emotions of his audience and to lift them out of the realm of emotional, violent responses and to provide an avenue for peaceful and positive outlets for their grief.

31. E. If you are familiar with rhetorical strategies, this question would be an easy one for you. The word "so" in the first line of paragraph 8 is your obvious indicator of cause and effect.

32. A. At first glance, this quotation seems quite obvious. However, careful consideration of its meaning is rather more difficult. Although each of the other choices contains words or an idea that is a single part of the quotation, none other than A takes into consideration sleep, time, pain, wisdom, and God.

33. D. There is nothing in the last two paragraphs that indicates that the purpose is to pay tribute to Martin Luther King, Jr. Therefore, D is the appropriate choice.

Explanations to the Questions Related to the Einstein Letter

34. D. This question requires your knowing the uses of the dash and your ability to recognize how it is actually used in a given text. Here, you should see that Einstein is making a statement that could have also been set off with commas or parentheses. It is a more informal device.

35. **C.** This is a rather obvious answer to a question that many would be tempted to "over-read." Don't try to make things more complicated. You should also be aware of the conventions of the letter form.
36. **B.** This is a process of elimination question. Checking for evidence to support each of the choices eliminates A, C, D, and E. Those who would point to C must look again at both paragraphs 6 and 7. Here, it becomes obvious that paragraph 6 does not contain any reference to funding. Remember, all parts of the answer must be supported by the text.
37. **C.** This is a diction and tone question. If you look carefully at Einstein's choice of words and phrases in paragraphs 3 and 6 and his selection of supporting details, you can only conclude correctly that he is being cautionary.
38. **A.** The very fact that Einstein does not include any introductory information about Fermi or Szilard indicates that FDR *is* cognizant of current scientific endeavors. Moreover, there is no evidence of neglect on the part of the President. Therefore, A is your correct choice.
39. **D.** An AP Comp student needs to be familiar with the modes of discourse and must be able to recognize them. This question asks the reader to note the persuasive nature of Einstein's letter. There is no storytelling, no directions, and no exposition or analysis.
40. **E.** If you know the meaning of gravity as used in this selection, you are led to look for words that are indicative of a seriousness of purpose. This is also a question that can be answered without necessarily returning to the text. A, B, C, and D contain a common urgency and a seriousness lacking in choice E.
41. **E.** Understatement, which Einstein uses because of his unwillingness to become an alarmist, is shown by his use of qualifying words and phrases rather than direct statements. Each of the choices provides room for Presidential ambivalence.
42. **A.** Einstein appeals to fear (paragraph 3), presents evidence (paragraph 4, line 1), makes predictions (paragraph 2), and offers a plan [paragraph 5, (a), (b), and (c)]. However, he does NOT discuss anything with members of the scientific community in this letter.
43. **D.** Assumptions are tricky questions to work with because they demand your own thoughts and conclusions from the piece. Read each choice carefully, making certain that nothing in a choice is contradictory or invalid. In this question, choice D is the only assumption NOT supported by the text. (*Note:* Information contained in previous questions could have been helpful in selecting your answer.)
44. **D.** Inferences are not the same as assumptions. To infer is to reach a conclusion based on facts or observations. To assume is to take something for granted, although it is not proved. In other words, if I see you carrying an opened, wet umbrella and wearing a wet raincoat, I can *infer* that it is rainy outside. I could *assume* that you are a person who listens to the weather forecasts. For this question involving an inference, the reader cannot find proof in the letter to support the conclusion that the Germans may have misunderstood the significance of the scientific discovery.

Explanations of the Photography Passage

45. **D.** This selection is not totally focused on the subject of history being spectacle. The ellipsis indicates that material preceded this given piece. The use of footnote 7 is indicative of previous research. The word *this* establishes the relationship between cause and effect. The word *suggests* and the phrase "takes on the characteristics of spectacle" are indications of opinion.
46. **B.** This is a close-reading question in which the student must be able to recognize antecedents.
47. **A.** A close reading of the footnote will reveal that the last edition was 1977. The date would not be listed if the earlier version had been used.
48. **E.** Each of the rhetorical questions (lines 1–12 and 28–29) asks for the reader's input.

49. B. The "initial context" is that photographs are, by their very nature, historical.

50. E. These several lines present to the reader the double-edged debate. Is photography only objective or only subjective, or is it a combination of the two?

51. E. Choices A, B, C, and D are all directly stated by the author in the passage.

52. C. This footnote directs the reader to other sources that present differing opinions on the subject of "visual history."

53. A. Every point, including the footnotes, supports the author's position, which is clearly stated in this excerpt.

54. E. The evidence for this response is found in lines 21–22: ". . . machine establishes the truth. . . ."

55. B. If you look closely at the paragraph, you will see multiple indications of comparison and contrast: *both*, *first path*, and *second path*, *former*, and *latter*.

56. D. Footnotes 8, 9, and 10 indicate the breadth of knowledge and confidence of the author. This is obvious from his sources and his recommendations to the reader.

Introduction to Chapters 5, 6, and 7

The essay part of the AP Language and Composition exam emphasizes three major skills:

- Analysis
- Argument
- Synthesis

The analysis prompt asks the student to analyze the author's purpose and how he achieves it. The argument prompt requires the student to take a position on an issue and develop it with appropriate evidence. The synthesis prompt directs the student to carefully read several sources related to a specific topic and to cite at least three of these sources to support his argument or analysis.

Because you must carefully read the subject prompt for the synthesis essay, Section II requires an additional 15 minutes. You will be able to read all three of the essay questions during this 15-minute period.

But, you will **not** be permitted to open and write in the actual test booklet. Once the **close reading** time has elapsed, you will be directed to open the test booklet and begin to write your three essays.

The heading of Section II looks something like this:

Section II

Number of questions—3

Percent of total grade—55

Each question counts one-third of the total section score.

You will have a total of 2 hours to write, which you may divide any way you choose. Because each essay carries the same weight, do **not** spend an inappropriate amount of time on any one question.

Chapters 5, 6, and 7 of this book introduce you to each of the three essay types.

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Chapter 2

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Preparing for the AP U.S. History Exam

IN THIS CHAPTER

Summary: The right preparation plan for you depends on your study habits and the amount of time you have before the test. This chapter provides some examples of plans you can use or adapt to your needs.

Key Idea

Key Ideas

- Choose the study plan that is right for you.
- Begin to prepare for the AP exam at the beginning of the school year. Developing historical analytical skills, evaluating themes in U.S. history, and studying important concepts take far more time and effort than by simply memorizing facts. The sooner you begin preparing for the test, the better.

Getting Started

You have made the decision to take AP U.S. History. Enjoy! You will be exposed to all of the fascinating stories that make up U.S. history. To be successful in this course, you will have to work much harder than you would in a "regular" high school U.S. history course. You will be required to read more, including reading and analyzing a wide variety of primary source documents throughout the year. In addition, you will be required to utilize historical thinking, to analyze history in a thematic way, and to be knowledgeable of specific concepts that help guide the study of American history. It cannot be stressed enough that the examination for this course that you will take in May is not a test that will simply measure what you "know" about U.S. history; instead, it is an examination that tests your ability to analyze major events, concepts, and themes in American history utilizing specific historical analytical skills.

Being able to utilize historical analytical skills, study history thematically, and develop conceptual thinking are not skills that develop overnight. In fact, it is difficult to develop these skills in the context of one specific course. If you are reading this before you are actually enrolled in an AP U.S. History course, you may want to take the most challenging history courses you can *before* you take AP U.S. History. Try to think conceptually in any history course that you take; a *real* study of history of any type is much more than just memorizing facts.

Creating a Study Plan

As has already been noted several times, preparing for this exam involves much more than just memorizing important dates, names, and events that are important in U.S. history. Developing historical analytical skills, evaluating themes in U.S. history, and studying important concepts take far more time and effort than by simply memorizing facts. Therefore, it is strongly suggested that you take a year-long approach to studying and preparing for the test.

However, for some students this is not possible. Therefore, some suggestions for students who have only one semester to prepare for the exam and students who have only six weeks to prepare for the exam are included. In the end, it is better to do *some* systematic preparation for the exam than to do none at all.

Study Groups

Many students who have gotten a 5 on the U.S. History exam reported that working in a study group was an important part of the successful preparation that they did for the test. In an ideal setting, three to five students get together, probably once a week, to review material that was covered in class the preceding week and to practice historical, thematic, and conceptual thinking. If at all possible, do this! A good suggestion is to have study groups set a specific time to meet every week and stick to that time. Without a regular meeting time, study groups usually meet fewer times during the year, often cancel meetings, and so on.

THREE PLANS FOR TEST PREPARATION

Plan A:
Yearlong Preparation for the AP U.S. History Exam

Strategy This is the plan we highly recommend. Besides doing all of the readings and assignments assigned by your teacher, also do the following activities. (Check off the activities as you complete them.)

IN THE FALL

_______ Create a study group and determine a regular meeting time for that group.
_______ Coordinate the materials in this manual with the curriculum of your AP U.S. History class.
_______ Study the review chapters in Step 4 of this book that coincide with the material you are studying in class.
_______ Begin to do outside reading on U.S. history topics (either topics of interest to you or topics that you know you need more background in).
_______ In your study group, emphasize historical analysis and thematic and conceptual thinking.

FROM DECEMBER TO MARCH

_______ Continue to meet with your study group and emphasize historical analysis and thematic and conceptual thinking.
_______ Continue to study the review chapters in Step 4 of this book that coincide with the material you are studying in class.
_______ Take the diagnostic test in Step 2 of this book to see what the test will be like and assess your strengths and weaknesses.
_______ Learn the strategies discussed in Step 3 of this book. Practice applying them as you study and review for the AP U.S. History exam.
_______ Carefully study the format and approach to document-based questions (DBQs). You will probably have one on your midterm exam in class.
_______ Using the eras of U.S. history you have studied in class, create your own DBQ for two of the units and try to answer it.
_______ Intensify your outside reading of U.S. history topics.
_______ Take two U.S. history textbooks and compare and contrast their handling of three events of U.S. history. What do these results tell you?

DURING APRIL AND MAY

_______ Continue to meet with your study group and emphasize historical analysis and thematic and conceptual thinking. Many study groups meet at additional times in the weeks leading up to the test.

_______ Continue to study the review chapters in Step 4 of this book that coincide with the material you are studying in class.

_______ Practice creating and answering multiple-choice and short-answer questions in your study group.

_______ Develop and review worksheets of essential historical content with your study group.

_______ Highlight material in your textbook (and in this manual) that you may not understand, and ask your teacher about it.

_______ Write two or three essays as they would appear on the exam under timed questions and have a member of your study group (or a classmate) evaluate them.

_______ Take the practice tests provided in Step 5 of this book. Set a timer and practice pacing yourself.

You are well prepared for the test. Go get it!

Plan B:
One-Semester Preparation for the AP U.S. History Exam

Strategy Besides doing all of the readings and assignments assigned by your teacher, you should do the following activities. (Check off the activities as you complete them.)

FROM JANUARY TO MARCH

_______ Establish a study group of other students preparing in the same way that you are. In your study group you should review essential factual knowledge, but also analyze the essential themes and concepts in the course.

_______ Study the review chapters in Step 4 of this book that coincide with the material you have studied or are studying in class.

_______ Take the diagnostic test in Step 2 of this book to familiarize yourself with the test and assess your strengths and weaknesses.

_______ Learn the strategies discussed in Step 3 of this book. Practice applying them as you study and review for the AP U.S. History exam.

_______ Write two or three document-based and sample multiple-choice questions.

_______ Read at least one outside source (historical essay or book) on a topic that you are studying in class.

_______ In your study group, practice creating and answering short-answer and multiple-choice questions.

DURING APRIL AND MAY

_______ Continue to meet with your study group, and review essential factual knowledge and essential themes and concepts of the course you have taken or are currently taking. Some study groups increase the amount of time that they meet together in the weeks right before the test.

_______ Study the review chapters in Step 4 of this book that coincide with the material you have studied or are presently studying in class. Focus on weak areas you identified in the diagnostic test of Step 2 of this book.

_______ Practice creating and answering sample essays with your study group.

_______ Develop and review worksheets of essential historical content with your study group.

_______ Ask your teacher to clarify things in your textbook or in this manual that you do not completely understand. If you have nagging questions about some specific historical details, get the answers to them!

_______ Take the practice tests provided in Step 5 of this book. Set a timer and practice pacing yourself.

Plan C:
Four- to Six-Week Preparation for the AP U.S. History Exam

Strategy Besides doing all of the reading and assignments assigned by your teacher, do the following activities. (Check off the activities as you complete them.)

IN APRIL

_______ Take the diagnostic test in Step 2 of this book to familiarize yourself with the test and assess your strengths and weaknesses.

_______ Study the review chapters in Step 4 of this book that coincide with any weak areas you identified from the diagnostic exam.

_______ Learn the strategies discussed in Step 3 of this book. Practice applying them as you study and review for the AP U.S. History exam.

_______ Write one sample document-based question (DBQ) as modeled by samples in this manual.

_______ Carefully review the sections of this manual that outline the essential content of each historical period.

_______ If possible, create or join a study group with other students to help prepare for the exam.

IN MAY

_______ Many teachers organize study sessions right before the actual exam. Go to them!

_______ With your study group or individually, review essential content from the course and major concepts of each unit.

_______ Complete another sample DBQ essay and analyze your results.

_______ Review the glossary of this manual another time to help review essential content.

_______ Be certain of the format of the test and the types of questions that will be asked.

_______ Take the practice tests provided in Step 5 of this book. Set a timer and practice pacing yourself.

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Chapter 15

Union Divided: The Civil War (1861–1865)

IN THIS CHAPTER

Summary: The Civil War was the culmination of nearly 40 years of tensions between the North and the South. Northern Abolitionists looked forward to the war with great anticipation: victory over the South would finally allow the dreaded institution of slavery to be eliminated. Northern industrialists saw the war as an opportunity, at long last, to expand their control of American industry. The majority of Southerners rejoiced at the onset of war; they perceived that victory would allow the "Southern way of life" to continue without constant criticism from the North. As in many wars, politicians and generals on both sides predicted a quick victory. Newspapers in both the North and the South declared that the war would be over by Christmas of 1861.

To state that the Civil War was just about slavery is an oversimplification. Certainly, criticism by Northern Abolitionists of the "peculiar institution" of slavery, and Southern responses to that criticism, were important factors. However, other tensions between the North and the South also existed. The future of the American economy as seen by Northern industrialists differed drastically from the desires and needs of the leaders of Southern plantation societies. Most important, the Southern view of "states' rights" differed most dramatically from the view of the Union held in the North. By 1861, many political leaders in the South fervently espoused the views that John C. Calhoun had formulated decades earlier. It was up to the individual state to decide on the validity of any federal law or federal action for that state. This position was intolerable to President Lincoln and most political leaders in the North. If anything, it was debate over the state's rights issue that made the Civil War inevitable.

Other factors increased the animosity between the North and the South. By this point, slavery was synonymous with Southern identity; in Southern eyes, any attack on slavery was an attack on the South as a whole. The fact that this struggle between the North and the South had gone on for 40 years served to harden positions on both sides. In addition, by this point the population of the North was greater than the population of the South, and the number of free states was greater than the number of slave states. As a result, Southerners knew that Northern antislave interests would control the Congress (and the ability to influence Supreme Court appointments) and the Electoral College for the foreseeable future.

Key Idea

Keywords

First Battle of Bull Run (1861): early Civil War engagement ending in defeat for the Union army; this battle convinced many in the North that victory over the Confederacy would not be as easy as they first thought it would be.

Emancipation Proclamation: January 1, 1863, proclamation that freed slaves in Southern territories was controlled by the Union army; this executive proclamation by President Lincoln also committed the Union to the abolition of slavery.

Battle of Gettysburg (1863): bloodiest overall battle of the Civil War; many historians claim that the Southern defeat in this battle was the beginning of the end for the Confederacy.

Appomattox: Virginia courthouse where General Robert E. Lee surrendered Confederate forces on April 9, 1865.

Advantages of the North and South in the War

Many Southerners were very excited when the Civil War finally began, yet there were some harsh realities facing them as the war commenced. Most of the nation's wealth was situated in the North; the industrialization of the North would give Northerners an advantage in producing guns, bullets, and other materials needed for warfare. The Northern railway system was far superior to the existing railways in the South. Most influential banks and financial markets were located in the North. More people (by a nearly 3-to-1 margin) lived in the North. The South could at least say that they were larger than the North; conquering the South would be a formidable task. At the outset of the war, Southerners also felt that their officer corps, led by men such as Robert E. Lee, was superior to the officer corps of the Union, led by Winfield Scott.

Aftermath of Secession

As mentioned in the previous chapter, South Carolina, Mississippi, Florida, Louisiana, Texas, Georgia, and Alabama all voted to secede from the Union in late 1860 or early 1861. In February 1861, the **Confederate States of America** was officially created. States in the Upper South (such as Virginia and Kentucky) were not eager to join the secessionist movement (there were fewer slaves in these states). Leaders of Kentucky and Maryland proposed that Congress in Washington enact legislation that would protect slavery in any territory or state where it already existed; the desire of these leaders was the preservation of the Union. President James Buchanan did little to aid the situation. Buchanan stated in December

1860 that secession from the Union was illegal, but that nowhere in the Constitution was it stated that any state could be forced to remain in the Union.

Politicians in South Carolina and elsewhere in the South interpreted Buchanan's statement as, in essence, stating that he would do nothing to bring back the seceded states and that they were now independent. Leaders in South Carolina demanded the surrender of **Fort Sumter**, a federal fort located in Charleston harbor. To test the will of the leaders of South Carolina, Buchanan sent an unarmed merchant ship to bring supplies to the fort in January 1861. When the ship was fired on, Buchanan did not send the navy in (which many in South Carolina was sure he would do); "patriots" in South Carolina and elsewhere in the South now felt certain that independence was theirs.

As the crisis continued at Fort Sumter, Senator John Crittenden of Kentucky emerged with a compromise plan. The **Crittenden Plan** proposed that the federal government guarantee the existence of slavery in any state where it existed, and that the line of the Missouri Compromise be extended all the way to the Pacific, with territories to the north of the line being free from slavery and those south of the line having slavery. Republicans in Congress rejected this plan, since it went away from the concept of "free soil" that president-elect Lincoln had just been elected on.

Attack on Fort Sumter and the Beginning of the War

Abraham Lincoln had to walk a political tightrope upon his inauguration in March 1861. It was necessary to maintain the authority of the federal government, but at the same time to do nothing that would provoke war with the South. Many of Lincoln's advisors thought that negotiations could bring at least some of the states that had seceded back into the Union. In his inauguration speech, however, Lincoln stated that force would be used if necessary to preserve the Union.

The skill of Lincoln as president was immediately called upon. In April 1861, Lincoln sent another ship to supply Fort Sumter. The government of South Carolina was informed that the ship would be arriving and that no troops would land unless the delivery of these supplies was interfered with. Jefferson Davis and the Confederate government saw this as an opportunity to strike against the Union. Confederate guns bombed Fort Sumter for two days, and on April 14 the fort surrendered. Davis was hopeful that early victory would force states in the Upper South to turn to the Confederate cause; Confederates also hoped to obtain British and French assistance. Any thought of compromise between North and South ended with the attack on Fort Sumter.

Three days after the surrender of the fort, Virginia passed a resolution favoring secession. On the same day, Robert E. Lee rejected an offer to command the Union army, resigned from the Union army, and took control of the Confederate army. In the end, Lincoln was able to keep four of the states of the Upper South in the Union (Kentucky, Missouri, Maryland, and Delaware).

War Aims and Strategies

From the beginning of the war, the Southern defense of the slave system was unrelenting. This position greatly undermined the possibility of the Confederacy receiving aid from the French and the English. Economically, European support of the Confederacy would have made sense; European nations were dependent on cotton cultivated in the American

South. However, both France and England firmly opposed slavery and had outlawed it in their countries decades earlier. The South also overestimated the British need for Southern cotton; Britain soon proved that it could get cotton elsewhere.

Both sides began recruiting armies in the spring and early summer of 1861. Lincoln was able to summon support in the Northern states not from speeches on slavery but from the simple claim that the actions of the South was an attack on the very principles of the republican form of government. Both sides predicted early victory. The capital of the Confederacy was moved to Richmond, Virginia, after Virginia joined the Confederacy; cries of "On to Richmond!" filled the Northern newspapers. For political reasons, Lincoln pushed for an early attack against the South (Winfield Scott presented an alternative proposal, stating that the best policy for the North would be to blockade all Southern ports and starve the South into submission). A Union army advanced on Richmond. On July 21, 1861, at the **First Battle of Bull Run**, Union forces retreated in chaos back toward Washington. After this battle, Northern political leaders and generals conceded that victory in this war would not be as easy as they initially thought it might be.

Effects of Bull Run

The Battle of Bull Run showed both sides that new tactics would be necessary for victory. The plan proposed by Winfield Scott, now referred to as the **Anaconda Plan**, was reviewed more carefully by Abraham Lincoln. Lincoln had the United States Navy blockade Southern ports; as the war wore on, this became increasingly important. Industrial goods that the South had imported from the industrial North in earlier years now could not be obtained from Europe either. Later in the war, Confederate states could not export cotton to Europe for very badly needed currency. Another part of the Anaconda Plan called for Northern naval forces to control the Mississippi River. The Union made major headway with this part of the plan in April 1862 when a Union naval force captured New Orleans.

The Confederacy also made a major foreign trading mistake in early 1862. Cotton-producing states were convinced not to export cotton to England and France. Confederate leaders thought that textile factory owners in those countries would be so affected by this that they would pressure their governments to help the Confederacy and get their cotton back. Instead, Europeans turned elsewhere for cotton (especially India). As stated previously, when the South wanted to export cotton later in the war, they couldn't because of the naval blockade. It also became obvious that the organization of the South into a confederacy during a period of war was a disadvantage; individual state governments had the constitutional right to block critical tax programs and requisitions. The decision of the Confederacy to print paper money with no secure backing also would prove to be detrimental.

Union Triumphant in the West

The Confederacy won several more battles in 1862, including the **Second Battle of Bull Run**. General George McClellan was named commander of the Union army and began formulating a plan to attack the Confederacy from the west. In February 1862, forces commanded by General Ulysses S. Grant captured Fort Henry and Fort Donelson, in Tennessee. Forces on both sides realized the importance of these victories. Grant continued to conquer Southern territory from this position. On April 6, 1862, the incredibly bloody but inconclusive **Battle of Shiloh** was fought. Up until this point, it was the bloodiest battle ever fought in America. McClellan began to develop the reputation as a commander who was afraid to enter his troops into battle, even though the situation warranted it.

The Confederacy attempted to use technology to defeat the Northern naval blockade. In March 1862, they presented their very first **ironclad ship**, the ***Merrimack***. Shortly after, the Union displayed the first Union ironclad, the ***Monitor***. The two ironclads met once in battle, with neither ship able to do much damage to the other.

Developments in the South and in the North

Being a nation founded on the principle of **states' rights** was often a disadvantage for the Confederacy. Many Confederate soldiers who enlisted for one year in 1861 appeared ready to return home in 1862. General Robert E. Lee insisted that a system of **conscription** be introduced to ensure a steady supply of soldiers. In April 1862, the Confederate legislature passed laws requiring three years in the army for all white men from ages 18 to 35 (after the horrible losses of Antietam, this was extended to 45). Many advocates of states' rights violently objected to these regulations. Three Southern governors tried to block the conscription law in their states, saying that only the individual states had the right to make such laws. In some sections of the South, nearly 60 percent of available manpower never served in the army. The Confederacy also adopted a plan to pay plantation owners who released their slaves to serve in the army; this was largely resisted because it was economically harmful to slave owners.

By late 1862, severe shortages of food and other materials began to spread throughout the South. Prices skyrocketed. Many soldiers deserted the army to return home to help their families through these difficult times. Large numbers of deserters and those who had resisted the draft became a problem in some sections of the South. The Confederacy instituted an income tax in order to get needed income for the government. Under existing circumstances, the actual collection of this money was sometimes difficult.

Many similar tensions existed in the North. In 1863, a system of conscription was introduced, requiring service of all men from ages 20 through 45. As in the South, draft dodgers could be found in the North. A provision of the Northern draft law that was very unpopular with many allowed a drafted person to avoid service by hiring a substitute or by paying the government $300; many of the "replacement" soldiers were Irish immigrants. Draft riots took place in New York City in July 1863, with nearly 200 people dying in these protests. Many taking part in the riots were Irish Americans, and many of those killed were black. Draft offices and other buildings were destroyed; Irish Americans did not want to take part in a war that would free the slaves, whom they perceived would be their competitors for jobs.

The North also had trouble financing the war. In 1861, a federal income tax was instituted. Still short of money, the government began issuing "**greenbacks**" in 1862; this money, not backed by gold, was considered official legal tender until the end of the war.

In every wartime setting in American history, the power of the chief executive was expanded. This was certainly true in the Civil War. President Lincoln assumed powers that no previous president had even considered. By executive order, parts of Kentucky were placed under **martial law** for much of the war. Some Democrats in the North, nicknamed **Copperheads**, vigorously opposed the war, stating that it would lead to masses of freed slaves coming North and taking jobs. Copperheads were sometimes arrested, and three of them were actually deported from the North. Over 14,000 who opposed the war were imprisoned without trial. In several cases, Lincoln ordered the **writ of habeas corpus** suspended.

Emancipation Proclamation

When he was elected president, Abraham Lincoln had no thought whatsoever of freeing the slaves; he repeatedly stated that he had no constitutional right to do that. However, on a practical level Lincoln realized that the continued existence of slavery in the South would make Northern victory harder; the existence of slavery allowed Southern landowners to leave their fields and fight in the Confederate army.

The **Emancipation Proclamation** was issued on January 1, 1863. The timing of this was a brilliant political move. Support for the war in the North had been waning; the Emancipation Proclamation gave Northerners a moral justification to continue fighting. This measure was received by different groups in predictable ways. Northern blacks were heartened by it, Southerners condemned it, and in Southern territories controlled by the Union army, slaves were actually freed. Many in England agreed with the proclamation; any last hopes that England might enter the war to aid the Confederacy were dashed at this point. Some whites in the North feared that ex-slaves would end up taking their jobs, and as a result, in the 1862 congressional elections, Democrats picked up seats.

Blacks were not accepted into the Union army at the beginning of the war. After the Emancipation Proclamation, many ex-slaves from Southern territories and free blacks from the North joined the Union army. By 1865, blacks made up almost 10 percent of the entire Union army. Black soldiers traditionally served in all-black units with white officers. (The heroism of the 54th Massachusetts Infantry can be seen in the movie *Glory*.)

1863: The War Tips to the North

The darkest days of the war for the Union occurred in late 1862 and early 1863. The Union army suffered major defeats at the **Battle of Fredericksburg** (December 13, 1862) and at the **Battle of Chancellorsville** (May 1 to 3, 1863). Competent leadership of the Union army remained a major problem.

Yet, time was an enemy of the Confederate army. As commander, General Robert E. Lee found it increasingly difficult to get men and resources (the Northern naval blockade definitely was affecting Southern military efforts by this point). In June 1863, Lee decided to move the Confederate army out of Virginia into Pennsylvania. At the **Battle of Gettysburg** (July 1 to 3, 1863), Lee was defeated by the Union army, commanded by General George Meade. This was the bloodiest overall battle of the war, with 24,000 casualties suffered by the North and 28,000 by the South. Lee's army was forced to retreat to Virginia and would never again be able to mount an attack into Northern territory. Some military historians claim that the fate of the Confederate army was sealed by their defeat at Gettysburg.

The tide of the war continued to swing to the North as a result of several victories by armies commanded by Ulysses S. Grant. On July 4, 1863, Grant completed his victory at the **Battle of Vicksburg**, ending a siege of the city that lasted six weeks. Victory at Vicksburg gave the Union virtual control of the Mississippi River. In November, Grant was victorious at the Battle of Chattanooga (November 23 to 25, 1863). Abraham Lincoln's **Gettysburg Address** had been given four days earlier. In January 1864, Grant was made commander of the Union army. At the same time, some in the Confederate government began speaking of the need for peace negotiations with the North.

Grant and the army of the Potomac began to advance toward Richmond in the spring of 1864, while an army commanded by William T. Sherman began to advance toward Atlanta.

War Weariness in the North and the South

In both the North and the South, the pressures of a long war were obvious by 1864. To many in the South, it was clear that the Confederacy would be defeated. Severe food and material shortages continued. In the North, the presidential campaign of 1864 produced little excitement. Lincoln's Democratic opponent was General George McClellan. In early September 1864, Lincoln confided to friends that he thought he would lose the presidency. However, word arrived that General Sherman had taken the key Confederate city of Atlanta. That, along with a lack of any real enthusiasm for (and by) McClellan, allowed Lincoln to easily win reelection.

End of the Confederacy

Sherman employed a scorched earth policy as he marched from Atlanta to Savannah, Georgia, in November and December 1864. In early April 1865, General Lee took the Confederate army from Richmond and tried to escape to the South. The Union army caught up to him, and he finally surrendered on April 9, 1865, at the courthouse in **Appomattox**, Virginia. By the first week of June, all other Confederate forces also surrendered and began to return to oftentimes devastated homelands.

Lincoln only had time to begin to plan for what a post–Civil War America would look like. On April 14, 1865, he was assassinated by John Wilkes Booth at Ford's Theater. Booth was a pro-Southerner. He and a group of co-conspirators also planned to kill Vice President Andrew Johnson and other members of the Lincoln cabinet. Booth was hunted down several days later and was killed by gunfire; several others conspiring with him were found and, after trials by military tribunals, hanged. The incredibly difficult task of reconstruction would have to be handled by the new president, Andrew Johnson, a Tennessee Democrat whom Lincoln had chosen to be his vice president.

Chapter Review

Rapid Review

Key Idea

To achieve the perfect 5, you should be able to explain the following:

- By 1861, various social, political, economic, and cultural factors made conflict between the North and the South inevitable.
- The North had numerous industrial, transportation, and financial advantages that they utilized throughout the Civil War.
- The Confederate States of America was created in February 1861; the fact that these states were organized as a confederacy had several disadvantages that would become obvious as the war progressed.
- Success for the Confederacy depended on European aid; Southerners overestimated the dependence of Europe on Southern crops.

- Confederate generals proved much more competent than their Union counterparts in several key battles in the first years of the war.
- By late 1862, the war had produced severe effects on the home fronts; food shortages were occurring in the South, and President Lincoln imposed martial law in several locations and suspended the writ of habeas corpus in the cases of some of his political opponents.
- The Emancipation Proclamation provided a moral justification for Northerners to continue the war.
- The war shifted decisively in favor of the North in 1863, with the battles at Gettysburg and Vicksburg proving to be critical victories for the North.
- The surrender of the Confederacy in April 1865 was caused by a severe lack of morale, manpower, and economic stability in the South.

Time Line

1860: Lincoln elected president
South Carolina secedes from Union
1861: Confederate States of America created
Attack on Fort Sumter
First Battle of Bull Run
Union begins blockade of Southern ports
1862: New Orleans captured by Union navy
Battle of Shiloh
Conscription begins in Confederate states
Emancipation of slaves in Southern states begins
Battle of Antietam
British announce they will not aid the Confederacy in any substantial way
1863: Emancipation Proclamation
Conscription begins in the North; draftees may hire "replacements"
First black soldiers enlist in Union army
Crucial Union victory at Gettysburg
Crucial Union victory at Vicksburg
Draft riots in New York City
1864: Abraham Lincoln reelected
General Sherman carries out his "march to the sea"
Desertion becomes a major problem in the Confederate army
1865: General Lee surrenders at Appomattox
Abraham Lincoln assassinated

› Review Questions

1. The North held many advantages at the beginning of the Civil War *except:*
A. the North occupied more territory than the South
B. the North had more railroad lines
C. the North had more factories
D. the North had a larger population

2. European states did not aid the Confederacy in the Civil War because ______________.
A. there were alternative sources of cotton and other crops that they could turn to
B. they opposed the Confederacy's position on slavery
C. they did not believe that the Confederacy could win
D. all of the above

3. The military draft was unpopular to many in the North because ______________.
   A. the draft allowed blacks to enter the armed forces
   B. the draft allowed Irish-American immigrants to enter the army
   C. the draft allowed those drafted to hire "replacements"
   D. martial law was needed in many locations to enforce the draft provisions

4. The Battle of Vicksburg was an important victory for the Union because ______________.
   A. it reversed several Union defeats in the same year
   B. it gave the Union a pathway to Atlanta
   C. it gave the Union virtual control of the Mississippi River
   D. it demonstrated that General Lee could, in fact, be beaten

5. Copperheads were ______________.
   A. Democrats in the North who opposed the war
   B. Republicans in the North who suggested that Lincoln be replaced
   C. Democrats in the North who switched alliance to Lincoln
   D. Southern Democrats who wanted negotiations with the North as early as 1863

Answers and Explanations

1. A. All of the others were major advantages for the Union war effort.

2. D. All of the reasons given helped convince the Europeans not to assist the Confederacy. The Confederacy's position on slavery proved to be especially troublesome, since slavery had long been outlawed in Europe.

3. C. The fact that replacement soldiers, usually immigrants, could be hired or that a payment of $300 to the government could get a man out of the draft made the system very unpopular to many.

4. C. The six-week Battle of Vicksburg occurred in 1863 and helped turn the war in the Union's favor. As a result of Vicksburg, the Mississippi River was virtually in the hands of the Union. Lee did not command the Confederate forces at Vicksburg.

5. A. Copperheads were Democrats in the North who claimed that the war would bring economic ruin to the North, with freed slaves taking jobs that whites now had. Some were arrested and deported.

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Chapter 2

How to Plan Your Time

IN THIS CHAPTER

Summary: Assess your own study patterns and preparation plans

Key Idea

Key Ideas

- ✪ Explore three approaches
- ✪ Choose a calendar that works for you

Three Approaches to Preparing for the AP Literature Exam

No one knows your study habits, likes, and dislikes better than you do. So you are the only one who can decide which approach you want and/or need to adopt to prepare for the Advanced Placement Literature exam. Look at the brief profiles below. These may help you to place yourself in a particular prep mode.

You're a full-year prep student (Approach A) **if:**

1. You're the kind of person who likes to plan for a vacation or the prom a year in advance.
2. You'd never think of missing a practice session, whether it's your favorite sport, musical instrument, or activity.
3. You like detailed planning and everything in its place.
4. You feel you must be thoroughly prepared.
5. You hate surprises.
6. You're always early for appointments.

You're a one-semester prep student (Approach B) **if:**

1. You begin to plan for your vacation or the prom 4–5 months before the event.
2. You are willing to plan ahead so that you will feel comfortable in stressful situations, but you are okay with skipping some details.
3. You feel more comfortable when you know what to expect, but a surprise or two does not floor you.
4. You're always on time for appointments.

You're a 4–6 week prep student (Approach C) **if:**

1. You accept or find a date for the prom a week before the big day.
2. You work best under pressure and tight deadlines.
3. You feel very confident with the skills and background you've gained in your AP Literature class.
4. You decided late in the year to take the exam.
5. You like surprises.
6. You feel okay if you arrive 10–15 minutes late for an appointment.

CALENDAR FOR EACH PLAN

Calendar for Approach A: Yearlong Preparation for the AP Literature Exam

Strategy Although its primary purpose is to prepare you for the AP Literature exam you will take in May, this book can enrich your study of literature, your analytical skills, and your writing skills.

SEPTEMBER–OCTOBER (Check off the activities as you complete them.)

_______ Determine the student mode into which you place yourself.
_______ Carefully read Chapters 1 and 2.
_______ Pay close attention to the walk through the Diagnostic exam.
_______ Get on the Web and take a look at the AP website(s).
_______ Skim the Comprehensive Review section.
_______ Buy a highlighter.
_______ Flip through the entire book. Break the book in. Write in it. Highlight it.
_______ Get a clear picture of what your own school's AP Literature curriculum is.
_______ Review the Bibliography and establish a pattern of outside reading in the literary genres.
_______ Begin to use the book as a resource.

NOVEMBER (The first 10 weeks have elapsed.)

_______ Write the free-response essay in the Diagnostic exam.
_______ Compare your essay with the sample student essays.
_______ Refer to the section on the free-response essay.
_______ Take five of our prompts and write solid opening paragraphs.

DECEMBER

_______ Maintain notes on literary works you studied in and out of class.
_______ Refine your analytical skills.
_______ Write the prose passage or poetry essay in the Diagnostic exam. (This will depend on the organization of your own curriculum.)
_______ Compare your essay with sample student essays.

JANUARY (20 weeks have elapsed.)

_______ Write the third essay in the Diagnostic exam. (This will depend on the one you did previously.)
_______ Compare your essay with sample student essays.

FEBRUARY

_______ Take the multiple-choice section of the Diagnostic exam.
_______ Carefully go over the explanations of the answers to the questions.
_______ Score yourself honestly.
_______ Make a note of terms, concepts, and types of questions that give you difficulty.
_______ Review troublesome terms in the Glossary.

MARCH (30 weeks have elapsed.)

_______ Form a study group.
_______ Outline or create a chart for full-length works that would be appropriate for the free-response essay.
_______ Choose a favorite poem and create an essay question to go with it, or use one of our suggested prompts.
_______ Choose a prose passage or essay and create an essay question to go with it, or choose one of our suggested prompts.
_______ Write the poetry essay.
_______ Write the prose essay.
_______ Compare essays and rate them with your study group. (Use our rubrics.)

APRIL

_______ Take Practice Test 1 in the first week of April.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct your weaknesses.
_______ Practice creating multiple-choice questions of different types with your study group.
_______ Develop and review worksheets for and with your study group.

MAY—First 2 Weeks (THIS IS IT!)

_______ Highlight only those things in the Glossary you are still unsure of. Ask your teacher for clarification. Study!
_______ Thoroughly prepare three to five complete, full-length works; include several quotations that you can work into various responses.
_______ Write at least three times a week under timed conditions.
_______ Take Practice Test 2.
_______ Score yourself.
_______ Give yourself a pat on the back for how much you have learned and improved over the past nine months.
_______ Go to the movies. Call a friend.
_______ Get a good night's sleep. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST.

Calendar for Approach B: Semester-Long Preparation for the AP Literature Exam

Strategy Working under the assumption that you've completed one semester of literature studies, apply those skills you've learned to prepare for the May exam.

You have plenty of time to supplement your course work by taking our study recommendations, maintaining literary notations, doing outside readings, and so on.

We divide the next 16 weeks into a workable program of preparation for you.

JANUARY–FEBRUARY

(Check off the activities as you complete them.)

_______ Carefully read Chapters 1 and 2.
_______ Write the three essays in the Diagnostic exam.
_______ Compare your essays with the sample student essays.
_______ Complete the multiple-choice section of the Diagnostic exam.
_______ Carefully go over the answers and explanations of the answers.
_______ Take a close look at the Bibliography for suggestions on possible outside readings.

MARCH (10 weeks to go.)

_______ Form a study group.
_______ Outline or create a chart for full-length works that would be appropriate for the free-response essay.
_______ Choose a favorite poem and create an essay question to go with it, or use one of our suggested prompts.
_______ Choose a prose passage or essay and create an essay question to go with it, or choose one of our suggested prompts.
_______ Write the poetry essay.
_______ Write the prose essay.
_______ Compare essays and rate them with your study group. (Use our rubrics.)

APRIL

_______ Take Practice Test 1 in the first week of April.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct your weaknesses.
_______ Practice creating multiple-choice questions of different types with your study group.
_______ Develop and review worksheets for and with your study group.

MAY First 2 Weeks (THIS IS IT!)

_______ Highlight only those things in the Glossary you are still unsure of. Ask your teacher for clarification. Study!
_______ Thoroughly prepare at least three to five complete, full-length works; include several quotations that you can work into various questions.
_______ Write at least three times a week under timed conditions.
_______ Take Practice Test 2.
_______ Score yourself.
_______ Give yourself a pat on the back for how much you have learned and improved over the past nine months.
_______ Go to the movies. Call a friend.
_______ Get a good night's sleep. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST.

Calendar for Approach C: 4–6 Week Preparation for the AP Literature Exam

Strategy

At this point, we are going to assume that you have been developing your literary, analytical, and writing skills in your English class for more than 6 months. You will, therefore, use this book primarily as a specific guide to the AP Literature exam.

Remember, there is a solid review section in this text to which you should refer.

Given the time constraints, now is not the time to try to expand your AP Literature background. Rather, it is the time to limit and refine what you already know.

APRIL

________ Skim through Chapters 1 and 2.
________ Carefully go over the "rapid reviews."
________ Strengthen, clarify, and correct areas you are weak in after taking the Diagnostic exam.
________ Write a minimum of three sample opening paragraphs for each of the three types of essays.
________ Write a minimum of two timed essays for each type of essay on the exam.
________ Complete Practice Test 1.
________ Score yourself and analyze your errors.
________ Refer to appropriate chapters to correct your weaknesses.
________ Refer to the Bibliography.
________ If you feel unfamiliar with specific poetic forms, refer to the list of suggested, appropriate works.
________ Create review sheets for three to five solid, full-length works.
________ Skim and highlight the Glossary.
________ Develop a weekly study group to hear each other's essays and to discuss literature.

MAY—First 2 Weeks (THIS IS IT!)

________ Complete Practice Test 2.
________ Score yourself and analyze your errors.
________ Refer to appropriate chapters to correct your weaknesses.
________ Go to the movies. Call a friend.
________ Get a good night's sleep. Fall asleep knowing you are well prepared.
________ GOOD LUCK ON THE TEST.

"One of the first steps to success on the AP exam is knowing your own study habits."
—Margaret R.
AP English teacher

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Chapter 5

Section II of the Exam: How to Approach the Prose Passage Essay

IN THIS CHAPTER

Key Idea

Summary: Complete explanation of the prose passage essay and its purpose as it is presented on the AP English Literature exam

Key Ideas

- ✪ Learn the types of prose passage prompts you might encounter on the AP English Literature exam
- ✪ Learn about the rubrics and rating of the AP English Literature prose passage essay
- ✪ Learn the basics of reading and notating a given passage
- ✪ Learn the basics of constructing your response to the prompt
- ✪ Examine the student models that respond to the diagnostic exam's prose passage essay prompt
- ✪ Learn how the rubrics were used to rate the student sample essays

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Introduction to the Prose Passage Essay

This section of the exam gives you an opportunity to read and analyze a prose piece of litera-ture. This is your chance to become personally involved in the text and to demonstrate your literary skills.

What Is an Ap Literature Prose Passage?

Generally, it is a one-page excerpt from a work of fiction. More often than not, the selection will be from a novel or short story. The nonfiction selection may include essays, biographies, autobiographies, and articles from periodicals. Be aware that the exam may also present an excerpt from a drama.

What Is the Purpose in Writing an Essay About a Prose Piece?

First, the people at the College Board want to determine your facility in reading and interpreting a sustained piece of literature. It requires you to understand the text and to analyze those techniques and devices the author uses to achieve his or her purpose.

Second, the AP exam is designed to allow you to demonstrate your ease and fluency with terminology, interpretation, and criticism. Also, the level of your writing should be a direct reflection of your critical thinking.

Third, the AP exam determines your ability to make connections between analysis and interpretation. For example, when you find a metaphor, you should identify and connect it to the author's intended purpose or meaning. You should not just list items as you locate them. You must connect them to your interpretation.

Tip

Before beginning to work with an actual prose passage, read the review of processes and terms in the Comprehensive Review section of this book. You should also have completed some of the activities in that section.

Types of Prose Passage Essay Prompts

Let's look at a few prose passage questions that have been asked on the AP Literature exam in the past:

Key Idea

- Analyze narrative and literary techniques and other resources of language used for characterization.
- How does a narrator reveal character? (i.e., tone, diction, syntax, point of view)
- How does the author reveal a character's predicament? (i.e., diction, imagery, point of view)
- Explain the effect of the passage on the reader.
- Compare/contrast two passages concerning diction and details for the effect on the reader.
- How does the passage provide characterization and evaluation of one character over another? (i.e., diction, syntax, imagery, tone)
- What is the attitude of the speaker toward a particular subject?
- Analyze the effect of revision when given both the original and the revised version of a text.
- Analyze style and tone and how they are used to explore the author's attitudes toward his or her subject.
- How is the reader prepared for the conclusion of the piece?

You should be prepared to write an essay based on any of these prompts. Practice. Practice. Practice anticipating questions. Keep a running list of the kinds of questions your teacher asks.

Tip

Don't be thrown by the complexity of a passage. Remember, **you** choose the references you wish to incorporate into your essay. So, even if you haven't understood everything, you are still able to write an intelligent essay—**as long as you address the prompt** and refer to the parts of the passage you do understand.

Watch out for overconfidence when you see what you believe to be an easy question with an easy passage. You are going to have to work extra hard to find the nuances in the text that will allow you to write a mature essay.

Rating the Prose Passage Essay

You will be relieved to know that the rating of your essay is *not* based on whether or not the reader likes you or agrees with your point of view.

How Do the AP Readers Rate My Essay?

It's important to understand just what it is that goes into rating your essay. This is called a "rubric," but don't let that word frighten you. A rubric is a word that simply refers to the rating standards that are set and used by the people who read the essays. These standards are fairly consistent, no matter what the prompt might be. The primary change is in the citing of the specifics in a particular prompt.

Let us assure you that, as experienced readers of the AP English exams, we know that the readers are trained to reward those things you do well in addressing the question. They are *not* looking to punish you. They are aware of the time constraints, and they read your essay just as your own instructor would read the first draft of an essay you wrote on a 40-minute exam. These readers look forward to an interesting, insightful, and well constructed essay.

So, let's take a look at the following rubric:

Key Idea

- A "**9** essay" has all the qualities of an 8 essay, *and* the writing style is especially impressive, as is the analysis and/or discussion of the specifics related to the prompt and the text.
- An "**8** essay" will effectively and cohesively address the prompt. It will analyze and/or discuss the stylistic elements called for in the question. And it will do so using appropriate evidence from the given text. The essay will also show the writer's ability to control language well.
- A "**7** essay" has all the properties of a 6, only with a more complete and well-developed analysis /discussion or a more mature writing style.
- A "**6** essay" adequately addresses the prompt. The analysis and/or discussion is on target and makes use of appropriate specifics from the text. However, these elements are less fully developed than scores in the 7, 8, and 9 range. The essay writer's ideas are expressed with clarity, but the writing may have a few errors in syntax and /or diction.
- A "**5** essay" demonstrates that the writer understands the prompt. The analysis / discussion is generally understandable, but the analysis /discussion is limited or uneven. The writer's ideas are expressed clearly with a few errors in syntax or diction.
- A "**4** essay" is not an adequate response to the prompt. The writer's analysis / discussion of the text indicates a misunderstanding, an oversimplification, or a misrepresentation of the given passage. The writer may use evidence that is not appropriate or not sufficient to support the analysis /discussion.
- A "**3** essay" is a lower 4 because it is even less effective in addressing the prompt. It is also less mature in its syntax and organization.
- A "**2** essay" indicates little success in speaking to the prompt. The writer may misread the question, only summarize the passage, never develop the required analysis/discussion, or simply ignore the prompt and write about another topic altogether. The writing may also lack organization and control of language and syntax. (*Note*: No matter how good a summary may be, it will never rate more than a 2.)
- A "**1** essay" is a lower 2 because it is even more simplistic, disorganized, and lacking in control of language.

Tip

Remember, the essay is really a first draft. The test readers know this and approach each essay with this in mind.

Timing the Prose Essay

Timing is crucial. With that in mind, here's a workable strategy:

Strategy

- 1–3 minutes reading and "working the prompt."
- 5 minutes reading and making marginal notes about the passage. Try to isolate 2 quotations that strike you. This may give you your opening and closing.
- 10 minutes preparing to write. (Choose one or two of the following methods that you feel comfortable with.)
  —Underlining, bracketing, circling
  —Marginal notations
  —Charts or key word/one word/line number outlining
- 20 minutes to write your essay, based on your preparation.
- 3 minutes for proofreading.

Using a highlighter is not allowed during the exam. However, it is a strong tool for practice in critical reading.

Note: Throughout this book, the term "highlight" will also refer to underlining, circling, or bracketing.

Tip

In the margin, note what time you should be finished with each essay. For example, the test starts at 1 P.M. You write 1:40 in the margin. Time to move on.

Working the Prompt

You can't write clearly unless you know **Why** you are writing and **What's Expected** of you. When you "Work the Prompt," you are maximizing both of these areas.

How Should I Go About Reading the Prose Prompt?

To bring the answer home to you, we will deconstruct a prompt for you now. (This is the same question that is in the Diagnostic exam.) Plan to spend 1–3 minutes carefully reading the question. This will give you time to really digest what the question is asking you to do.

Here's the prompt:

> In the following passage from the short story "The Dead," James Joyce presents an insight into the character of Gabriel. Write a well-organized essay in which you discuss various aspects of Gabriel's character that Joyce reveals to the reader and to Gabriel himself. Refer to such techniques and devices as imagery, point of view, motif, diction, and syntax.

Here are three reasons why you should do a 1–3-minute careful analysis of the prompt.

Key Idea

1. Once you know what is expected, you will read in a more directed manner.
2. Once you internalize the question, you will be sensitive to those details that will apply.
3. Once you know all the facets that need to be addressed, you will be able to write a complete essay demonstrating adherence to the topic.

Tip Topic adherence, which means sticking to the question, is key to achieving a high score.

Let's begin. Underline, circle, or bracket the essential terms and elements in the prompt. Time yourself. How long did it take you? Don't worry if it took you longer than 1–3 minutes in this first attempt. You will be practicing this technique throughout this review, and it will become almost second nature to you.

Compare our working of the prompt with yours.

> In the following passage from the short story "The Dead," James Joyce presents an insight into the character of Gabriel. Write a well-organized essay in which you discuss various aspects of Gabriel's character that Joyce reveals to the reader and to Gabriel himself. Refer to such techniques and devices as imagery, point of view, motif, diction, and syntax.

Tip When the question uses the expression "such as," you are **not** required to use only those ideas presented; you are free to use your own selection of techniques and devices. Notice that the prompt requires more than one technique. One will not be enough. You **must** use more than one. If you fail to use more than one technique, no matter how well you present your answer, your essay will be incomplete.

Reading and Notating the Prose Passage

Depending on your style and comfort level, choose one of these approaches to your reading:

Strategy

1. Read quickly to get the gist of the passage. Reread, using the visual and marginal notes approach.
2. Read slowly, using highlighting and making marginal notes. Reread to confirm that you understand the full impact of the passage.

Note: In both approaches, you **must** highlight and make marginal notes. There is no way to avoid this. Ignore what you don't immediately understand. It may become clear to you after you finish reading the passage. Practice. Practice. Concentrate on those parts of the passage that apply to what you highlighted in the prompt.

There are many ways to read and interpret any given passage. You have to choose which one to use and which specifics to include for support. Don't be rattled if there is leftover material.

We've reproduced the passage for you below so that you can practice both the reading and the process of deconstructing the text. Use highlighting, arrows, circles, underlining, notes, numbers, and whatever you need to make the connections clear to you.

Let's begin. Spend 8–10 minutes working the material. **Do not skip this step.** It is time well spent and is a key to the high-score essay.

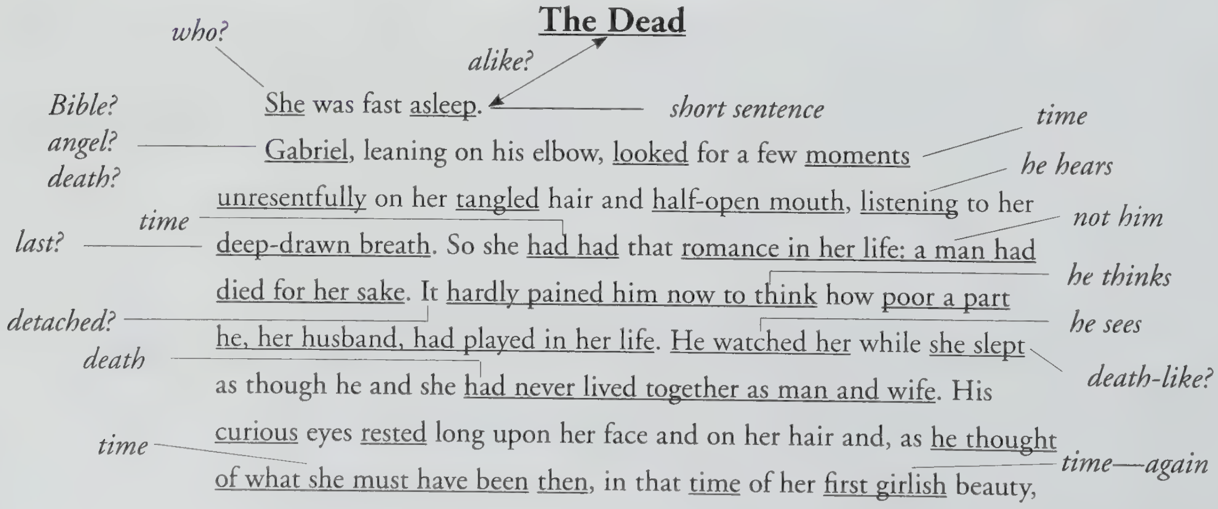
The Dead

She was fast asleep.

Gabriel, leaning on his elbow, looked for a few moments unresentfully on her tangled hair and half-open mouth, listening to her deep-drawn breath. So she had had that romance in her life: a man had died for her sake. It hardly pained him now to think how poor a part he, her husband, had played in her life. He watched her while she slept as though he and she had never lived together as man and wife. His curious eyes rested long upon her face and on her hair and, as he thought of what she must have been then, in that time of her first girlish beauty, a strange friendly pity for her entered his soul. He did not like to say even to himself that her face was no longer beautiful but he knew that it was no longer the face for which Michael Furey had braved death.

Perhaps she had not told him all the story. His eyes moved to the chair over which she had thrown some of her clothes. A petticoat string dangled to the floor. One boot stood upright, its limp upper fallen down: the fellow of it lay upon its side. He wondered at his riot of emotions of an hour before. From what had it proceeded? From his aunt's supper, from his own foolish speech, from the wine and dancing, the merry-making when saying good-night in the hall, the pleasure of the walk along the river in the snow. Poor Aunt Julia! She, too, would soon be a shade with the shade of Patrick Morkan and his horse. He had caught that haggard look upon her face for a moment when she was singing *Arrayed for the Bridal.* Soon, perhaps, he would be sitting in the same drawing-room dressed in black, his silk hat on his knees. The blinds would be drawn down and Aunt Kate would be sitting beside him, crying and blowing her nose and telling him how Julia had died. He would cast about in his mind for some words that might console her, and would find only lame and useless ones. Yes, yes: that would happen very soon.

Key Idea Now, compare your reading notes with what we've done below. Yours may vary from ours, but the results of your note-taking should be similar in scope.



The Dead

She was fast asleep.

Gabriel, leaning on his elbow, looked for a few moments unresentfully on her tangled hair and half-open mouth, listening to her deep-drawn breath. So she had had that romance in her life: a man had died for her sake. It hardly pained him now to think how poor a part he, her husband, had played in her life. He watched her while she slept as though he and she had never lived together as man and wife. His curious eyes rested long upon her face and on her hair and, as he thought of what she must have been then, in that time of her first girlish beauty,

he's kind? a strange friendly pity for her entered his soul. He did not like to say *back to death*
weak even to himself that her face was no longer beautiful but he knew that it *time*
coward? was no longer the face for which Michael Furey had braved death. *fury?* *not a coward*
self-image *time*
Perhaps she had not told him all the story. His eyes moved to the
chair over which she had thrown some of her clothes. A petticoat string *weak?*
dangled to the floor. One boot stood upright, its limp upper fallen down: *lack of control?*
time the fellow of it lay upon its side. He wondered at his riot of emotions of *afraid to let go?*
an hour before. From what had it proceeded? From his aunt's supper,
self-image from his own foolish speech, from the wine and dancing, the merry-making
death? when saying good-night in the hall, the pleasure of the walk along the river *time*
cold death? in the snow. Poor Aunt Julia! She, too, would soon be a shade with the
ghost? *cold* shade of Patrick Morkan and his horse. He had caught that haggard look *cold*
time upon her face for a moment when she was singing *Arrayed for the Bridal.*
time Soon, perhaps, he would be sitting in the same drawing-room dressed in *mourning*
death black, his silk hat on his knees. The blinds would be drawn down and *closed off*
Aunt Kate would be sitting beside him, crying and blowing her nose and *mourning*
death *weak?* telling him how Julia had died. He would cast about in his mind for some *self-image* *detached*
uncertain? words that might console her, and would find only lame and useless ones. *weakness*
Yes, yes: that would happen very soon. ——— *time*

Strategy

After you have marked the passage, review the prompt. You are asked to look for ways Joyce reveals Gabriel's character. When you consult your notes, certain categories will begin to pop out at you. These can be the basis for the development of the body of your essay. For example, we saw details and images that support the concepts of:

- Death
- Passivity
- Time
- Detachment
- Insecurity

In addition, stylistically, we noticed the use of short sentences.

Here's how one category developed using the notations made on the passage. Notice that we have ignored notes that did not apply to the prompt.

Concept: Time

Words/Phrases from the Text:

"a few moments"
"had had that romance"
"he thought of what she must have been then"
"girlish beauty"
"no longer beautiful"
"no longer the face"
"an hour before"
"soon be a shade"
"for a moment"
"soon"
"that would happen very soon"

Conclusion: Gabriel moves from the distant past to the near future. He becomes aware of the change in the marital relationship with his wife and with his own passage.

Your Turn

Now you choose a concept you are able to explore and defend that reveals Gabriel's character.

Concept:

Words/Phrases from the Text:

Conclusion:

In response to the prompt, we have decided that the techniques/devices we will analyze are:

- Imagery
- Diction
- Style
- Motif

Here's one technique/device and how it is developed in the passage. Again, notice that we use our margin notes to trace this development.

Technique/Device: Imagery

Words/Phrases from the Text:

the title "The Dead"
"entered his soul"
"fallen down"
"in the snow"
"dressed in black"
"crying and blowing her nose"
Gabriel's very name
"braved death"
"good-night"
"would soon be a shade"
"the blinds would be drawn"

Conclusion: These images foreshadow and emphasize that there is a coldness and a loss in relationships and self-image.

Your Turn

Now you choose the technique/device you are able to explore and defend that reveals Gabriel's character.

Technique/Device:

Words/Phrases from the Text:

Conclusion:

Key Idea If you expand the above techniques/devices and categories into interpretive statements and support those statements with appropriate details that you've already isolated, you will be writing a detailed essay.

Writing the Opening Paragraph

Your opening statement is the one that sets the tone of your essay and possibly raises the expectations of the reader. Spend time on your first paragraph to maximize your score.

Make certain that your topic is very clear. This reinforces the idea that you fully understand what is expected of you and what you will communicate to the reader. Generally, identify both the text and its author in this first paragraph.

A suggested approach is to relate a direct quotation from the passage to the topic.

Tip Consider the "philosophy of firsts." It is a crucial strategy to spend focused time on the first part of the question and on the first paragraph of the essay because:

1. It establishes the direction and tone of your essay.
2. It gives you the guidelines for what to develop in your essay.
3. It connects you to the reader.

Remember our philosophy: **In the Beginning:** if you focus on the beginning, the rest will fall into place. A wonderful thing happens after much practice, highlighting, and note-taking. Your mind starts to focus automatically. Trust us on this. It is the winning edge that can take an average essay and raise it to a higher level.

Let's begin. Take 5 minutes to write your opening paragraph for "The Dead" prompt. Write quickly, referring to your notes.

Let's check what you've written:

Highlight these points to see if you've done them. You may be surprised at what is actually there.

- Have you included author and title? _____ Yes _____ No
- Have you addressed the character of Gabriel? _____ Yes _____ No

- Have you specifically mentioned the techniques you will refer to in your essay?

_____ Yes _____ No

Here are four sample opening paragraphs that address all of the criteria:

A

In "The Dead" by James Joyce, the character Gabriel is revealed through diction, point of view, and imagery as he watches his wife sleep.

B

Poor Gabriel! Who would have thought he knew so little about himself and his life. And yet, in "The Dead," James Joyce, through diction, point of view, and imagery, makes it clear to the reader and to Gabriel that there is much to reveal about his character.

C

"Yes, yes: that would happen very soon." And, yes, very soon the reader of the excerpt from Joyce's "The Dead" gets to know the character of Gabriel. Through diction, point of view, and imagery, we are introduced to Gabriel and what he thinks of himself.

D

"The Dead." How apt a title. James Joyce turns his reader into a fly on the wall as Gabriel is about to realize the many losses in his life. Death pervades the passage, from his sleeping wife to his dying aunt.

Key Idea

Each of these opening paragraphs is an acceptable beginning to an AP Literature exam essay. Note what each of these paragraphs accomplishes:

- Each has identified the title and author.
- Each has stated which techniques/devices will be used.
- Each has stated the purpose of analyzing these techniques /devices.

Now, note what is different about each opening paragraph:

- Sample **A** restates the question without anything extra. It is to the point, so much so that it does nothing more than repeat the question. It's correct, but it does not really pique the reader's interest. (Use this type of opening if you feel unsure of or uncomfortable with the prompt.)
- Sample **B** reveals the writer's attitude toward the subject. The writer has already determined that Gabriel is flawed and indicates an understanding of how Gabriel's character is revealed in the passage.
- Sample **C**, with its direct quotation, places the reader immediately into the passage. The reader quickly begins to hear the writer's voice through his or her choice of words (diction).
- Sample **D**, at first glance, reveals a mature, confident writer who is not afraid to *imply* the prompt's criteria.

Note: There are many other types of opening paragraphs that could do the job as well. The paragraphs above are just a few samples.

Into which of the above samples would you classify your opening paragraph?

____

Writing the Body of the Essay

When you write the body of your essay, take only 15–20 minutes. Time yourself and try your best to finish within that time frame.

Since this is practice, don't panic if you can't complete the essay within the allotted time. You will become more and more comfortable with the tasks presented to you as you gain experience with this type of question.

Strategy

What Should I Include in the Body of the Prose Passage Essay?

- This is where you present your interpretation and the points you wish to make that are related to the prompt.
- Use specific references and details from the passage.
    — Don't always paraphrase the original; refer directly to it.
    — Place quotation marks around those words and phrases that you extract from the passage.
- Use "connective tissue" in your essay to establish adherence to the question.
    — Use the repetition of key ideas from your opening paragraph.
    — Try using "echo words" (i.e., synonyms, such as death/loss/passing or character/persona/personality).
    — Create transitions from one paragraph to the next.

To understand the process, carefully read the following sample paragraphs. Each develops one of the categories and techniques/devices asked for in the prompt. Notice the specific references and the "connective tissue." Also, notice that details that do not apply to the prompt have been ignored.

A

This paragraph develops **imagery**.

Joyce creates imagery to lead his reader to sense the cloud of death that pervades Gabriel's world. From its very title "The Dead," the reader is prepared for loss. Just what has Gabriel lost: his wife, his confidence, his job, a friend, a relative, what? As his "wife slept," Gabriel sees her "half-open mouth" and "listens" to her "deep-drawn breath." The reader almost senses this to be a death watch. The images about the room reinforce this sense of doom. One boot is "limp" and the other is "upon its side." Picturing the future, Gabriel sees a "drawing-room dressed in black" with blinds "drawn down" and his Aunt Kate "crying" and "telling him how Julia had died." And to underscore his own feelings of internal lifelessness, he can only find "lame and useless" words of comfort.

B

This paragraph develops the **motif of time**.

Time is a constant from the beginning to the end of the passage. In the first paragraph, Gabriel is in the present while thinking of the past. He is an observer, watching his wife as he, himself, is observed by the narrator, and as we, as readers, observe the entire scene. Time moves the reader and Gabriel through the experience. Immediately, we spend a "few moments" with Gabriel as he goes back and forth in time assessing his relationship with his wife. He recognizes she "had had romance in her life." But, "it hardly pains him now." He thinks of what she had been "then" in her "girlish" beauty, which may indicate his own aging. His "strange friendly pity," because she is "no longer beautiful," may be self-pity, as well. In the next paragraph, we are with Gabriel as he reflects on his emotional "riot" only an hour before. However, he jumps to the future because he can't sustain self-examination. He chooses to allow himself to jump to this future and a new subject—Aunt Julia's death. In this future, he continues to see only his inability and incompetence. For Gabriel, all this will happen "very soon."

C

This paragraph develops **diction**.

Gabriel appears to be a man who is on the outside of his life. Joyce's diction reveals his passive nature. Gabriel "looked on" and "watched" his wife sleeping. He spent time "listening to her breath" and was "hardly pained by his role in her life." His eyes "rest" on her, and he "thinks of the past." All of Gabriel's actions are as weak as a "limp" and "fallen down" boot, "inert in the face of life." He is in direct contrast to Michael Furey, who has "braved death." And he knows this about himself. The narrator's diction reveals that Gabriel "did not like to say even to himself," implying that he is too weak to face the truth.

Later in the text, Gabriel's word choice further indicates his insecurity. He is troubled by his "riot of emotions," his "foolish speech." It is obvious that Gabriel will not take such risks again.

D

This passage develops **style**.

Joyce's very straightforward writing style supports the conclusions he wishes the reader to draw about the character of Gabriel. Most sentences are in the subject/verb, simple sentence form, reflecting the plain, uncomplicated character of Gabriel.

Joyce employs a third person narrator to further reinforce Gabriel's detachment from his own circumstances. We watch him observing his own life with little or no connection on his part. He wonders at his "riot of emotions." All this is presented without Joyce using obvious poetic devices. This punctuates the lack of "romance" in Gabriel's life when compared with that of Michael Furey.

Tip ❯ Start a study group. Approach an essay as a team. After you've deconstructed the prompt, have each person write a paragraph on a separate area of the question. Then come together and discuss what was written. You'll be amazed at how much fun this is because the work will carry you away. This is a chance to explore exciting ideas.

Again, sharing your writing with members of your class or study group will allow you to gain experience and find a comfort zone with requirements and possibilities.

We urge you to spend more time developing the body paragraphs than worrying about a concluding paragraph, especially one that begins with "In conclusion," or "In summary." In such a brief essay, the reader will have no problem remembering what you have already stated. It is not necessary to repeat yourself in a summary-type final paragraph.

If you want to make a final statement, try to link your ideas to a particularly effective line or image from the passage.

Note: Look at the last line of Sample **B** on motif. For Gabriel, all this will happen "very soon." This final sentence would be fine as the conclusion to the essay. A conclusion does not have to be a paragraph. It can be the writer's final remark or observation presented in a sentence or two.

Let's begin. Write the body of your essay. Time yourself. Allow 15–20 minutes to complete this task.

Sample Student Essays

Following are two actual student essays followed by a rubric and comments on each. Read both of the samples in sequence to clarify the differences between "high" and "mid-range" essays.

Student Essay A

A picture is worth a thousand words, but James Joyce manages to paint a pretty vivid one in only two short paragraphs. Joyce offers tremendous insight into the character of Gabriel in the short story "The Dead." He captures the essence of a scene laden with death and laced with tones of despair and hopelessness. By employing third person narration alternating with a stream of consciousness, Joyce demonstrates his abilities to delve deep into Gabriel's mind, illustrating this somewhat detached disposition and low self-image.

The passage takes us through Gabriel's reflections upon past, present, and future events while his inner character unfolds. Joyce's careful use of diction suggests that Gabriel has emotionally closed himself off to the world as he tries to cope with some aforementioned incident. He was "hardly pained" to think about a situation which caused a "riot of emotions" just a little earlier on that evening. Here, Joyce is emphasizing Gabriel's way of coping with an unfavorable event by blocking it out. He continues to "unresentfully" reflect upon what had occurred, closing himself off from any pain he obviously experienced a short while ago.

With the powerful omniscience of a third-person narrator, Joyce is able to describe the workings of Gabriel's inner consciousness without writing from the first-person point of view. Gabriel further detaches himself as he thinks about his wife. He watches her from the point of view of an outsider, as if they were never married. The mere fact that Gabriel is able to do this suggests that he and his wife do not have a truly loving relationship. This assertion is underscored by the "friendly" pity Gabriel feels for his wife, emphasizing the lack of true love in their relationship. Gabriel later questions his wife's honesty, further emphasizing a troubled relationship. The reader may be inclined to infer that Gabriel is completely devoid of compassion; however, this idea is refuted. Gabriel proceeds to express an element of sorrow when he thinks back to his wife's youth and beauty.

The evening's events had evidently triggered some type of emotional outburst which Gabriel cannot stop thinking about. His mental state is paralleled by the chaotic state of disorder in the room he is in. With a masterful control of language and syntax, Joyce describes in short, choppy sentences the array of clothing strewn around the room. This is followed by one of the longest sentences in the passage. Joyce reveals this series of events all at once, paralleling Gabriel's release of a multitude of emotions at once.

Joyce weaves a motif of darkness and death into the story. His aunt's "haggard" appearance ironically catches Gabriel's attention during the recitation of Arrayed for the Bridal, a seemingly happy song. This image of happiness and marriage is further contrasted with images of the woman's funeral and a detailed description of how Gabriel will mourn for her. Joyce also takes time to underscore Gabriel's low self-esteem, in that he will only think of "lame and useless" words at a time when comforting tones are necessary. He is essentially describing himself, since it has been established that he failed as a husband and that he is emotionally distraught even though he blocks out the pain he feels. "The blinds would be drawn down," Gabriel says, as he describes both the room at his aunt's funeral and his mental state of affairs.

The true originator of "stream-of-consciousness" techniques, Joyce delves deep into Gabriel's mind, describing his wide range of emotions and state of mind. His powerful diction reveals a great deal about Gabriel's character while his implied insights penetrate into the reader's mind, reinforcing the abstract meanings behind the actions and events that transpire throughout the course of his story.

Student Essay B

In the excerpt from the short story "The Dead" from Dubliners by James Joyce, the author describes some personality traits of the character Gabriel as he sits watching a sleeping woman. The point of view from which this excerpt is expressed helps the reader to get to know Gabriel because the narrator is omniscient and knows how Gabriel perceives things and what he is thinking. With the use of many literary devices such as imagery, diction, and syntax, the reader is able to see that Gabriel is an observant and a reflective person, but he is also detached.

Gabriel comes across as observant, because throughout the entire passage he is observing a woman, his wife, sleeping. He scans the room looking over everything and taking note of everything. An example of this is looking at "her tangled hair and half-open mouth, listening to her deep-drawn breath." The author uses the technique of syntax ("deep-drawn breath" and "half-open mouth") in the above quotation to show us exactly what Gabriel is seeing. Gabriel notices many details, and they are described so that the reader can clearly formulate a picture of what he is gazing at. This imagery can be seen in lines such as the one where the woman's boots are being described. "One boot stood upright, its limp upper fallen down: the fellow of it lay upon the side." The diction used such as "limp" and "upright," are concrete words that create clear pictures. Another reason that Gabriel comes across as observant is because he catches and notices little things. For example, he "caught" the "haggard look" on his Aunt Julia's face.

Resulting from the fact that Gabriel is observant, he is also reflective. He thinks over past events that had happened and wonders what caused them and why he did what he did. In the first paragraph he reflects on his wife's "fading beauty," what she used to look like, and the story of the death of Michael Furey. He realizes that it is a possibility that she had not told him the entire story concerning the boy's death. He further reflects when he is thinking about his emotional outburst. He asks himself many questions including "From what had it proceeded?"

A feeling of detachment is also present. The way he looks at his wife "as though he and she had never lived together as man and wife" shows that he is viewing his own life from an objective standpoint. He is able to look at his own life as though it wasn't his. The sentence that reads "it hardly pained him now to think how poor a part he, her husband, had played in her life," further exemplifies this feeling of detachment. Feelings that he used to feel no longer even touched him. He was able to recognize them, yet remain separate. In the second paragraph Gabriel continues to come across as remote. He is able to picture and describe in great detail the death and funeral of his Aunt Julia. He narrates the future drastic event in a matter-of-fact way. Gabriel goes so far as to describe what he will be thinking at the time of his Aunt Julia's death which is "he would cast about in his mind for some words that might console her (his Aunt Kate), and would find only lame and useless ones." This statement finalizes the idea that Gabriel is a person who is, at least to some degree, detached from his own life.

Even though the passage is fairly short, the author is able to impart a fair amount of information concerning the character Gabriel. It becomes apparent that he possesses the qualities of observance, reflection, and detachment. These qualities are all interconnected because of the fact that he is observant leading to his ability to reflect on his actions and actions of others. This in turn leads to his detachment, because when he reflects on his life he does it from the standpoint of a third-person narrator. The author's use of literary techniques helps to convey these personality traits of Gabriel to a reader.

Let's take a look at a set of rubrics for this prose passage essay. (If you want to see actual AP rubrics as used in a recent AP Lit exam, log on to the College Board website: <www.collegeboard.org/ap>.) As you probably know, essays are rated on a 9–1 scale, with 9 the highest and 1 the lowest. Since we are not with you to personally rate your essay and to respond to your style and approach, we will, instead, list the criteria for high-, middle-, and low-range papers. These criteria are based on our experience with rubrics and reading AP Literature exam essays.

A "high-range" essay can be a 9 or an 8. "Middle" refers to essays in the 7, 6, 5 range. And the "low-scoring" essays are rated 4, 3, 2, 1.

After reading the following rubrics, evaluate the two essays that you have just read.

Note: We all can recognize a 9 essay. It sings, and we wish we had written it. It's wonderful that the essays don't all have to sing the same song with the same words and rhythm. Conversely, we can, unfortunately, recognize the 1 or 2 paper, which is off key, and we are relieved not to have written one like it.

Rating the Student Essays

High-Range Essay (9–8)

Key Idea

- Indicates complete understanding of the prompt.
- Distinguishes between what Gabriel acknowledges about himself and what the reader comes to know about him.
- Explores the complexity of Gabriel's character.
- Identifies and analyzes Joyce's literary techniques, such as imagery, diction, point of view, motif, and style.
- Cites specific references to the passage.
- Illustrates and supports the points being made.
- Is clear, well-organized, and coherent.
- Reflects the ability to manipulate language at an advanced level.
- Contains only minor errors or flaws, if any.

Note: Rarely, a 7 essay can make the jump into the high range because of its more mature style and perception.

Middle-Range Essay (7–6–5)

- Refers accurately to the prompt.
- Refers accurately to the literary devices used by Joyce.
- Provides a less thorough analysis of Gabriel's character than the higher-rated paper.
- Is less adept at linking techniques to the purpose of the passage.
- Demonstrates writing that is adequate to convey the writer's intent.
- May not be sensitive to the implications about Gabriel's character.

Note: The 7 paper demonstrates a more consistent command of college-level writing than does the 5 or 6 paper.

A 5 paper does the minimum required. It relies on generalizations and sketchy analysis. It is often sidetracked by **plot**, and the references may be limited or simplistic.

Low-Range Essay (4–3–2–1)

- Does not respond adequately to the prompt.
- Demonstrates insufficient and/or inaccurate understanding of the passage.
- Does not link literary devices to Gabriel's character.
- Underdevelops and/or inaccurately analyzes literary techniques.
- Fails to demonstrate an understanding of Gabriel's character.
- Demonstrates weak control of the elements of diction, syntax, and organization.

Note: A 4 or 3 essay may do no more than paraphrase sections of the passage rather than analyze Gabriel's character.

A 2 essay may merely summarize the passage. **(No matter how well written, a summary can never earn more than a 2.)**

A 1–2 essay indicates a major lack of understanding and control. It fails to comprehend the prompt and/or the passage. It may also indicate severe writing problems.

How would you rate these essays?

Now, compare your evaluation of the two student essays with ours.

Student Essay A

This is a high-range paper for the following reasons:

Key Idea

- Is on task.
- Shows complete understanding of the prompt and the passage.
- Indicates perceptive, subtle analysis (line 8).
- Maintains excellent topic adherence (lines 9, 17, 28, 39).
- Uses good "connective tissue" (repetition of key words).
- Chooses good specific references (lines 11, 12, 21, 35).
- Knows how to distinguish between the author and the narrator.
- Understands **point of view** well.
- Makes suggestions and inferences (lines 7, 20).
- Demonstrates good critical thinking.
- Is perceptive about syntax and the style of the author (lines 27–33).

- Links techniques with character (line 34).
- Demonstrates mature language manipulation (line 34).
- Understands function of diction and motif (lines 40–44).

Tip It's best to omit extraneous judgmental words from your essay (line 45).

This is obviously a mature, critical reader and writer. Using subtle inferences and implications, the writer demonstrates an understanding of the character of Gabriel both as Joyce presents him and as Gabriel views himself. There is nothing extraneous or repetitious in this essay. Each point leads directly and compellingly to the next aspect of Gabriel's character.

This is definitely a strong, high-range essay.

Student Essay B

This is a middle-range essay for the following reasons:

Key Idea

- Sets up an introduction which indicates the techniques that will be developed, but neglects to clearly set up the required discussion of how Gabriel views himself.
- Immediately establishes that the essay will address Gabriel's character as drawn by the narrator and seen by the reader.
- Addresses three aspects of Gabriel's character without fully developing the analysis of literary techniques.
- Adheres to the essay's topic.
- Uses "connective tissue" (lines 21, 28).
- Uses "echo words" (lines 8, 9, 10).
- Uses citations from the passages.
- Isolates some details to illustrate Gabriel's character (lines 31–32, 39).
- Confuses syntax with diction (lines 12–13).
- Lacks development of literary technique in paragraph 4.
- Displays faulty diction and syntax.
- Does not develop an important part of the prompt—how Gabriel views himself.
- Incorporates faulty logic at times (lines 44–49).

This essay is a solid, middle-range paper. The writer has a facility with literary analysis. Even though there are flashes of real insight, they are not sustained throughout the essay. There is a strong opening paragraph which makes it clear to the reader what the topic of the paper is. The writer obviously grasps Gabriel's character and the needed details to support the character analysis. But the weakness in this paper is the writer's incomplete development of the relationship of literary techniques to character analysis.

Note: Both essays have concluding paragraphs that are repetitive and largely unnecessary. It is best to avoid this type of ending.

Your Turn

How about sharing these samples with members of your class or study group and discussing possible responses?

Try a little reverse psychology. Now that you are thoroughly familiar with this passage, construct two or three alternative prompts. (Walk a little in the examiner's shoes.) This will help you gain insight into the process of test-making. Create two questions of your own. (See the Types of Prose Passage Essay Questions section of this chapter for ideas.)

1st alternative prompt for "The Dead"

2nd alternative prompt for "The Dead"

Rapid Review

After you've absorbed the ideas in this chapter, the following points will provide you with a quick refresher when needed:

- Familiarize yourself with the types of prose questions (prompts).
- Highlight the prompt and understand all the required tasks.
- Time your essay carefully.
- Spend sufficient time "working the passage" before you begin writing.
- Mark up the passage.
- Create a strong opening paragraph.
- Refer often to the passage.
- Use concrete details and quotes to support your ideas.
- Always stay on topic.
- Avoid plot summary.
- Include transitions and echo words.
- Check the models and rubrics for guidance for self-evaluation.
- Practice—vary the question and your approach.
- Share ideas with others.

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AP Calculus AB

Chapter

2

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How to Plan Your Time

IN THIS CHAPTER

Summary: The right preparation plan for you depends on your study habits and the amount of time you have before the test.

Key Idea

Key Idea

✪ Choose the study plan that is right for you.

2.1 Three Approaches to Preparing for the AP Calculus AB Exam

Overview of the Three Plans

No one knows your study habits, likes, and dislikes better than you. So you are the only one who can decide which approach you want and/or need to adopt to prepare for the Advanced Placement Calculus exam. Look at the brief profiles below. These may help you to place yourself in a particular prep mode.

You are a full-year prep student (Approach A) if:

1. You are the kind of person who likes to plan for everything far in advance . . . and I mean far . . . ;
2. You arrive at the airport 2 hours before your flight because, "you never know when these planes might leave early . . . ";
3. You like detailed planning and everything in its place;
4. You feel you must be thoroughly prepared;
5. You hate surprises.

You are a one-semester prep student (Approach B) if:

1. You get to the airport 1 hour before your flight is scheduled to leave;
2. You are willing to plan ahead to feel comfortable in stressful situations, but are okay with skipping some details;
3. You feel more comfortable when you know what to expect, but a surprise or two is cool;
4. You're always on time for appointments.

You are a 6-week prep student (Approach C) if:

1. You get to the airport just as your plane is announcing its final boarding;
2. You work best under pressure and tight deadlines;
3. You feel very confident with the skills and background you've learned in your AP Calculus class;
4. You decided late in the year to take the exam;
5. You like surprises;
6. You feel okay if you arrive 10–15 minutes late for an appointment.

2.2 Calendar for Each Plan

A Calendar for Approach A:
A Year-Long Preparation for the AP Calculus AB Exam

Although its primary purpose is to prepare you for the AP Calculus AB Exam you will take in May, this book can enrich your study of calculus, your analytical skills, and your problem-solving techniques.

SEPTEMBER–OCTOBER (Check off the activities as you complete them.)

_______ Determine into which student mode you would place yourself.
_______ Carefully read Steps 1 and 2.
_______ Get on the Web and take a look at the AP website(s).
_______ Skim the Cvnvmbujwf Review section. (These areas will be part of your year-long preparation.)
_______ Buy a few highlighters.
_______ Flip through the entire book. Break the book in. Write in it. Toss it around a little bit . . . Highlight it.
_______ Get a clear picture of what your own school's AP Calculus curriculum is.
_______ Begin to use the book as a resource to supplement the classroom learning.
_______ Read and study Chapter 5—Review of Precalculus.
_______ Read and study Chapter 6—Limits and Continuity.
_______ Read and study Chapter 7—Differentiation.

NOVEMBER (The first 10 weeks have elapsed.)

_______ Read and study Chapter 8—Graphs of Functions and Derivatives.
_______ Read and study Chapter 9— Applications of Derivatives.

DECEMBER

_______ Read and study Chapter 10—More Applications of Derivatives.
_______ Read and study Chapter 11—Integration.
_______ Review Chapters 6–8.

JANUARY (20 weeks have now elapsed.)

_______ Read and study Chapter 12—Definite Integrals.
_______ Review Chapters 9–11.

FEBRUARY

_______ Read and study Chapter 13—Areas and Volumes.
_______ Take the Diagnostic Test.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct weaknesses.

MARCH (30 weeks have now elapsed.)

_______ Read and study Chapter 14—More Applications of Definite Integrals.
_______ Review Chapters 12–14.

APRIL

_______ Take Practice Exam 1 in first week of April.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct weaknesses.
_______ Review Chapters 6–14.

MAY—First Two Weeks (THIS IS IT!)

_______ Take Practice Exams 2 and 3.
_______ Score yourself.
_______ Study appropriate chapters to correct weaknesses.
_______ Get a good night's sleep the night before the exam. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST!

A Calendar for Approach B:
A Semester-Long Preparation for the AP Calculus AB Exam

Working under the assumption that you've completed one semester of calculus studies, the following calendar will use those skills you've been practicing to prepare you for the May exam.

JANUARY

_______ Carefully read Steps 1 and 2.
_______ Read and study Chapter 6—Limits and Continuity.
_______ Read and study Chapter 7—Differentiation.
_______ Read and study Chapter 8—Graphs of Functions and Derivatives.
_______ Read and Study Chapter 9—Applications of Derivatives.

FEBRUARY

_______ Read and study Chapter 5—Review of Precalculus.
_______ Read and study Chapter 10—More Applications of Derivatives.
_______ Read and study Chapter 11—Integration.
_______ Take the Diagnostic Test.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct weaknesses.
_______ Review Chapters 6–9.

MARCH (10 weeks to go.)

_______ Read and study Chapter 12—Definite Integrals.
_______ Read and study Chapter 13—Areas and Volumes.
_______ Read and study Chapter 14—More Applications of Definite Integrals.
_______ Review Chapters 10–12.

APRIL

_______ Take Practice Exam 1 in first week of April.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct weaknesses.
_______ Review Chapters 6–14.

MAY—First Two Weeks (THIS IS IT!)

_______ Take Practice Exams 2 and 3.
_______ Score yourself.
_______ Study appropriate chapters to correct weaknesses.
_______ Get a good night's sleep the night before the exam. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST!

A Calendar for Approach C:
A Six-Week Preparation for the AP Calculus AB Exam

At this point, we are going to assume that you have been building your calculus knowledge base for more than six months. You will, therefore, use this book primarily as a specific guide to the AP Calculus AB Exam.

Given the time constraints, now it is not the time to try to expand your AP Calculus curriculum. Rather, it is the time to limit and refine what you already do know.

APRIL 1st–15th

_______ Skim Steps 1 and 2.
_______ Skim Chapters 6–10.
_______ Carefully go over the "Rapid Review" sections of Chapters 6–10.
_______ Take the Diagnostic Test.
_______ Evaluate your strengths and weaknesses.
_______ Study appropriate chapters to correct weaknesses.

APRIL 16th–May 1st

_______ Skim Chapters 11–14.
_______ Carefully go over the "Rapid Review" sections of Chapters 11–14.
_______ Complete Practice Exam 1.
_______ Score yourself and analyze your errors.
_______ Study appropriate chapters to correct weaknesses.

MAY—First Two Weeks (THIS IS IT!)

_______ Complete Practice Exams 2 and 3.
_______ Score yourself and analyze your errors.
_______ Study appropriate chapters to correct weaknesses.
_______ Get a good night's sleep. Fall asleep knowing you are well prepared.

GOOD LUCK ON THE TEST!

Summary of the Three Study Plans

MONTH	APPROACH A: SEPTEMBER PLAN	APPROACH B: JANUARY PLAN	APPROACH C: 6-WEEK PLAN
September–October	Chapters 5–7		
November	Chapters 8 and 9		
December	Chapters 10 and 11 Review Chapters 6–8		
January	Chapter 12 Review Chapters 9–11	Chapters 6–9	
February	Chapter 13 Diagnostic Test	Chapters 5, 10–11 Diagnostic Test Review Chapters 6–9	
March	Chapter 14 Review Chapters 12–14	Chapters 12–14 Review Chapters 10–12	
April	Practice Exam 1 Review Chapters 5–14	Practice Exam 1 Review Chapters 5–14	Diagnostic Test Review Chapters 5–10 Practice Exam 1 Review Chapters 11–14
May	Practice Exams 2 and 3	Practice Exams 2 and 3	Practice Exams 2 and 3

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Chapter 9

Applications of Derivatives

IN THIS CHAPTER

Summary: Two of the most common applications of derivatives involve solving related rate problems and applied maximum and minimum problems. In this chapter, you will learn the general procedures for solving these two types of problems and to apply these procedures to examples. Both related rate and applied maximum and minimum problems appear often on the AP Calculus AB exam.

Key Ideas

Key Idea

- General Procedure for Solving Related Rate Problems
- Common Related Rate Problems
- Inverted Cone, Shadow, and Angle of Elevation Problems
- General Procedure for Solving Applied Maximum and Minimum Problems
- Distance, Area, Volume, and Business Problems

9.1 Related Rate

Main Concepts: General Procedure for Solving Related Rate Problems, Common Related Rate Problems, Inverted Cone (Water Tank) Problem, Shadow Problem, Angle of Elevation Problem

General Procedure for Solving Related Rate Problems

Strategy

1. Read the problem and, if appropriate, draw a diagram.
2. Represent the given information and the unknowns by mathematical symbols.
3. Write an equation involving the rate of change to be determined. (If the equation contains more than one variable, it may be necessary to reduce the equation to one variable.)

4. Differentiate each term of the equation with respect to time.
5. Substitute all known values and known rates of change into the resulting equation.
6. Solve the resulting equation for the desired rate of change.
7. Write the answer and indicate the units of measure.

Common Related Rate Problems

Example 1

When the area of a square is increasing twice as fast as its diagonals, what is the length of a side of the square?

Let z represent the diagonal of the square. The area of a square is $A=\frac{z^2}{2}$.

$$\frac{dA}{dt}=2z\frac{dz}{dt}\left(\frac{1}{2}\right)=z\frac{dz}{dt}$$

Since $\frac{dA}{dt}=2\frac{dz}{dt}$, $2\frac{dz}{dt}=z\frac{dz}{dt}\Rightarrow z=2$.

Let s be a side of the square. Since the diagonal $z=2$, then $s^2+s^2=z^2$
$\Rightarrow 2s^2=4\Rightarrow s^2=4\Rightarrow s^2=2$ or $s=\sqrt{2}$.

Example 2

Find the surface area of a sphere at the instant when the rate of increase of the volume of the sphere is nine times the rate of increase of the radius.

Volume of a sphere: $V=\frac{4}{3}\pi r^3$; Surface area of a sphere: $S=4\pi r^2$.

$$V=\frac{4}{3}\pi r^3;\quad \frac{dV}{dt}=4r^2\frac{dr}{dt}.$$

Since $\frac{dV}{dt}=9\frac{dr}{dt}$, you have $9\frac{dr}{dt}=4\pi r^2\frac{dr}{dt}$ or $9=4\pi r^2$.

Since $S=4\pi r^2$, the surface area is $S=9$ square units.

Note: At $9=4\pi r^2$, you could solve for r and obtain $r^2=\frac{9}{4\pi}$ or $r=\frac{3}{2}\frac{1}{\sqrt{\pi}}$. You could then substitute $r=\frac{3}{2}\frac{1}{\sqrt{\pi}}$ into the formula for surface area $S=4\pi r^2$ and obtain 9. These steps are of course correct but not necessary.

Example 3

The height of a right circular cone is always three times the radius. Find the volume of the cone at the instant when the rate of increase of the volume is twelve times the rate of increase of the radius.

Let r, h be the radius and height of the cone respectively.

Since $h=3r$, the volume of the cone $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi r^2(3r)=\pi r^3$.

$$V=\pi r^3; \quad \frac{dV}{dt}=3\pi r^2\frac{dr}{dt}.$$

When $\frac{dV}{dt}=12\frac{dr}{dt}$, $12\frac{dr}{dt}=3\pi r^2\frac{dr}{dt} \Rightarrow 4=\pi r^2 \Rightarrow r=\frac{2}{\sqrt{\pi}}$.

Thus, $V=\pi r^3=\pi\left(\frac{2}{\sqrt{\pi}}\right)^3=\pi\left(\frac{8}{\pi\sqrt{\pi}}\right)=\frac{8}{\sqrt{\pi}}$.

Tip

- Go with your first instinct if you are unsure. Usually that is the correct one.

Inverted Cone (Water Tank) Problem

A water tank is in the shape of an inverted cone. The height of the cone is 10 meters and the diameter of the base is 8 meters as shown in Figure 9.1-1. Water is being pumped into the tank at the rate of 2 m^3/min. How fast is the water level rising when the water is 5 meters deep? (See Figure 9.1-1.)

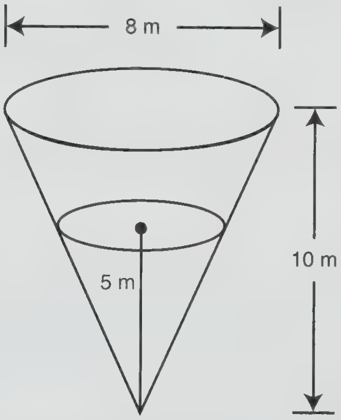


Figure 9.1-1

Solution:

Step 1: Define the variables. Let V be the volume of water in the tank; h be the height of the water level at t minutes; r be the radius of surface of the water at t minutes; and t be the time in minutes.

Step 2: Given: $\frac{dV}{dt}=2$ m^3/min. Height $=10$ m, diameter $=8$ m.

Find: $\frac{dh}{dt}$ at $h=5$.

Step 3: Set up an equation: $V=\frac{1}{3}\pi r^2 h$.

Using similar triangles, you have $\frac{4}{10}=\frac{r}{h} \Rightarrow 4h=10r$; or $r=\frac{2h}{5}$. (See Figure 9.1-2.)

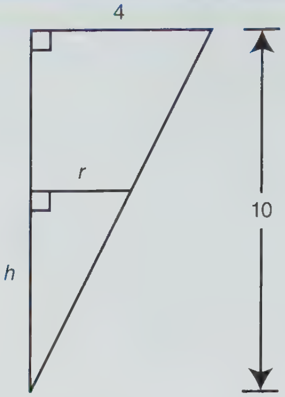


Figure 9.1-2

Thus, you can reduce the equation to one variable:

$$V = \frac{1}{3}\pi\left(\frac{2h}{5}\right)^2 h = \frac{4}{75}\pi h^3.$$

Step 4: Differentiate both sides of the equation with respect to *t*.

$$\frac{dV}{dt} = \frac{4}{75}\pi(3)h^2\frac{dh}{dt} = \frac{4}{25}\pi h^2\frac{dh}{dt}$$

Step 5: Substitute known values.

$$2 = \frac{4}{25}\pi h^2\frac{dh}{dt};\ \frac{dh}{dt} = \left(\frac{25}{2}\right)\frac{1}{\pi h^2}\ \text{m/min}$$

$$\text{Evaluating } \frac{dh}{dt} \text{ at } h=5;\ \left.\frac{dh}{dt}\right|_{h=5} = \left(\frac{25}{2}\right)\frac{1}{\pi(5)^2}\ \text{m/min}$$

$$= \frac{1}{2\pi}\ \text{m/min.}$$

Step 6: Thus, the water level is rising at $\frac{1}{2\pi}$ m/min when the water is 5 m high.

Shadow Problem

A light on the ground 100 feet from a building is shining at a 6-foot-tall man walking away from the light and toward the building at the rate of 4 ft/sec. How fast is his shadow on the building becoming shorter when he is 40 feet from the building? (See Figure 9.1-3.)

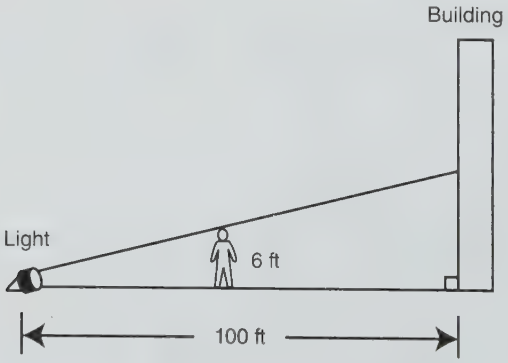


Figure 9.1-3

Solution:

Step 1: Let s be the height of the man's shadow; x be the distance between the man and the light; and t be the time in seconds.

Step 2: Given: $\frac{dx}{dt}=4$ ft/sec; man is 6 ft tall; distance between light and building = 100 ft.
Find $\frac{ds}{dt}$ at $x=60$.

Step 3: (See Figure 9.1-4.) Write an equation using similar triangles, you have:

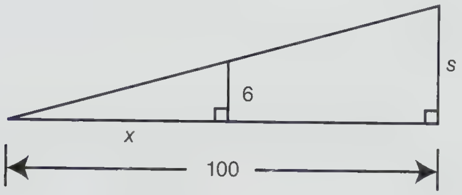


Figure 9.1-4

$$\frac{6}{s}=\frac{x}{100};\ s=\frac{600}{x}=600x^{-1}$$

Step 4: Differentiate both sides of the equation with respect to t.

$$\frac{ds}{dt}=(-1)(600)x^{-2}\frac{dx}{dt}=\frac{-600}{x^2}\frac{dx}{dt}=\frac{-600}{x^2}(4)=\frac{-2400}{x^2}\text{ ft/sec}$$

Step 5: Evaluate $\frac{ds}{dt}$ at $x=60$.
Note: when the man is 40 ft from the building, x (distance from the light) is 60 ft.

$$\left.\frac{ds}{dt}\right|_{x=60}=\frac{-2400}{(60)^2}\text{ ft/sec}=-\frac{2}{3}\text{ ft/sec}$$

Step 6: The height of the man's shadow on the building is changing at $-\frac{2}{3}$ ft/sec.

Tip

- Indicate units of measure, e.g., the velocity is 5 m/sec *or* the volume is 25 in^3.

Angle of Elevation Problem

A camera on the ground 200 meters away from a hot air balloon records the balloon rising into the sky at a constant rate of 10 m/sec. How fast is the camera's angle of elevation changing when the balloon is 150 m in the air? (See Figure 9.1-5.)

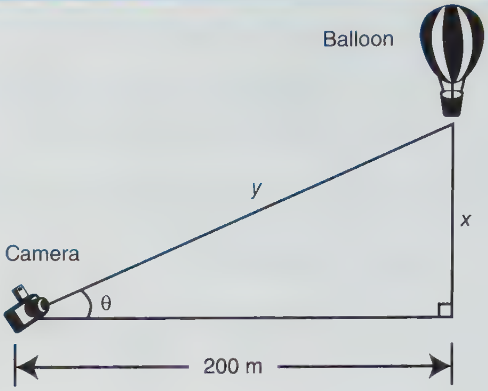


Figure 9.1-5

Step 1: Let x be the distance between the balloon and the ground; θ be the camera's angle of elevation; and t be the time in seconds.

Step 2: Given: $\frac{dx}{dt} = 10$ m/sec; distance between camera and the point on the ground where the balloon took off is 200 m, $\tan\theta = \frac{x}{200}$.

Step 3: Find $\frac{d\theta}{dt}$ at $x = 150$ m.

Step 4: Differentiate both sides with respect to t.

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{200}\frac{dx}{dt}; \quad \frac{d\theta}{dt} = \frac{1}{200}\left(\frac{1}{\sec^2\theta}\right)(10) = \frac{1}{20\sec^2\theta}.$$

Step 5: $\sec\theta = \frac{y}{200}$ and at $x = 150$.

Using the Pythagorean Theorem: $y^2 = x^2 + (200)^2$

$$y^2 = (150)^2 + (200)^2$$

$$y = \pm 250.$$

Since $y > 0$, then $y = 250$. Thus, $\sec\theta = \frac{250}{200} = \frac{5}{4}$.

$$\text{Evaluating } \left.\frac{d\theta}{dt}\right|_{x=150} = \frac{1}{20\sec^2\theta} = \frac{1}{20\left(\frac{5}{4}\right)^2} \text{ radian/sec}$$

$$= \frac{1}{20\left(\frac{5}{4}\right)^2} = \frac{1}{20\left(\frac{25}{16}\right)} = \frac{1}{\frac{125}{4}} = \frac{4}{125} \text{ radian/sec}$$

or .032 radian/sec

$= 1.833$ deg/sec.

Step 6: The camera's angle of elevation changes at approximately 1.833 deg/sec when the balloon is 150 m in the air.

9.2 Applied Maximum and Minimum Problems

Main Concepts: General Procedure for Solving Applied Maximum and Minimum Problems, Distance Problem, Area and Volume Problems, Business Problems

Strategy

General Procedure for Solving Applied Maximum and Minimum Problems

Steps:

1. Read the problem carefully and if appropriate, draw a diagram.
2. Determine what is given and what is to be found, and represent these quantities by mathematical symbols.
3. Write an equation that is a function of the variable representing the quantity to be maximized or minimized.
4. If the equation involves other variables, reduce the equation to a single variable that represents the quantity to be maximized or minimized.
5. Determine the appropriate interval for the equation (i.e., the appropriate domain for the function) based on the information given in the problem.
6. Differentiate to obtain the first derivative and to find critical numbers.
7. Apply the First Derivative Test or the Second Derivative Test by finding the second derivative.
8. Check the function values at the end points of the interval.
9. Write the answer(s) to the problem and, if given, indicate the units of measure.

Distance Problem

Find the shortest distance between the point A (19, 0) and the parabola $y = x^2 - 2x + 1$.

Solution:

Step 1: Draw a diagram. (See Figure 9.2-1.)

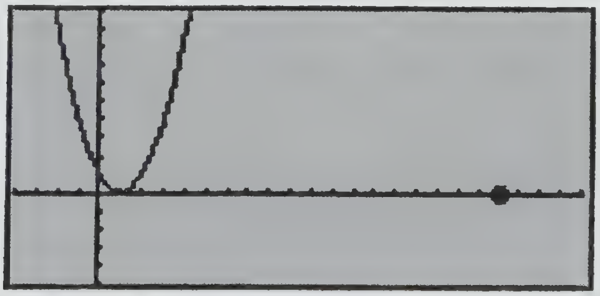


Figure 9.2-1

Step 2: Let $P(x, y)$ be the point on the parabola and let Z represent the distance between points $P(x, y)$ and $A(19, 0)$.

Step 3: Using the distance formula,

$$Z=\sqrt{(x-19)^2+(y-0)^2}=\sqrt{(x-19)^2+(x^2-2x+1-0)^2}$$
$$=\sqrt{(x-19)^2+\left((x-1)^2\right)^2}=\sqrt{(x-19)^2+(x-1)^4}.$$

(Special case: In distance problems, the distance and the square of the distance have the same maximum and minimum points.) Thus, to simplify computations, let $L=Z^2=(x-19)^2+(x-1)^4$. The domain of L is $(-\infty,\infty)$.

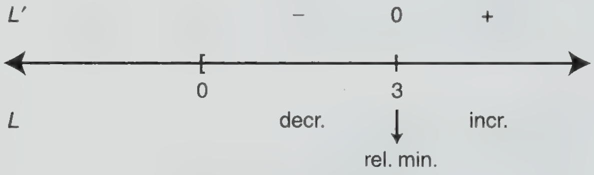
Step 4: Differentiate: $\frac{dL}{dx}=2(x-19)(1)+4(x-1)^3(1)$
$$=2x-38+4x^3-12x^2+12x-4=4x^3-12x^2+14x-42$$
$$=2(2x^3-6x^2+7x-21).$$

$\frac{dL}{dx}$ is defined for all real numbers.

Set $\frac{dL}{dx}=0$; $2x^3-6x^2+7x-21=0$. The factors of 21 are $\pm1, \pm3, \pm7$, and ±21.

Using Synthetic Division, $2x^3-6x^2+7x-21=(x-3)(2x^2+7)=0 \Rightarrow x=3$.
Thus the only critical number is $x=3$.
(Note: Step 4 could have been done using a graphing calculator.)

Step 5: Apply the First Derivative Test.



Step 6: Since $x=3$ is the only relative minimum point in the interval, it is the absolute minimum.

Step 7: At $x=3$, $Z=\sqrt{(3-19)^2+(3^2-2(3)+1)^2}=\sqrt{(-16)^2+(4)^2}$
$=\sqrt{272}=\sqrt{16}\sqrt{17}=4\sqrt{17}$. Thus, the shortest distance is $4\sqrt{17}$.

Tip

- Simplify numeric or algebraic expressions only if the question asks you to do so.

Area and Volume Problems

Example—Area Problem

The graph of $y=-\frac{1}{2}x+2$ encloses a region with the x-axis and y-axis in the first quadrant. A rectangle in the enclosed region has a vertex at the origin and the opposite vertex on the graph of $y=-\frac{1}{2}x+2$. Find the dimensions of the rectangle so that its area is a maximum.

Solution:

Step 1: Draw a diagram. (See Figure 9.2-2.)

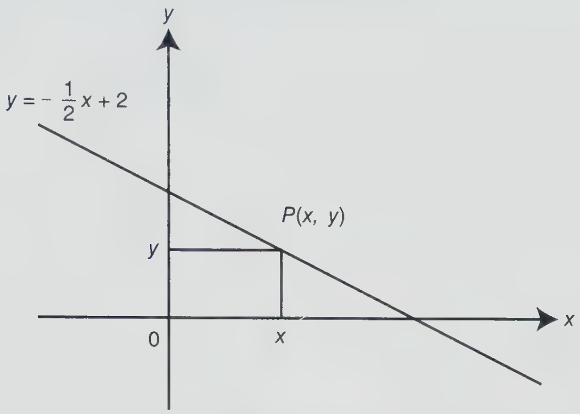


Figure 9.2-2

Step 2: Let $P(x, y)$ be the vertex of the rectangle on the graph of $y = -\frac{1}{2}x + 2$.

Step 3: Thus, the area of the rectangle is:

$$A = xy \text{ or } A = x\left(-\frac{1}{2}x + 2\right) = -\frac{1}{2}x^2 + 2x.$$

The domain of A is $[0, 4]$.

Step 4: Differentiate:

$$\frac{dA}{dx} = -x + 2.$$

Step 5: $\frac{dA}{dx}$ is defined for all real numbers.

Set $\frac{dA}{dx} = 0 \Rightarrow -x + 2 = 0$; $x = 2$.

$A(x)$ has one critical number $x = 2$.

Step 6: Apply the Second Derivative Test:

$\frac{d^2A}{dx^2} = -1 \Rightarrow A(x)$ has a relative maximum point at $x = 2$; $A(2) = 2$.

Since $x = 2$ is the only relative maximum, it is the absolute maximum. (Note that at the endpoints: $A(0) = 0$ and $A(4) = 0$.)

Step 7: At $x = 2$, $y = -\frac{1}{2}(2) + 2 = 1$.

Therefore, the length of the rectangle is 2, and its width is 1.

Example—Volume Problem (with calculator)

If an open box is to be made using a square sheet of tin, 20 inches by 20 inches, by cutting a square from each corner and folding the sides up, find the length of a side of the square being cut so that the box will have a maximum volume.

Solution:

Step 1: Draw a diagram. (See Figure 9.2-3.)

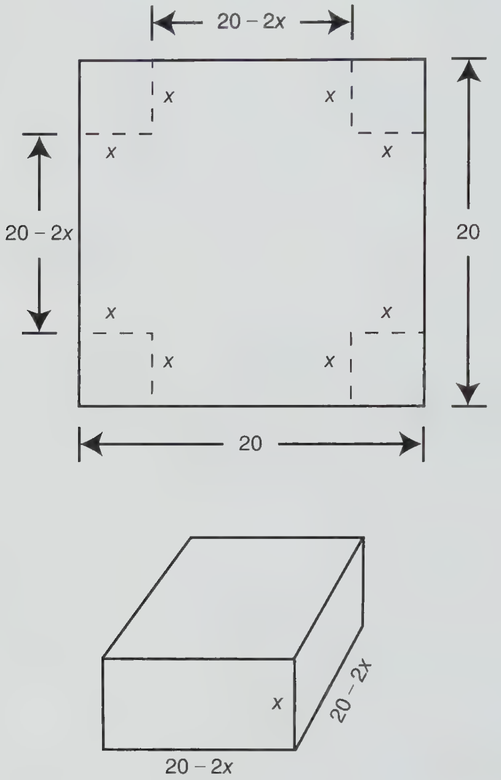


Figure 9.2-3

Step 2: Let x be the length of a side of the square to be cut from each corner.

Step 3: The volume of the box is $V(x) = x(20 - 2x)(20 - 2x)$.
The domain of V is $[0, 10]$.

Step 4: Differentiate $V(x)$.
Enter $d(x * (20 - 2x) * (20 - 2x),\ x)$ and we have $4(x - 10)(3x - 10)$.

Step 5: $V'(x)$ is defined for all real numbers:

Set $V'(x) = 0$ by entering: [*Solve*] $(4(x - 10)(3x - 10) = 0,\ x)$, and obtain $x = 10$ or $x = \frac{10}{3}$. The critical numbers of $V(x)$ are $x = 10$ and $x = \frac{10}{3}$. $V(10) = 0$ and $V\left(\frac{10}{3}\right) = 592.59$. Since $V(10) = 0$, you need to test only $x = \frac{10}{3}$.

Step 6: Using the Second Derivative Test, enter $d(x*(20-2x)*(20-2x),\ x,\ 2)|x=\dfrac{10}{3}$ and obtain -80. Thus, $V\left(\dfrac{10}{3}\right)$ is a relative maximum. Since it is the only relative maximum on the interval, it is the absolute maximum. (Note at the other endpoint $x=0,\ V(0)=0$.)

Step 7: Therefore, the length of a side of the square to be cut is $x=\dfrac{10}{3}$.

Tip

- The formula for the average value of a function f from $x=a$ to $x=b$ is $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$.

Business Problems

Summary of Formulas

1. $P=R-C$: Profit = Revenue − Cost
2. $R=xp$: Revenue = (Units Sold)(Price Per Unit)
3. $\overline{C}=\dfrac{C}{x}$: Average Cost $=\dfrac{\text{Total Cost}}{\text{Units produced/Sold}}$
4. $\dfrac{dR}{dx}$: Marginal Revenue ≈ Revenue from selling one more unit
5. $\dfrac{dP}{dx}$: Marginal Profit ≈ Profit from selling one more unit
6. $\dfrac{dC}{dx}$: Marginal Cost ≈ Cost of producing one more unit

Example 1

Given the cost function $C(x)=100+8x+0.1x^2$, (a) find the marginal cost when $x=50$; and (b) find the marginal profit at $x=50$, if the price per unit is \$20.

Solution:

(a) Marginal cost is $C'(x)$. Enter $d(100+8x+0.1x^2,\ x)|x=50$ and obtain \$18.

(b) Marginal profit is $P'(x)$
$P=R-C$
$P=20x-(100+8x+0.1x^2)$. Enter $d(20x-(100+8x+0.1x\text{^}2,\ x)|x=50$ and obtain 2.

Tip

- Carry all decimal places and round only at the final answer. Round to 3 decimal places unless the question indicates otherwise.

Example 2

Given the cost function $C(x)=500+3x+0.01x^2$ and the demand function (the price function) $p(x)=10$, find the number of units produced in order to have maximum profit.

Solution:

Step 1: Write an equation.
Profit = Revenue – Cost
$P=R-C$
Revenue = (Units Sold)(Price Per Unit)
$R=xp(x)=x(10)=10x$
$P=10x-(500+3x+0.01x^2)$

Step 2: Differentiate.
Enter $d(10x-(500+3x+0.01x^\wedge 2, x))$ and obtain $7-0.02x$.

Step 3: Find critical numbers.
Set $7-0.02x=0 \Rightarrow x=350$.
Critical number is $x=350$.

Step 4: Apply Second Derivative Test.
Enter $d(10x-(500+3x+0.01x^\wedge 2), x, 2)|x=350$ and obtain -0.02.
Since $x=350$ is the only relative maximum, it is the absolute maximum.

Step 5: Write a solution.
Thus, producing 350 units will lead to maximum profit.

9.3 Rapid Review

1. Find the instantaneous rate of change at $x=5$ of the function $f(x)=\sqrt{2x-1}$.

Answer: $f(x)=\sqrt{2x-1}=(2x-1)^{1/2}$

$$f'(x)=\frac{1}{2}(2x-1)^{-1/2}(2)=(2x-1)^{-1/2}$$

$$f'(5)=\frac{1}{3}$$

2. If h is the diameter of a circle and h is increasing at a constant rate of 0.1 cm/sec, find the rate of change of the area of the circle when the diameter is 4 cm.

Answer: $A=\pi r^2=\pi\left(\frac{h}{2}\right)^2=\frac{1}{4}\pi h^2$

$$\frac{dA}{dt}=\frac{1}{2}\pi h\frac{dh}{dt}=\frac{1}{2}\pi(4)(0.1)=0.2\pi \text{ cm}^2/\text{sec}.$$

3. The radius of a sphere is increasing at a constant rate of 2 inches per minute. In terms of the surface area, what is the rate of change of the volume of the sphere?

Answer: $V=\frac{4}{3}\pi r^3$; $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ since $S=\pi r^2$, $\frac{dV}{dt}=28$ in.^{3}/min.

4. Using your calculator, find the shortest distance between the point (4, 0) and the line $y = x$. (See Figure 9.3-1.)

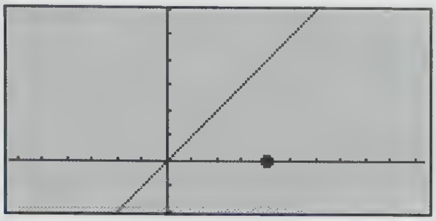

[−6.3, 10] by [−2, 6]

Figure 9.3-1

Answer:

$S = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(x-4)^2 + x^2}$

Enter $y1 = ((x-4)\hat{}2 + x\hat{}2)\hat{}(.5)$ and $y2 = d(y1(x), x)$.

Use the [*Zero*] function for $y2$ and obtain $x = 2$. Use the [*Value*] function for $y1$ at $x = 2$ and obtain $y1 = 2.82843$. Thus, the shortest distance is approximately 2.828.

9.4 Practice Problems

Part A—The use of a calculator is not allowed.

1. A spherical balloon is being inflated. Find the volume of the balloon at the instant when the rate of increase of the surface area is eight times the rate of increase of the radius of the sphere.

2. A 13-foot ladder is leaning against a wall. If the top of the ladder is sliding down the wall at 2 ft/sec, how fast is the bottom of the ladder moving away from the wall when the top of the ladder is 5 feet from the ground? (See Figure 9.4-1.)

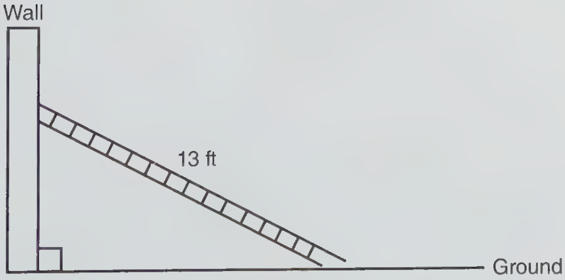


Figure 9.4-1

3. Air is being pumped into a spherical balloon at the rate of 100 cm^3/sec. How fast is the diameter increasing when the radius is 5 cm?

4. A woman 5 feet tall is walking away from a streetlight hung 20 feet from the ground at the rate of 6 ft/sec. How fast is her shadow lengthening?

5. A water tank in the shape of an inverted cone has a height of 18 feet and a base radius of 12 feet. If the tank is full and the water is drained at the rate of 4 ft^3/min, how fast is the water level dropping when the water level is 6 feet high?

6. Two cars leave an intersection at the same time. The first car is going due east at the rate of 40 mph and the second is going due south at the rate of 30 mph. How fast is the distance between the two cars increasing when the first car is 120 miles from the intersection?

7. If the perimeter of an isosceles triangle is 18 cm, find the maximum area of the triangle.

8. Find a number in the interval (0, 2) such that the sum of the number and its reciprocal is the absolute minimum.

9. An open box is to be made using a piece of cardboard 8 cm by 15 cm by cutting a square from each corner and folding the sides up. Find the length of a side of the square being cut so that the box will have a maximum volume.

10. What is the shortest distance between the point $\left(2, -\frac{1}{2}\right)$ and the parabola $y = -x^2$?

11. If the cost function is $C(x) = 3x^2 + 5x + 12$, find the value of x such that the average cost is a minimum.

12. A man with 200 meters of fence plans to enclose a rectangular piece of land using a river on one side and a fence on the other three sides. Find the maximum area that the man can obtain.

Part B—Calculators are allowed.

13. A trough is 10 meters long and 4 meters wide. (See Figure 9.4-2.) The two sides of the trough are equilateral triangles. Water is pumped into the trough at 1 m^3/min. How fast is the water level rising when the water is 2 meters high?

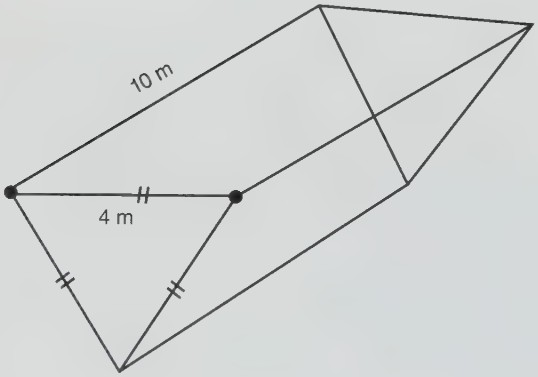


Figure 9.4-2

14. A rocket is sent vertically up in the air with the position function $s = 100t^2$ where s is measured in meters and t in seconds. A camera 3000 m away is recording the rocket. Find the rate of change of the angle of elevation of the camera 5 sec after the rocket went up.

15. A plane lifts off from a runway at an angle of 20°. If the speed of the plane is 300 mph, how fast is the plane gaining altitude?

16. Two water containers are being used. (See Figure 9.4-3.)

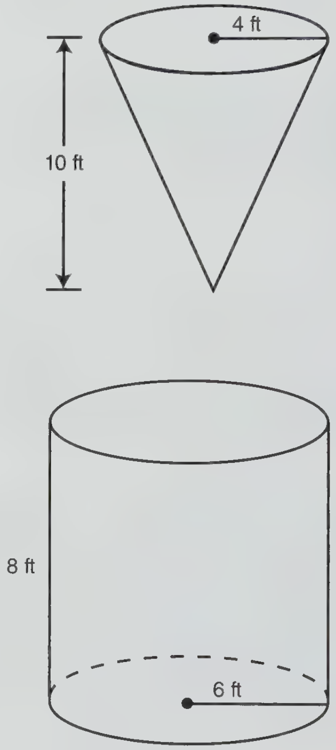


Figure 9.4-3

One container is in the form of an inverted right circular cone with a height of 10 feet and a radius at the base of 4 feet. The other container is a right circular cylinder with a radius of 6 feet and a height of 8 feet. If water is being drained from the conical container into the cylindrical container at the rate of 15 ft^3/min, how fast is the water level falling in the conical tank when the water level in the conical tank is 5 feet high? How fast is the water level rising in the cylindrical container?

17. The wall of a building has a parallel fence that is 6 feet high and 8 feet from the wall. What is the length of the shortest ladder that passes over the fence and leans on the wall? (See Figure 9.4-4.)

18. Given the cost function $C(x)=2500+0.02x+0.004x^2$, find the product level such that the average cost per unit is a minimum.

19. Find the maximum area of a rectangle inscribed in an ellipse whose equation is $4x^2+25y^2=100$.

20. A right triangle is in the first quadrant with a vertex at the origin and the other two vertices on the x- and y-axes. If the hypotenuse passes through the point (0.5, 4), find the vertices of the triangle so that the length of the hypotenuse is the shortest possible length.

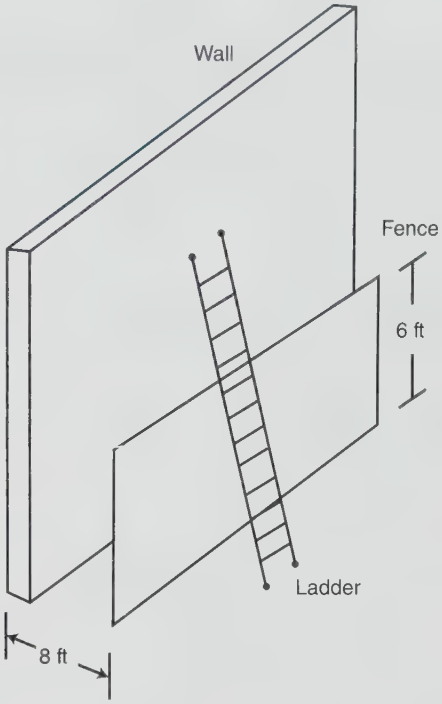


Figure 9.4-4

9.5 Cumulative Review Problems

(Calculator) indicates that calculators are permitted.

21. If $y=\sin^2(\cos(6x-1))$, find $\dfrac{dy}{dx}$.

22. Evaluate $\displaystyle\lim_{x\to\infty}\frac{100/x}{-4+x+x^2}$.

23. The graph of f' is shown in Figure 9.5-1. Find where the function f: (a) has its relative extrema or absolute extrema; (b) is increasing or decreasing; (c) has its point(s) of inflection; (d) is concave upward or downward; and (e) if $f(3)=-2$, draw a possible sketch of f. (See Figure 9.5-1.)

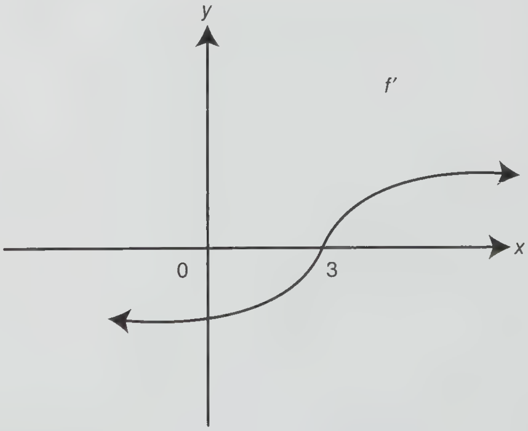


Figure 9.5-1

24. (Calculator) At what value(s) of x does the tangent to the curve $x^2 + y^2 = 36$ have a slope of -1.

25. (Calculator) Find the shortest distance between the point (1, 0) and the curve $y = x^3$.

9.6 Solutions to Practice Problems

Part A—The use of a calculator is not allowed.

1. Volume: $V = \frac{4}{3}\pi r^3$;

   Surface Area: $S = 4\pi r^2 \; \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

   Since $\frac{dS}{dt} = 8\frac{dr}{dt}$,

   $8\frac{dr}{dt} = 8\pi r \frac{dr}{dt} \Rightarrow 8 = 8\pi r$

   or $r = \frac{1}{\pi}$.

   At $r = \frac{1}{\pi}$, $V = \frac{4}{3}\pi\left(\frac{1}{\pi}\right)^3 = \frac{4}{3\pi^2}$ cubic units.

2. Pythagorean Theorem yields $x^2 + y^2 = (13)^2$.

   Differentiate: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$.

   At $x = 5$, $(5)^2 + y^2 = 13^2 \Rightarrow y = \pm 12$, since $y > 0$, $y = 12$.

   Therefore,

   $\frac{dy}{dt} = -\frac{5}{12}(-2)$ ft/sec $= \frac{5}{6}$ ft/sec. The ladder is moving away from the wall at $\frac{5}{6}$ ft/sec when the top of the ladder is 5 feet from the ground.

3. Volume of a sphere is $V = \frac{4}{3}\pi r^3$.

   Differentiate: $\frac{dV}{dt} = \left(\frac{4}{3}\right)(3)\pi r^2$

   $\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$.

   Substitute: $100 = 4\pi(5)^2$

   $\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi}$ cm/sec.

   Let x be the diameter. Since

   $x = 2r$, $\frac{dx}{dt} = 2\frac{dr}{dt}$.

   Thus, $\left.\frac{dx}{dt}\right|_{r=5} = 2\left(\frac{1}{\pi}\right)$ cm/sec

   $= \frac{2}{\pi}$ cm/sec. The diameter is increasing at $\frac{2}{\pi}$ cm/sec when the radius is 5 cm.

4. (See Figure 9.6-1.) Using similar triangles, with y the length of the shadow you have:

   $\frac{5}{20} = \frac{y}{y+x} \Rightarrow 20y = 5y + 5x \Rightarrow$

   $15y = 5x$ or $y = \frac{x}{3}$.

   Differentiate:

   $\frac{dy}{dt} = \frac{1}{3}\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{1}{3}(6)$

   $= 2$ ft/sec.

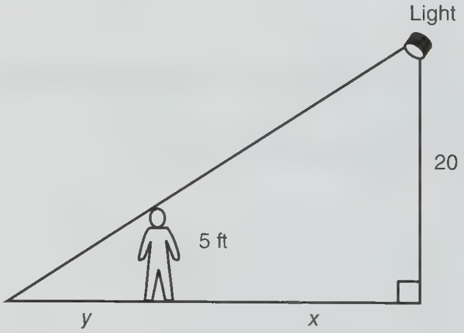


Figure 9.6-1

5. (See Figure 9.6-2.) Volume of a cone

   $V = \frac{1}{3}\pi r^2 h$.

   Using similar triangles, you have

   $\frac{12}{18} = \frac{r}{h} \Rightarrow 2h = 3r$ or $r = \frac{2}{3}h$, thus reducing the equation to

   $V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2(h) = \frac{4\pi}{27}h^3$.

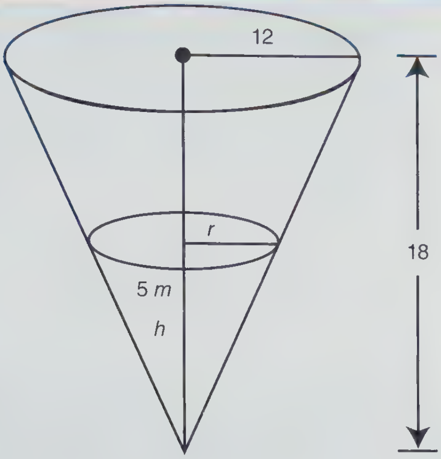


Figure 9.6-2

Differentiate: $\frac{dV}{dt}=\frac{4}{9}\pi h^2\frac{dh}{dt}$.

Substituting known values:

$-4=\frac{4\pi}{9}(6)^2\frac{dh}{dt} \Rightarrow -4=16\pi\frac{dh}{dt}$ or

$\frac{dh}{dt}=-\frac{1}{4\pi}$ ft/min. The water level is

dropping at $\frac{1}{4\pi}$ ft/min when $h=6$ ft.

6. (See Figure 9.6-3.)

Step 1: Using the Pythagorean Theorem, you have $x^2+y^2=z^2$. You also have $\frac{dx}{dt}=40$ and $\frac{dy}{dt}=30$.

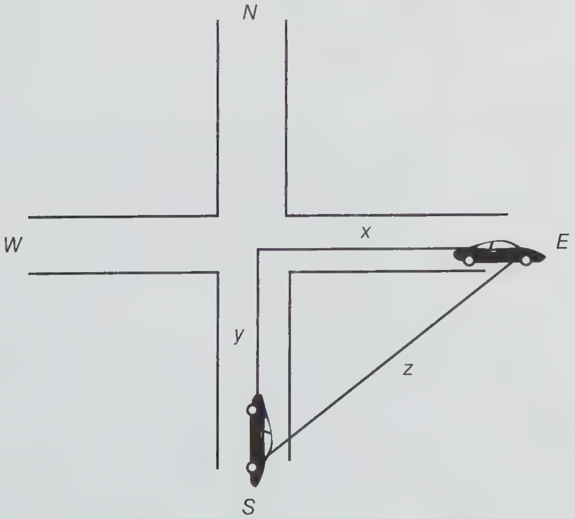


Figure 9.6-3

Step 2: Differentiate:

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}.$$

At $x=120$, both cars have traveled 3 hours and thus, $y=3(30)=90$. By the Pythagorean Theorem, $(120)^2+(90)^2=z^2 \Rightarrow z=150$.

Step 3: Substitute all known values into the equation:

$$2(120)(40)+2(90)(30)=2(150)\frac{dz}{dt}.$$

Thus $\frac{dz}{dt}=50$ mph.

Step 4: The distance between the two cars is increasing at 50 mph at $x=120$.

7. (See Figure 9.6-4.)

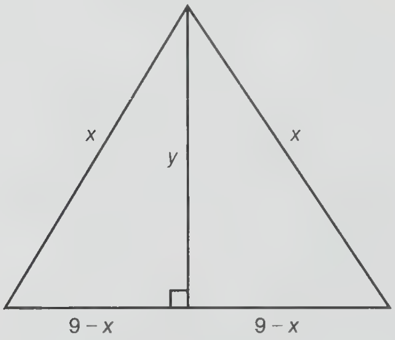


Figure 9.6-4

Step 1: Applying the Pythagorean Theorem, you have

$x^2=y^2+(9-x)^2 \Rightarrow y^2=$

$x^2-(9-x)^2=$

$x^2-\left(81-18x+x^2\right)=$

$18x-81=9(2x-9)$, or

$y=\pm\sqrt{9(2x-9)}=$

$\pm 3\sqrt{(2x-9)}$ since $y>0$,

$y=3\sqrt{(2x-9)}$.

The area of the triangle

$A=\frac{1}{2}\left(3\sqrt{2x-9}\right)(18-2x)=$

$\left(3\sqrt{2x-9}\right)(9-x)=$

$3(2x-9)^{1/2}(9-x)$.

Step 2: $\dfrac{dA}{dx} = \dfrac{3}{2}(2x-9)^{-1/2}(2)(9-x)$

$+(-1)(3)(2x-9)^{1/2}.$

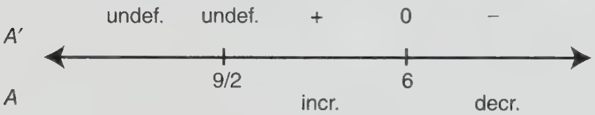
$= \dfrac{3(9-x)\ -3(2x-9)}{\sqrt{2x-9}}$

$= \dfrac{54-9x}{\sqrt{2x-9}}$

Step 3: Set $\dfrac{dA}{dx} = 0 \Rightarrow 54-9x=0;\ x=6.$

$\dfrac{dA}{dx}$ is undefined at $x=\dfrac{9}{2}$. The critical numbers are $\dfrac{9}{2}$ and 6.

Step 4: First Derivative Test:



Thus at $x=6$, the area A is a relative maximum.

$A(6) = \left(\dfrac{1}{2}\right)(3)(\sqrt{2(6)-9})(9-6)$

$= 9\sqrt{3}$

Step 5: Check endpoints. The domain of A is $[9/2, 9]$. $A(9/2)=0$; and $A(9)=0$. Therefore, the maximum area of an isosceles triangle with the perimeter of 18 cm is $9\sqrt{3}$ cm^2. (Note that at $x=6$, the triangle is an equilateral triangle.)

8. Step 1: Let x be the number and $\dfrac{1}{x}$ be its reciprocal.

Step 2: $s = x + \dfrac{1}{x}$ with $0 < x < 2$.

Step 3: $\dfrac{ds}{dx} = 1 + (-1)x^{-2} = 1 - \dfrac{1}{x^2}$

Step 4: Set $\dfrac{ds}{dx} = 0 \Rightarrow 1 - \dfrac{1}{x^2} = 0$
$\Rightarrow x = \pm 1$, since the domain is $(0, 2)$, thus $x=1$.

$\dfrac{ds}{dx}$ is defined for all x in $(0, 2)$.
Critical number is $x=1$.

Step 5: Second Derivative Test:
$\dfrac{d^2s}{dx^2} = \dfrac{2}{x^3}$ and $\left.\dfrac{d^2s}{dx^2}\right|_{x=1} = 2.$
Thus at $x=1$, s is a relative minimum. Since it is the only relative extremum, at $x=1$, it is the absolute minimum.

9. (See Figure 9.6-5.)

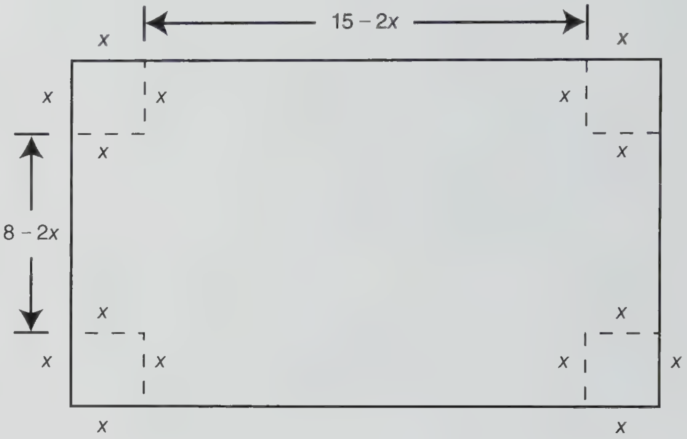


Figure 9.6-5

Step 1: Volume: $V = x(8-2x)(15-2x)$ with $0 \le x \le 4$.

Step 2: Differentiate: Rewrite as
$V = 4x^3 - 46x^2 + 120x$
$\dfrac{dV}{dx} = 12x^2 - 92x + 120.$

Step 3: Set $V=0 \Rightarrow 12x^2 - 92x + 120 = 0$
$\Rightarrow 3x^2 - 23x + 30 = 0$. Using the quadratic formula, you have $x=6$ or $x=\dfrac{5}{3}$ and $\dfrac{dV}{dx}$ is defined for all real numbers.

Step 4: Second Derivative Test:

$\dfrac{d^2V}{dx^2} = 24x - 92;\ \left.\dfrac{d^2V}{dx^2}\right|_{x=6}$

$= 52$ and $\left.\dfrac{d^2V}{dx^2}\right|_{x=\frac{5}{3}} = -52.$

Thus at $x=\frac{5}{3}$ is a relative maximum.

Step 5: Check endpoints. At $x=0$, $V=0$ and at $x=4$, $V=0$. Therefore, at $x=\frac{5}{3}$, V is the absolute maximum.

10. (See Figure 9.6-6.)

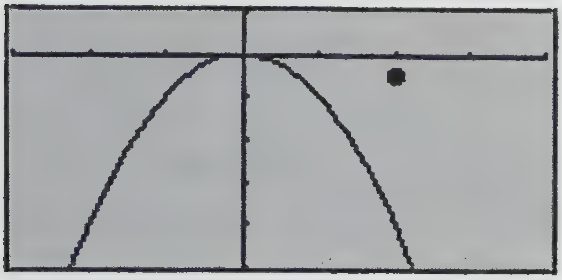


Figure 9.6-6

Step 1: Distance Formula:

$$Z=\sqrt{(x-2)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2}$$

$$=\sqrt{(x-2)^2+\left(-x^2+\frac{1}{2}\right)^2}$$

$$=\sqrt{x^2-4x+4+x^4-x^2+\frac{1}{4}}$$

$$=\sqrt{x^4-4x+\frac{17}{4}}$$

Step 2: Let $S=Z^2$, since S and Z have the same maximums and minimums.

$S=x^4-4x+\frac{17}{4}$; $\frac{dS}{dx}=4x^3-4$

Step 3: Set $\frac{dS}{dx}=0$; $x=1$ and $\frac{dS}{dx}$ is defined for all real numbers.

Step 4: Second Derivative Test:

$$\frac{d^2S}{dx^2}=12x^2 \text{ and } \left.\frac{d^2S}{dx^2}\right|_{x=1}=12.$$

Thus at $x=1$, Z has a minimum, and since it is the only relative extremum, it is the absolute minimum.

Step 5: At $x=1$,

$$Z=\sqrt{(1)^4-4(1)+\frac{17}{4}}$$

$$=\sqrt{\frac{5}{4}}.$$

Therefore, the shortest distance is $\sqrt{\frac{5}{4}}$.

11. Step 1: Average Cost:

$$\overline{C}=\frac{C(x)}{x}=\frac{3x^2+5x+12}{x}$$

$$=3x+5+\frac{12}{x}.$$

Step 2: $\frac{d\overline{C}}{dx}=3-12x^{-2}=3-\frac{12}{x^2}$

Step 3: Set $\frac{d\overline{C}}{dx}=0 \Rightarrow 3-\frac{12}{x^2}=0 \Rightarrow$

$3=\frac{12}{x^2} \Rightarrow x=\pm 2$. Since $x>0$, $x=2$ and $\overline{C}(2)=17$. $\frac{d\overline{C}}{dx}$ is undefined at $x=0$ which is not in the domain.

Step 4: Second Derivative Test:

$$\frac{d^2\overline{C}}{dx^2}=\frac{24}{x^3} \text{ and } \left.\frac{d^2\overline{C}}{dx^2}\right|_{x=2}=3$$

Thus at $x=2$, the average cost is a minimum.

12. (See Figure 9.6-7.)

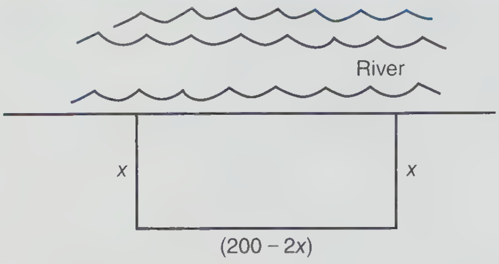


Figure 9.6-7

Step 1: Area:
$A = x(200 - 2x) = 200x - 2x^2$
with $0 \le x \le 100$.

Step 2: $A'(x) = 200 - 4x$

Step 3: Set $A'(x) = 0 \Rightarrow 200 - 4x = 0$;
$x = 50$.

Step 4: Second Derivative Test:
$A''(x) = -4$; thus at $x = 50$, the area is a relative maximum.
$A(50) = 5000 \text{ m}^2$.

Step 5: Check endpoints.
$A(0) = 0$ and $A(100) = 0$; therefore at $x = 50$, the area is the absolute maximum and 5000 m^2 is the maximum area.

Part B—Calculators are allowed.

13. Step 1: Let h be the height of the trough and 4 be a side of one of the two equilateral triangles. Thus, in a 30–60 right triangle, $h = 2\sqrt{3}$.

Step 2: Volume:
$V = (\text{area of the triangle}) \cdot 10$

$$= \left[\frac{1}{2}(h)\left(\frac{2}{\sqrt{3}}h\right)\right]10 = \frac{10}{\sqrt{3}}h^2.$$

Step 3: Differentiate with respect to t.

$$\frac{dV}{dt} = \left(\frac{10}{\sqrt{3}}\right)(2)h\frac{dh}{dt}$$

Step 4: Substitute known values:

$$1 = \frac{20}{\sqrt{3}}(2)\frac{dh}{dt};$$

$$\frac{dh}{dt} = \frac{\sqrt{3}}{40} \text{ m/min.}$$

The water level is rising $\frac{\sqrt{3}}{40}$ m/min when the water level is 2 m high.

14. (See Figure 9.6-8.)

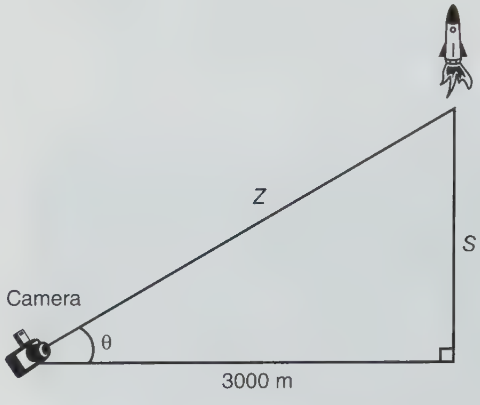


Figure 9.6-8

Step 1: $\tan\theta = S/3000$

Step 2: Differentiate with respect to t.

$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{3000}\frac{dS}{dt};$$

$$\frac{d\theta}{dt} = \frac{1}{3000}\left(\frac{1}{\sec^2\theta}\right)\frac{dS}{dt}$$

$$= \frac{1}{3000}\left(\frac{1}{\sec^2\theta}\right)(200t)$$

Step 3: At $t = 5$; $S = 100(5)^2 = 2500$; Thus, $Z^2 = (3000)^2 + (2500)^2 = 15{,}250{,}000$. Therefore, $Z = \pm 500\sqrt{61}$, since $Z > 0$, $Z = 500\sqrt{61}$. Substitute known values into the equation:

$$\frac{d\theta}{dt} = \frac{1}{3000}\left(\frac{1}{\frac{500\sqrt{61}}{3000}}\right)^2(1000),$$

since $\sec\theta = \dfrac{Z}{3000}$.

$\frac{d\theta}{dt}=0.197$ radian/sec. The angle of elevation is changing at 0.197 radian/sec, 5 seconds after liftoff.

15. (See Figure 9.6-9.)

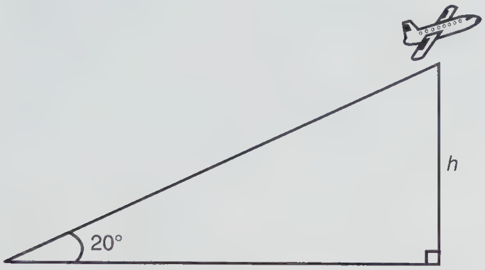


Figure 9.6-9

$\text{Sin } 20^\circ=\frac{h}{300t}$

$h=(\sin 20^\circ)300t;$

$\frac{dh}{dt}=(\sin 20^\circ)(300)\approx 102.606$ mph. The plane is gaining altitude at 102.606 mph.

16. $V_{\text{cone}}=\frac{1}{3}\pi r^2 h$

Similar triangles: $\frac{4}{10}=\frac{r}{h}\Rightarrow 5r=2h$ or $r=\frac{2h}{5}$.

$V_{\text{cone}}=\frac{1}{3}\pi\left(\frac{2h}{5}\right)^2 h=\frac{4\pi}{75}h^3;$

$\frac{dV}{dt}=\frac{4\pi}{75}(3)h^2\frac{dh}{dt}.$

Substitute known values:

$-15=\frac{4\pi}{25}(5)^2\frac{dh}{dt};$

$-15=4\pi\frac{dh}{dt};\ \frac{dh}{dt}=\frac{-15}{4\pi}\approx -1.19$ ft/min.

The water level in the cone is falling at $\frac{-15}{4\pi}$ ft/min ≈ -1.19 ft/ min when the water level is 5 feet high.

$V_{\text{cylinder}}=\pi R^2H=\pi(6)2H=36\pi H.$

$\frac{dV}{dt}=36\pi\frac{dH}{dt};\ \frac{dH}{dt}=\frac{1}{36\pi}\frac{dV}{dt};\ \frac{dH}{dt}$

$=\frac{1}{36\pi}(15)=\frac{5}{12\pi}$ ft/min

≈ 0.1326 ft/min or 1.592 in/min.

The water level in the cylinder is rising at $\frac{5}{12\pi}$ ft/min $=0.133$ ft/min.

17. Step 1: Let x be the distance of the foot of the ladder from the higher wall. Let y be the height of the point where the ladder touches the higher wall. The slope of the ladder is $m=\frac{y-6}{0-8}$ or $m=\frac{6-0}{8-x}$. Thus,

$\frac{y-6}{-8}=\frac{6}{8-x}\Rightarrow (y-6)(8-x)$

$=-48$

$\Rightarrow 8y-xy-48+6x=-48$

$\Rightarrow y(8-x)=-6x\Rightarrow y=\frac{-6x}{8-x}.$

Step 2: Pythagorean Theorem:

$l^2=x^2+y^2=x^2+\left(\frac{-6x}{8-x}\right)^2$

Since $l>0$, $l=\sqrt{x^2+\left(\frac{-6x}{8-x}\right)^2}$,

$x>8$.

Step 3: Enter

$y1=\sqrt{\{x{}^\wedge 2+[(-6*x)/(8-x)]{}^\wedge 2\}}.$

The graph of y_1 is continuous on the interval $x>8$. Use the [*Minimum*] function of the calculator and obtain $x=14.604$; $y=19.731$. Thus the minimum value of l is 19.731 or the shortest ladder is approximately 19.731 feet.

18. Step 1: Average Cost $\overline{C}=\frac{C}{x}$; thus, $\overline{C}(x)$

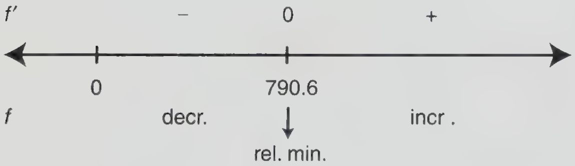
$=\frac{2500+0.02x+0.004x^2}{x}$

$=\frac{2500}{x}+0.02+0.004x.$

Step 2: Enter: $y1 = \dfrac{2500}{x} + .02 + .004 * x$

Step 3: Use the [*Minimum*] function in the calculator and obtain $x = 790.6$.

Step 4: Verify the result with the First Derivative Test. Enter $y2 = d(2500/x + .02 + .004x, x)$; Use the [*Zero*] function and obtain $x = 790.6$. Thus $\dfrac{dC}{dx} = 0$; at $x = 790.6$.
Apply the First Derivative Test:



Thus the minimum average cost per unit occurs at $x = 790.6$. (The graph of the average cost function is shown in Figure 9.6-10.)

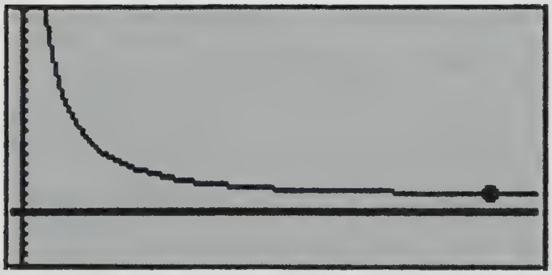


Figure 9.6-10

19. (See Figure 9.6-11.)

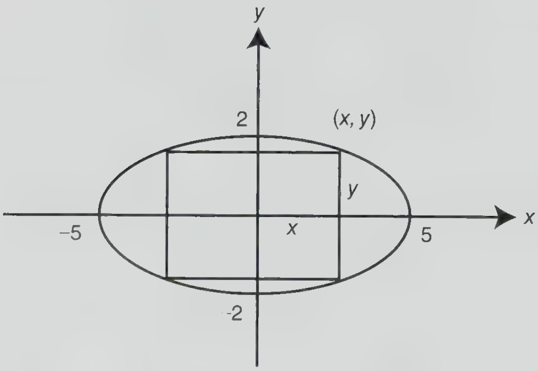


Figure 9.6-11

Step 1: Area $A = (2x)(2y)$; $0 \le x \le 5$ and $0 \le y \le 2$.

Step 2: $4x^2 + 25y^2 = 100$;
$25y^2 = 100 - 4x^2$.

$$y^2 = \frac{100 - 4x^2}{25} \Rightarrow y = \pm\sqrt{\frac{100 - 4x^2}{25}}$$

Since $y \ge 0$

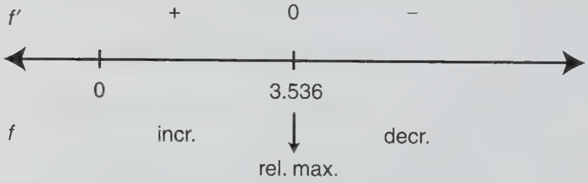
$$y = \sqrt{\frac{100 - 4x^2}{25}} = \frac{\sqrt{100 - 4x^2}}{5}.$$

Step 3: $A = (2x)\left(\dfrac{2}{5}\right)\left(\sqrt{100 - 4x^2}\right)$

$$= \frac{4x}{5}\sqrt{100 - 4x^2}$$

Step 4: Enter $y1 = \dfrac{4x}{5}\sqrt{100 - 4x^2}$
Use the [*Maximum*] function and obtain $x = 3.536$ and $y_1 = 20$.

Step 5: Verify the result with the First Derivative Test.
Enter
$y2 = d\left(\dfrac{4x}{5}\sqrt{100 - 4x^2}, x\right)$. Use the [*Zero*] function and obtain $x = 3.536$.
Note that:



The function f has only one relative extremum. Thus, it is the absolute extremum. Therefore, at $x = 3.536$, the area is 20 and the area is the absolute maxima.

20. (See Figure 9.6-12.)

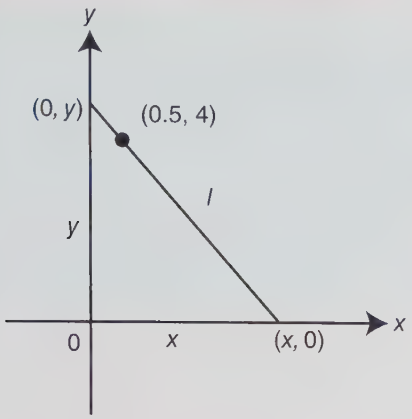


Figure 9.6-12

Step 1: Distance formula:
$l^2 = x^2 + y^2$; $x > 0.5$ and $y > 4$.

Step 2: The slope of the hypotenuse:

$$m = \frac{y-4}{0-0.5} = \frac{-4}{x-0.5}$$

$$\Rightarrow (y-4)(x-0.5) = 2$$

$$\Rightarrow xy - 0.5y - 4x + 2 = 2$$

$$y(x-0.5) = 4x$$

$$y = \frac{4x}{x-0.5}.$$

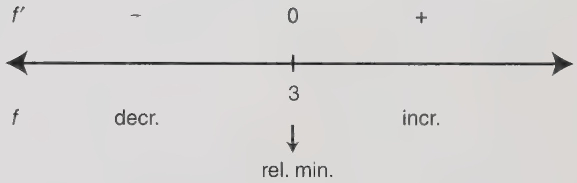
Step 3: $l^2 = x^2 + \left(\frac{4x}{x-0.5}\right)^2$;

$$l = \pm\sqrt{x^2 + \left(\frac{4x}{x-0.5}\right)^2}$$

Since $l > 0$, $l = \sqrt{x^2 + \left(\frac{4x}{x-0.5}\right)^2}$.

Step 4: Enter $y1 = \sqrt{x^2 + \left(\frac{4x}{x-0.5}\right)^2}$ and use the [*Minimum*] function of the calculator and obtain $x = 2.5$.

Step 5: Apply the First Derivative Test. Enter $y2 = d(y1(x), x)$ and use the [*Zero*] function and obtain $x = 2.5$. Note that:



Since f has only one relative extremum, it is the absolute extremum.

Step 6: Thus, at $x = 2.5$, the length of the hypotenuse is the shortest. At $x = 2.5$, $y = \frac{4(2.5)}{2.5-0.5} = 5$. The vertices of the triangle are (0, 0), (2.5, 0), and (0, 5).

9.7 Solutions to Cumulative Review Problems

21. Rewrite: $y = [\sin(\cos(6x-1))]^2$

Thus, $\frac{dy}{dx} = 2\,[\sin(\cos(6x-1))]$

$$\times [\cos(\cos(6x-1))]$$

$$\times [-\sin(6x-1)]\,(6)$$

$$= -12\sin(6x-1)$$

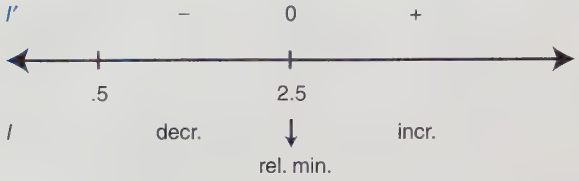
$$\times [\sin(\cos(6x-1))]$$

$$\times [\cos(\cos(6x-1))].$$

22. As $x \to \infty$, the numerator $\frac{100}{x}$ approaches 0 and the denominator increases without bound (i.e., ∞).

Thus, the $\lim_{x\to\infty} \frac{100/x}{-4+x+x^2} = 0$.

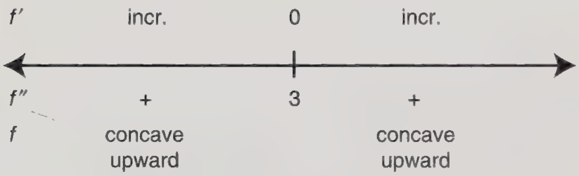
23. (a) Summarize the information of f' on a number line.



Since f has only one relative extremum, it is the absolute extremum. Thus, at $x=3$, it is an absolute minimum.

(b) The function f is decreasing on the interval $(-\infty, 3)$ and increasing on $(3, \infty)$.

(c)



No change of concavity ⇒ No point of inflection.

(d) The function f is concave upward for the entire domain $(-\infty, \infty)$.

(e) Possible sketch of the graph for $f(x)$. (See Figure 9.7-1.)

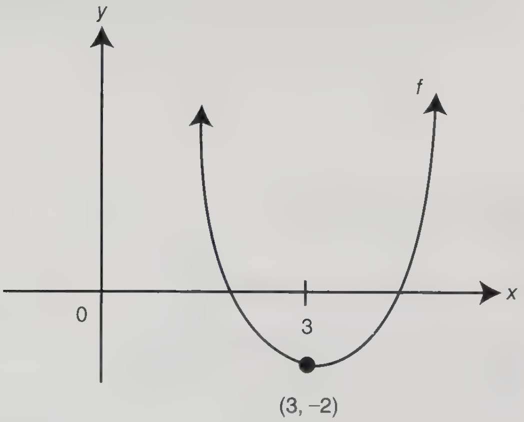


Figure 9.7-1

24. (Calculator) (See Figure 9.7-2.)

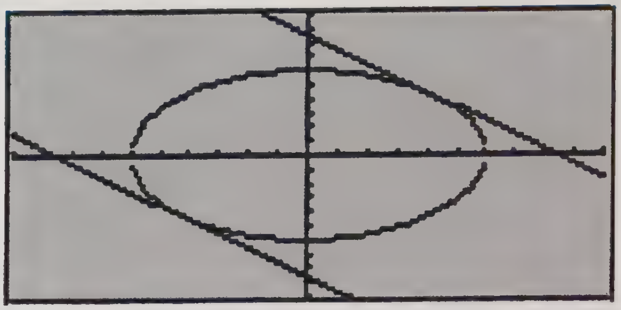


Figure 9.7-2

Step 1: Differentiate:
$2x+2y\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=-\dfrac{x}{y}$.

Step 2: Set $\dfrac{dy}{dx}=-1 \Rightarrow \dfrac{-x}{y}=-1 \Rightarrow$
$y=x$.

Step 3: Solve for y: $x^2+y^2=36 \Rightarrow$
$y^2=36-x^2$;
$y=\pm\sqrt{36-x^2}$.

Step 4: Thus, $y=x \Rightarrow \pm\sqrt{36-x^2}=$
$x \Rightarrow 36-x^2=x^2 \Rightarrow$
$36=2x^2$ or $x=\pm 3\sqrt{2}$.

25. (Calculator) (See Figure 9.7-3.)

Step 1: Distance formula:
$z=\sqrt{(x-1)^2+(x^3)^2}=$
$\sqrt{(x-1)^2+x^6}$.

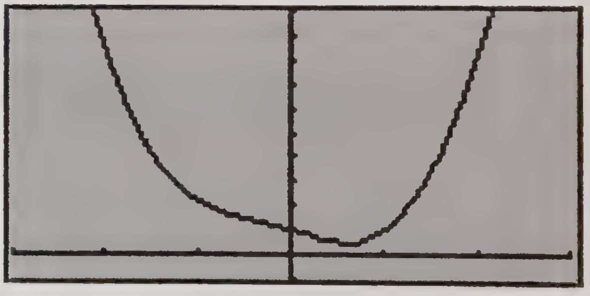


Figure 9.7-3

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Chapter 2

How to Plan Your Time

IN THIS CHAPTER

Summary: The right preparation plan for you depends on your study habits and the amount of time you have before the test.

Key Idea

Key Idea

✪ Choose the study plan that's right for you.

Three Approaches to Preparing for the AP U.S. Government and Politics Exam

What kind of preparation program for the AP exam should you follow? Should you carefully follow every step, or are there perhaps some steps you can bypass? That depends not only on how much time you have, but also on what kind of student you are. No one knows your study habits, likes, and dislikes better than you do, so you are the only one who can decide which approach you want or need to adapt. This chapter presents three possible study plans, labeled A, B, and C. Look at the brief profiles below. These may help you determine which of these three plans is right for you.

> *"Study groups helped me focus."*
> —DA, AP student

You're a full-year prep student if:

- You are the kind of person who likes to plan for everything very far in advance.
- You buy your best friend a gift two months before his or her birthday because you know exactly what to choose, where you will buy it, and how much you will pay for it.
- You like detailed planning and everything in its place.
- You feel that you must be thoroughly prepared.
- You hate surprises.

If you fit this profile, consider **Plan A**.

You're a one-semester prep student if:

"My teacher held review sessions right before the exam—they really help!"
—TG, AP student

- You buy your best friend a gift one week before his or her birthday because it sort of snuck up on you, yet you have a clear idea of exactly what you will be purchasing.
- You are willing to plan ahead to feel comfortable in stressful situations, but are okay with skipping some details.
- You feel more comfortable when you know what to expect, but a surprise or two is cool.
- You're always on time for appointments.

If you fit this profile, consider **Plan B**.

You're a six-week prep student if:

"Relax! Knowing the makeup of the test and how to take the test really helps."
—JB, AP student

- You buy your best friend a gift for his or her birthday, but you need to include a belated card because you missed it by a couple of days.
- You work best under pressure and tight deadlines.
- You feel very confident with the skills and background you have learned in your AP U.S. Government and Politics class.
- You decided late in the year to take the exam.
- You like surprises.
- You feel okay if you arrive 10 to 15 minutes late for an appointment.

If you fit this profile, consider **Plan C**.

Calendar for Each Plan

Plan A:
You Have a Full School Year to Prepare

Although its primary purpose is to prepare you for the AP U.S. Government and Politics exam you will take in May, this book can enrich your study of government and politics, your analytical skills, and your essay writing skills.

SEPTEMBER–OCTOBER (Check off the activities as you complete them.)

— Determine the student mode (A, B, or C) that applies to you.
— Carefully read the Introduction and Chapter 1 of this book.
— Take the Diagnostic Exam in Chapter 3.
— Get on the Web and take a look at the AP Web site(s).
— Skim the Comprehensive Review section. (Reviewing the topics covered in this section will be part of your year-long preparation.)
— Buy a few color highlighters.
— Flip through the entire book. Break the book in. Write in it. Highlight it.
— Get a clear picture of your own school's AP Government and Politics curriculum.
— Begin to use the book as a resource to supplement the classroom learning.
— Read and study Chapters 4 and 5.

NOVEMBER (The first 10 weeks have elapsed.)

— Read and study Chapter 6, Architecture and Development of United States Government.
— Read and study Chapter 7, Federalism.

DECEMBER

— Read and study Chapter 8, Political Culture.
— Read and study Chapter 9, Political Parties.
— Review Chapters 6 and 7.

JANUARY (20 weeks have elapsed.)

— Read and study Chapter 10, Voting and Elections.
— Read and study Chapter 11, Interest Groups and the Mass Media.
— Review Chapters 6 to 9.

FEBRUARY

— Read and study Chapter 12, The Legislative Branch.
— Read and study Chapter 13, The Executive Branch and the Bureaucracy.
— Read and study Chapter 14, The National Judiciary.
— Review Chapters 6 to 11.

MARCH (30 weeks have now elapsed.)

— Take Practice Exam 1 in the first week of March.
— Read and study Chapter 15, Civil Liberties and Civil Rights.
— Read and study Chapter 16, Politics and Public Policymaking.
— Review Chapters 6 to 14.

APRIL

— Take Practice Exam 2 in the first week of April.
— Evaluate your strengths and weaknesses.
— Study appropriate chapters to correct your weaknesses.
— Review Chapters 6 to 16.

MAY (First 2 weeks) (THIS IS IT!)

— Review the Introduction and Chapters 1 to 16—all the material.
— Take Practice Exam 3.
— Score yourself.
— Get a good night's sleep before the exam. Fall asleep knowing that you are well prepared.

GOOD LUCK ON THE TEST!

Plan B:
You Have One Semester to Prepare

Working under the assumption that you've completed or are taking one semester of government and politics studies, the following calendar will use the skills you've been practicing to prepare you for the May exam.

JANUARY–FEBRUARY

— Carefully read the Introduction and Chapters 1 to 5 of this book.
— Take the Diagnostic Exam.
— Read Chapters 4 and 5.
— Read and study Chapter 6, Architecture and Development of U.S. Government.
— Read and study Chapter 7, Federalism.
— Read and study Chapter 8, Political Culture.
— Read and study Chapter 9, Political Parties.
— Read and study Chapter 10, Voting and Elections.
— Read and study Chapter 11, Interest Groups and the Mass Media.
— Review Chapters 6 to 11.

MARCH (10 weeks to go)

— Read and study Chapter 12, The Legislative Branch.
— Review Chapters 6 and 7.
— Read and study Chapter 13, The Executive Branch and the Bureaucracy.
— Review Chapters 8 and 9.
— Read and study Chapter 14, The National Judiciary.
— Review Chapters 10 to 11.
— Read and study Chapter 15, Civil Liberties and Civil Rights.
— Read and study Chapter 16, Politics and Public Policymaking.

APRIL

— Take Practice Exam 1 in the first week of April.
— Evaluate your strengths and weaknesses.
— Study appropriate chapters to correct your weaknesses.
— Take Practice Exam 2 in the third week of April.
— Review Chapters 6 to 11.
— Review Chapters 12 to 16.

MAY (First 2 weeks) (THIS IS IT!)

— Review the Introduction and Chapters 1 to 16—all the material.
— Take Practice Exam 3.
— Score yourself.
— Get a good night's sleep before the exam. Fall asleep knowing that you are well prepared.

GOOD LUCK ON THE TEST!

SOUTHWESTERN ADVANTAGE

Chapter 12

The Legislative Branch

IN THIS CHAPTER

Summary: Article I of the United States Constitution creates a **bicameral**, or two-house, legislature consisting of the House of Representatives and the Senate. The current structure of the Congress was the result of the Connecticut (Great) Compromise, reached at the Constitutional Convention. The Founding Fathers based their compromise in part on the belief that each house would serve as a check on the power of the other house. The House of Representatives was to be based on the population in the states, representative of the people, with its members chosen by popular vote. The Senate was to represent the states, with each state having the same number of senators, chosen by the state legislatures.

Key Terms

bicameral
apportionment
reapportionment
congressional districting
gerrymandering
incumbency effect
casework
constituents
Speaker of the House
majority leader
president pro tempore
floor leaders
seniority system
standing committee
select committee
joint committee
conference committee
caucuses
trustee
franking privilege
oversight
bills
rules committee
filibuster
cloture
pork barrel legislation
logrolling
riders
amendments
lobbying
legislative veto

Structure of Congress

Figure 12-1 shows the structure of the two arms of Congress.

Structure of Congress: A Comparison of the House and Senate

	House of Representatives	**Senate**
Membership	435 members (apportioned by population)	100 members (two from each state)
Term of office	2 years; entire House elected every 2 years	6 years; staggered terms with one-third of the Senate elected every 2 years
Qualifications	at least 25 years of age; citizen for 7 years; must live in state where district is located	at least 30 years of age citizen for 9 years must live in state
Constituencies	Smaller, by districts	Larger, entire state
Prestige	Less prestige	More prestige

Figure 12-1

Organization of Congress

- Two houses meet for terms of two years beginning on January 3 of odd-numbered years; each term is divided into two one-year sessions
- The president may call special sessions in cases of national emergency
- Each house of Congress chooses its own leadership and determines its own rules

Election to Congress

Getting Elected to the House of Representatives

The Constitution guarantees each state at least one representative. Members are chosen from districts within each state. Some practices related to determining congressional representation are:

Key Idea

- ***apportionment***—distribution among the states based on the population of each of the states
- ***reapportionment***—the redistribution of Congressional seats after the census determines changes in population distribution among the states
- ***congressional districting***—the drawing by state legislatures of congressional districts for those states with more than one representative
- ***gerrymandering***—drawing congressional districts to favor one political party or group over another

Getting Elected to the Senate

The Constitution guarantees that "no state, without its consent, shall be deprived of its equal suffrage in the Senate" (Article V).

- Members were originally chosen by the state legislatures in each state.
- Since 1913, the Seventeenth Amendment allows for the direct election of senators by the people of the state.

Incumbency Effect

Key Idea

The **incumbency effect** is the tendency of those already holding office to win reelection. The effect tends to be stronger for members of the House of Representatives and weaker for the Senate. Advantages may include:

- *name recognition*—Voters are more likely to recognize the office holder than the challenger.
- *credit claiming*—The office holder may have brought government projects and money into the state or district.
- ***casework** for **constituents***—Office holders may have helped constituents solve problems involving government and the bureaucracy.
- *more visible to constituents*—Members can use the "perks" of the office to communicate with constituents. Franking, the privilege of sending official mail using the incumbent's signature as postage, provides communication with constituents.
- *media exposure*—Incumbents are more likely to gain "free" publicity during a campaign through the media.
- *fund-raising abilities*—It is generally greater for incumbents.
- *experience in campaigning*—Incumbents have already experienced the campaign process.
- *voting record*—Voters can evaluate their performance based on their record.

Term Limits

Although several states have passed legislation establishing term limits for members of Congress, the Supreme Court has ruled that neither the states nor Congress may impose term limits without a constitutional amendment. Therefore, today, there are no limitations on the number of terms a member of Congress may serve.

Leadership of Congress

The majority political party in each house controls the leadership positions of Congress.

House of Representatives

Key Idea

- The **Speaker of the House** is the presiding officer and most powerful member of the House. Major duties include assigning bills to committee, controlling floor debate, and appointing party members to committees. The Speaker is elected by members of his or her political party within the House.
- Majority and minority leaders
  - — The **majority leader** serves as the major assistant to the speaker, helps plan the party's legislative program, and directs floor debate.
  - — The minority floor leader is the major spokesperson for the minority party and organizes opposition to the majority party.
- Whips help floor leaders by directing party members in voting, informing members of impending voting, keeping track of vote counts, and pressuring members to vote with the party.

Senate

- The U.S. vice president, although not a Senate member, is the presiding officer of the Senate, according to the Constitution. The vice president may not debate and only votes to break a tie.

- The **president pro tempore** is a senior member of the majority party chosen to preside in the absence of the Senate president. This is a mostly ceremonial position lacking real power.
- Majority and minority **floor leaders**
  - — The majority floor leader is the most influential member of the Senate and often the majority party spokesperson.
  - — The minority floor leader performs the same role as the House minority leader.
- Whips serve the same role as whips in the House of Representatives.

The Committee System

Most of the work of Congress is accomplished through committees. Committees permit Congress to divide the work among members, thus allowing for the study of legislation by specialists and helping speed up the passage of legislation.

Leadership of Committees

Committee chairpersons are members of the majority party in each house chosen by party caucus. They set agendas, assign members to subcommittees, and decide whether the committee will hold public hearings and which witnesses to call. They manage floor debate of the bill when it is presented to the full House or Senate. Traditionally chairpersons were chosen based on the **seniority system**, with the majority party member having the longest length of committee service chosen as chairperson. Today, reforms allow for the selection of chairpersons who are not the most senior majority-party member on the committee. However, most are long-standing members of the committee.

Membership on Committees

The percentage of each committee's membership reflects the overall percentage of Democrats and Republicans in each house. Members try to serve on committees where they can influence public policy relating to their district or state (for example, a Kansas senator on the agriculture committee) or influence important national public policy.

Types of Committees

- A **standing committee** is a permanent committee that deals with specific policy matters (agriculture, energy and natural resources, veterans' affairs).
- A **select committee** is a temporary committee appointed for a specific purpose. Most are formed to investigate a particular issue, such as the Senate Watergate Committee.
- A **joint committee** is made up of members of both houses of Congress. It may be a select committee (Iran-Contra Committee) or perform routine duties (Joint Committee on the Library of Congress).
- A **conference committee** is a temporary committee of members from both houses of Congress, created to resolve the differences in House and Senate versions of a bill. It is a compromise committee.

Caucuses

Caucuses are informal groups formed by members of Congress who share a common purpose or set of goals (Congressional Black Caucus, Women's Caucus, Democratic or Republican Caucus).

Congressional Staff and Support

- Personal staff work directly for members of Congress in Washington, D.C., and their district offices in their home states.
- Committee staff work for committees and subcommittees in Congress, researching problems and analyzing information.
- Support agencies provide services to members of Congress (Library of Congress, Government Printing Office).

Roles of Members of Congress

Members of Congress have several roles.

- *policymaker*—make public policy through the passage of legislation
- *representative*—represent constituents
  - — delegate—members vote based on the wishes of constituents, regardless of their own opinions
  - — **trustee**—after listening to constituents, members vote based on their own opinions
- *constituent servant*—help constituents with problems
- *committee member*—serve on committees
- *politician/party member*—work to support their political party platform and get reelected

Privileges of Members of Congress

Members of Congress enjoy several privileges, including:

- allowances for offices in their district or home state
- travel allowances
- the **franking privilege** allows members of Congress to send mailings to constituents postage free
- immunity from arrest while conducting congressional business
- immunity from libel or slander suits for their speech or debate in Congress

Powers of Congress

Congress has legislative and nonlegislative powers.

1. Legislative powers—power to make laws

Key Idea

   - *expressed powers*—powers specifically granted to Congress, mostly found in Article I, Section 8 of the Constitution
   - *implied powers*—powers that may be reasonably suggested to carry out the expressed powers; found in Article I, Section 8, Clause 18; "necessary and proper" or elastic clause; allows for the expansion of Congress's powers (expressed power to raise armies and navy implies the power to draft men into the military)
   - *limitations on powers*—powers denied Congress by Article I, Section 9 and the Tenth Amendment

2. Nonlegislative powers—duties other than lawmaking

   - *electoral powers*—selection of the president by the House of Representatives and/or vice president by the Senate upon the failure of the electoral college to achieve a majority vote

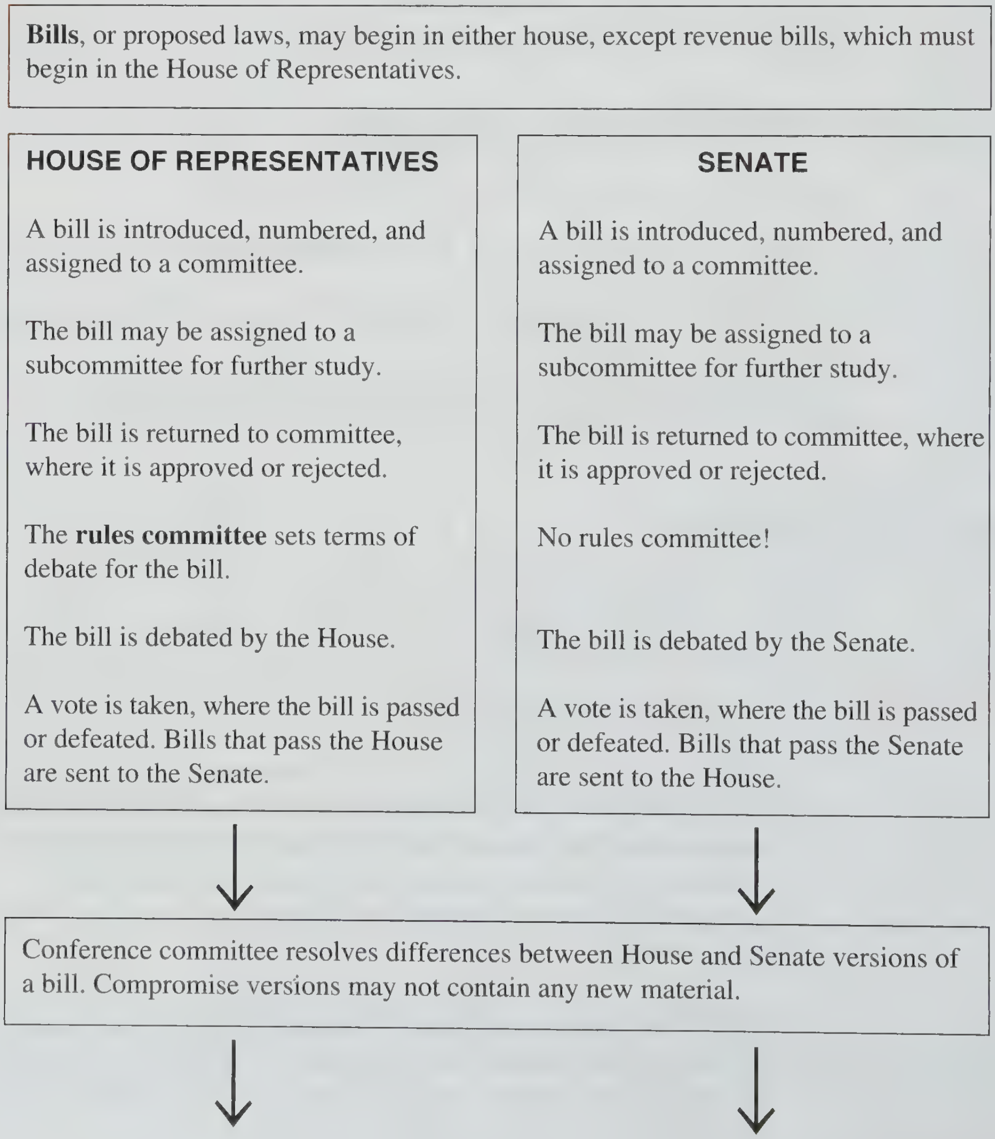
- *amendment powers*—Congress may propose amendments by a two-thirds vote of each house or by calling a national convention to propose amendments if requested by two-thirds of the state legislatures
- *impeachment*—the House may bring charges, or impeach, the president, vice president, or any civil officer; case is tried in the Senate with the Senate acting as the jury (Andrew Johnson and Bill Clinton were both impeached by the House but not convicted by the Senate)
- *executive powers of Senate*—the Senate shares the appointment and treaty-making powers with the executive branch; the Senate must approve appointments by majority vote and treaties by two-thirds vote
- *investigative/**oversight** powers*—investigate matters falling within the range of its legislative authority; often involves the review of policies and programs of the executive branch

The Lawmaking Process

Figure 12-2 shows the steps involved for a bill to become a law.

Key Idea

How a Bill Becomes a Law



Bills, or proposed laws, may begin in either house, except revenue bills, which must begin in the House of Representatives.

HOUSE OF REPRESENTATIVES

A bill is introduced, numbered, and assigned to a committee.

The bill may be assigned to a subcommittee for further study.

The bill is returned to committee, where it is approved or rejected.

The **rules committee** sets terms of debate for the bill.

The bill is debated by the House.

A vote is taken, where the bill is passed or defeated. Bills that pass the House are sent to the Senate.

SENATE

A bill is introduced, numbered, and assigned to a committee.

The bill may be assigned to a subcommittee for further study.

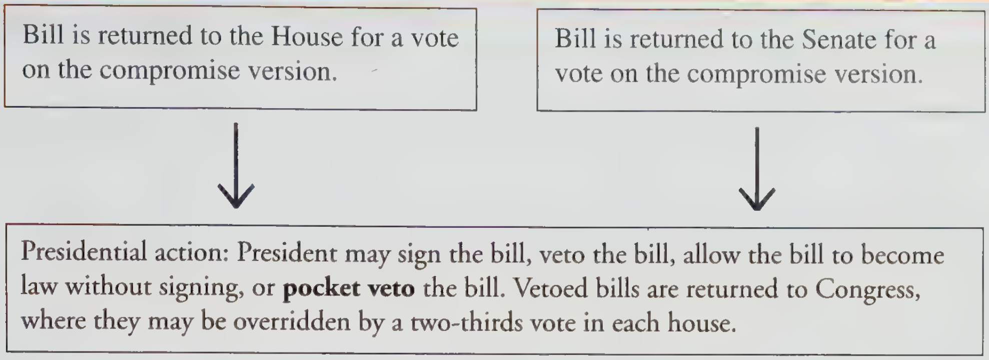
The bill is returned to committee, where it is approved or rejected.

No rules committee!

The bill is debated by the Senate.

A vote is taken, where the bill is passed or defeated. Bills that pass the Senate are sent to the House.

Conference committee resolves differences between House and Senate versions of a bill. Compromise versions may not contain any new material.



Bill is returned to the House for a vote on the compromise version.

Bill is returned to the Senate for a vote on the compromise version.

Presidential action: President may sign the bill, veto the bill, allow the bill to become law without signing, or **pocket veto** the bill. Vetoed bills are returned to Congress, where they may be overridden by a two-thirds vote in each house.

Figure 12-2

Legislative Tactics

Legislative tactics are the strategies and devices used by Congress and others in an attempt to block legislation or to get legislation passed.

Key Idea

- *caucuses*—may form voting blocs.
- *the committee system*—Plays a major role in the passage of legislation; bills may die if committees fail to act upon them or reject them.
- ***filibuster** and **cloture***—Filibuster is unlimited debate in an attempt to stall action on a bill. It occurs in the Senate only, and is possible because the Senate's rules for debate are almost unrestricted. Cloture is the method by which the Senate limits a filibuster. It involves a petition to end debate and requires the vote of at least 60 senators.
- ***pork barrel legislation***—an attempt to provide funds and projects for a member's home district or state.
- ***logrolling***—an attempt by members to gain the support of other members in return for their support on the member's legislation; "I'll support your bill, if you will support mine."
- ***riders***—additions to legislation which generally have no connection to the legislation; generally legislation that would not pass on its own merit; when a bill has lots of riders it becomes a "Christmas tree bill."
- ***amendments***—additions or changes to legislation that deal specifically with the legislation
- ***lobbying***—Trying to influence members of Congress to support or reject legislation.
- *conference committees*—May affect the wording and therefore the final intent of the legislation.
- ***legislative veto***—The rejection of a presidential or executive branch action by a vote of one or both houses of Congress, used mostly between 1932 and 1980 but declared unconstitutional by the Supreme Court in 1983 (Immigration and Naturalization Service v. Chadha) stating that Congress cannot take any actions having the force of law unless the president agrees.

Influences on Congress

Various individuals and groups influence Congress members.

- *constituents*—Members, especially those who hope to win reelection, often take into consideration the opinions of their constituents and voters back home in their district or state.
- *other lawmakers and staff*—More senior members often influence newer members; committee members who worked on legislation often influence other members; and staff often research issues and advise members.

- *party influences*—Each party's platform takes a stand on major issues, and loyal members often adhere to the "party line." Members in the House are more likely to support the party position than are Senators.
- *president*—Presidents often lobby members to support legislation through phone calls, invitations to the White House, or even appeals to the public to gain support from voters to bring pressure on members.
- *lobbyists and interest groups*—often provide members with information on topics relating to their group's interest or possible financial support in future campaigns.

Review Questions

1. After a national census has been taken, changes in population distribution cause the changing of congressional seats among the states. This effect is commonly called
   (A) congressional districting
   (B) apportionment
   (C) gerrymandering
   (D) reapportionment
   (E) census apportionment

2. Which of the following is NOT true of the Congress?
   (A) Each house determines its own leadership and rules.
   (B) Terms of Congress last for two years.
   (C) Congress is unicameral in nature.
   (D) Only the president may call special sessions of Congress.
   (E) A session of Congress begins on January 3 of odd-numbered years.

3. Which of the following is true about the officers of the House of Representatives?
   (A) It is the responsibility of the party whip to keep track of vote counts and pressure members to vote with the party.
   (B) The presiding officer is the majority leader.
   (C) Minority leaders direct floor debates.
   (D) The major duty of the majority leader is to assign bills to committee.
   (E) The whip serves as the major assistant to the speaker in determining the party's legislative program.

4. Temporary committees appointed for the specific purpose of investigating a particular issue are called
   (A) joint committees
   (B) select committees
   (C) standing committees
   (D) investigating committees
   (E) compromise committees

5. Which of the following is NOT true of members of Congress?
   (A) Members of Congress act as policymakers and make public policy through the passage of legislation.
   (B) Members of Congress are constituent servants.
   (C) Members of Congress work to support their political party platform.
   (D) After listening to constituents, members vote based on the opinions of those constituents, that is, they become the trustee of the constituent.
   (E) Members of Congress serve on committees.

6. Which of the following best describes a nonlegislative power of Congress?
   (A) power to declare war
   (B) power to tax
   (C) impeachment power
   (D) power to regulate commerce
   (E) power to make all laws which are necessary and proper

7. Which of the following would not be a step that a proposed bill would encounter on its journey through Congress?
   (A) The bill is debated.
   (B) The bill is voted on, at which time it may pass or be defeated.
   (C) The bill may be assigned to a subcommittee.
   (D) The bill is numbered.
   (E) The bill is introduced by a member of the Congress or by a member of the executive department.

8. After receiving a bill the president may
    - I. sign the bill
    - II. veto the bill
    - III. return the bill to Congress for clarification on some point within the bill
    - IV. allow the Congress to pocket veto the bill, thereby allowing the bill to become law without his signature
        - (A) I only
        - (B) II only
        - (C) I and II only
        - (D) I, II, III only
        - (E) I, II, III, and IV

9. Which legislative tactic is most effectively used only in the Senate?
    - (A) filibustering
    - (B) logrolling
    - (C) caucuses
    - (D) riders
    - (E) pork barrel legislation

10. Which of the following is not considered to be a contributing factor to the incumbency effect?
    - (A) name recognition
    - (B) voting record
    - (C) educational background
    - (D) experience in campaigning
    - (E) visibility to constituents

Answers and Explanations

1. **D.** Reapportionment is the change in the number of congressional seats per state based on state population changes. Gerrymandering (C) is the drawing of congressional electoral districts in order to give an advantage to a group or party.

2. **C.** Congress is a bicameral legislature composed of the House of Representatives and the Senate. The other responses are correct descriptions of Congress.

3. **A.** The minority and majority whips are responsible for keeping track of vote counts and pressuring members to support the party vote. The presiding officer is the speaker of the House (B). The speaker directs floor debates (C) and largely controls the assignment of bills to committee (D). The majority leader serves as the major assistant to the speaker (E).

4. **B.** Select committees are temporary committees responsible for investigating specific issues. Some select committees investigate issues (D). Joint committees (A) are those made up of members of both houses of Congress. Standing committees (C) are permanent committees that deal with specific policy matters. A compromise committee (E) is a description of a conference committee.

5. **D.** Members of Congress who vote according to constituent wishes are delegates of the constituents. The remaining answer choices accurately reflect the duties of members of Congress.

6. **C.** The impeachment power of Congress is a nonlegislative power, having nothing to do with the passage of legislation. The powers to declare war (A), to tax (B), to regulate commerce (D), and to make all laws that are necessary and proper (E) deal with the passage of legislation. They are, therefore, among the legislative powers of Congress.

7. **E.** Although bills may be suggested by members of the executive branch, they may be introduced only by members of Congress. The remaining answer choices are correct and occur in the following order: D, C, A, B.

8. **C.** The president may not return bills to Congress for clarification. Only the president has the power of the pocket veto.

9. **A.** Filibustering is a stalling tactic used only in the Senate. Logrolling (B), caucuses (C), riders (D), and pork barrel legislation (E) are used in both houses of Congress.

10. **C.** The incumbency effect is the tendency of those already holding office to win reelection because of advantages of holding that office. A candidate's educational background does not contribute to the incumbency effect. Name recognition (A), voting record (B), campaign experience (D), and visibility to constituents (E) are factors that contribute to the incumbency effect.

How to Use Math Advantage

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- ALGEBRA
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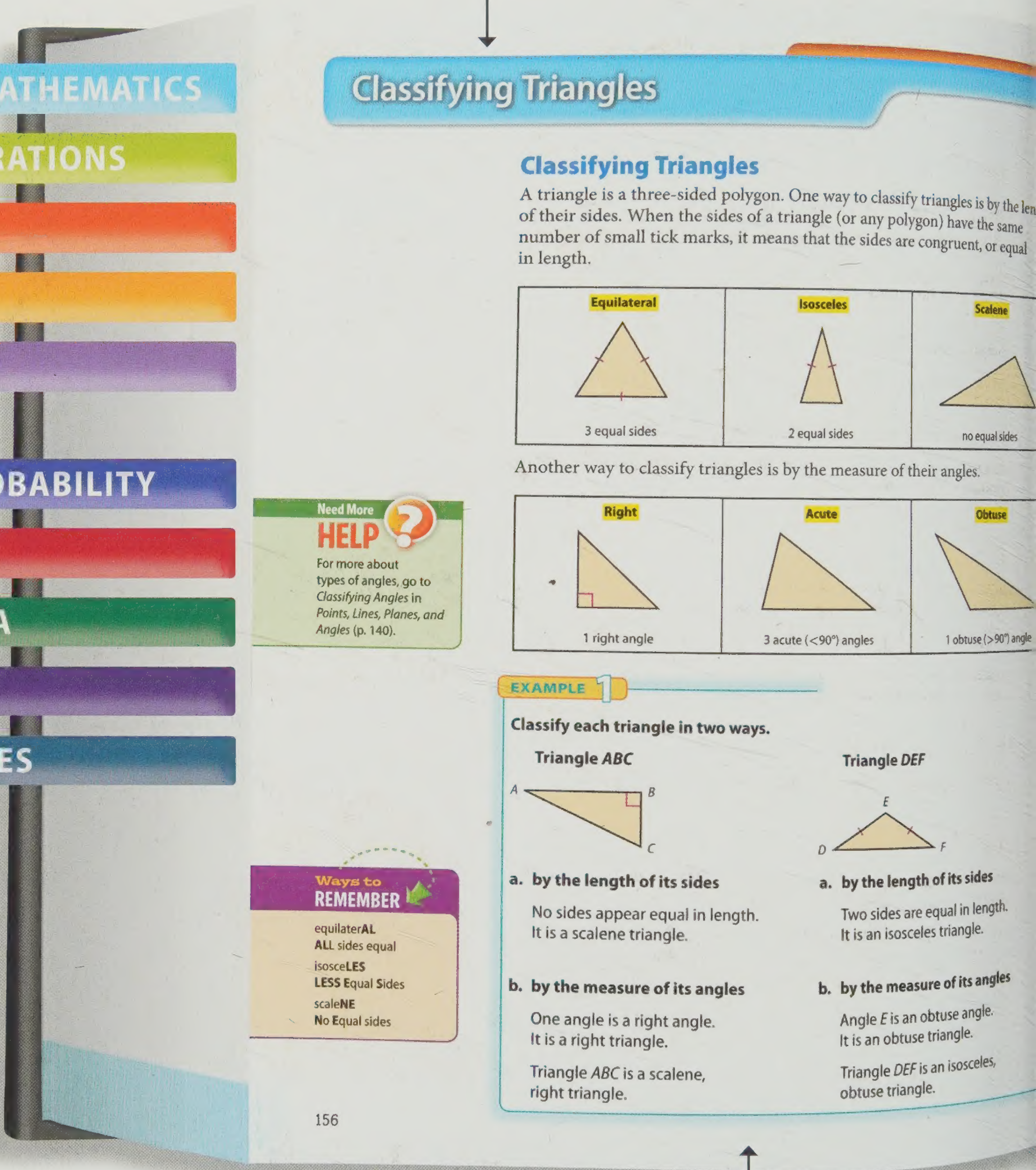
- CALCULUS
- SPECIAL ADVANTAGES

Strand/Subject color bar



Classifying Triangles

Classifying Triangles

A triangle is a three-sided polygon. One way to classify triangles is by the len of their sides. When the sides of a triangle (or any polygon) have the same number of small tick marks, it means that the sides are congruent, or equal in length.

Equilateral	Isosceles	Scalene
3 equal sides	2 equal sides	no equal sides

Another way to classify triangles is by the measure of their angles.

Right	Acute	Obtuse
1 right angle	3 acute (<90°) angles	1 obtuse (>90°) angle

Need More **HELP**

For more about types of angles, go to *Classifying Angles* in *Points, Lines, Planes, and Angles* (p. 140).

EXAMPLE 1

Classify each triangle in two ways.

Triangle *ABC*

a. by the length of its sides

No sides appear equal in length. It is a scalene triangle.

b. by the measure of its angles

One angle is a right angle. It is a right triangle.

Triangle *ABC* is a scalene, right triangle.

Triangle *DEF*

a. by the length of its sides

Two sides are equal in length. It is an isosceles triangle.

b. by the measure of its angles

Angle *E* is an obtuse angle. It is an obtuse triangle.

Triangle *DEF* is an isosceles, obtuse triangle.

Ways to **REMEMBER**

equilateral**AL**
ALL sides equal
isosce**LES**
LESS **E**qual **S**ides
scale**NE**
No **E**qual sides

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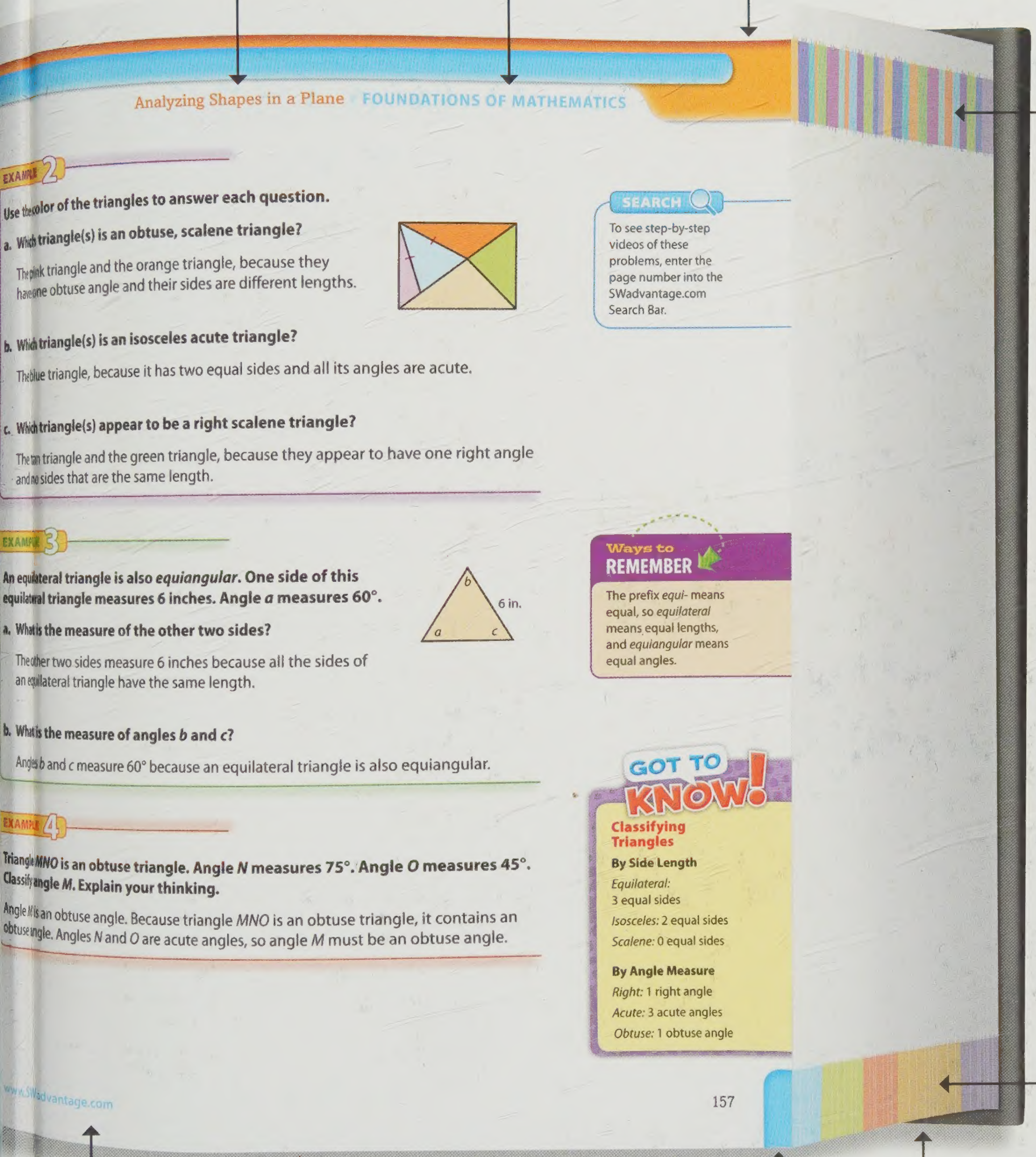
Step-by-step solutions

The unit within the Strand is in the same color as the unit bar.

The name of the Strand is in the same color as the Strand bar.

Unit color bar



Analyzing Shapes in a Plane FOUNDATIONS OF MATHEMATICS

EXAMPLE 2

Use the color of the triangles to answer each question.

a. Which triangle(s) is an obtuse, scalene triangle?

The pink triangle and the orange triangle, because they have one obtuse angle and their sides are different lengths.

b. Which triangle(s) is an isosceles acute triangle?

The blue triangle, because it has two equal sides and all its angles are acute.

c. Which triangle(s) appear to be a right scalene triangle?

The tan triangle and the green triangle, because they appear to have one right angle and no sides that are the same length.

EXAMPLE 3

An equilateral triangle is also *equiangular*. One side of this equilateral triangle measures 6 inches. Angle *a* measures 60°.

a. What is the measure of the other two sides?

The other two sides measure 6 inches because all the sides of an equilateral triangle have the same length.

b. What is the measure of angles *b* and *c*?

Angles *b* and *c* measure 60° because an equilateral triangle is also equiangular.

EXAMPLE 4

Triangle *MNO* is an obtuse triangle. Angle *N* measures 75°. Angle *O* measures 45°. Classify angle *M*. Explain your thinking.

Angle *M* is an obtuse angle. Because triangle *MNO* is an obtuse triangle, it contains an obtuse angle. Angles *N* and *O* are acute angles, so angle *M* must be an obtuse angle.

SEARCH

To see step-by-step videos of these problems, enter the page number into the SWadvantage.com Search Bar.

Ways to REMEMBER

The prefix *equi-* means equal, so *equilateral* means equal lengths, and *equiangular* means equal angles.

GOT TO KNOW!

Classifying Triangles

By Side Length

Equilateral: 3 equal sides

Isosceles: 2 equal sides

Scalene: 0 equal sides

By Angle Measure

Right: 1 right angle

Acute: 3 acute angles

Obtuse: 1 obtuse angle

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Unit color bar shows on edge

Unit color bars

Strand/Subject color tab shows on edge and bottom

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